TRANSACTIONS
OF THE
American Institute of Electrical Engineers

Vol. 47 APRIL, 1928 No. 2

TABLE OF CONTENTS

Surge Impulse Breakdown of Air, by J. J. Torok.. 349
Discussion........ 355
Approximate Solution for Electrical Networks When These are Highly Oscillatory, by E. A. Guillemin........ 361
Discussion........ 365
The Boltzmann-Hopkinson Principle of Superposition as Applied to Dielectrics, by F. D. Murnaghan........ 366
Discussion........ 368
Symposium on Interconnection........372-422
Conowingo Hydroelectric Project with Particular Reference to Interconnection, by W. C. L. Eglin........ 372
Progress and Problems from Interconnection in Southeastern States, by W. E. Mitchell..... 382
Some Aspects of Pacific Coast Interconnections, by P. M. Downing........ 393
Interconnection and Power Development in Chicago and the Middle West, by H. B. Gear 399
Discussion........ 408
Saturation Permeameter, by S. L. Gokhale...... 423
Discussion........ 427
Compressed Powdered Permalloy, Manufacture and Magnetic Properties, by W. J. Shackelton and I. G. Barber........ 429
Discussion........ 436
The Effect of Humidity on the Dry Flashover Potential of Pin-Type Insulators, by J. T. Littleton, Jr. and W. W. Shaver........ 438
Discussion........ 441
The Cathode Oscillograph as Used in the Study of Lightning and Other Surges on Transmission Lines, by Harald Norinder........ 446
Discussion........ 455
Synchronous Machines—IV, by R. E. Doherty and C. A. Nickle........ 457
Discussion........ 487
The Calculation of the Armature Reactance of Synchronous Machines, by P. L. Alger...... 493
Discussion........ 512
The Reactances of Synchronous Machines, by R. H. Park and B. L. Robertson........ 514
Discussion........ 535
The Thermal Volume Meter, by G. W. Penney and C. J. Fechheimer........ 537
Discussion........ 546
Recent Improvements in Turbine Generators, by Stuart L. Henderson and C. Richard Soderberg........ 549
Discussion........ 570
Design and Application of Two-Pole Synchronous Motors, by D. W. McLenegan and Ivan H. Summers........ 579
Discussion........ 586
Heat Losses in D-C. Armature Conductors, by W. V. Lyon, E. Wayne, and M. L. Henderson 589
Discussion........ 597
Effect of Transient Conditions on Application of D-C. Compound Motors, by Leon R. Ludwig 599
Discussion........ 606
Loudspeakers of High Efficiency and Load Capacity, by C. R. Hanna........ 607
Discussion........ 614
Certain Topics in Telegraph Transmission Theory, by H. Nyquist........ 617
Automatic Control of Edison Systems as Applied in the St. Charles Street Substation of the Union Electric Light and Power Company, by O. J. Rotty and E. L. Hough........ 645
Discussion........ 653
Protection of Supervisory Control Lines Against Over-Voltage, by Edward F. W. Beck....... 655
Discussion........ 665
1926 Lightning Experience on 132-Kv. Transmission Lines, by Philip Sporn........ 668
Discussion........ 674
Vacuum Tube Synchronizing Equipment, by T. A. E. Belt and N. Hoard........ 678
Condenser-Type Bushing Used with Synchronizing Equipment, by E. E. Spracklen, D. E. Marshall, and P. O. Langguth........ 684
Discussion........ 696
Metallic Arc Welding Electrodes, A Study of the Effects of Surface Materials, by J. B. Green.. 700
Discussion........ 704
The Stability of the Welding Arc, by P. Alexander 706
Discussion........ 710
Arc-Welded Structures and Bridges, by A. M. Candy........ 711
Discussion........ 716
Welding and Manufacturing of Large Electrical Apparatus, by A. P. Wood........ 717
Discussion........ 720

PUBLISHED QUARTERLY BY THE AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
IN JANUARY, APRIL, JULY, AND OCTOBER
33 West 39th St., New York, N. Y.

Cloth Covers, Subscription $12.00 per year, $3.50 per copy
Copyright 1928. By A. I. E. E.
Printed in U. S. A.

Surge Impulse Breakdown of Air

BY J. J. TOROK*

Associate, A. I. E. E.

Synopsis.—*This paper describes a phenomenon which is used as a method of studying the development of high-voltage sparkover. It has been found that if a voltage which rises very rapidly to a value more than sufficient to cause breakdown if continuously applied is impressed upon a sphere-gap, and this is very quickly reduced to a low value, luminous streamers are produced instead of the usual intense spark. These streamers can be observed with the eye and can be recorded on a photographic plate by the use of a quartz lens.*

The phenomenon described and recorded is of intense interest because it presents the intermediate stages between the initiation of and the actual flashover between the spheres.

The effect on time lag of flashover with special reference to the state of ionization of the gas previous to flashover is also discussed.

Besides giving information on the mechanism of development of spark-over, this method of "suppressed discharge" has been used to determine the nature of the electrostatic field about insulators and between electrodes of various shapes.

* * * * *

In studying spark discharges, various investigators have tried to obtain the relation between voltages applied, lengths of gap, shape of electrodes, humidity, pressure, state of ionization, and various other factors in gaseous dielectric breakdown.

H. Hertz[1] discovered that the spark passed more easily when the electrodes were exposed to ultra violet light and later, E. Wiedemann and A. Ebert[2] proved that the spark was affected only when the light was projected on the cathode. G. Jaumann[3] concluded that the sparking voltage was determined also by the rate of rise of voltage dv/dt. E. Warburg[4] concluded from experiments that ultra violet light reduces only the time lag of the spark because of the photoelectric effect, *i. e.*, the liberation of electrons from the cathode by light.

Townsend,[5] who developed the theory of ionization by collison, summarizes his views as follows: "In order to start a discharge it is necessary that there should be some ions in the gas initially, and when the force in the field between the electrodes is raised to a certain point, these are multiplied by collison to a sufficient extent to maintain the discharge.[6] At first the negative ions alone generate others as they move through the gas, but as the force increases and the sparking potential is approached, the positive ions also acquire the property of producing others to an appreciable extent."[7] In his theories, Townsend does not attempt, quantitatively, to account for time lag of spark discharges. He merely states that time lag is a direct function of the initial ionization of the gas and an inverse function of the overvoltage. Attempts have been made, however, to extend Townsend's theory to account for time lag of sparks, but none has been successful in accounting for time lags of a shorter period than 10^{-7} seconds which experimenters, such as Pedersen,[8] Peek,[9] and Burawoy,[10] find to exist at atmospheric pressures and large gap lengths.

W. Rogowski[11] has made a thorough analysis of the time lag problem, and has applied Townsend's theory quantitatively to a homogeneous field. In his calculations he assumes that the field contains at least one free electron which starts the ionization process. The time of breakdown was taken to be the time for the current to reach twice the value which would have been obtained if there was no ionization by collison of positive ions. This time is found from the determination of the rate of ionization, the respective velocities, and the distance traveled by both the positive and negative ions. The calculations are carried to a fair degree of refinement. The results he obtained show that the doubling of the current is reached after a time interval of d/u in which d is the distance between electrodes (in his case 1 cm.) and u the velocity of the positive ion (which he found to be approximately 10^5 cm/sec.). The time required to double the current would be of the order of 10^{-5} seconds. A discrepancy arises between the calculated and experimental time lag of spark formation.

Rogowski points out that the electron moves rapidly enough to transverse the entire field (1 cm. in length) many times during 10^{-6} seconds. The sluggishness of the positive ion causes much difficulty and is probably the weak point in Townsend's theory. Rogowski believes that space charge plays an important part in the breakdown, which would enable Townsend's theory to hold for surge breakdown by merely changing the number of electrons produced by positive ions in one cm. of travel.

APPARATUS

For the study of impulse characteristics of gaps and other apparatus, it is necessary to have a controllable source of high-voltage impulses. Surges are usually produced artificially by discharging condensers through a resistance by means of a sphere-gap. The voltage piles up across the resistance which gradually discharges the condenser. The discharge may be a damped oscillation or a d-c. impulse, depending upon the value of the constants of the circuit. A surge generator of this nature, which the author used in studying time

---

*Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

1. For references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

lags of sparks, was fully described by D. F. Miner.[12] Fig. 1 is a schematic diagram of the surge generator. It consists of ten groups of condensers, nine of which are charged in parallel through one-megohm resistors, and the first group of the series is charged directly without current limiting resistors. The purpose of eliminating the resistance in the first stage is to fix the potential of the charging apparatus to the voltage of the first group during discharge. G_2 is set to break down at a given voltage, while the other intermediate gaps, G_1, G_3, G_4, ... G_{10} are set for voltages approximately 25 per cent higher than G_2. When the condensers become charged to the voltage E, for which G_2 is set, the gap G_2 breaks down throwing groups two and three in series. The voltage now across the ends of these two groups is $2\,E$, which is above the breakdown voltage of the gaps at the terminals, consequently they too will break down. Thus, all the gaps break down in succession. Since there are ten groups charged to a potential of E, the final voltage across the terminals of the condenser groups will be $10\,E$. The rated voltage of each condenser group is 100 kv. Thus the maximum surge voltage obtainable without reflections is 1000 kv. By reducing the gap spacings a lower surge peak is obtained.

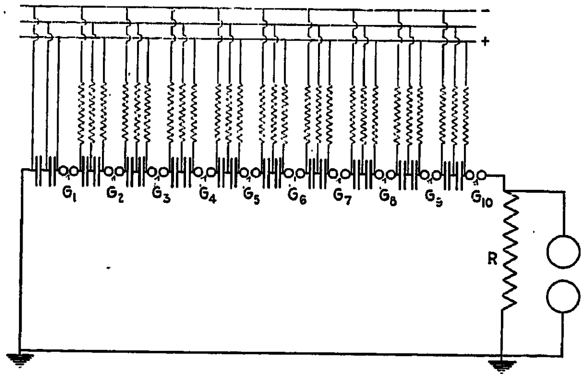


FIG. 1—DIAGRAM OF CONNECTIONS FOR 10-STAGE SURGE GENERATOR

VISUAL EVIDENCE OF TIME LAG

The study of time lag demands a knowledge of wave form of the impulse obtained from the surge generator. While attempting to determine the wave front of steep waves by reflection methods, the author noticed that occasionally a glow resembling a corona discharge occurred between the 75-cm. spheres used as a measuring gap. It had been generally accepted in low frequency work that the sphere-gap flashover takes place without the initial corona formation that precedes a needle-gap flashover. The observance of this glow discharge led to a special study of the conditions under which it occurred.

The surge generator was connected directly to a transmission line 50 ft. long as shown in Fig. 2. The end of the line was grounded, thus causing the voltage at G_3 (75-cm. spheres) to be suddenly reduced when the wave reflected from the grounded end reached G_3; consequently the voltage impressed on G_3 rose very steeply and then dropped very sharply as a result of the reflection at the end of the line. Under these conditions, when trying to measure the crest of the voltage with a 75-cm. sphere-gap at G_3, a glow, instead of a spark, appeared between the two spheres. This phenomenon showed that the voltage was so quickly reduced to a negligible value after ionization began that a complete spark was prevented from forming.

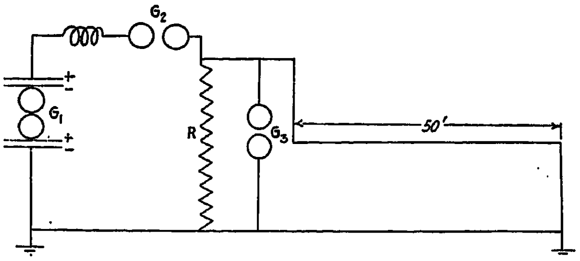


FIG. 2—DIAGRAM FOR USING TRANSMISSION LINE

IMPROVEMENT OVER TRANSMISSION LINE

The voltage wave impressed upon the sphere-gap was then of the nature shown in Fig. 3. The ionization by collision would probably start before E, but the developed spark might not occur until point f was reached due to the time lag of the sphere-gap. If the voltage was brought to zero somewhat faster, the developed spark might not occur at all, even though the air between the spheres would be sufficiently ionized to cause a luminous glow. Since the wave shape obtained in this way had such a flat top, conditions were quite critical, and therefore any slight overvoltage or decrease in spacing of the spheres caused a complete breakdown, while if the voltage did not quite come up to that for which the spheres were set, no luminous discharge would occur at all. A less critical arrangement was found by using a second sphere-gap, G_4 Fig. 4, instead of the grounded transmission line. This circuit was a great improvement over the original one as it was much more flexible. By varying the setting of the second sphere-gap G_4, it was possible to obtain a photograph of the discharge in the first gap

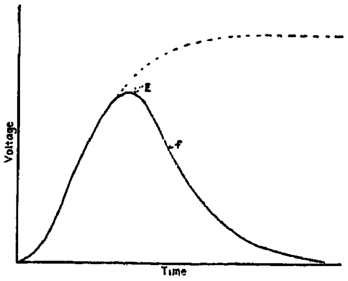


FIG. 3—GENERAL SHAPE OF WAVE IMPRESSED ON SPHERE-GAPS

G_3 in any stage of its development. The second sphere-gap reduced the voltage to a negligible value much more rapidly than was possible with the transmission line, where the wave front was the controlling factor. An example of a discharge across the first sphere-gap obtained in this way is shown in Fig. 5.

From the known characteristics of the surge front, a fair conception of the time required for a streamer to bridge a 40-cm. gap was obtained. This time using 75-cm. spheres was found to be of the order of 10^{-8} seconds. The wave used had a rate of rise of approximately 5000 kv. per microsecond. Thus, knowing the sphere-gap settings (and their corresponding static sparkover voltages) at which suppressed discharges first appear and the maximum separation at which complete sparkover is obtained, the time elapsing between the two periods can easily be found.

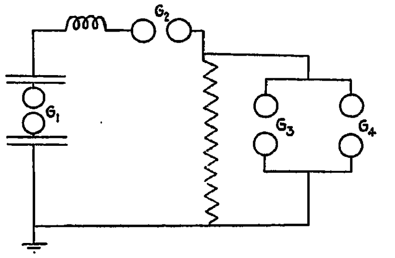


FIG. 4—METHOD FOR UTILIZING SPHERE-GAP INSTEAD OF TRANSMISSION LINE

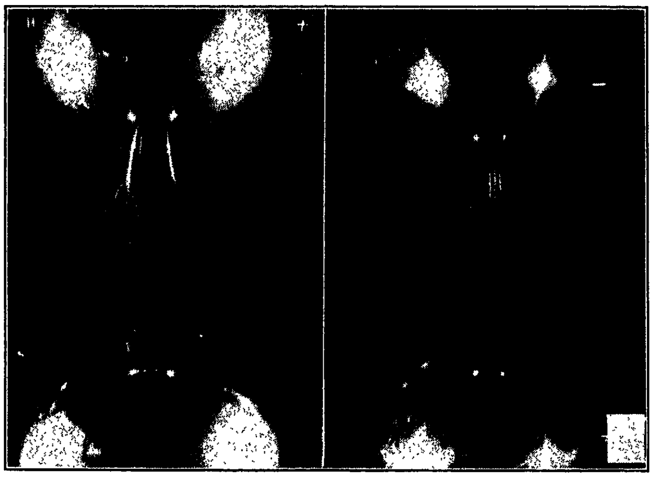


FIG. 5 FIG. 6

FIG. 4—A TYPICAL SUPPRESSED DISCHARGE IN WHICH CATHODE IS GROUNDED

FIG. 5—SUPPRESSED DISCHARGE WITH ANODE GROUNDED

Discharges of the nature shown in Fig. 5, or "suppressed discharges" as they might be termed, give much information concerning the process of breakdown. The bright streamers on the ungrounded positive sphere in Fig. 5 span only a portion of the distance between the spheres. The rest of the gap is bridged by a light purple haze. Differences in intensities of luminosity of various portions of the field indicate in a rough way that the state of ionization is not uniform throughout. It is probable that the bright anode streamers have already the properties of arcs[13] while the haze extending from the ends of the streamers to the cathode is still a glow discharge. Evidently then, the breakdown starts at the most intense portion of the field (assuming a uniform density of free electrons throughout the field) and progresses into the field until it meets a streamer developing from the electrode of opposite polarity. In the case of sphere-gaps, when one sphere is grounded, the gradient at the surface of the ungrounded sphere is much higher than the gradient on the grounded sphere. This fact is clearly illustrated by Figs. 5 and 6. In Fig. 5 the cathode is grounded while in Fig. 6 the anode is grounded. The streamers on the grounded sphere have developed but little, showing a weak field at that point. These figures indicate rather clearly that a spark develops from a dark current to a glow discharge which in turn assumes the properties of an arc at points where the current densities are highest.

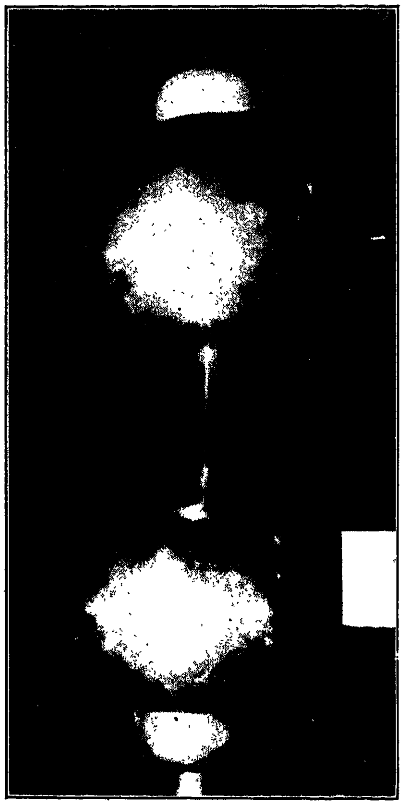


FIG. 7—STREAMERS AFFECTED BY SPACE CHARGE

SPACE CHARGE

Under atmospheric conditions comparatively low voltage is necessary to maintain arcs. Thus when a streamer develops into an arc it has the effect of a needle extending into the electrostatic field. The field form under these conditions will change, resulting in the development of new streamers and an acceleration in their rate of development.

The manner in which the development of slower streamers is affected by the change in field form is shown in Fig. 7. The upper or cathode streamers are concave outwards. The center streamer, having probably started first and developed faster than the rest, reduced the gradient between the tip and the sphere from which it developed, and caused the lines of force of the electrostatic field to be concave outwards.

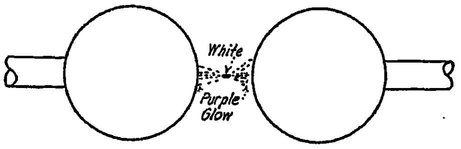


Fig. 8—Formation of Streamers in Space Between Electrodes

Most theoretical determinations of time lag of sparks assume a fixed homogeneous field throughout the entire process. These assumptions do not hold beyond the time when a streamer forms. As pointed out herein, these streamers are in effect conductors extending into the electrostatic field. Since the entire potential appears at the tips of these streamers, the spacing between electrodes will be materially reduced, producing an increase in the gradient between the anode and cathode streamers. The maximum gradient appears at the tips of the streamers because they can be considered as needle electrodes. As the streamers grow the gradient increases, and since the rate of ionization is a function of the gradient, the streamers must develop more rapidly in the more advanced stages. Thus, the development of the streamers cannot be considered as uniform.

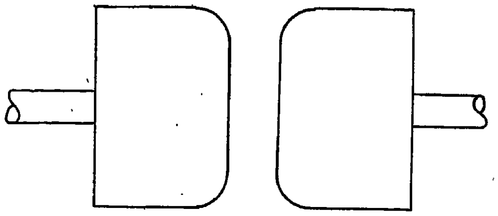


Fig. 9—Electrodes Used to Obtain Homogeneous Field

Suppressed discharges have been obtained on gaps varying from 0.25 cm. to 75 cm. spacing. For voltages ranging below 100-kv. crest, 6.25-cm. spheres were used; then for potential ranging between 100 kv. and 500 kv., 50-cm. spheres were utilized. For all voltages above 500 kv., 75 cm. spheres were used. While observing these discharges, on small spacings (0.25 cm. to 2-cm.) several discharges occurred which had their intense streamers in the center with a faint glow extending from both ends to the electrodes. Fig. 8 illustrates this phenomenon.

This type of discharge was obtained only after several lower voltage surges, not resulting in breakdown of the air, were impressed on the spheres. Several explanations for the phenomenon were considered, the following two being the most logical; first, that the location of the first streamer is a matter of probability, and the resulting variability in its location becomes more marked as the initial number of free electrons in the field diminishes; second, that a current of air carries out the air ionized by application of voltage below the breakdown value, replacing it with air containing the normal number of free electrons. The surface friction prevented the air next to the electrodes from being swept out. When a surge of higher amplitude than the rest was applied, the air in the center of the gap became ionized to a higher degree than that at the surface, due to the greater initial number of free electrons at that point. As ionization progressed, a streamer formed in the center, but before it could reach either electrode, the voltage was reduced by the suppressing gap.

Should the second explanation be correct, it would mean that the breakdown (at least with very steep surges and normally ionized field) is not produced by electrons traversing the entire field but that each portion of the field is ionized separately by the local ions, these sections being joined together as the streamer progresses.

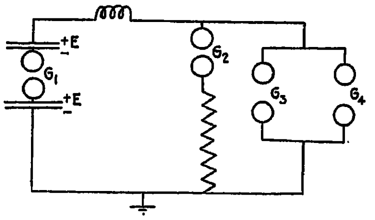


Fig. 10—Scheme for De-Ionizing Air Between Spheres by Condenser Potential

Using spherical electrodes this phenomenon was reproduced quite often, *i. e.*, 1 out of 10 times. Again when the field distribution is considered it is difficult to attribute this type of suppressed discharge to probability. The highest gradient is at the surface of the spheres, while the gradient at the center is appreciably lower; thus the velocity of the electrons and positive ions will be higher near the surface than in the center and more electrons will be produced there. With higher density of free ions near the surface, the probability of the arc originating at the center would be much less. However, when electrodes of the type shown in Fig. 9 were used, the phenomenon occurred 3 out of 10 times, the gradient throughout the field being more nearly homogeneous than with spheres, which gives the probability theory more weight.

The Effect of Varying the Initial Density of Ions

The original number of ions in the field can be varied by two means,—by controlling a source of ionization and by drawing the free ions out of the field. The latter can be effected by impressing a constant voltage below breakdown value on the electrodes. In applying the first method, soft X-rays were used to ionize the field. The set-up used is shown schematically in Fig. 10. Gaps G_3 and G_4 had a constant potential impressed upon them, which in this case is always the condenser charging potential E. When G_1 broke down the two banks of condensers were thrown in series, impressing $2\,E$ on gaps G_2, G_3, and G_4. In this manner surges were applied to sphere gaps which were constantly excited. Under these conditions the number of ions in the field just before breakdown was reduced to a very small quantity. G_3 was set slightly above the breakdown value of G_1 so that G_1 would break down before G_3. G_4 was used to measure the voltage and under suitable conditions would show suppressed discharges appearing across G_3. The results of this test are tabulated in Table I. Under these conditions, suppressed discharge at G_1 could rarely be obtained, and even then the glow at the ends of the streamers was very faint. The maximum voltage indicated by G_4 was 66 kv. X-rays were directed on G_4 Fig. 10 to increase the ionization in the gap. Now, however, the spacing in G_4 had to be increased to 3.18 cm. before complete breakdown ceased. This setting corresponds to 81 kv. Suppressed discharge appeared up to a separation of 3.8 cm. or 89 kv. The glow at the tips of the streamers was now very pronounced. These two tests indicate rather strongly that the time lag of sphere-gaps is a function of the number of ions in the field when the surge is applied. The prominence of the glow at the ends of the streamers

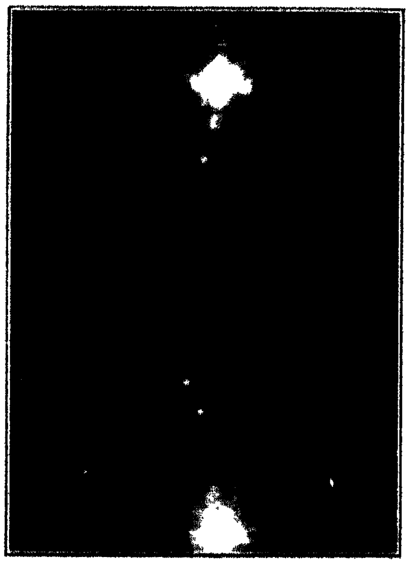


FIG. 11—RARE FORM OF SUPPRESSED DISCHARGE

TABLE I

Conditions	G_1		G_2		G_4		Remarks
	Sep. in. cm.	Kv.	Sep. in. cm.	Kv.	Sep. in. cm.	Kv.	
Excited No X-rays..	1.68	50	1.9	57	2.33	66	Complete flashover on G_4 ceased at 2.33-cm. spacing and suppressed discharge occurred very seldom above that value.
Excited X-rays on G_4	1.68	50	1.9	57	3.18 3.8	81	Complete flashover ceased on G_4 for separations above 3.18 cm. but suppressed disharge appeared from 3.18 cm. to 3.8 cm.

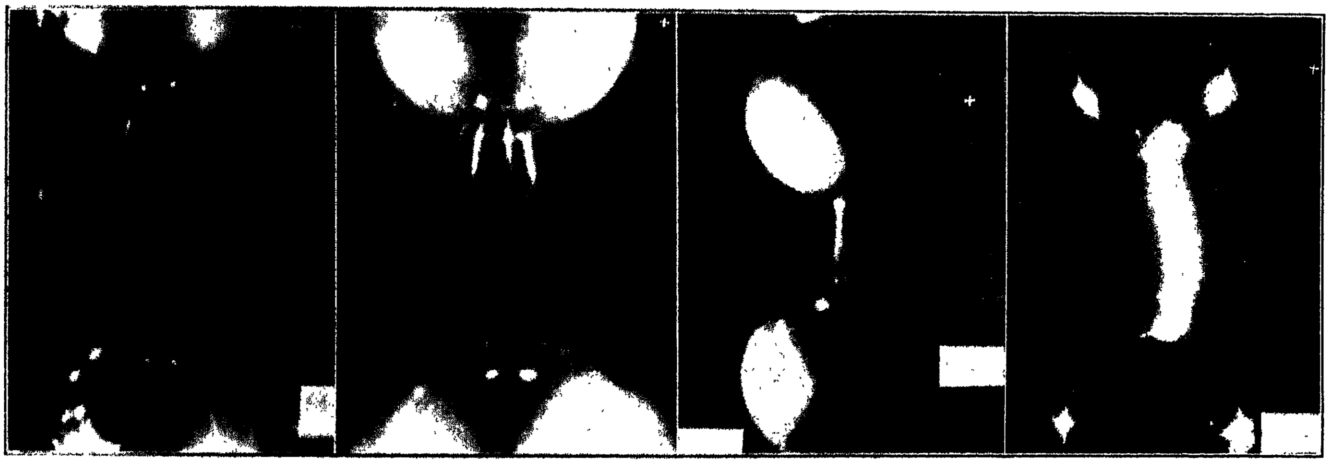


FIG. 12 FIG. 13 FIG. 14 FIG. 15

FIGS. 12, 13, 14, 15—SUCCESSIVE STAGES OF BREAKDOWN OF AIR.

12. Early stage
13. Secondary stage
14. Space almost spanned by streamers
15. Complete discharge

in the case of previously ionized gaps together with the increase of speed of breakdown is in full accord with the assumption that the free ions originally in the field produce numerous ionized sections which finally join to form streamers. The rapidity of development of these streamers might be taken as a function of the number of these ionized sections.

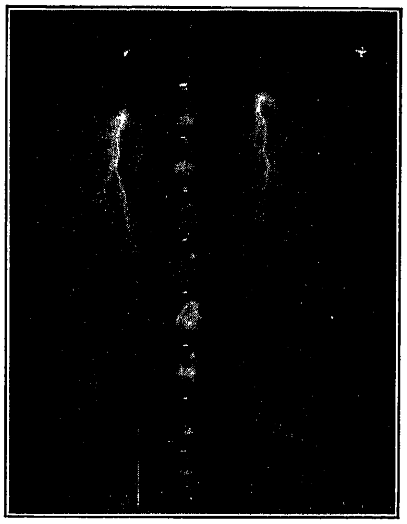


Fig. 16—Electrostatic Field Around Insulators Modified by Arcing Ring and Arcing Horn

ELECTROSTATIC FIELD DETERMINATIONS

It seems possible to study the progressive stages of breakdown by the *suppressed discharge method.* This will yield important information as to the character of breakdown with various type of apparatus. Fig. 11 portrays a rare type of streamer development. The streamer starts at the side rather than in the center and develops according to the electrostatic field at that point. This cut reveals the effect of the earth upon the electrostatic field between the spheres.

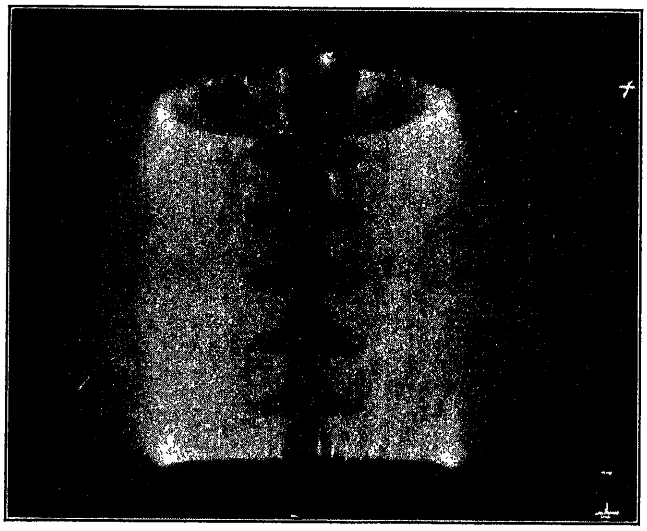


Fig. 17—Field Around Insulators as Improved by Circular Arcing Rings

Figs. 12, 13, 14, 15 were selected from a large number of tests as representing progress of breakdown from initial streamer formation to arc-over.

One application of the suppressed discharge which has proved useful in engineering work is shown in Figs. 16, 17, and 18. A picture of the electrostatic field surrounding an insulator string at the time of impulse application is produced and shows the corrective or destructive effect caused by use of special transmission fittings. In Fig. 16 a horn was used at the top and a ring at the bottom of an eight unit string. The concentration at the horn tips is shown and also the drawing-in of the field at the top of the string. This leads frequently to cascading flashover. In Fig. 17 two arcing rings were used and the field is much im-

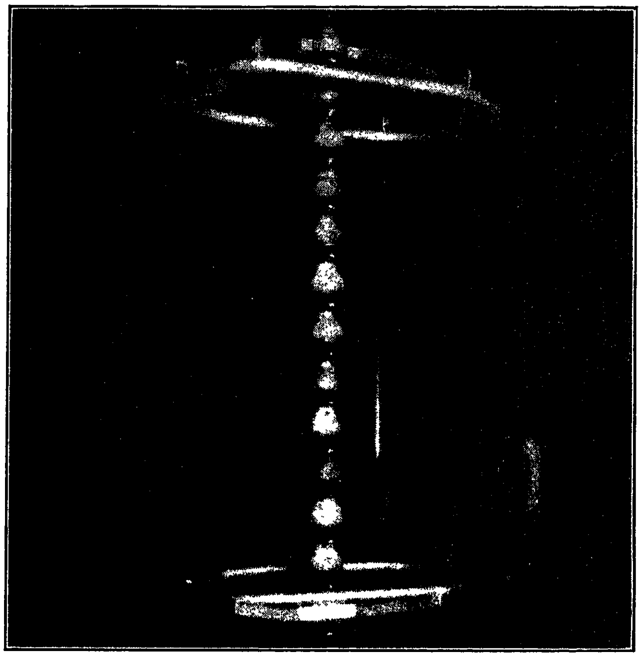


Fig. 18—Electrostatic Field as Modified by Elliptical Rings

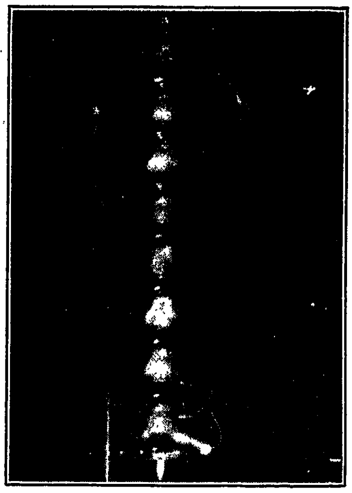


Fig. 19—The Early Stages of Insulator Flashover

proved. A flashover clear of the string will result from this field. Special elliptical rings were attached in Fig. 18 and the resulting field is well distributed.

Fig. 19 reveals the manner in which a string of insulators cascade. The bottom insulator is already shunted by a streamer, while the one above it is just in the process of cascading. A streamer is forming from the cap of each of the two bottom insulators. The streamer progressing from the cap of the one next to the bottom is barely visible. Unfortunately the streamers develop much more rapidly in the latter stages making it very difficult to obtain suppressed discharge after the streamers have spanned a third of the spacing between the electrodes. Thus a complete story of dielectric breakdown by this method is almost impossible, but much can be learned from the earlier stages of development.

The writer wishes to express his appreciation to Dr. J. Slepian for many suggestions and comments which have been of great assistance in carrying on this work and in preparing this paper, and also to Messrs. Fortescue and Tenney for their valuable criticisms.

Bibliography

1. H. Hertz, *Wied. Ann.* 31, 1887, p. 983.

2. E. Wiedemann and A. Ebert, *Wied. Annalen* 33, 1884, pp. 241-269.

3. G. Jaumann, *Wied. Ann.* 55, 1895, p. 656; *d. k. Akad, d. Wiss. Wien. Abt.* II a. 1888, p. 765-805.

4. E. Warburg, Sitzungsber, *d. k. pr. Akad, d. Wiss. zu Berlin*, p. 223-236, 1896 (1): b) 1897, p. 128-136. (I) *Wied. Ann.* 59 S. 1-16, 1896:d) 63, 1897, p. 285-395:e) *Verh. Deutch. Phys. Ges.* 2, p. 212-217.

5. J. S. Townsend, Electricity in Gases, Oxford, 1915.

6. J. S. Townsend, Electricity in Gases, p. 338.

7. J. S. Townsend, Electricity in Gases, p. 261.

8. Pedersen, *Ann. d. Phys.* 4th Series, Vol. 71, p. 317.

9. F. W. Peek, Jr., Dielectric Phenomena in High Voltage Engineering, 1915.

10. O. Burawoy, *Archiv fur Elektrotechnik*, XVI Band, 14 June, 1926, 3 Heft.

11. W. Rogowski, *Archiv fur Elektrotechnik*, Aug. 25, 1926.

12. D. F. Miner, *Electric Journal*, June 1927.

13. J. Slepian, *Journal of the Franklin Institute*, Jan. 1926.

Discussion

C. L. Fortescue: I will bring out some important practical deductions from the results Mr. Torok obtained with the suppressed flashover of spheres.

First I want to review what he found for these spheres. The stages of breakdown are: 1, free electrons are drawn into the field; 2, free electrons are increased by ionization by collision, recombination taking place also; 3, a critical stage is reached where the rate of ionization and volume density of the free electrons has reached a critical point, which shows itself in the emission of ultra-violet rays. This results in the final stage when it is believed that molecular dissociation of the air molecules takes place, which is shown by intense white streamers which form at the electrode having the highest intensity, and these streamers grow outward from the electrode and finally grow together and cause the breakdown. The growth of these streamers takes place progressively from the electrode outward, and it apparently has a velocity of propagation which is proportionate to the difference of potential between the electrodes, so that from this point on the breakdown takes place as a function of time. It is not instantaneous but takes an appreciable time.

This is the factor which is responsible for the fact that with very steep wave fronts an insulator, or a string of insulators, has what is known as an impulse factor. That is to say, it breaks down at a higher value than the normal 60-cycle flashover.

I wish to show the reason for this by means of the accompanying Fig. 1. The steepness of the wave fronts is shown by straight lines, and the slope determines the steepness. The surges we consider are surges which would keep on rising if the insulator or spark-gap did not flash over. The horizontal line shows the voltage at which the insulating string or sphere-gap will have reached the critical stage, where, you might say, it is just at the point of breaking down.

Breaking down is not an instantaneous process; it takes time. So during the time while the breakdown is taking place the voltage is still rising, and the actual arc-over will take place at some higher voltage, the magnitude of which will depend upon the character of the insulator or spark-gap. Now if we were to have a wave which just reached the 60-cycle flashover or critical point, and then remained at that voltage for an appreciable length of time, and then came down, the flashover would take place at some appreciable time from the point at which it reached that voltage, and the potential recorded would be the same as the 60-cycle flashover potential or slightly higher. However, if the same wave had been still going up, the voltage recorded would not be the 60-cycle voltage but the flashover would take place at some time after this stage had been reached and at a higher value. This time can be calculated by the velocity at which these streamers come together.

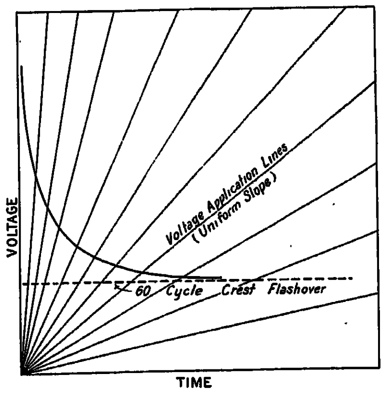


Fig. 1—Time-Voltage Flashover of Insulation

The steeper the wave front the higher the voltage that will be reached before those streamers come together. But the velocity increases with the potential, and the actual time that elapses between the critical point and the point of flashover becomes somewhat less as the steepness increases, so that we can figure out the points for each steepness and we will get a curve like that shown in Fig. 1. That is what Mr. Torok's results indicate, and as a result of his work we have been able to calculate the flashover curve for different steepnesses of wave front of strings of insulators, and have been able to duplicate those results by actual observations very closely. In fact the curves check so well that the discrepancies are within the errors of observation.

Another important thing that this shows is that the sphere-gap always has a time lag, though this happens to be very small. The sphere-gap is not instantaneous. From the results of Mr. Torok's tests I have made some calculations on the probable impulse factor of the sphere-gap. For instance, for a wave front which has a steepness of about 10,000 kv. per microsecond, the impulse factor would be about 1.05. The steepest wave we get in lightning does not go much above 4000 kv. per microsecond, and at that value the impulse factor is probably not more than 1 or 2 per cent above unity at the most, so that we can always rely on the sphere-gap to give the correct surge potential for all waves that we are likely to get in service, and therefore for all practical waves in the laboratory. We can, of course, produce waves in the laboratory that have a much greater steepness than those that are encountered in lightning.

Now one more point. These results of Mr. Torok also throw a great deal of light on the phenomena of lightning. Mr. Simpson, one of the British authorities on lightning, has shown, at least he has satisfied himself, that certain experiments which he has made indicate that when a cloud is negatively charged the lightning stroke will start from the earth instead of the cloud. His experiments have convinced him that this is true, and he has also shown photographs supporting this point of view.

In Fig. 2 herewith, is shown a cloud about 1 km. above the earth's surface. Simpson says that the lightning discharge starts in the earth and goes up and then branches out into a portion of the cloud and discharges this portion of the cloud. He claims that when the cloud is positively charged the stroke does not branch out and only a very small portion of the cloud is discharged. And therefore, he says that the severe discharges from the clouds are all negative. Apparently this is true, for we have always found in our field investigations in lightning that all the severe discharges were negative.

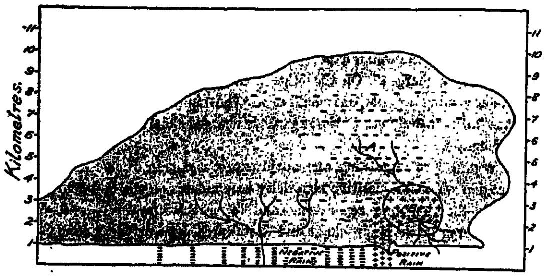


FIG. 2

Let me point out what probably occurs as a result of Mr. Torok's investigations with spheres. These streamers do form at the earth and they are propagated toward the cloud at a certain velocity which is quite fast. We don't know just what it is between cloud and earth but between spheres it happens to be about one-tenth the velocity of light. The cloud is not discharged when the streamer reaches the cloud. That is only the beginning of the discharge. All that has happened is that a conducting path has been formed between the earth and the cloud. After this is formed, a part of the cloud discharges to the earth in the form of a surge. In other words a dynamic surge is formed along the path that has been made, conducting by this first, initial streamer, a condition which Mr. Torok describes. So that when this portion of the cloud is discharged a wave or surge is set up which, if it strikes a transmission line, is already dynamic and divides into two ways forming the surge encountered in lightning. This surge, being dynamic, does not reach as high values as if it were simply a case of a certain charged capacity in the cloud combining with the capacity of the transmission line to produce a potential on the line, but the dynamic surge divides and is reduced thereby. Otherwise it would be very hard to account for the fact that a great deal more damage is not done by lightning. The potentials of the clouds are so high that one would expect very much higher potentials in surges than actually appear, if the phenomena were the same as that of the discharge between two condensers, one of which is charged to a high potential and the other is uncharged.

The foregoing gives, I believe, a correct scientific explanation of the lightning discharge, and one which will account for the surge potentials which are encountered in the field.

J. A. Duncan: Mr. Torok refers to Rogowski's analysis of the time lag problem on the basis of the Townsend Ionization by Collision Theory. Rogowski has, as Mr. Torok says, been rather careful with his mathematics and I shall assume that it is correct. After reading his paper, my understanding of his conclusions is as follows:

A few electrons are assumed to be present in the air previous to the application of the electric field. A field of 30,000 volts per cm. is suddenly applied. The electrons acquire velocities which on the average are of the order of magnitude of $V = 4.3 \times 10^7$ cm./second. This means that an electron will move completely across the gap, which in his case was one centimeter, in about 2.3×10^{-8} seconds. In so doing, it will ionize a great number of molecules, each of which gives rise to one new electron and one positive ion. These new electrons will all immediately proceed to move towards the anode and acquire velocities comparable to 4.3×10^7 cm./sec. and will in turn each produce a great many more new ions by collision as they go. The new positive ions, on the other hand, move toward the cathode with velocities only of the order of $U = 0.96 \times 10^5$ cm./sec. and will take several hundred times as long to get under way to the extent of producing further new ions by their collisions with molecules.

Breakdown is assumed to have occurred when the current reaches a value twice as great as that which would result from the ions produced by collision of negative electrons only. In other words, it occurs after the positive ions begin to have an appreciable effect, according to his picture. The time is too long in comparison with experiment simply because the positive ions move relatively slowly.

The question naturally arises whether there is some other secondary agency which begins to produce new ions and hence aid in the increase or doubling of the current long before the positive ions begin to have an effect.

According to Haas, (Introduction in Theoretical Physics, Vol. 11, Par. 119) it requires several times as much energy to ionize an atom into positive and negative ions as it does to excite it to a condition such that it can radiate light. This excitation consists of knocking an electron out of its normal orbit to an outer orbit, from which it may later fall back. The electron remains in the outer orbit for a time whose order of magnitude is 10^{-8} sec., according to Haas, and then falls all or part of the way back to its original orbit and in so doing, emits light. This light will be scattered in all directions, some falling on each electrode. Suppose some of this is ultra-violet light. In this case, new electrons will be emitted by the photoelectric effect at each electrode. Those emitted from the cathode are available for a very marked increase in the current which increase can take place in less than 10^{-7} seconds. The elapsed times occurring are as follows, for the one cm. gap:

Time for 1st set of electrons to each anode.... = 2.3×10^{-8} sec.
Time allowed for emission of light........... = 1×10^{-8} sec.
Time for passage of light to cathode

$$= \frac{1}{3 \times 10^{10}} = 0.33 \times 10^{-10} \text{ sec.}$$

Time for second set of electrons to reach anode............................... = 2.3×10^{-8}
Total elapsed time........................ = 5.6×10^{-8} sec.

In this connection it is interesting to note that the photoelectric effect, *i. e.*, the emission of electrons from the surface of metals on which ultra-violet light is allowed to fall, was discovered by Hertz when he observed that the light from one spark-gap between aluminum electrodes in air had an appreciable effect on the ease of discharge of a second gap if this light fell on the electrodes.

What I am now suggesting is simply that it seems conceivable to me that in a single gap there might exist a similar effect even in a time shorter than 10^{-7} sec.

Mr. Torok has made a much deeper study of breakdown phenomena than I have and I trust he will be so kind as to tell us whether such a supposition, in the light of his experience, is conceivable to him.

Joseph Slepian: Mr. Torok in his paper brings out that the Townsend theory of breakdown in spark-gaps apparently fails at atmospheric pressures when the speed with which the spark-gap breaks down is considered. In the mechanism required by the Townsend theory, time of the order of 10^{-5} or 10^{-4} sec. would be required for breakdown. The pictures which Mr. Torok has obtained seem to point out also pretty definitely that the Townsend mechanism does not hold at atmospheric pressure, and they suggest what may be the way out of the difficulty. The Townsend mechanism apparently works very well at low pressures. It requires electrons moving toward the anode to ionize by collision, and also requires the positive ions generated by collision to return towards the cathode. The return of these positive ions is necessary in order to bring out by collision a new and increased supply of electrons to carry on the breakdown process. But it is the slowness of the positive ions in returning toward the cathode that causes the difficulty with the application of the Townsend theory to breakdown at atmospheric pressures.

According to the Townsend mechanism it would seem that the conductivity should develop over the whole breakdown path in a rather uniform manner. That is, the whole path should first become slightly conducting, and then more and more highly conducting. The pictures of suppressed discharges should then give either faint breakdown paths or more intense breakdown paths. The pictures actually show the path completely broken down in one part and not at all broken down over the remainder. They show that the breakdown progresses rapidly across the gap, being almost complete over part of the gap before it begins in other parts of the gap. The pictures strongly suggest that the single passage of the initiating electron across the gap is enough to give breakdown. According to the Townsend theory when the single electron goes across the gap and is accompanied by its tremendous progeny of electrons that it generates on the way, the initiating electron and its descendants all travel together and leave behind them only a trail of slow-moving positive ions, which could hardly be called conducting.

But Mr. Torok's pictures suggest that the trail which the initiating electron does leave behind is highly conducting. A possibility which occurred to me is that at atmospheric pressures the multiplication by collision of the initiating electron is so rapid and the multiplication takes place in a path of such small section, that the mere passage of it, with its accompanying new electrons, is enough to raise the temperature of the path to a high degree and to leave behind a path that is at such high temperature that it is thermally ionized, and continues to be ionized by virtue of high temperature.

On this basis, I have calculated what the temperature would be in the path traversed by the initiating electron, and was rather surprised to have it come out sufficiently high. In fact, on this basis I was able to check quite closely the values for breakdown of spark-gaps at atmospheric pressure, and also to get the right order of magnitude for the time of breakdown.

I believe that a very valuable part of Torok's work is in the suggestions which it leaves for future work, such, for example, as taking into account the temperature rise in the path of an electron, as I have attempted to do.

F. W. Peek, Jr.: I began to study this subject about 1913 and in 1915 presented a paper before the Institute on the *Effect of Transient Voltages on Dielectrics*. (See TRANS. A. I. E. E., Vol. XXXIV, part II, 1915, p. 1857. Cathode-ray oscillographs that could record transients of short duration were not available so it was necessary to devise an impulse generator for which the waves could be calculated. Such a generator was devised and we were able to determine time and time-lags for all types of gaps, insulation, insulators, etc. It was found that these time-lags came conveniently in millionths of seconds. For this reason the term "microsecond" was used and found to be an exceedingly convenient unit.

The early work with this lightning generator showed that the impulse sparkover or breakdown voltage was always higher than the 60-cycle or continuously applied breakdown voltage due to the time-lag. The ratio between the impulse breakdown voltage and the 60-cycle voltage was called the "impulse ratio." This term is still used to indicate the relative values of insulation for lightning or impulse breakdown. The higher this ratio under given conditions the greater the lag. This research showed that for electrodes giving an approximately uniform field, as for instance the sphere gap, the lag was small or the impulse ratio was approximately unity over a large range of impulses. It also showed that for irregular fields the lag was appreciable and the impulse ratio increased with the steepness of the wave front. In other words, it was found that the lag was not constant but varied with the duration or rate of application of the voltage as well as with the type of electrode used. Since the impulse ratio of the sphere-gap approximated unity except for the very steepest waves, it offered a means of measuring transient voltages and has been an almost indispensable tool in such investigations.

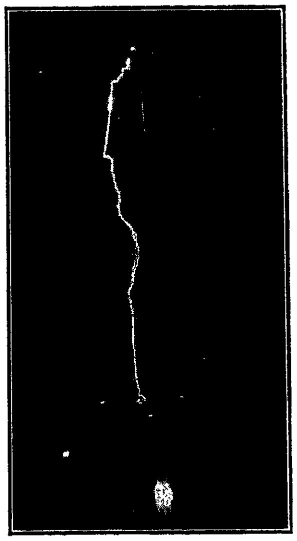


FIG. 3

This early investigation was carried up to about 200,000 volts. Since then the work has been carried on to considerably over 2,000,000 volts. Recently I have been able to obtain actual oscillograms of the calculated time-lags and waves of the original investigation. These measurements were made with a cathode-ray oscillograph of the Dufour type developed in our general engineering laboratory at Schenectady.

The oscillograms checked our early work very well. Similar oscillograms have also been made of the very high-voltage waves with a similarly good check of the calculated values.

A great deal of practical value came out of this early investigation. For instance, since the sphere-gap had practically no time-lag it offered an ideal lightning-arrester gap. On the other hand the information made it possible to design bushings and insulation with high time-lag or great lightning strength.

The more recent work at the higher voltages has also been of practical value in determining the best means of building transformers and other apparatus to resist lightning. One instance that might be mentioned is the grading shield for insulators which is of particular value at high voltages, since it not only prevents cascading but also increases the lightning arc-over.

Mr. Torok's pictures showing the value of the grading ring are quite interesting and confirm my own work.

I have also observed the partial or incomplete lightning arcs due to transients of short duration. The accompanying Fig. 3 is a good example of this. In this figure the completed arc is shown as well as other incomplete arcs along the lines of force between the gap. In the course of my investigations I have obtained similar photographs of bushings, insulators, etc. I hope that Mr. Torok will be able to continue his very interesting work.

H. J. Ryan: (by letter) Mr. Torok's unique experiments made by a new mode of procedure yielded important results which are placed in the light of a clear understanding. These are the pioneer group of a long line of experiments that will follow quickly, now that the value of the "arrested" method is so well demonstrated. The role of the space charge in the Townsend theory of ionization by collision whereby the heavy positive ions also attain velocities high enough to produce other ions by collision, has been clearly emphasized. The participation of the positive as well as the negative ions is necessary in the process of sufficiently augmenting ionization by collision to retain the Townsend theory in accounting for the enormous conductivity that is known to develop with great suddenness in sparkover. In addition to local citations by Mr. Torok, see the brief paper on "Ionization by Positive Ions," in the December 23, 1927 issue of *Science* by Dr. Leonard B. Loeb and his more extended paper on "Theory of Electrical Breakdown of Gases at Atmospheric Pressure," soon to appear in the *Journal* of the Franklin Institute of Pennsylvania. In concluding the latter paper, Dr. Loeb says "experiments should be undertaken in laboratories equipped for the purpose to look for both the spark-lag and the non-uniform field."

It is of great practical as well as theoretical value to know the facts about the time-lag and the other essential physical characteristics of sphere-gap discharges. The result recorded and discussed in connection with Fig. 8 well authenticates the important fact that the blue-white channel which grows into a sparkover with enormous suddenness is formed of dissociated air without the presence, necessarily, of electron-laden metallic vapor drawn from the electrodes. When not too explosive the spark merges later into an arc pervaded by metallic vapor. The time factor in the rapid production of the high conductivity of the spark channel is well established in the paper. The impedance of all ordinary high-voltage sources having small terminal capacitances is too great to accord sufficient current acceleration, *di/dt*, to develop strong sparks. Only when the capacitances of the electrodes are relatively large can the current acceleration be great enough to develop sparks that are strong enough to be easily distinguished from the arcs that follow.

To date, then, it appears that sparkovers are due to highly conducting channels of electrically broken down or dissociated air produced when the intensity of migration of ions formed in corona exceeds a critical value. The voltage consumed in producing this critical intensity of migration of corona ions in the air at 25 deg. cent. and 76 cm. pressure is approximately 4 kv. eff. per cm. or 10 kv. eff. per in. At higher driving voltages the ionized air is electrically dissociated so as to become suddenly enormously more conductive. The magnitude of the conducting channels thus established is largely determined by the strength of the current that the impedance of the rest of the higher-voltage-source circuit will permit to flow. Many able physicists are at work upon the intimate characteristics of this greatly enhanced ionization that results in sparkover and which for present purposes we are calling *electrical dissociation*.

That air may become electrically dissociated and rendered enormously conductive can be conveniently demonstrated in the high-voltage laboratory by using nitrogen-filled incandescent lamps as discharging electrodes. The lamp bulbs are mounted axially with their convex walls face to face and separated by a short distance, 1 to 3 in., according to size. On applying to them gradually increasing voltage, a value of voltage is soon found at which the nitrogen goes into corona and an abundant supply of ions is thus produced. The space charges that result within the bulbs migrate to and are bound at the inner faces of the opposing bulbs to form a strong field between them through the open air. Such air then goes into corona too, copiously forming ions that are made simultaneously to migrate with an intensity that exceeds the critical migrating intensity resulting in the dissociation which forms the highly conducting blue-white channels characteristic of sparkover. The channels separate and convey the ions to respective outer bulb-wall to be bound to the opposing charges existing on the inner bulb-walls. Thus sparkovers are produced between glass walls as electrodes with no possibility of electron-laden metallic vapor taking part. In performing this experiment care must be taken to shut off the supply voltage quickly after the sparks between the bulbs have been started, otherwise the pyroelectric effect that follows will produce thermal punctures. It is well to remember that additional condensers connected between the electrodes or better, perhaps, between each electrode and ground will increase the available current acceleration necessary for well defined sparkover action.

This sparkover experiment with the nitrogen-filled incandescent lamps was suggested by Bradley Cozzens of our present high-voltage laboratory staff.

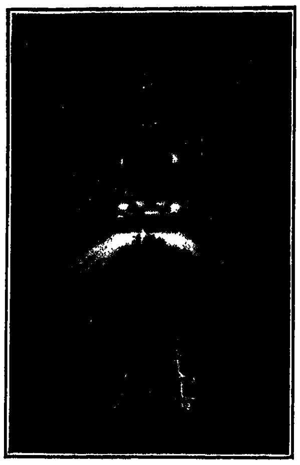


Fig. 4—Electrostatic Field Outlined by Corona during Impulse Tests on a 73,000-Volt Oil Filled Bushing Subjected to Impulses of 850,000 Volts

E. D. Eby: (communicated after adjournment) Mr. Torok's observations concerning the study of the electrostatic field by means of the suppressed discharge as illustrated in Figs. 16 to19 of his paper remind me of a similar study which I made in 1924.

At that time we were investigating the impulse flashover of high-voltage bushings of the oil-filled type. These bushings offer a splendid means for study of impulse phenomena because they are immune from damage under such impulses and may be tested repeatedly without failure. Among the interesting tests we made were some arranged to show the distribution of the electrostatic field around the bushing under impulse conditions. The distribution under 60-cycle conditions had been observed repeatedly and had actually been measured and plotted. It was, of course, out of the question to measure the distribution with impulse voltages, but we found we could photograph the electrostatic field and thus secure a very fair idea of its distribution. By applying impulses of different magnitude up to and including the flashover voltage, and by photographing the corona formed by these impulses with a special lens and rapid film, we were able to secure a very good idea of the electrostatic field under impulses.

In Fig. 4, herewith, there is shown a photograph of a 73,000-

volt oil-filled bushing subjected to about 850,000 volts. This is below the flashover voltage of the bushing, and the discharges at the most intense points of the field reach only a short distance into space. The shape of the field is well outlined by the corona. A similar view taken during the same series of tests was published in the *Electrical World* June 12, 1926. The more intense illumination of the film was produced in that instance by applying ten successive impulses during the exposure.

In Fig. 5, herewith, there is shown a view of the electrostatic field surrounding an 88,000-volt oil-filled bushing when subjected to impulses of about 1,000,000 volts. This view is particularly interesting in showing the concentration of the luminous field at points on the electrodes. Just above the porcelain shell, shafts of light are seen emerging from the surface of the clamping ring, these spots probably indicating small beads on the surface of the casting. Likewise, below the porcelain shell the concentration of the field upon the heads of the clamping bolts is very conspicuous.

During the same series of tests, impulses were applied to the bushing, subjected to a spray of water, in accordance with usual wet-test practise. The voltage required to produce flashover under these conditions was approximately the same on the average as required when the bushing was dry. This fact is of special importance in a discussion of the performance of bushings under service conditions. It demonstrates that the wet flashover voltage of a bushing is of interest only in terms of the system frequency. If the wet flashover is of such value that it exceeds any low-frequency potential that can occur on the system, there is nothing gained by increasing it further. Since voltages of the order of magnitude of flashover of bushings of good design occur in service only under impulse conditions where the wet and dry flashover voltages are the same, it becomes of prime importance to relate properly the impulse flashover of the bushing to other insulations on the system and particularly to the puncture strength of the bushing itself under impulses. Beyond question, a bushing should be self-protecting under impulse voltages as well as at system frequency, so that it may withstand impulse flashover without puncture in service.

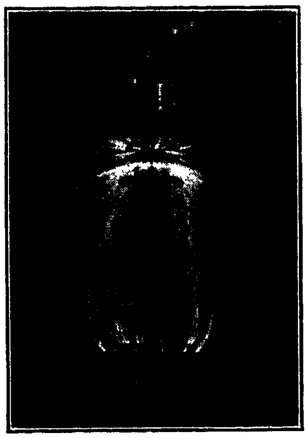


FIG. 5—ELECTROSTATIC FIELD OUTLINED BY CORONA DURING IMPULSE TESTS ON AN 88,000-VOLT OIL FILLED BUSHING SUBJECTED TO IMPULSES OF 1,000,000 VOLTS

The illustrations in Mr. Torok's paper in which insulators are shown, and the accompanying pictures of similar tests on oil-filled bushings illustrate also why the impulse ratio of these devices is approximately the same as that of the needle-gap. The wide departure from uniformity in the distribution of the electrostatic field permits portions of the space near the electrodes to become completely conducting before the intervening region has become sufficiently stressed to permit discharge. If a design of bushing or insulator approaching a sphere-gap in characteristics were to be subjected to impulses, the impulse flashover voltage would be relatively lower. Such a bushing or insulator having a normal 60-cycle flashover voltage would invite premature flashover under impulses. For this reason, a bushing with a high impulse ratio is desirable—that is, one having an impulse voltage comparable with that of other gap spacings on the system. While points on the electrodes of a bushing will reduce its 60-cycle flashover, they serve the more important function of maintaining a high impulse ratio.

K. B. McEachron: (communicated after adjournment). The method of studying the breakdown of air under impulse conditions given by Mr. Torok, sheds further light on the mechanism of the propagation of lightning through the earth's atmosphere. A more complete understanding of how the breakdown between the cloud and earth is accomplished would aid materially in determining the effects on electrical circuits located within the cloud field. It has been stated, based on Simpson's theory, that the discharge of a positive cloud is slow and therefore not harmful to the electric circuits and apparatus involved. On the other hand, discharges from the negative cloud are abrupt and give rise to dangerous voltages on transmission circuits. These statements are said to be determined from the results of klydonograph studies on transmission lines in which the highest voltages are found to be negative, indicating direct strokes, and the smaller voltages positive indicating induced charges, but from negative clouds.

If it were known that the negative discharges were the result of direct strokes and conversely that all of the direct strokes produced negative figures, the case would be somewhat more definite, but unfortunately such correlation of data is lacking. Positive and negative surges have both been found of magnitudes indicating potentials on conductors of 1800 kv. or more. On some of the systems studied, almost no difference in potential was found between the positive and negative surges.

In the very nature of the case one would expect nearly always to have many more induced surges than direct strokes and consequently find one polarity predominating. Although on some systems this has been true, there have been notable exceptions. In view of the nature of the data yet available, it does not appear that any definite statement can be made as to the polarity of clouds which give rise to dangerous surges. The same cloud may have, of course, different polarities at different points within the cloud.

Studies similar to that of Mr. Torok's coupled with an investigation of the lightning discharge itself will do much toward reaching the true determination of the mechanism of the lightning discharge.

From the paper and the ensuing discussion, I gather the impression that the effects due to polarity are masked by the fact that one of the two spheres was grounded. If Figs. 5 and 6 of the paper are comparable, one would be inclined to the belief that with ungrounded spheres, a discharge would appear having a greater volume on the positive sphere than on the negative sphere.

When viewed through a stroboscope, 60-cycle discharges appear almost entirely on the positive conductor. A study was made of this discharge a few years ago at Purdue University.[1] Using a synchronous camera the corona discharge from a pair of needles was spread along a time axis. Occasionally when a critical potential was reached, a spark would occur which was associated with the positive needle but not always with the negative. The negative point during these tests always showed a very intense point of light. The investigation was conducted in two ways; one in which the field was alternately positive and negative, and the other in which the needle was moved synchronously so that any given location was subjected to a pulsating field always of the same polarity. With either method, it was found that the streamers and corona discharge were associated with the positive needle.

---

[1] "A Photographic Study of High-Voltage Discharges," Bulletin No. 19, Engineering Experiment Station, Purdue University, Lafayette, Ind.

Two Photographic Methods of Studying High-Voltage Discharges, K. B. McEachron, TRANS. A. I. E. E., Vol. XLII, 1923, p. 947.

I believe it would be of value if Mr. Torok would give us the benefit of his ideas and experience with relation to the effect of the polarity of the sphere on the discharge so that his results may be correlated with the facts relating to 60-cycle discharges.

J. J. Torok: I wish to emphasize a few points that probably have not been made clear in my presentation.

Each photograph portrays the effects of a single surge on the apparatus under test. Mr. Peek and Mr. Eby have described tests in which they have made similar photographs of electrostatic fields. However, in their determinations, the voltage applied was always below breakdown value, and for that reason only the corona discharges were recorded. In our tests the voltage was raised well above the continuously applied breakdown potential and then was reduced to a negligible value before the developing streamers spanned the entire gap. This method has many advantages since it furnished: 1, a reliable determination of conditions throughout the entire field, 2, information on the process of breakdown at any stage of development, and 3, an indication of the process involved in the breakdown of air.

Mr. Duncan has suggested that the ionizing properties of ultra-violet light might be used as a basis for explaining this phenomenon. The ultra-violet light theory has been given consideration, but it seemed inapplicable in this case. The recombination of the free positive and negative ions liberates a given quantity of energy, only a part of which appears in the form of ultra-violet light. If all this energy were used in producing new electrons in the field (which is hardly possible), the maximum number of ions generated could not exceed the original number that had recombined. Thus from an energy standpoint, it is hard to justify this theory. Ultra-violet light may aid in the formation of the pink glow in the dark portions of the field, but the white streamers are probably formed by intensive thermal ionization, by which means a much greater proportion of electrons is produced than by ultra-violet light.

The time lags of sphere gaps and other apparatus can be attributed to the slowness of development of the white streamers. A study of the development of these streamers in various types of fields was made. It was found that the development depended upon the intensity of the field. In the near future some of these results as well as the methods by which they were obtained will be published.

Mr. Peek pointed out that his investigations showed that calculated waves checked very closely those recorded by the cathode-ray oscillograph. The surge-generator circuit used in producing these suppressed discharges contained many gaps which changed their resistances sufficiently on breakdown to modify the wave front. In some types of surge generators, there are fewer gaps, which reduces the arc-resistance variation.

As to the theory involved in the breakdown of air, the calculations that Dr. Slepian has made will probably eliminate many of the discrepancies and make it clearer than I could.

Regarding Mr. McEachron's question on polarization of the spheres, we are now working on this and it will be reported later.

Approximate Solution for Electrical Networks
When These are Highly Oscillatory

BY E. A. GUILLEMIN[1]
Associate, A. I. E. E.

Synopsis.—The general solution to the slightly damped network is expressed in terms of the undamped solution by means of series expansions. The first part of the paper gives a method for evaluating the complex roots of the determinantal equation, and the second part shows how the expansions of the first part may be correlated with the Heaviside formula to form the complete approximate solution. An example illustrates the application to a simple network.

IN working out the general solution of an electrical network, subjected to suddenly applied e. m. fs., by the classical method, there are two steps which involve a disagreeable amount of calculation—the determination of the natural angular velocities of the system, and the evaluation of integration constants or transient current amplitudes. The use of Heaviside's expansion formula does considerable toward simplifying—at least systematizing—the latter; but there still remains the necessity of solving high powered algebraic equations, which, when the network contains more than two meshes, becomes a tedious process. Primary among the factors which contribute toward the unwieldiness of the solution are the dissipative terms introduced by the presence of ohmic resistances. The solution for an idealized system with no ohmic losses is much simpler both analytically and in consequent numerical manipulation. In the first place, the determinantal equation for this case contains only even powers of the independent variable, say p, and, when solved for p^2, contains only negative real roots which may be evaluated by Newton's or Lagrange's approximation methods when the degree is above the third in p^2. Secondly, only purely imaginary or purely real quantities enter into the manipulations involved in evaluating integration constants instead of complex quantities. As soon as resistance enters into the problem, and in actual cases it always does, the work increases manifold. In practise, however, it is very frequently the case that although resistances are present, they are very small and cause only a relatively slow attenuation. We call such circuits "highly oscillatory." Practically every network used in connection with radio telephony comes under this head. It would seem, therefore, both useful and very logical to approach this case from the standpoint of the perturbation problem, and consider the slightly damped network as perturbed out of its undamped condition. It is the object of this paper to show how, by means of expansion in series, this standpoint may be carried out. The same idea may be applied to the distributed constant problem and many others; and it is hoped that interest may be stimulated in this general direction.

If we consider the meshes in our electrical network numbered from 1 to n, and let λ_{ii}, ρ_{ii}, σ_{ii} be the total inductance, resistance, and elastance, respectively, in mesh i, and further let $-\lambda_{ik}$, $-\rho_{ik}$, $-\sigma_{ik}$ be the total inductance, resistance, and elastance, respectively, common to meshes i and k (λ_{ik} being the sum of mutual and common self inductances), then the system of homogeneous differential equations describing the natural mode of oscillation of the system, when written for the mesh charges, takes the form:[2]

$$\left.\begin{array}{l} a_{11}\,x_1 + a_{12}\,x_2 + \;.\;\;.\;\;.\;\;.\; + a_{1n}\,x_n = 0 \\ a_{21}\,x_1 + a_{22}\,x_2 + \;.\;\;.\;\;.\;\;.\; + a_{2n}\,x_n = 0 \\ .\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;. \\ a_{n1}\,x_1 + a_{n2}\,x_2 + \;.\;\;.\;\;.\;\;.\; + a_{nn}\,x_n = 0 \end{array}\right\} \qquad (1)$$

$$\left.\begin{array}{ll} \text{with} & a_{ik} = \lambda_{ik}\dfrac{d^2}{d\,t^2} + \rho_{ik}\dfrac{d}{d\,t} + \sigma_{ik} \\ \text{and} & x_k = \displaystyle\int_0^t i_k\,d\,t \end{array}\right\} \qquad (2)$$

The use of mesh charges x_k instead of mesh currents i_k avoids the appearance of integrals in the system (1) and also brings out a closer analogy between the electrical system and the equivalent mechanical system in which displacements take the place of electrical charges. If we assume as solutions to system (1) the normal functions:

$$x_k = X_k\,\epsilon^{pt} \qquad (3)$$

there results the system of simultaneous homogeneous algebraic equations:

$$\left.\begin{array}{l} b_{11}\,X_1 + b_{12}\,X_2 + \;.\;\;.\;\;.\;\;.\; + b_{1n}\,X_n = 0 \\ b_{21}\,X_1 + b_{22}\,X_2 + \;.\;\;.\;\;.\;\;.\; + b_{2n}\,X_n = 0 \\ .\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;.\;\;. \\ b_{n1}\,X_1 + b_{n1}\,X_2 + \;.\;\;.\;\;.\;\;.\; + b_n\,X_n = 0 \end{array}\right\} \qquad (4)$$

$$\text{where:}\quad b_{ik} = b_{ik}(p) = \lambda_{ik}\,p^2 + \rho_{ik}\,p + \sigma_{ik} \qquad (5)$$

The assumptions (3) will be useful if system (4) leads to non-trivial solutions for the amplitudes X_k. This will be the case if the determinant of the system vanishes, *i. e.*, if:

---

[1] Instructor in Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

[2] See Carson's "Electric Circuit Theory and Operational Calculus," McGraw-Hill, 1926. Our system of equations (1) is identical with Carson's system (14), p. 10, with the substitution: $i_k = \dfrac{d\,x_k}{d\,t}$, allowing for differences of notation, of course. Our following equations (4) and (6) correspond in the same way to Carson's equations (15) and (16), pp. 10 and 11

$$D(p) = \begin{vmatrix} b_{11} & \dots\dots & b_{1n} \\ b_{21} & \dots\dots & b_{2n} \\ \dots & \dots\dots & \dots \\ b_{n1} & \dots\dots & b_{nn} \end{vmatrix} = 0 \qquad (6)$$

For the solution to be unique it is further necessary that D have the rank $n - 1$, *i.e.*, that at least one of its first minors shall not vanish. Equation (6) is in compact form the determinantal equation of the oscillatory system. Since each element is in general a polynomial of the second degree in p, D will be a polynomial of the $2\,n$th degree in p, and according to the fundamental law of algebra (6) can be satisfied in $2\,n$ ways. Hence $2\,n$ solutions (3) exist with $2\,n$ arbitrary amplitudes X_k to satisfy the $2\,n$ initial charges and currents in the given electrical system.

Consistent with our major premise, we proceed to expand (6) in powers of ρ_{ik} by Taylor's theorem. We have:[3]

$$D(p) = \Big[D(p) \Big]_{\rho_{ik}=0} + \frac{1}{1!} \sum_{1}^{n}{}^{ik} \left[\frac{\partial D(p)}{\partial \rho_{ik}} \right]_{\rho_{ik}=0} \cdot \rho_{ik} + \dots \qquad (7)$$

But according to the rule for differentiating determinants:[4]

$$\frac{\partial D(p)}{\partial \rho_{ik}} = \frac{\partial D(p)}{\partial b_{ik}} \frac{\partial b_{ik}}{\partial \rho_{ik}} = B_{ik}(p) \frac{\partial b_{ik}}{\partial \rho_{ik}}$$

$$= B_{ik}(p) \cdot p \qquad (8)$$

where $B_{ik}(p)$ is the minor of b_{ik} in (6). Since $D(p)$ is a whole rational function in ρ_{ik}, the expansion (7) will contain a finite number of terms. From (8) it is evident[5] that successive terms will rapidly become smaller if $\rho_{ik} << p$. We will assume this and consequently neglect all powers of the ρ_{ik}'s above the first. If now we adopt the notation:

$$\left.\begin{aligned} \Big[D(p) \Big]_{\rho_{ik}=0} &= D^*(p) \\ \text{and} \quad \Big[B_{ik}(p) \Big]_{\rho_{ik}=0} &= B_{ik}^*(p) \end{aligned}\right\} \qquad (9)$$

our determinantal equation (6) becomes:

$$D(p) = D^*(p) + p \sum_{1}^{n}{}^{ik} B_{ik}^*(p) \cdot \rho_{ik} = 0 \qquad (10)$$

A first approximation to the roots of (10) is given by:

$$D^*(p) = 0 \qquad (11)$$

which is the determinantal equation for neglected attenuation. As already pointed out, it contains only even powers of p, the roots for p^2 being negative real, and hence those for p purely imaginary. Let the roots of (11) be given by:

$$p = p^* \qquad (12)$$

According to the Newtonian method[6] we then put for the roots of $D(p)$:

$$p = p^* + \delta \qquad (13)$$

and substitute this value into $D^*(p)$ of equation (10). For the second term in (10) it is sufficient to substitute (12). We then have:

$$D(p) = D^*(p^* + \delta) + p^* \sum_{1}^{n}{}^{ik} B_{ik}^*(p^*) \cdot \rho_{ik} = 0 \qquad (14)$$

Expanding the first term in a Taylor's series about $p = p^*$ we get:

$$D^*(p^* + \delta) = D^*(p^*) + \frac{1}{1!} \left(\frac{\partial D^*}{\partial p} \right)_{p=p^*} \cdot \delta + \dots \qquad (15)$$

Now if we note that

$$\begin{aligned} D^*(p) &= C_n p^n + C_{n-2} p^{n-2} + \dots + C_0 \\ &= C_n (p^* + \delta)^n + C_{n-2} (p^* + \delta)^{n-2} + \dots + C_0 \end{aligned}$$

and $(p^* + \delta)^n$

$$= p^{*n} \left\{ 1 + \frac{n}{1!} \frac{\delta}{p^*} + \frac{n(n-1)}{2!} \left(\frac{\delta}{p^*} \right)^2 + \dots \right\}$$

we see that terms of (15) above the first degree in δ may be neglected if:

$$\frac{n-1}{2} \frac{\delta}{p^*} << 1 \qquad (16)$$

which is the criterion for a "highly oscillatory" network in the sense in which that term is used in this paper. Substituting (15) into (14) and noting that:

$$D^*(p^*) = 0,$$

and also that

$$\left[\frac{\partial D^*}{\partial p} \right]_{p=p^*} = \sum_{1}^{n}{}^{ik} \left[\frac{\partial D^*}{\partial b_{ik}} \frac{\partial b_{ik}}{\partial p} \right]_{p=p^*}$$

$$= \sum_{1}^{n}{}^{ik} 2\,\lambda_{ik}\, p^* B_{ik}^*(p^*) \qquad (17)$$

we get $\delta \displaystyle\sum^{n}{}^{ik} 2\,\lambda_{ik} B_{ik}^*(p^*) + \sum^{n}{}^{ik} \rho_{ik} B_{ik}^*(p^*) = 0$

---

[3] See Wood's "Advanced Calculus," Ginn & Co., 1926, p. 49, equation (1), or any similar text covering Taylor's theorem.

[4] See, for example, R. F. Scott's "Theory of Determinants," Cambridge University Press, 1880, p. 35.

[5] This will be clear if we note that B_{ik} is two powers lower in p than D. Hence the second term of (7) divided by the first will be of the order: ρ_{ik}/p.

[6] See, for example, Woods & Bailey, "A Course in Mathematics," Ginn & Co., 1907, p. 114. The method is there shown applied to the solution of numerical equations, but the general idea involved is the same as that used in this paper.

$$\text{or } \delta = - \frac{\sum_{1}^{n}{}^{ik} \rho_{ik} B_{ik}^{*} (p^{*})}{\sum_{1}^{n}{}^{ik} 2 \lambda_{ik} B_{ik}^{*} (p^{*})} \qquad (18)$$

From the form of (18) it is clear that δ will always be real, and hence represents the damping factor associated with the frequency given by p^*. The complete complex "natural angular velocity" so-called, is given by (13). The form of (18) is in itself interesting. The damping factors appear as the negative quotient of the averaged mesh resistances weighted by the unperturbed minors, and double the mesh inductances averaged in the same way. The dimension of δ and its similarity to that for the simple R, L, C circuit are evident. The latter may serve as a simple illustration of our result. Here,

$$D(p) = b(p) = L p^2 + R p + \frac{1}{C} = 0.$$

$$D^*(p) = L p^2 + \frac{1}{C} = 0; \; p^* = \frac{\pm j}{\sqrt{L C}}.$$

The B_{ik}^* are unity in this case and we have,

$$\delta = - \frac{R}{2 L},$$

$$\text{and hence, } p = - \frac{R}{2 L} \pm j \frac{1}{\sqrt{L C}}.$$

The exact root differs from this only in that the damped angular frequency,

$$\pm j \sqrt{\frac{1}{L C} - \frac{R^2}{4 L^2}}$$

appears in place of the undamped.

Although equation (18) is applicable more directly to the mathematical representation of the network itself, the same idea can be carried out on the determinantal equation in its polynomial form. If we have given:

$$D(p) = A_n p^n + A_{n-1} p^{n-1} + \ldots + A_0 = 0 \qquad (19)$$

it can easily be shown that the corresponding determinantal equation for the undamped case is approximately[7] given by simply striking the second, fourth, etc., terms, *i. e.*, that:

$$D^*(p) = A_n p^n + A_{n-2} p^{n-2} + \ldots + A_0 = 0. \qquad (20)$$

Then if we let,

$$G(p) = A_{n-1} p^{n-1} + A_{n-3} p^{n-3} + \ldots + A_1 p \qquad (21)$$

$$\text{it will be seen that } \left[G(p) \right]_{\rho_{ik}=0} = 0,$$

and hence that (21) can take the place of the double sum in (7). Applying the Newtonian method as before we find:

$$\delta = - \frac{G(p^*)}{\left(\frac{\partial D^*}{\partial p} \right)_{p=p^*}} \qquad (22)$$

To illustrate the application of (22), let us consider the numerical quartic:

$$D(p) = p^4 + 50 p^3 + 8.91 \times 10^5 p^2 + 2.03 \cdot 10^7 p + 1.601 \cdot 10_n = 0$$

$$\text{Here } D^*(p) = p^4 + 8.91 \cdot 10^5 p^2 + 1.601 \cdot 10^{11} = 0$$

$$\text{and } \quad p^* = \pm j\, 800 \text{ and } \pm j\, 500.$$

For the first of these:

$$G(p^*) = \mp j\, 9.36 \cdot 10^9; \left(\frac{\partial D^*}{\partial p} \right)_{p=p^*} = \mp j\, 6.23 \cdot 10^8$$

$$\text{and hence} \qquad \delta = - 15.$$

For the second:

$$G(p^*) = \pm j\, 3.90 \cdot 10^9; \left(\frac{\partial D^*}{\partial p} \right)_{p=p^*} = \pm j\, 3.90 \cdot 10^8,$$

$$\text{and hence} \qquad \delta = - 10.$$

The final roots are:

$$- 10 \pm j\, 500 \text{ and } - 15 \pm j\, 800$$

which are exactly the ones used in making up the above equation, the differences due to the approximate method of solution not being noticeable on the slide rule used in the calculation.

It frequently happens that due to the absence of independent inductance or elastance in one of the meshes, the determinantal equation comes out to be of an odd degree. The number of natural angular frequencies contained in the system in that case will be $\frac{n-1}{2}$ where n is the degree of the equation. The odd root will be purely real. The fact that it possesses a normal coordinate[8] as well as the rest of the frequencies but vanishes for vanishing resistances, seems to indicate physically that in the latter case a regrouping of the meshes can be carried out so as to reduce their number by one—there being always as many normal coordinates as there are meshes, (physical dimensions in the equivalent mechanical system). It would seem as though for this real root our method would fail, since its imaginary part is zero. However, it can be shown that so long as this real root is within the same order of magnitude as the real portions of the rest of the roots, the method still holds with fair accuracy. Let us suppose we have an equation of even degree given by:

$$D(p) = 0;$$

---

[7] For the preceding simple example this happens to be exact; but in general the resistance terms will contribute a small ashre to the coefficients of even powers of p.

[8] See paper by the writer entitled "Making Normal Coordinates Coincide with the Meshes of an Electrical Network," *Proceedings* of the I. R. E., Nov. 1927, p. 935.

and we introduce an extra real root $p = -\alpha$. Then the equation of odd degree becomes:

$$D_0(p) = (p + \alpha)\, D(p) = 0. \qquad (23)$$

The equation for neglected resistance would then be:

$$D_0^*(p) = p\, D^*(p) = 0,$$

which contains the zero root mentioned above. Hence, substituting $p = 0 + \delta$ into (23) and expanding we get:

$$D_0(\delta) = D_0(0) + \frac{1}{1!}\left(\frac{\partial D_0}{\partial p}\right)_{p=0} \cdot\cdot + \ldots = 0,$$

$$\text{and} \quad \delta = -\frac{D_0(0)}{\left(\frac{\partial D_0}{\partial p}\right)_{p=0}}.$$

But: $\quad D_0(0) = \alpha\, D(0),$

$$\text{and} \quad \left(\frac{\partial D_0}{\partial p}\right)_{p=0} = \alpha\left(\frac{\partial D}{\partial p}\right)_{p=0} + D(0),$$

$$\text{so that:} \quad \delta = -\frac{\alpha\, D(0)}{D(0) + \alpha\left(\frac{\partial D}{\partial p}\right)_{p=0}},$$

from which it is clear that the method still gives satisfactory results if:

$$\alpha\left(\frac{\partial D}{\partial p}\right)_{p=0.} < < D(0);$$

or by using the notation of equation (19), if:

$$\alpha < < \frac{a_0}{a_1}.$$

In our numerical example above, this means that an additional real root $(p + \alpha)$ may be introduced and re-evaluated by formula (22) with good results as long as

$$\alpha < < \frac{1.601 \cdot 10^{11}}{2.03 \cdot 10^{7}} = 7890,$$

which condition is easily fulfilled if α is not much larger than the real portions of the other roots. Note that for the evaluation of this real root equation (22) simply becomes equal to the negative quotient of the last two coefficients of the odd powered determinantal equation in the form of (19). After this odd root has thus been determined, it is best to divide it out and then treat the remainder as in the case where $D(p)$ is of even degree. To illustrate let us take our previous numerical example, and introduce the root $p = -100$. It then becomes:

$$D(p) = p^5 + 150\, p^4 + 8.96 \cdot 10^6\, p^3 + 1.094 \cdot 10^8\, p^2 + 1.62 \cdot 10^{11}\, p + 1.601 \cdot 10^{13} = 0.$$

Here: $\quad D^*(p) = p^5 + 8.96 \cdot 10^6\, p^3 + 1.62 \cdot 10^{11}\, p,$

and $\quad G(p) = 150\, p^4 + 1.094 \cdot 10^8\, p^2 + 1.601 \cdot 10^{13}.$

By applying (22) for $p^* = 0$ we get:

$$\delta = -\frac{1.601 \cdot 10^{13}}{1.62 \cdot 10^{11}} = -98.8$$

which is the root correct to within 0.2 per cent.

Our next step is to show how nicely the above method of arriving at the approximate complex roots fits in with an approximate determination of the transient current amplitudes by means of Heaviside's well-known expansion formula. The latter may be put into the form:[9]

$$i_{ik} = E \sum_{\nu} \frac{B_{ik}(p_\nu)\, \epsilon^{p_\nu t}}{\left(\frac{\partial D}{\partial p}\right)_{p=p_\nu}} \qquad (24)$$

when written for the current in mesh i due to a suddenly applied direct voltage E in mesh k or vice versa. $D(p)$ is the determinant given by (6), and B_{ik} the respective minor of this determinant. The summation is to extend over all the roots of the determinantal equation:

$$D(p) = 0.$$

For the case of slight damping, however, the solution is given approximately by:

$$i_{ik} = E \sum_{\nu} \frac{B_{ik}^*(p_\nu^*)\, \epsilon^{p_\nu t}}{\left(\frac{\partial D^*}{\partial p}\right)_{p=p_\nu^*}}. \qquad (25)$$

But the denominators in this sum are already known from equation (17), so that we merely have to substitute these same values into (25) in order to get the complete solution. The approximate solution to the determinantal equation dovetails nicely with the process of evaluation of the transient current amplitudes. Substituting (17) into (25), we can write for Heaviside's expansion formula in approximate form:

$$i_{ik} = \frac{E}{2} \sum_{\nu} \frac{B_{ik}^*(p_\nu^*)\, \epsilon^{p_\nu t}}{p_\nu^* \sum_{1}^{n}{}^{ik}\, \lambda_{ik}\, B_{ik}^*(p_\nu^*)} \qquad (26)$$

If the determinantal equation (6) has no zero root, it means that the mesh in which the e. m. f. is impressed contains a condenser so that the steady state becomes

---

[9] It will be seen with a little investigation that our:

$$\frac{D(p)}{B(p)} = p\, Z(p)$$

where $Z(p)$ is the transient impedance as usually defined in connection with Heaviside's expansion formula. Now:

$$\frac{d}{dp}\left[\frac{D}{B}\right]_{p=p_\nu} = \left[\frac{B \cdot \frac{dD}{dp} - D\frac{dB}{dp}}{B^2}\right]_{p=p_\nu} = \frac{\left(\frac{dD}{dp}\right)_{p=p_\nu}}{B(p^\nu)}$$

$$= \left[p\frac{dZ}{dp} + Z\right]_{p=p_\nu} = p^\nu \left(\frac{dZ}{dp}\right)_{p=p_\nu} + Z(p^\nu).$$

Comparing with the usual form of Heaviside's formula, the form (24) will be seen to be identical with it. For additional information on this point, see "Notes on Operational Calculus," by V. Bush, obtainable from the Electrical Engineering Dept. of the Massachusetts Institute of Technology, Cambridge, Mass.

zero. Since the terms in (26) are conjugate pairs, the current will be twice the real portion, or:

$$i_{ik} = E \sum_{1,3,5..}^{n} {}^{\nu} \frac{B_{ik}^* (p_\nu^*)\, \epsilon^{\delta t}}{| p_\nu^* | \sum_{1}^{n} {}^{ik} \lambda_{ik} B_{ik}^* (p_\nu^*)} \sin | p_\nu^* I t | \quad (27)$$

In order to illustrate the entire process of solution outlined in this paper, let us consider a simple network consisting of two inductively coupled meshes with a suddenly applied constant potential of E volts in the first,—the circuit constants being given by:

$\lambda_{11} = 0.3$ henries; $\lambda_{12} = -.1$ henries; $\lambda_{22} = 0.2$ henries.

$\rho_{11} = 5$ ohms; $\rho_{12} = 0$; $\rho_{22} = 5$ ohms.

$\sigma_{11} = 10^6$ darafs; $\sigma_{12} = 0$; $\sigma_{22} = 10^8$ darafs.

Then,

$$D^* (p) = \begin{vmatrix} .3\, p^2 + 10^6 & -.1\, p^2 \\ -.1\, p^2 & .2\, p^2 + 10^8 \end{vmatrix}$$

$$= p^4 + 6.04 \cdot 10^8 p^2 + 2 \cdot 10^{15} = 0.$$

From which $p_{1,2}^* = \pm j\, 2.45 \cdot 10^4$; $p_{3,4}^* = \pm j\, 1.87 \cdot 10^3$.

We then find

$B_{11}^*(p_{1,2}^*) = -2 \cdot 10^7$; $B_{12}^*(p_{1,2}^*) = -6 \cdot 10^7$; $B_{22}^*(p_{1,2}^*) = -1.79 \cdot 10^8$.

and $B_{11}^*(p_{3,4}^*) = 9.93 \cdot 10^7$; $B_{12}^*(p_{3,4}^*) = -3.5 \cdot 10^5$; $B_{22}^*(p_{3,4}^*) = -5 \cdot 10^4$.

From which $\sum_{1}^{n} {}^{ik} \rho_{ik} B_{ik}^* (p_{1,2}^*) = -9.95 \cdot 10^8$;

$$\sum_{1}^{n} {}^{ik} \rho_{ik} B_{ik}^* (p_{3,4}^*) = 4.965 \cdot 10^8,$$

and $\sum_{1}^{n} {}^{ik} 2\, \lambda_{ik} B_{ik}^* (p_{1,2}^*) = -5.96 \cdot 10^7$;

$$\sum_{1}^{n} {}^{ik} 2\, \lambda_{ik} B_{ik}^* (p_{3,4}^*) = 5.97 \cdot 10^7.$$

and substituting into (18) we have:

$$\delta_{1,2} = -\frac{9.95 \cdot 10^8}{5.96 \cdot 10^7} = -16.7$$

$$\delta_{3,4} = -\frac{4.96 \cdot 10}{5.97 \cdot 10^7} = -8.33$$

Then substituting into (27) we get for the current in mesh one:

$$i_{11} = E\, (2.74 \cdot 10^{-5}\, \epsilon^{-16.7t} \sin 24500\, t + 1.78 \cdot 10^{-3}\, \epsilon^{-8.33t} \sin 1870\, t).$$

The current in mesh two is found from this by simply multiplying the first term by $\dfrac{B_{12}^* (p_{1,2}^*)}{B_{11}^* (p_{1,2}^*)}$,

and the second by $\dfrac{B_{12}^* (p_{3,4}^*)}{B_{11}^* (p_{3,4}^*)}$.

We find $i_{12} = E\, (8.22 \cdot 10^{-5}\, \epsilon^{-16.7t} \sin 24500\, t - 6.28 \cdot 10^{-6}\, \epsilon^{-8.33t} \sin 1870\, t).$

It is quite apparent that the saving in tedious manipulation over the usual method is considerable.

Discussion

V. Bush: Unfortunately, every time we try to apply Heaviside's expansion theorem to any network of moderate complication we run into insurmountable difficulties in computation. Those appear specifically in an equation of a higher order which we must solve for the roots. Dr. Guillemin has shown us how in general, in any case where the dissipation is small, we can arrive at those roots and at a form of the expansion theorem which is sufficiently accurate by methods which are much shorter than the complete methods. The object is the exposition of modified forms of treatment for oscillatory cases of low damping.

E. A. Guillemin: There is an additional point in connection with the approximate method of solution, which was not mentioned explicitly in the paper, but which may be of interest to those who wish to apply it to specific problems.

Two methods of attack for the evaluation of the complex roots were presented: The first was based upon the determinantal equation in its determinant form (Equation 6), and the second, upon the determinantal equation in its polynominal form (Equation 19). The correlation of this process of evaluation of roots with the Heaviside formula (Equation 25), however, was carried through only on the determinant basis, and brought into the explicit form given by Equation 27. The same correlation may, of course, be carried through in connection with the determinantal equation in its polynomial form, as follows:

Equation 22 may be solved for $\left(\dfrac{\partial D^*}{\partial p} \right) p = p^*$ and the result substituted in Equation 25. Grouping pairs of conjugate terms, the final result is:

$$i_{in} = -2\, E \sum_{1,3,5}^{n} \nu \frac{\delta_\nu B_{in}^* (p^*_\nu) \,.\, \epsilon^{\delta_\nu t}}{| G (p^*_\nu) |} \sin | p_\nu^* |\, t\, .$$

On applying this form it should be noted that $G(p^*_\nu)$ is a purely imaginary quantity and of the form $\pm j$ times an absolute magnitude which may be numerically positive or negative in itself. It is this numerical quantity (with proper sign) which is to be substituted in the above formula.

The Boltzmann-Hopkinson Principle
of Superposition as Applied to Dielectrics

BY F. D. MURNAGHAN[1]
Non-member

Synopsis.—*The principle of superposition, discussed first for dielectrics by Hopkinson, has been shown for a long time in experiment. However, its validity for the several theories which have been advanced for dielectric behavior has been shown in only a few cases. One of the exceptions is the important theory of Maxwell.*[2] *This paper shows that the principle of superposition is a necessary consequence of Maxwell's theory. It goes somewhat further and shows that the principle is valid for any theory which leads to a system of linear differential equations with constant coefficients.*

* * * * *

If we have a plane layer dielectric with n layers and denote the thickness of the rth layer by a_r, its specific inductive capacity by K_r, and its specific resistance by r_r then the common current density u in each layer is given by

$$u = (D + b_r) f_r \qquad (1)$$

when D denotes time differentiation,[3] f_r stands for the displacement in the rth layer, and b_r is an abbreviation for the ratio $4\,\pi/K_r\,r_r$. Furthermore, the displacements f are connected with the applied e. m. f. E by the equation

$$\alpha_1 f_1 + \alpha_2 f_2 + \ldots + \alpha_n f_n = E \qquad (2)$$

where α_r is an abbreviation for the ratio $4\,\pi\,a_r/K_r$. These equations lead to a linear differential equation of order $n - 1$ for each of the displacements f, the equation satisfied by f_r, being

$$\left[\frac{\alpha_1}{D + b_1} + \frac{\alpha_2}{D + b_2} \ldots + \frac{\alpha_n}{D + b_n}\right] f_r = \frac{E}{D + b_r} \qquad (3)$$

it being understood, however, that this is merely a convenient way of writing the differential equation which results when both sides are cleared of fractions by multiplying through by the product of all the denominators $(D + b_r)$. The dielectric being supposed initially uncharged we have to find solutions of (3), consistent with (1), which have all the same value $E(0)/\alpha_1 + \ldots + \alpha_n)$ when $t = 0$.

The differential equation (3) has on the left hand side a differential expression $F\,(D)$ of order $n - 1$, the coefficient of the highest order derivative D^{n-1} being $(\alpha_1 + \alpha_2 + \ldots + \alpha_n)$, and we shall denote this coefficient by A, for convenience. The right hand side of (3) is the result of differentiating the function E of t in the manner indicated by the product of all the $(D + b)$ save the single one $(D + b_r)$. It will be convenient first to consider the auxiliary equation

$$F\,(D)\,y = E\,(t) \qquad (4)$$

since if we have a solution y of this equation the function f_r obtained by operating on y by the product of all the $(D + b)$ save the single one $(D + b_r)$ will satisfy the equation (3). In fact since the coefficients in the polynominal $F\,(D)$ are constants we may interchange the order of the differentiations indicated by $F\,(D)$ and by the product of the factors $(D + b)$. Hence

$$\begin{aligned} F\,(D)\,.\,(D + b_1)\,(\ldots)\,(D + b_n)\,y &= (D + b_1)\,(\ldots)\,(D + b_n)\,.\,F\,(D)\,y \\ &= (D + b_1)\,(\ldots)\,(D + b_n)\,E\,(t) \end{aligned}$$

We now proceed to find, by a method due to Cauchy, that particular solution of the equation (4) which vanishes, together with all its derivatives up to the n − 2nd inclusive when $t = 0$. The homogeneous equation corresponding to (4), *i. e.*,

$$F\,(D)\,z = 0 \qquad (5)$$

being of order $n - 1$, has $n - 1$ distinct solutions which we may denote by $(z_1, z_2, \ldots z_{n-1})$, and the general solution of (5) is, in accordance with the theory of linear differential equations, a combination, with constant coefficients, of these n distinct solutions. Thus the general solution of (5) may be written in the form

$$z = C_1 z_1 + C_2 z_2 + \ldots + C_{n-1} z_{n-1} \qquad (6)$$

The constants C are entirely arbitrary and we choose them so that the function z of t defined by (6) vanishes together with its derivatives up to the n − 3rd inclusive when $t = \tau$; the n − 2nd derivative, on the contrary, assuming the value $E\,(\tau)/A$. The prescribing of the values of z and its derivatives up to the n − 2nd, inclusive, in this way for a particular value τ of t gives us exactly $n - 1$ linear equations for the determination of the values of the constants C which occur in (6). These constants will depend on the particular value τ of t chosen to enable us to determine them by means of the conditions stated and they will, in addition, be proportional to $E\,(\tau)$ since the right hand members of the $n - 1$ linear equations for the C's are all zero save one which is precisely $E\,(\tau)/A$. We shall indicate these facts by writing z of (6) in the form

---

[1] Johns Hopkins University.

[2] See Maxwell's Theory of the Layer Dielectric, F. D. Murnaghan, TRANS. A. I. E. E., Vol. XLVI, 1927, p. 259.

[3] The symbol D stands for $d/d\,t$ and is used throughout the paper symbolically as an algebraic quantity. For the use of the symbolic operator $D + b$ in the solution of linear differential equations with constant coefficients see any standard treatise on the subject, for example, Cohen's Differential Equations (Heath & Co), p. 96.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-17, 1928.

$$z = E\,(\tau)\; v\,(t,\,\tau) \qquad (7)$$

The fact that the coefficients in the differential operator $F\,(D)$ of (5) are constants tells us further that the function v of the two arguments t and τ does not really involve them separately but is rather a function of their difference $(t-\tau)$. In fact the functions z_1, etc., are exponential functions of the time and these exponential functions reproduce themselves, multiplied by a constant factor, when differentiated with respect to the time. Hence each C occurs, in the $n-1$ linear equations which serve to determine them, multiplied by the same factor $e^{\delta_r \tau}$, say, in each of the equations. This tells us that the $v\,(t,\,\tau)$ of (7) is of the type

$$v\,(t,\,\tau) = B_1\, e^{\delta_1(t-\tau)} + \,.\;.\;.\, + B_{n-1}\, e^{\delta_{n-1}(t-\tau)} \qquad (8)$$

where the B's are pure numbers independent of both t and τ. The δ's are the zeros of the polynominal $F\,(D)$. We have found, then, a function

$$z = E\,(\tau)\,.\,v\,(t-\tau) \qquad (9)$$

which satisfies the homogeneous equation (5) and which vanishes together with its derivatives with respect to t up to the $n-3$rd inclusive when $t = \tau$ (*i. e.*, when the argument $t-\tau$ of the function v is zero); the $n-2$nd derivative assuming the value $E\,(\tau)/A$ when the argument of v is zero. We now proceed to show that the function y of the single variable t, defined by the integral

$$y = \int_0^t E\,(\tau)\,.\,v\,(t-\tau)\,d\,\tau \qquad (10)$$

is that particular solution of the equation (4) which vanishes together with its time derivatives up to the $n-2$nd, inclusive, when $t = 0$; in fact, since the variable t occurs in two places—the integrand and the upper limit—in the expression for y we have for the time derivative of y

$$D\,y = \int_0^t E\,(\tau)\,.\,D\,v\,(t-\tau)\,d\,\tau + E\,(t)\,.\,v\,(0)$$

The second member on the right hand side vanishes, since the function v has been so arranged as to vanish when its argument is zero. Proceeding to the second time derivative we find in the same way that

$$D^2\,y = \int_0^t E\,(\tau)\,.\,D^2\,v\,(t-\tau)\,d\,\tau$$

and so on up to the $n-1$st time derivative, which is slightly different; here the second term in the expression for the time derivative of the integral does not vanish, since the $n-2$nd time derivative of v does not vanish when the argument of v is zero but has the value $1/A$ then. We have, therefore,

$$D^{n-1}\,y = \int_0^t E\,(\tau)\,.\,D^{n-1}\,v\,(t-\tau)\,d\,\tau + E\,(t)/A$$

Upon combining these results and remembering that the coefficient of the highest order derivative, D^{n-1}, in $F\,(D)$ is A, we have

$$F\,(D)\,y = \int_0^t E\,(\tau)\,.\,F\,(D)\,v\,(t-\tau)\,d\,\tau + E\,(t)$$

The first member of the expression on the right vanishes since the function v of t is a solution of the homogeneous equation (5), *i. e.*, $F\,(D)\,v\,(t-\tau) = 0$ for all values of the argument $(t-\tau)$. This shows that y, as defined by (10) is a solution of (4), and since all the derivatives up to the $n-2$nd, inclusive, are expressed as integrals, the range of integration being from zero to t, all of these derivatives and the function y itself vanish when $t = 0$.

The various displacements f_r are now obtained from the auxiliary function y by operating on it in turn by the product of all the $(D+b)$'s but the single one $(D+b_r)$. From the expressions just given for the various time derivatives of y up to the $n-1$st we have, for example,

$$f_1 = \int_0^t E\,(\tau)\,.\,(D+b_2)\,(.\;.\;.)\,(D+b_n)\,v\,(t-\tau)\,d\,\tau + E\,(t)/A \qquad (11)$$

showing that the common initial value of all the displacements is $E\,(0)/A$. It will be convenient to introduce the notation

$$h_1\,(t-\tau) = (D+b_2)\,(.\;.\;.)\,(D+b_n)\,v\,(t-\tau) \qquad (12)$$

etc., the function $h_r\,(t-\tau)$ being the result of operating on the function $v\,(t-\tau)$ by the product of all the $(D+b)$'s but $(D+b_r)$. We have, then, the various displacements given by the formulas

$$f_r = \int_0^t E\,(\tau)\,.\,h_r\,(t-\tau)\,d\,\tau + E\,(t)/A \qquad (13)$$

The n functions h_r all have the same value $1/A$ when the argument $(t-\tau)$ is zero. The formulas (13) become more significant, from a physical point of view, if we change their appearance by an integration by parts. On introducing the functions $\varphi_r\,(t-\tau)$ by the equations of definition

$$\varphi_r\,(t-\tau) = \int_\tau^t h_r\,(t-s)\,d\,s + 1/A \qquad (14)$$

so that

$$\frac{d}{d\,\tau}\,\varphi_r\,(t-\tau) = -\,h_r\,(t-\tau)$$

we have from (13)

$$f_r = E\,(0)\,\varphi_r\,(t) - E\,(t)\,\varphi_r\,(0) + \int_0^t \frac{d\,E}{d\,\tau}\,\varphi_r\,(t-\tau)\,d\,\tau + E\,(t)/A$$

The second term in the expression on the right hand cancels the fourth, since when the argument of φ_r is zero, the two limits of the integral of (14) coincide so that $\varphi_r\,(0) = 1/A$. Then we have the final definite result

$$f_r = E\,(0)\,\varphi_r\,(t) + \int_0^t \frac{d\,E}{d\,\tau}\,\varphi_r\,(t-\tau)\,d\,\tau \qquad (15)$$

This equation gives a physical interpretation to the functions φ; for, on writing $E\,(t) = 1$ in (**15**), we see that $\varphi_r\,(t)$ is the displacement in the rth layer when a constant unit e. m. f. is applied at time $t = 0$ to the dielectric supposed uncharged.

The result given in (**15**) is the mathematical expression of the Boltzmann-Hopkinson Principle of Superposition as applied to the displacements. It is evident from its mode that the principle of superposition is valid not only for Maxwell's theory of dielectric absorption but for any theory which leads to a system of differential equations which are linear and have constant coefficients. For example a theory in which the current density was connected with the displacement by a second order equation of the type $(D^2 + b_r\,D + c_r)\,f_r = u$ would have the principle of superposition as a consequence.

Once the displacements have been found the current density u follows at once from (**1**). Writing

$$\psi\,(t-\tau) = (D + b_r)\,\varphi_r\,(t-\tau) \qquad (16)$$

(it is easily verified that the expression on the right has the same value for all the layers) we find

$$u\,(t) = E\,(0)\,\psi\,(t) + \int_0^t \frac{d\,E}{d\,\tau}\,\psi\,(t-\tau)\,d\,\tau + \frac{d\,E}{d\,t}\Big/ A \qquad (17)$$

The physical meaning of the function $\psi\,(t)$ is evident from this formula; it is the current at time t due to a constant unit e. m. f. applied to the uncharged dielectric at time $t = 0$. The principle of superposition is not valid for the entire current but the first two terms of (**17**) may be found by means of that principal and then the complete current follows by adding the displacement current which is given by the third term.

We have given elsewhere[4] the expression for the function ψ in terms of the constants of the layer dielectric and the formula (**17**) furnishes a ready method for calculating the current due to any type of applied e. m. f.

Discussion

J. B. Whitehead: (by letter) Most electrical engineers who are concerned with the behavior of dielectrics and insulation are aware that Maxwell's theory of dielectric absorption or residual charge cannot be applied for the predetermination of the properties of composite dielectrics and insulation. Yet Maxwell's theory is the only rational theory which has been proposed in this field, and the only one which seems to offer the possibility of experimental verification. Furthermore, the present indifference on the part of physicists to this important and highly interesting phenomenon is explained in many instances by their feeling that Maxwell has given a sufficient explanation.

The Committee on Electrical Insulation of the National Research Council several years ago set for itself the task of reviewing, collecting, and coordinating the existing knowledge as to the various properties of dielectrics. In considering Maxwell's theory it was realized that in several particulars it was incomplete as originally presented by the author. Among the most important of these was the absence of the complete expression for the charging current in the most general case of n layers, and the absence of a formal statement or proof that the Boltzmann-Hopkinson principle of superposition was a necessary consequence of Maxwell's theory of dielectric absorption. On the first of these questions Maxwell contented himself with a general statement, and he doubtless was convinced of the certainty of the second, because he was intimately acquainted with Hopkinson's work and indeed discussed with him the experimental evidence of the presence of the principle of superposition in Hopkinson's experiments on dielectrics. Responding to questions on the mathematical aspects of these problems, Dr. Murnaghan was kind enough to become deeply interested in them and has given us his paper[1] of last year, clearing up the first of the problems mentioned above. He now presents a second paper analyzing the relationship of superposition and Maxwell's theory of dielectric absorption. As will be clear in the concluding paragraphs of the paper, it is shown that the principle of superposition is valid for the displacements in a layer dielectric, but not for the total current. The final term in Equation **17**, indicating the important addition, might be called the variation of the average geometrical displacement.

Whatever may be said as to the difficulties encountered in an effort to apply Maxwell's theory, Dr. Murnaghan's two papers constitute important commentaries on the theory, and will be of great assistance to all students in rounding out the picture conceived by Maxwell.

V. Bush: I have found it difficult indeed to get at the point of this paper, and hence I should like to give my interpretation of it in engineering language.

In applying the Boltzmann-Hopkinson extension theorem, or the superposition principle, we are passing from the d-c. transient of a network to the a-c. transient, or, more generally, we are passing from the response to a unit e. m. f. to the response to any applied voltage whatever. This can be done whenever the network is linear, that is, where the effect of two volts is exactly twice the effect of one volt. If we cannot thus superpose results we cannot use the superposition principle, and the reverse is also true. Now, as far as this is concerned with dielectrics it doesn't lead us very far, because in the most interesting cases results do not add simply. If we assume Maxwell's theory of dielectrics, they do; but unfortunately practical dielectrics do not work that way. I see, however, in Dr. Murnaghan's paper a matter of more general interest than this and which I do not think he brought out so that it is immediately apparent. There is difficulty in applying the superposition principle even in certain linear circuits. Sometimes the response to a unit applied e. m. f. is impulsive. If we suddenly apply a unit e. m. f. to a simple condenser the current is impulsive; that is, it is mathematically infinite and lasts for an infinitesimal time. In such a case it is difficult to apply the principle and we require some further treatment. Such networks are of course only mathematical fictions, as there is always in practical cases some series resistance present in the circuit or the source. Wherever such difficulties are encountered we can use the last equation in Dr. Murnaghan's paper and arrive at a result. An engineering way of doing the same thing is to insert a small series resistance to make the circuit behave normally. That will always avoid impulsive response.

But if we wish to treat the simpler circuit, and without thus avoiding the discontinuity, we can do it in the way there suggested, which in brief is this: We apply the superposition principle using the response to the unit e. m. f. ignoring the sudden impulse, and then we add on another term which takes account of the effect of the impulsive part.

---

[4] A. I. E. E. JOURNAL, February 1927 p. 132.

[1] *Maxwell's Theory of the Layer Dielectrics*, F. D. Murnaghan, TRANS. A. I. E. E., Vol. XLVI, 1927, p. 259.

The superposition principle has been often treated in Institute papers, and the mathematical use of it is extensive. I should like to refer to the 1926 paper of Vladimir Karapetoff, and of J. B. Whitehead, and particularly to the discussion of those papers by Joseph Slepian; and I should like more especially to refer to the work of J. R. Carson in a number of papers where he has treated this matter in condiderable detail. Carson's proofs and modified forms furnish, in my opinion, an excellent and complete guide to its validity and use.

E R. LeGhait: One reason why the subject treated by Mr. Murnaghan is of a great practical interest is because the equation expressing the principle of superposition offers a ready means for finding the relationship between the absorption and the a-c. losses of a dielectric.

For instance if in Equation 17 of Mr. Murnaghan's paper, the memory function ψ (t) has been determined by absorption measurements and if one expresses that E is an alternating electromotive force of a certain frequency one is able to deduce therefrom the losses and dielectric power factor at that frequency. One sees at the same time the influence of the nature of the memory function on the shape of the power-factor frequency curve and one grasps very clearly the fact that absorption and a-c. losses are nothing but two different aspects of the same underlying phenomenon.

Moreover the following fact is put in evidence, namely that for any dielectric following the principle of superposition the a-c. losses vary as the square of the voltage. This means that for such a dielectric the power factor is independent of voltage.

In practise the power factor has been found to remain constant with voltage for all substances for which the principle of superposition was found to hold and in general for most solid dielectrics, at least when the conditions of test are such that no secondary phenomena are involved.

In the case of non-impregnated paper we were able to check experimentally the relationship between absorption and a-c. losses derived theoretically from the expression of the principle of superposition. This was found possible provided the memory function ψ (t) is well determined experimentally for the values of t that are of the order of magnitude of the duration of a cycle of the alternating current for which the losses are measured.

For complete dielectrics like oil-impregnated paper the conditions are very different. It is well known that the anomalous absorption current in liquid dielectrics does not follow the principle of superposition. In correlation with this the power factor of oil-impregnated paper even, and especially at very low stresses, does not remain constant with voltage. It varies widely on the contrary and its variations are markedly affected by frequency and temperatures.

In this case we have not at our disposal the so convenient expression of the principle of superposition and it is very difficult to find a relation between absorption and a-c. losses. This has however been done to some extent qualitatively at least. Moreover the cause of the variations of power factor with voltages has been traced back to the fact that the anomalous currents in the minute particles of oil present in the insulation do not follow the principle of superposition.

We believe that a close study of the phenomena presented by composite dielectrics that are in contradiction with the principle of superposition is capable of throwing new light on the mechanism of losses in that type of insulation and of determining the part played in their formation by the different elements entering in its constitution.

M. G. Malti: Since no physical statement of the Boltzmann-Hopkinson principle of superposition is given in Dr. Murnaghan's paper, I should like to supply a statement of this principle and then show that it holds true within a very limited range of potential.

Let three condensers A, B, and C, be identical in every respect as to the thickness and material of dielectric, as well as to the area and material of plates. Let continuous potentials E_a, E_b, and E_c be applied to condensers A, B, and C respectively such that $E_a = E_b + E_c$ and let the charges Q_a, Q_b, and Q_c, acquired by these condensers, be determined as defined by the writer.[2] The principle of superposition states that, under these conditions, $Q_a = Q_b + Q_c$.

This principle assumes a linear relation between Q and E as shown by (a) in Fig. 1 herewith. As a matter of fact the relation is linear only up to a certain value E_a of the applied potential. For values higher than E_a the relation follows the curve b.[3]

While Dr. Murnaghan's paper is a fine piece of mathematical work its utility is limited to potentials lying within the initial potential range $o - E_a$ because the principle of superposition fails beyond this range.

Joseph Slepian: It has always seemed to me that undue prominence has been given to the Maxwell multi-layer theory of dielectrics. It is true that a dielectric built up of layers, as Maxwell postulates, will show the phenomenon of absorption and that by using enough layers an absorption curve may be obtained which will approach as near as is desired to actual observed absorption curves. But it does not follow that if a dielectric shows an absorption curve it must be built of layers.

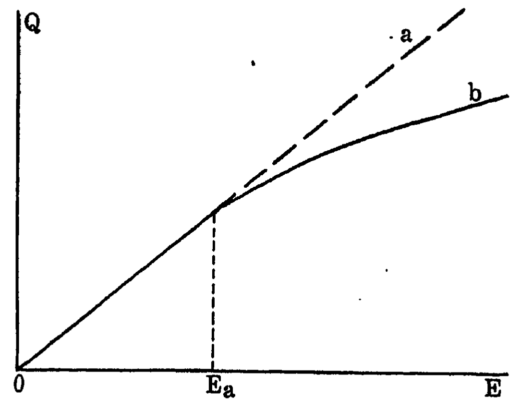


FIG. 1—RELATION BETWEEN CHARGE AND VOLTAGE IN SOLID DIELECTRICS

The situation is something like this: Given a box with a pair of terminals which when measured from the terminals acts as if it contained a resistor whether the measurements are made with continuous current, or with transients of the most varied sort. Can we then conclude that there is a resistor in the box? Clearly not. Networks can be set up in many ways which will perfectly imitate a resistor if measurements are confined to the terminals, as for example, in Figs. 2 and 3 shown herewith. So far as the terminals go these two circuits act exactly like ohmic resistors.

---

[2] See Eq. (9) in *A Theory of Imperfect Solid Dielectrics*, by M. G. Malti, TRANS. A. I. E. E., Vol. XLVI, 1927, p. 616

[3] Ibid, bibliography Nos. 54-21, 109-21, 107-24 and the following:

3/93 "Hysteresis and Viscosity of Mica for Rapid Oscillations," P. Janet. C. R. Vol. 116, p. 373, 1893. *J de Phys.* 3, 2, p. 337, 1893.

34/95 "On the Question of Dielectric Hysteresis," A. W. Potter and D. K. Morris. *Proc. Roy. Soc.* 57, p. 469, 1895.

35/95 "Energy Loss in Dielectrics," P. Granier, *Electrician* 36, p. 7, 1895.

18/97 "Conversion of Electrical Energy in Dielectrics," R. Threlfall. *Phys. Rev.* 4, p. 457 and 5 p. 21, 65, 1897.

52/04 and 53/04 "Solid Dielectrics," V. Crémieu and L. Malcles C. R. 139, p. 790 and 969, 1904.

60/11 "Measurement of Losses in Fibrous Dielectrics," H. Jordan. *Elek. Zeits.*, Vol. 32, p. 127, 1911.

8/14 "Passage of Electricity through Paraffin," G. G. deVillemontée, C. R. 158, pp. 1414, and 1571, 1914.

102/21 "Viscosity of Dielectrics," J. Granier, *Rev. Gen. de Elec.* Vol. 10, p. 219, 1921.

13/23 "Reversible Inductivity of Rochelle Salt Crystals," J. G. Frayne, *Phys. Rev.* 21, p. 348, 1923.

No amount of mathematical manipulation of the terminal quantities is ever going to give exact information as to what is inside the box. The obvious thing to do is to get into the box, open it up, and examine what is going on there.

In my discussion of Dr. Murnaghan's paper of a year ago I mentioned, in this connection, the work of the Russian, Joffe, who was actually getting into the box, but probing the interior of insulation under stress. If we can't get into the box, then the thing to do is to find the conditions where the simple Ohm's Law fails in some way. In the case of the network shown in Fig. 2, by going to such voltages that the condenser breaks down something will be learned of the network structure. In the case of the transmission line Fig. 3, the current might be raised until the resistor burned out, and then by examining the characteristics of what was left something might be learned. To obtain real information about dielectrics, we must go beyond the Boltzmann-Hopkinson principle, out of the range where it applies.

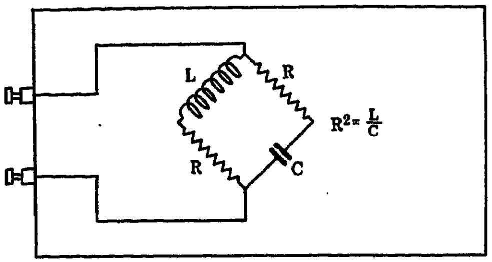


Fig. 2

Professor Murnaghan recognizes this situation and brings it out in the last part of the abstract which to my mind is the most important result of his paper. He says it goes somewhat further and shows that the principle is valid for any theory which leads to a system of linear differential equations with constant coefficients.

This overthrows any peculiarly preeminent place for the Maxwell Theory.

Let me now raise the question—over what range is the Boltzmann-Hopkinson principle valid? In almost any physical system, if small enough values of the variables are considered, as small velocities or small forces, we find that the system is described by means of linear differential equations. Hence the Boltzmann-Hopkinson principle or an equivalent principle should apply to almost any physical system if only the variables are kept small enough. Since this is true of almost every physical system, we say no more about a dielectric when we say that it satisfies the Boltzmann-Hopkinson principle for small values of electric forces, than if we say that it satisfies the law of conservation of energy.

Now, over what range does experiment show the superposition principle to be true? We do not need to go back very far in our own literature to get many cases where the Boltzmann-Hopkinson principle fails. In fact it seems that if any reasonable range of voltage gradient is considered, the principle is not valid. Curtis, in his very excellent paper of last summer on the "Conductivity of Dielectrics" says that below three to four hundred volts per cm. the conductivity currents in liquid dielectrics are proportional to the voltage, but for higher voltages, in some cases a saturation current is obtained and in other cases the current may increase more rapidly than in proportion to the voltage. Just recently a paper by Toryama has appeared in the *Archiv fur Elecktrotechnik* giving curves for the conductivity of transformer oil. These curves of current against voltage are convex downwards for all voltages from zero to breakdown. Of course if we limit ourselves to a small enough portion next to the origin, that portion may be regarded as straight, and that, I believe, is all there is to the Boltzmann-Hopkinson principle applied to dielectrics.

V. Karapetoff: This discussion sounds somewhat like an obituary of the Boltzmann-Hopkinson principle, and probably even the author will agree that the principle is inadequate, and yet like anything else in the progress of science it was necessary at one time until it has been worn threadbare. We see now that any hypothesis that leads to a system of linear differential equations with constant coefficients will give similar results, but essentially the shortcoming of this principle lies in the fact that it tries to build an ultra-microscopic structure and behavior of dielectrics on the basis of macroscopic images. In other words, this is one of those theories which may be called "bulk theories," that is, trying to account for the ultimate structure of matter by using images borrowed from the experience of our senses.

Seeing that we are now in a transition state to something more refined, and even ideational rather than material, I have felt a need for a word which would characterize the present outgoing theories, and so I have coined the adjective "macrocosmomorphic." "Macro" means "big;" "cosmos" means "the world;" and "morphic" means "of the shape of." Thus the word means built like the big world. The Boltzmann-Hopkinson principle is macrocosmo-morphic and therefore is destined to fall. We know from studies of X-Rays that simple crystalline bodies contain atomic lattices. For instance, rock salt contains alternate atoms of sodium and of chlorine in a definite arrangement, and so both the results of a mechanical stress or of an electrical stress applied to a crystal of rock salt must be expressed in terms of deformation of lattices and not of fictitious layers. Much of the advanced work done in Europe by Joffe, Rogowski, Smekal, and others, is directed toward a study of these lattices. Perhaps they are also macrocosmo-morphic, but at any rate a little more refined than plain layers. So we shall have to suck this new theory dry and then go to something still more refined.

E. R. LeGhait: In the discussion of the comparative advantages of micro and macro structures of dielectrics as bases for the explanation of the phenomena, I think it is well to bear in mind the wide range covered by the latter.

These phenomena are caused fundamentally by displacements of charges that lag behind the variations of the electromotive force. To explain this fact, some theories assume a certain viscosity within the atom that impedes the displacement of the electrons and causes an actual lag in polarization. Maxwell and his followers consider on the contrary that the lag of the displacements is not due to a lag in polarization but to the fact that it is the voltage distribution inside of the dielectric that lags behind the variations of the applied electromotive force. They thus look upon the dielectric as a conglomerate of particles that separately would possess only the properties of specific inductive capacity and conductivity and would not have any absorption. Of these two points of view, one based on a micro-structure of the dielectric and the other on a macro structure, which is to be preferred?

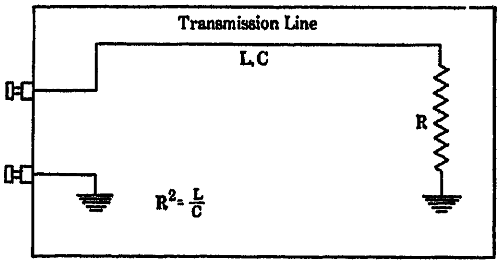


Fig. 3

I do not think it is either possible or desirable that all the anomalous phenomena of a given dielectric be explained on the basis of the same structure. I think that for the phenomena involving a considerably long duration of time, like losses at low frequencies, a macro structure ought to be considered, and that for those where very short durations of time are involved like losses at high frequencies, it is a micro structure that is the most suitable.

I believe that when one studies the anomalous properties of insulation and in particular the dielectric losses of impregnated paper at commercial frequencies, one finds great experimental evidence in favor of Maxwell's theory, or at least in favor of the idea at the basis of that theory, which is that the phenomena are due to the presence in the dielectric of materials of different specific inductive capacities and conductivities.

The observed variations of dielectric power factor of such insulation with frequency and temperature as well as absorption measurements point out in favor of the opinion that these effects are due to extra-molecular currents inside of the dielectric in accordance with the Maxwell theory.

Dr. Slepian expressed the opinion that the study of the anomalous effects was incapable of giving any indication on their ultimate cause and deplored the impossibility of opening up the dielectric and seeing what was inside of it. In connection with this it is interesting to make tests on paper while it is being dried out. Such tests show very clearly the influence of the presence of particles of a material of high conductivity (moisture in this case, imbedded in the dielectric) on its anomalous properties, and are a further confirmation of the value of the Maxwell theory.

However, I think that this theory as well as any assumed structure of the dielectric in accordance with it ought to be considered as merely explaining the anomalous properties within a certain range with a sufficient degree of approximation. In view of this, I think that one ought not to consider the heterogeneous dielectric as a conglomerate of particles of different materials that separately would only possess the properties of specific inductive capacity and conductivity with the exclusion of any kind of anomalous property. Let us remember that a perfect dielectric having only the property of specific inductive capacity can be looked upon as composed of atoms inside of which electric charges may be displaced, imbedded in a medium of infinitely high resistivity, the inter-atomic spaces. Such a dielectric can be considered as the limit case of a dielectric that would show only anomalous properties involving infinitely high and infinitely low time constants.

I think that instead of considering the composing materials of a heterogeneous dielectric as not having any kind of anomalous property one should merely consider them as not capable of showing alone any anomalous phenomena involving time constants of the same order of magnitude as those entering in the phenomena caused by their juxtaposition in the dielectric.

F. D. Murnaghan: There seems to be an impression in the minds of some readers that my paper is an advocate's brief pleading for a favorable view of Maxwell's Theory of Dielectric Absorption, but this impression is not correct. Professor Whitehead wished to give the theory a clear test and in order to do so found it necessary to complete in some points the mathematical argument and this was solely what I intended to do. Dr. Slepian's quotation from Professor Wilson's paper is, therefore, quite à propos; my paper is intentionally vacuous of physics and those who watch with interest the ever shifting sands of physical speculation may therefore justly charge me with trying to give my paper a permanent rather than a temporary value.

In the interest of frank discussion I must disagree with Professor Bush's comment that "Carson's proofs and modified forms furnish an excellent and complete guide to its (*i. e.*, the superposition principle) validity and use." When I first became interested in the present question I had access to Carson's interesting papers but found that my rather unintelligent use of his results led me into deep and troubled waters. What was the trouble? Carson was dealing with solutions of certain linear differential equations so that, in the first place, his work had to do *not* with the *current* but with the *displacement* in the various layers of the layer dielectric. In the second place a differential equation, by itself, is not a well set problem; one must know the initial conditions or values of the unknowns and their derivatives. The initial values of the displacements in our problem are not zero and I presume that Carson had in mind only the case when the initial values of the unknowns are zero. Be that as it may, his formulas do not apply to the present case which was the statement and proof of the superposition principle for the *current*. I feel it necessary to make these remarks as it would otherwise seem that I was thrashing very old straw indeed. The fact is that, in my opinion, even the men in high places quote incorrectly the superposition principle; to be explicit I have not seen my formula (17) in the literature although I have seen others which, if I read them aright, are incorrect.

Symposium on Interconnection

Conowingo Hydroelectric Project with Particular Reference to Interconnection

BY W. C. L. EGLIN[1]

Fellow, A. I. E. E.

Synopsis.—This paper discusses the interconnection between three of the leading power supply companies in the eastern part of the country, the Philadelphia Electric Co., the Pennsylvania Power & Light Co., and the Public Service Electric & Gas Co. It outlines the physical aspects of the tie-in between the companies which is a 220-kv. ring of high-load capacity, triangular in shape with sides respectively 49 mi., 82 mi., and 77 mi. long. The advantages of the interconnection are enumerated as well as some of the operating problems.

Preliminary to the discussion of the interconnection the paper discusses the Conowingo hydroelectric project, an important factor in the interconnection scheme. The plant has an initial installation of 280,000 kv-a. in generators.

* * * * *

As a sound basis for developing the relation between the Conowingo project and the three-party interconnection agreement recently completed between The Philadelphia Electric Co., the Pennsylvania Power & Light Co., and the Public Service Electric & Gas Co., it seems advisable to begin by restating briefly the more important features of the Conowingo project. However, it is not the intention to duplicate in this paper the excellent work which has been done by many writers in presenting detailed information on all phases of the preliminary investigation, design, construction, and proposed operation of the Conowingo project.

CONOWINGO PROJECT—GENERAL

The total drainage area of the Susquehanna River basin, as indicated in Fig. 1 is 27,400 sq. mi. divided as follows:

In Pennsylvania, 21,060 sq. mi. (77 per cent of the drainage area and 47 per cent of the area of the State)
In New York, 6080 sq. mi. (22 per cent of the drainage area and 13 per cent of the area of the State)
In Maryland, 260 sq. mi. (1 per cent of the drainage area and 2 per cent of the area of the State)

The boundaries of the drainage area in Pennsylvania are quite familiar to most persons, but the extension of the watershed in New York, northward almost to Utica, and eastward to about 40 mi. from the Hudson River near Albany, is probably less well known.

The Conowingo dam, on account of its location a few miles above tide water, intercepts the run-off from practically this entire area. United States Weather Bureau records over a long period of years indicate the precipitation to vary from 31.4 to 44.3 in. per year, averaging 39.4 in.

In making preliminary studies on the economic desirability of proceeding with the project, a long series of river-flow records was required. Since such records were available for Harrisburg, and the added drainage area between Harrisburg and Conowingo is almost exactly 12.5 per cent of that above Harrisburg, it has been assumed throughout that the flows at Conowingo are 12.5 per cent greater than at Harrisburg. According to the flow records from 1891 to 1926, the average annual discharge at Harrisburg varied from 21,600 sec. ft., (a run-off of 12.2 in. over the drainage area), to 52,600 sec. ft., (a run-off of 29.7 in.). The average for the period was 35,300 sec. ft., (a run-off of 19.9 in.), or about 50.5 per cent of the average precipitation.

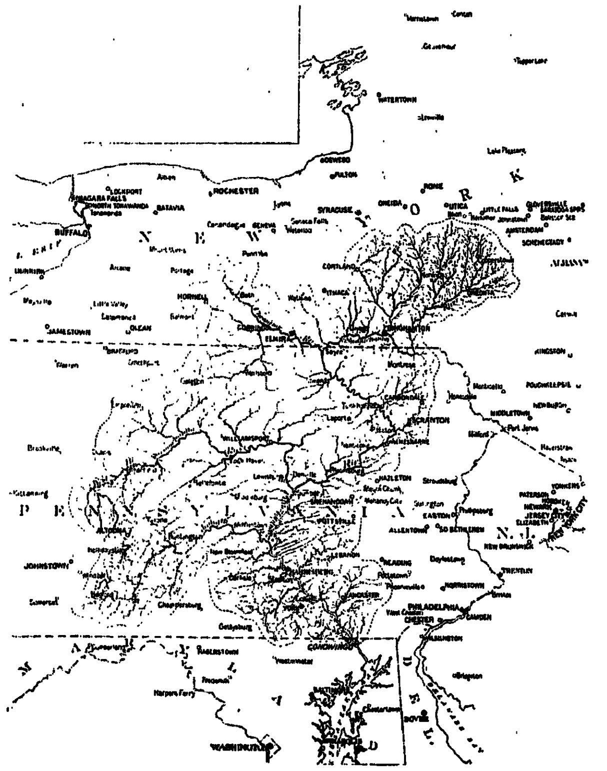


FIG. 1—SUSQUEHANNA RIVER DRAINAGE AREA

The extremely variable flow, both seasonally and from day to day, of the Susquehanna River is not sufficiently emphasized in the foregoing average values. Fig. 2, containing superimposed hydrographs plotted from Harrisburg data, with an auxiliary scale for correction to Conowingo, brings out more clearly the variation of the monthly average between the winter and spring high flows, and the low flows of summer and fall, but even this figure cannot illustrate the extent of the low-flow periods in any particular year. About 6000 sec. ft. of water at full load is required for each of the seven units initially installed. The records indicate that several times the average flow of the river at Conowingo was less than 6000 sec. ft. for four months or more, while in one period of 4½ months (concluding with the first half of December, during which the annual peak of The Philadelphia Electric Co. System frequently occurs) the average flow did not exceed 5000 sec. ft.

---

[1] Vice-president and Chief Engineer, The Philadelphia Electric Company.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

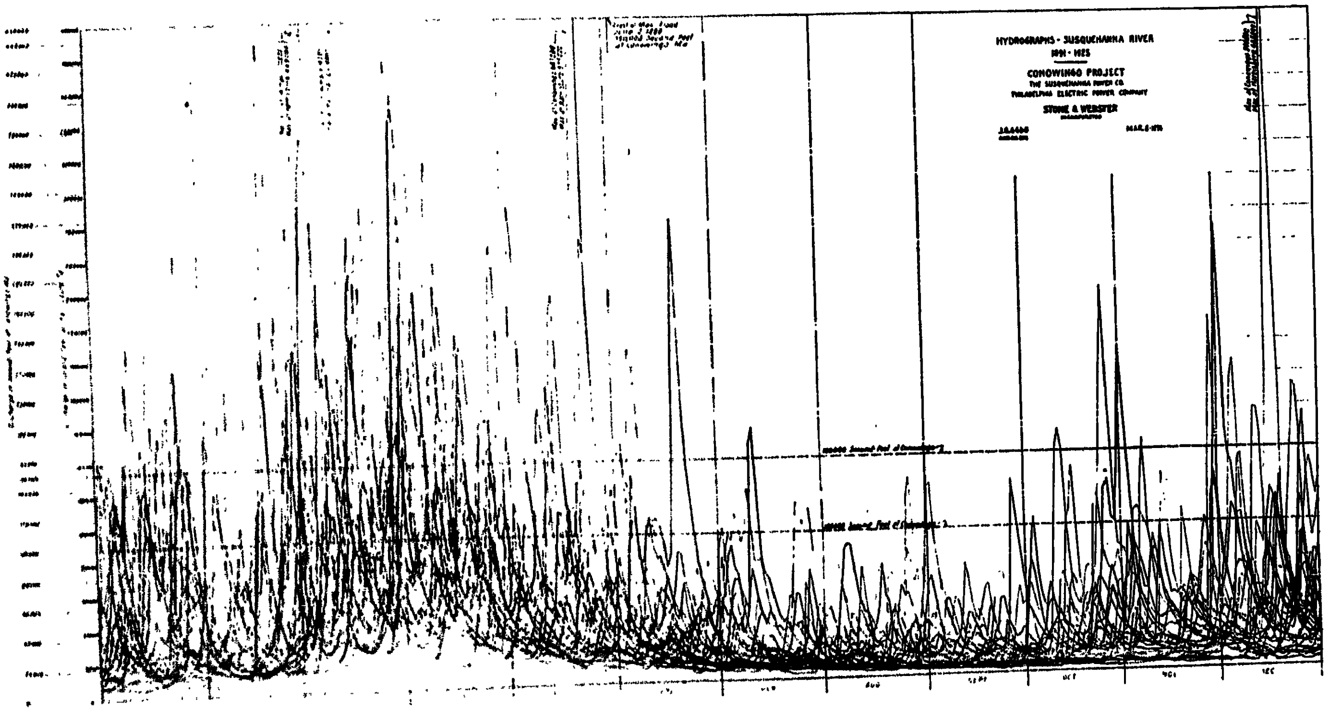


FIG. 2—COMPOSITE HYDROGRAPH OF THE SUSQUEHANNA RIVER

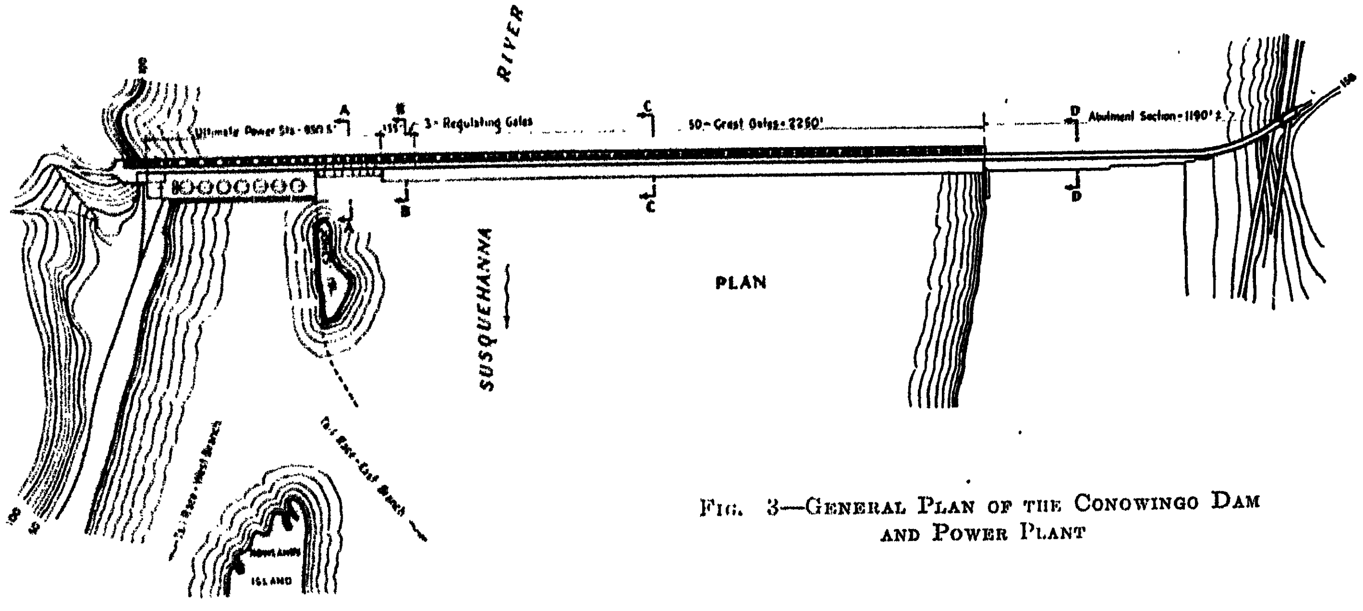


FIG. 3—GENERAL PLAN OF THE CONOWINGO DAM AND POWER PLANT

It is this feature, more than any other, which has prevented the construction of a power plant in this reach of the river earlier, since the project has been studied for about forty years almost continuously by various competent organizations. High-capacity, initial installation was imperative on account of the size of the dam. This precluded independent operation and building up of local load and involved transmission to existing load centers together with the provision of expensive, relay, steam-generating capacity for use in the low-flow periods.

That the project has now become economically justified is because The Philadelphia Electric Co. system load is large enough to absorb nearly the entire output of the plant at all times; while operation in connection with the existing steam stations in Philadelphia (utilizing Conowingo Station on the base of the load in high flows and on the peak in low), makes relay steam capacity unnecessary.

Conowingo Dam. Five possible sites for the dam and power house were subjected to a careful detailed study, which indicated that the selected site—about two miles below the former village of Conowingo—best satisfied the various conflicting requirements. Construction was begun in March 1926, and since that time weather and river-flow conditions have been so favorable that the original schedule for completion has been materially bettered. The dam is completed, and the pool full at present. It is expected that the first two units will be in regular operation early this year.

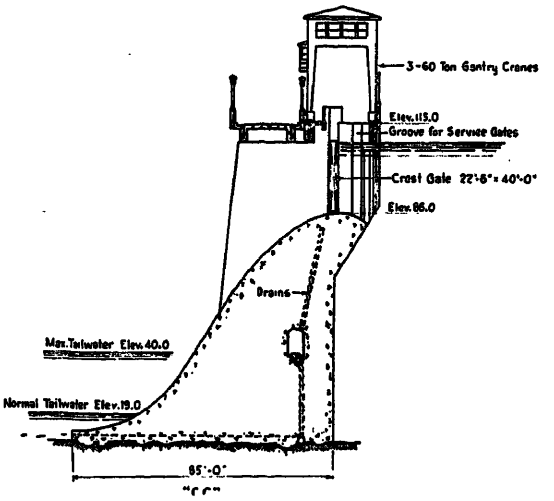


FIG. 4—CROSS-SECTION OF MAIN SPILLWAY SECTION CONOWINGO DAM

A lake has been formed, extending approximately 14 mi. upstream to the tail-race of the Holtwood Plant of the Pennsylvania Water & Power Co. At normal and maximum elevation of 108.5 ft. above sea level, the area of the pond will be about 8600 acres. With 7 ft. draw-down, storage is available to the amount of 2,500,000,000 cu. ft., which will be utilized in dry periods to supplement the river flow.

Since the former location of the Columbia and Port Deposit Branch of the Pennsylvania Railroad along the east bank of the Susquehanna River was to be submerged, it was necessary to provide a new road bed above the 108.5 ft. pond level from just below Holtwood to the dam and, below the dam, dropping down at a maximum of 0.35 per cent to the original grade at Port Deposit—about 16 mi. in all.

The Philadelphia-Baltimore Pike has been relocated so as to cross the river on a bridge placed on top of the dam structure. This bridge superseded the old steel bridge upon its demolition and flooding.

The dam is a gravity-section, concrete structure. As indicated in the plan, Fig. 3, starting from the west (left) side, an abutment section approximately 100 ft. long adjoins the power station section, 950.5 ft. long. The headworks structures for the four future units, as well as for the seven to be installed at this time, have been completed. The next section of the dam, 135 ft. in length, contains three regulating gates; then follows the main spillway section extending 2250 ft. A cross-section at this point is shown in Fig. 4. The fixed crest at elevation 86 is surmounted by 50 crest gates, supported by concrete piers rising from the dam on 45-ft. centers. The piers serve the further purpose of carrying the highway bridge already mentioned and the gate-crane bridge.

With all gates open and all wheels at full draft, about 880,000 sec. ft. will be discharged without raising the pool level above the normal of 108.5 ft. In comparison, the greatest recorded flood (in June 1889, coincident with the Johnstown Flood) was 750,000 sec. ft.

Beyond the main spillway section is the east abutment section about 1200 ft. long.

Conowingo Power Station. Fig. 5 is a cross section and Fig. 6 a plan of the power station. Seven generating units are initially installed, each consisting of a turbine of 54,000 hp. at full gate and 89 ft. net head, 81.8 rev. per min., direct connected to a 36,000-kw., 40,000-kv-a., 13,800-volt, three-phase, 60-cycle generator, and also to a 715 kv-a., 70 per cent power factor auxiliary alternator for supplying the more essential auxiliaries, namely the ventilating fans, governor pump, and unit exciter.

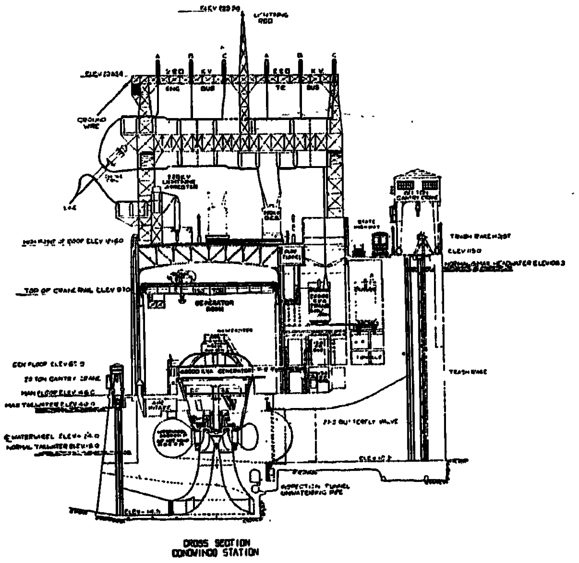


FIG. 5—CONOWINGO POWER STATION CROSS-SECTION

The generators, coupled in pairs, feed four 80,000-kv-a., 13.8/220-kv., transformer banks placed in open top compartments immediately above the 13.8-kv., switch structures. One extra 26,667-kv-a. transformer is provided as a spare for the four banks.

The output of the step-up transformers is controlled by the 220-kv. switching station on the roof of the power plant. The single line diagram, Fig. 7, indicates the four transformer banks and two 220-kv. lines now installed, and also the two banks and one line to be added upon completion of the ultimate installation of the power station.

Fig. 8 is a late view of the dam and power station from the west bank down stream.

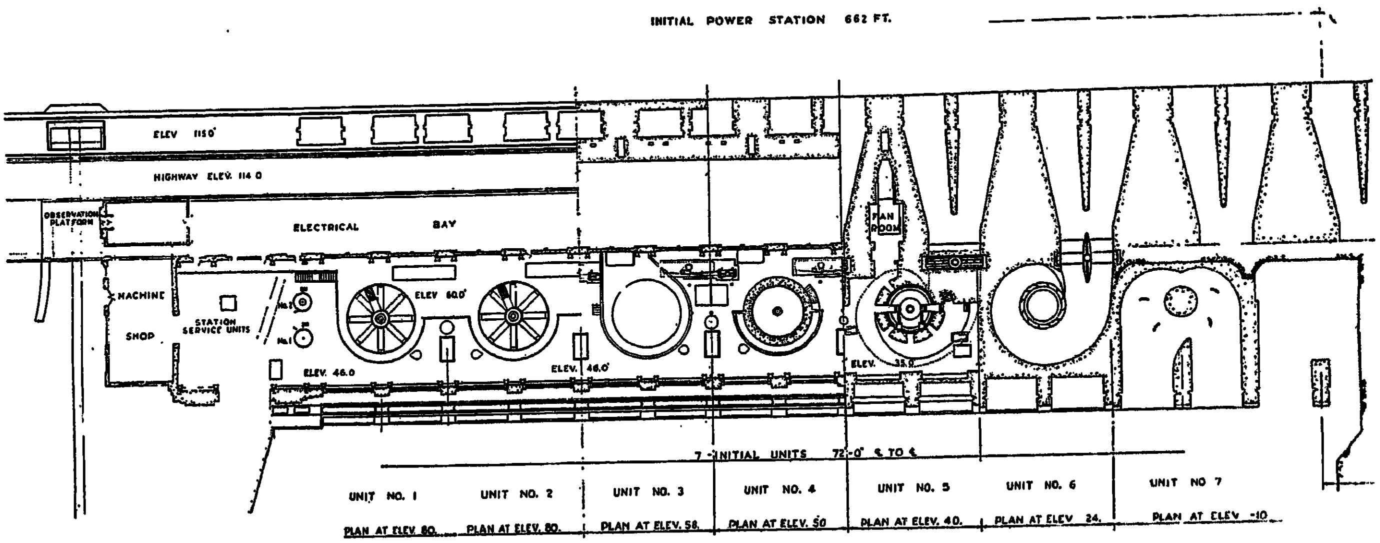


Fig. 6—Conowingo Power Station Plan

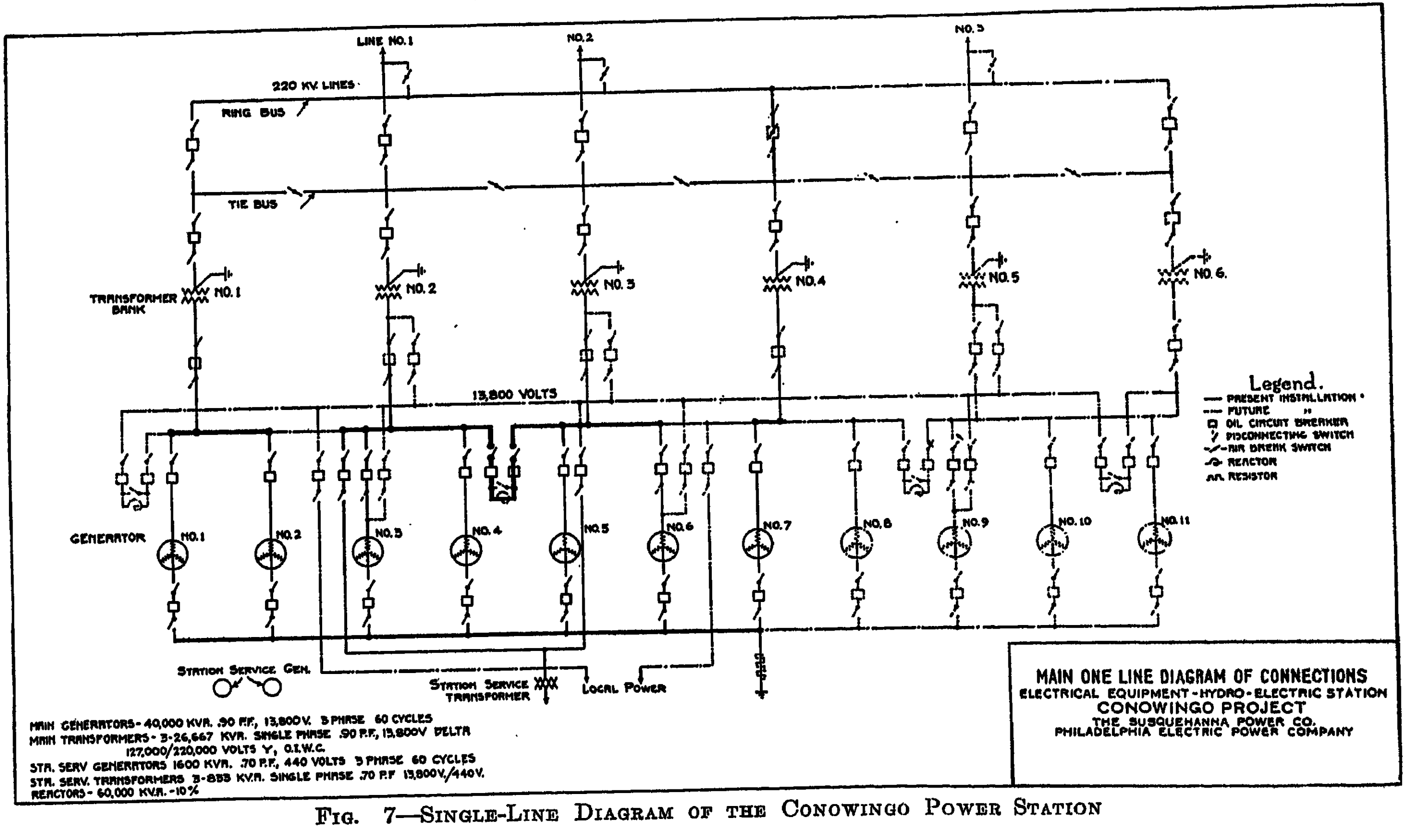

LINE NO. 1
NO. 2
NO. 3
220 KV LINES
RING BUS
TIE BUS
TRANSFORMER BANK
NO. 1
NO. 2
NO. 3
NO. 4
NO. 5
NO. 6.
13,800 VOLTS
GENERATOR
NO. 1
NO. 2
NO. 3
NO. 4
NO. 5
NO. 6
NO. 7
NO. 8
NO. 9
NO. 10
NO. 11
STATION SERVICE GEN.
STATION SERVICE TRANSFORMER
LOCAL POWER
Legend.
PRESENT INSTALLATION
FUTURE "
OIL CIRCUIT BREAKER
DISCONNECTING SWITCH
AIR BREAK SWITCH
REACTOR
RESISTOR
MAIN GENERATORS - 40,000 KVA. .90 P.F., 13,800V. 3 PHASE 60 CYCLES
MAIN TRANSFORMERS - 3-26,667 KVA. SINGLE PHASE .90 P.F., 13,800V DELTA
127,000/220,000 VOLTS Y, O.I.W.C.
STA. SERV GENERATORS 1600 KVA. .70 P.F., 440 VOLTS 3 PHASE 60 CYCLES
STA. SERV. TRANSFORMERS 3-833 KV.A. SINGLE PHASE .70 P.F 13,800V./440V.
REACTORS - 60,000 KV.A. -10%
MAIN ONE LINE DIAGRAM OF CONNECTIONS
ELECTRICAL EQUIPMENT - HYDRO - ELECTRIC STATION
CONOWINGO PROJECT
THE SUSQUEHANNA POWER CO.
PHILADELPHIA ELECTRIC POWER COMPANY

Fig. 7—Single-Line Diagram of the Conowingo Power Station

220-Kv. Lines

The 220-kv. transmission lines from Conowingo Power Station to Plymouth Meeting Substation (approximately 65 mi. long) are carried on steel towers with a normal span length of approximately 1100 ft. The right-of-way width is 315 ft. which provides for three single-circuit tower lines spaced 85 ft. apart. The two outer tower lines have been built initially, resulting in a present spacing of 170 ft. between lines. The three conductors of each line are 795,000-cir. mils aluminum steel, reinforced, and are arranged horizontally with a spacing of 25½ ft. Each tower line also carries two 183,000-cir. mils aluminum steel, reinforced, ground wires. The conductors and ground wire on suspension type towers are carried on a new type of clamp which releases the conductor, and thereby relieves the tangent towers of excessive longitudinal stresses in the event of wire failure. The insulators are of a high-strength type with fourteen units in suspension strings and sixteen units in strings in strain position. Fig. 9 shows the crossing span from Rowland's Island to the east bank of the Susquehanna River, with the access cableway from the west bank to Rowland's Island in the foreground. Fig. 10 illustrates typical tangent-tower construction.

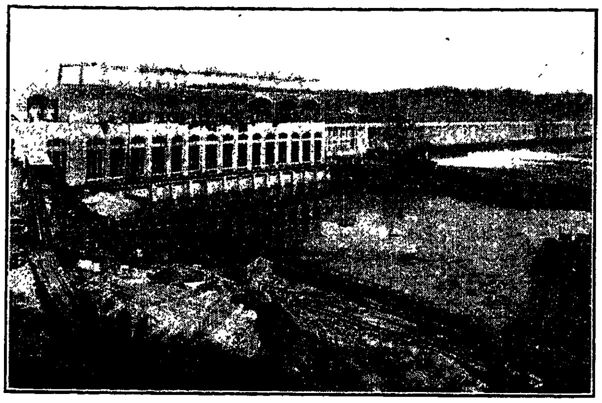


FIG. 8—CONOWINGO DAM AND POWER STATION, FROM WEST BANK—DOWNSTREAM SIDE

Interconnection—General. The term "interconnection" is used to denote many gradations of physical connection between separate systems, ranging from ties of very small capacity between outlying points of adjoining or interpenetrating systems (which, if utilized to the utmost, could not materially affect the operating economies of the system taken as a whole), up to trunk lines connecting principal centers of load and generation, involving a comprehensive mutual program with large investment, and promising commensurate return. To the latter class belongs the interconnection between The Philadelphia Electric Co., the Pennsylvania Power & Light Co. and the Public Service Electric & Gas Co., the first step of which is now practically ready for operation. It is the purpose of the three companies by this interconnection so to coordinate their construction and operation programs as to result in a maximum saving of investment and operating expense. The studies made preliminary to the final agreement indicate that the present contemplated construction, namely, a complete 220-kv. ring between the systems with suitable facilities for connecting each system to the ring, will, when carefully operated, utilize to the best advantage all of the diversity occurring between the system loads for the coming ten years.

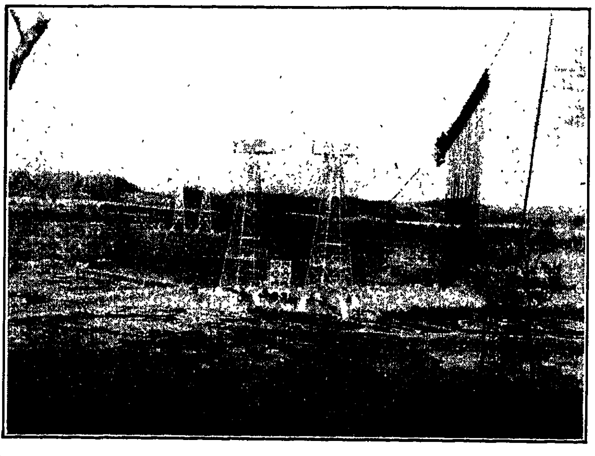


FIG. 9—SUSQUEHANNA RIVER CROSSING SPAN. ROWLAND'S ISLAND TO EAST BANK. ACCESS CABLEWAY FROM WEST BANK TO ROWLAND'S ISLAND IN FOREGROUND

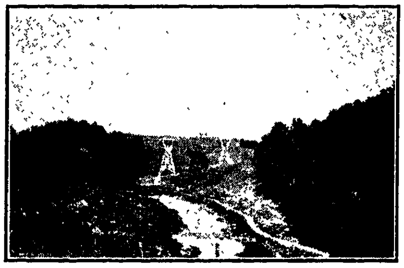


FIG. 10—CONOWINGO 220-KV. LINE TOWERS AT PORTERS BRIDGE

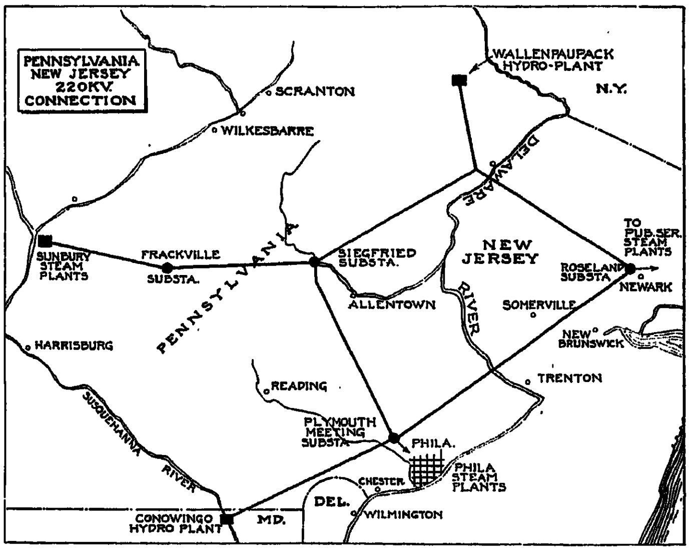


FIG. 11—DIAGRAMMATIC MAP OF PENNSYLVANIA-NEW JERSEY INTERCONNECTION

The points selected as terminals of the 220-kv. ring, forming roughly, an equilateral triangle, are: Plymouth Meeting near Norristown, for The Philadelphia Electric Co.; Siegfried near Allentown, for the Pennsylvania Power & Light Co.; and Roseland near Newark, for the Public Service Electric & Gas Co. The actual distances (line mileage) are: Plymouth Meeting to Siegfried, 49 mi.; Siegfried to Roseland, 82 mi.; Roseland to Plymouth Meeting, 77 mi.

Fig. 11 shows a diagrammatic map of this interconnection. A portion of the existing line from Wallenpaupack Hydroelectric Station of the Pennsylvania Power & Light Co. will be utilized as a portion of the Siegfried-Roseland line when this is built.

The line now practically completed, is that from Plymouth Meeting to Siegfried. It is of the same general type of construction as the Conowingo Line, though a number of details relating to foundation, tower design, and insulators were not identical.

In the interconnection plan, each company designs and constructs the lines lying within its own operating territory, although all three have agreed upon the climatic loading conditions and the general basis of design.

There are many points at which saving can be made, both in generating-capacity investment and in operating expenses, when two or more large systems of approximately equal system loads are interconnected by tie lines heavy enough to interchange any amount of energy which the economics of the situation may require. The principal saving in this case results from load diversity, in that the peak load of the combined systems is smaller than the sum of the individual peaks. A second important factor is reserve diversity, since the companies by pooling the reserve capacities which they maintain are able to dispense with a certain portion of this reserve without loss of reliability. A third factor is the staggering of construction among the companies, which permits the installation of two or three generating units in any particular year, for example, where four or five might be required with the three systems operating independently.

The distribution of combined load, in the most economical manner utilizing the more efficient stations of the combined systems for base load and the less efficient as peak stations, is a source of material savings in operating expenses. The furnishing of emergency power in excess of normal reserve requirements when this is available in the other companies results in no actual money saving but does improve reliability of operation.

Load Diversity. Load diversity between two or more systems may be analyzed into several distinct elements. In this case the principal portion of the load diversity is due to the difference in the shape of the individual daily load curves on any selected day during the peak period of the combined systems. Further contributions are made by the difference in the season of annual peak and by the non-coincidence of those accidental high days which cause weekly or monthly peaks on the individual system. As an instance, throughout the year the daily peak load of the Pennsylvania Power & Light Co. is in the morning, with the afternoon load slightly lower and the evening load materially reduced. Their annual peak load occurs in October.

The load curves of the Public Service Electric & Gas Co. and The Philadelphia Electric Co. are quite similar, both containing large evening loads throughout the year, and in particular a high peak near 5:00 p. m. throughout the winter months. The yearly maximum load of both usually is found in December.

Reserve Diversity. The experience of the three individual systems with calls upon their own reserve generating capacity indicates that these are relatively infrequent and no loss in reliability is expected, under the assumption that the interconnecting companies will not all require their share of the total reserve diversity simultaneously.

Economic Distribution of Combined System Load. In a single system of steam generating stations, especially when serving a compact territory, the proper apportionment of load among stations seldom offers serious difficulties. It is usually found economical, as the system grows, to build up one station at a time from initial to ultimate capacity rather than to have two or more under way, simultaneously. Thus each station in its operating performance reflects current practise during its design and construction period, and the cost margin between stations is sufficient to render obvious the correct distribution of load. Transmission limitations and minimum loading requirements of stations may enforce departures from this ideal distribution. In the case of a system of wide area and low concentration of load, it may prove more economical to establish two or more centers of supply, and increase them in capacity according to the requirements of their districts, rather than to provide the transmission facilities required to permit all the growth to take place at one station.

No matter to which class the concerned systems belong, the interconnection of two or more systems which have grown up entirely independently, and which are therefore likely to include stations of approximately equal-operating performance, offers many delicate problems of load apportionment.

Load Dispatching. The interconnection headquarters for general administration and load dispatching is to be established at Siegfried. The load dispatchers at this point will be in direct communication at all times, through the load dispatchers of the individual systems, with the steam and hydrogenerating stations of the three companies. They will be advised of the expected outputs from the various hydro plants of the system as far in advance as estimates can reasonably be made, and will plan the operation of the steam stations to carry most economically the remaining load. Load records, metering, etc., will all center here. The preliminary studies of the interconnection project have demonstrated conclusively that an opportunity for large savings exists, but only the most alert and skilled load dispatching will realize all of the possible benefits.

The detailed technique of interconnection operation has not been entirely worked out as yet, but the two companies which are immediately to be connected by the Plymouth Meeting-Siegfried line are in close touch with one another on these matters, and before operation commences will have worked out a complete schedule.

Stability and Reliability. While a detailed discussion of the stability and reliability of the 220-kv. interconnection lines is out of place in the present paper, it can be stated that these considerations were among the foremost governing the entire study and design. It is expected that adequate stability under all loading conditions will be afforded the 220-kv. ring by the synchronous condensers located at the terminal substations (in the case of The Philadelphia Electric Co., initially 90,000 kv-a.).

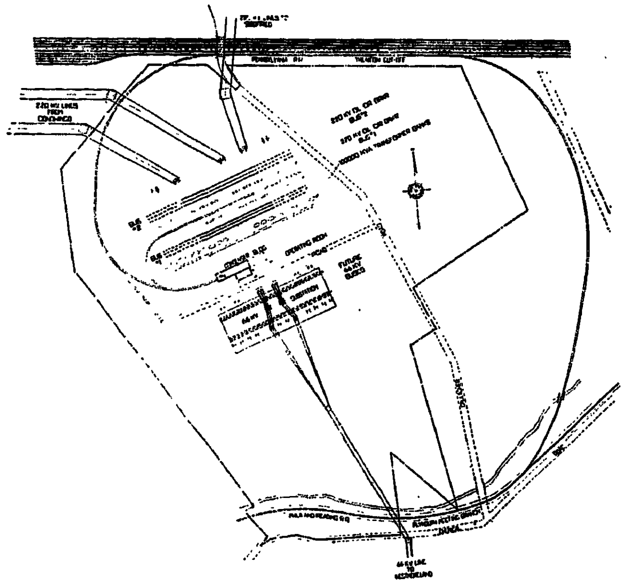
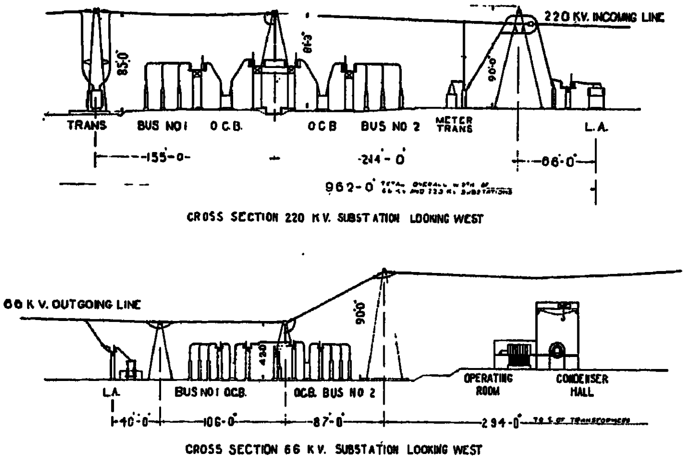


Fig. 12—Plot Plan of the Plymouth Meeting Substation



CROSS SECTION 220 K.V. SUBSTATION LOOKING WEST

CROSS SECTION 66 K.V. SUBSTATION LOOKING WEST

Fig. 13—Plymouth Meeting Substation Cross-Section

Connections from 220-Kv. Lines to The Philadelphia Co. System

Plymouth Meeting Substation. Plymouth Meeting Substation, the junction point for the 220-kv. Conowingo and Interconnection lines, and the 66-kv. lines to Philadelphia, was laid out on a tract of about 40 acres, as shown by Fig. 12. The Conowingo and Interconnection lines terminate at the 220-kv. buses, to which are also connected the 220/69-kv. transformers tying in with the 66-kv. lines to Westermoreland Substation in Philadelphia.

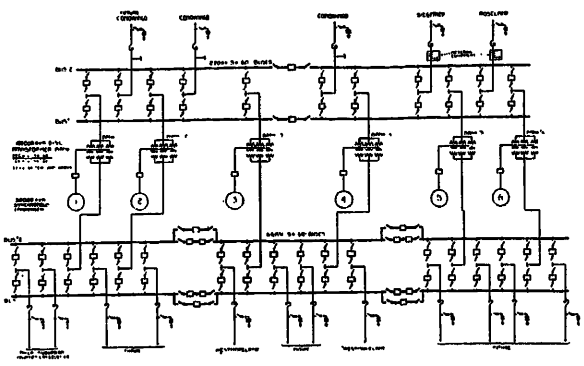


Fig. 14—Single-Line Diagram of Plymouth Meeting Substation

As the cross-section (Fig. 13) and the single-line diagram (Fig. 14) indicate, two 220-kv. buses are installed, provision being made for an ultimate of five 220-kv. lines and six transformer banks, with each bus sectionalized by an oil-circuit breaker. Any bank or line may be connected to either 220-kv. bus, through breakers. The initial installation consists of three lines, two from Conowingo and one from the Siegfried Substation of the Pennsylvania Power and Light Co., and two transformer banks, together with a portion of both buses.

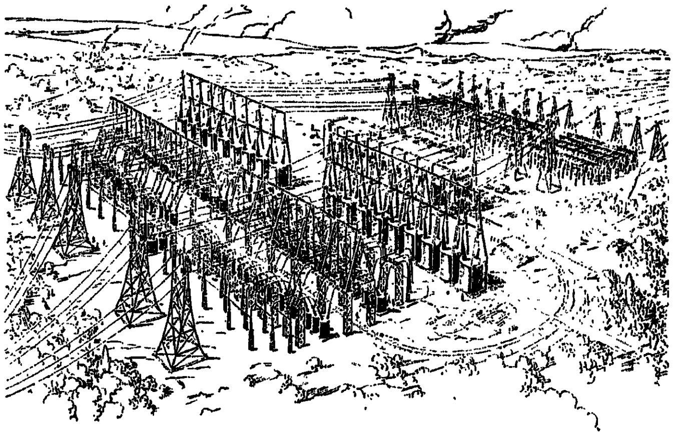


Fig. 15—Perspective Sketch: Plymouth Meeting Substation

A more comprehensive view of the substation, although not accurate in all details, is given by the perspective sketch, Fig. 15. On the 66-kv. side of the substation, two buses will be installed with provision for an ultimate of twelve 66-kv. lines, together with the six transformer banks. Sectionalizing is provided for at two points in each bus. The initial installation consists of two lines to Philadelphia, and two to the Philadelphia Suburban Counties Gas and Electric Co., and parts of both buses.

A condenser building and a control room are located between the 220-kv. and 66-kv. sections of the station.

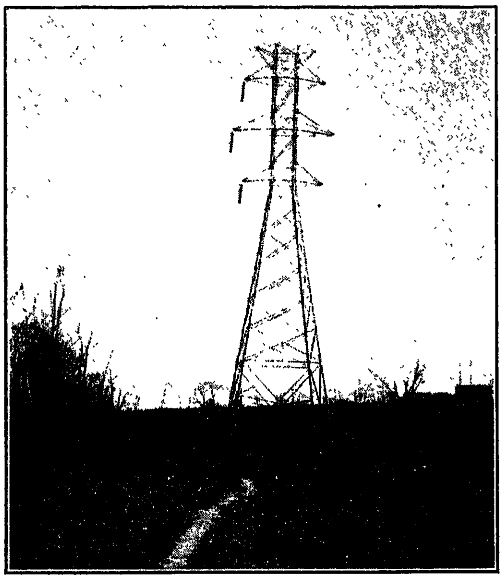


FIG. 16—TWO-CIRCUIT TOWER—PLYMOUTH MEETING—WESTMORELAND 66-KV. LINE

Three condensers are being installed initially, with provision for an ultimate of six units.

The transformer banks are self-cooled with a nominal rating of 100,000 kv-a.; however, with the air-jet radiator cooling to be installed, their capacity will be increased to approximately 130,000 kv-a. These transformers are of the three-winding type, with the 220-kv. and 66-kv. windings connected star and solidly grounded. The delta, tertiary winding is rated at 13,300 volts. Ratio changing under load equipment, giving a voltage range of 15 per cent, is provided with these transformers.

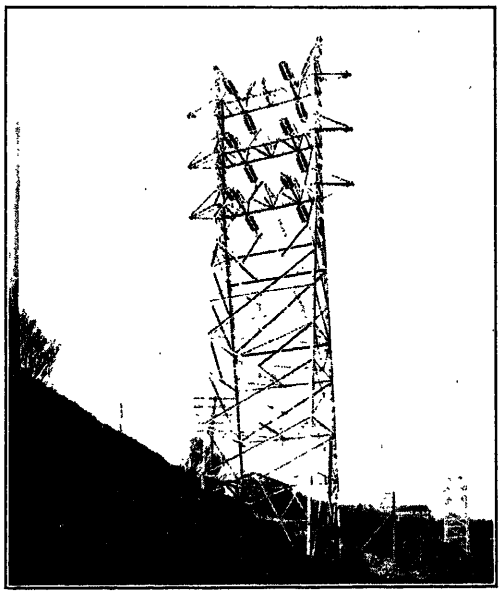


FIG. 17—FOUR-CIRCUIT TOWER—PLYMOUTH MEETING WESTMORELAND 66-KV. LINE

The synchronous condensers, which are connected to the transformer tertiary windings, are each rated at 30,000 kv-a. and are equipped with extra high-speed excitation.

Relay protection on the 220-kv. lines consists essentially of special type *C Z* relays and directional, ground-current relays. Transformer banks and condensers have the usual differential protection. The 66-kv. lines are also protected by *C Z* and ground-current relays.

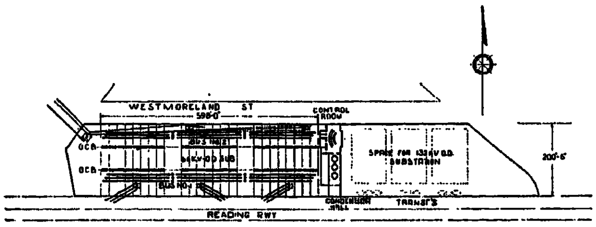


FIG. 18—WESTMORELAND SUBSTATION PLAN

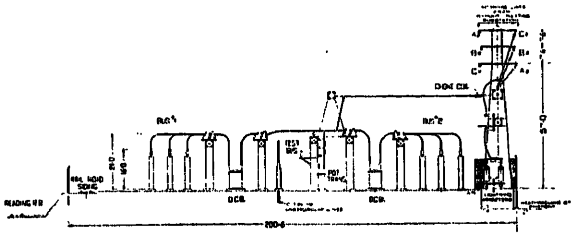


FIG. 19—CROSS-SECTION OF WESTMORELAND SUBSTATION

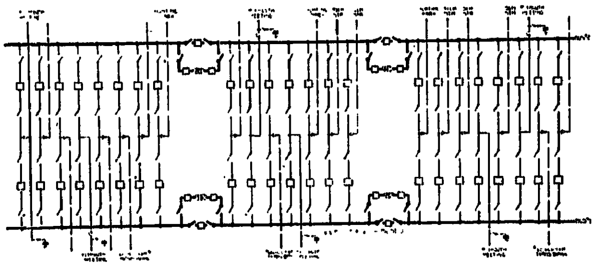


FIG. 20—SINGLE-LINE DIAGRAM OF WESTMORELAND SUBSTATION

The 220-kv. oil-circuit breakers have an interrupting capacity rating of 2,500,000 kv-a., and the 66-kv. line breakers a rating of 2,000,000 kv-a.

66-kv. Lines. The transmission lines, for connection from the 220-kv. system to the existing 66-kv. Philadelphia Electric Co. transmission system, are carried from the Plymouth Meeting Substation approximately ten miles to Westmoreland Substation in northern Philadelphia, generally following along the banks of the Schuylkill River. For approximately one-half of this distance the conventional two-circuit tower construction is used, with an average span length of 800 ft. (Fig. 16). On account of the limited space available for the remainder of the distance, a special type of four-circuit towers has been utilized; and some of these bridge the railroad right-of-way of the Reading Co. On this portion of the line, a normal tower spacing of 600 ft. has been adopted; and the towers located with a view to the requirements of future electrification catenary structure (Fig. 17). Two circuits each, of 500,000 cir. mils copper, have been constructed initially and provision has been made for an ultimate of six. Two ground wires are installed for each of the two circuit towers.

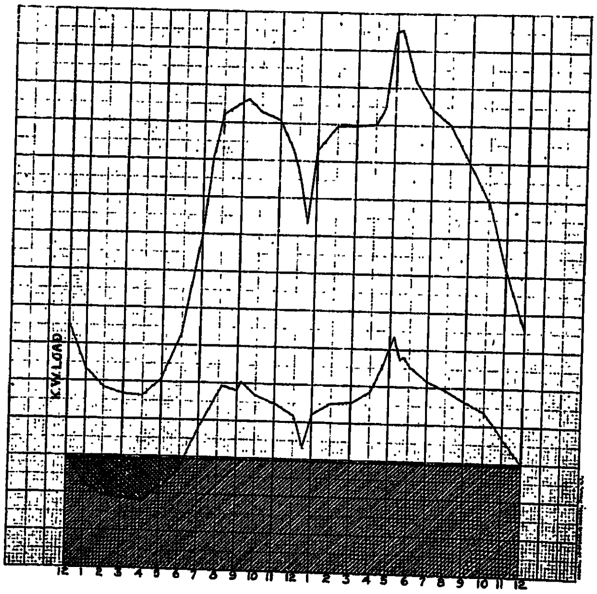


Fig. 21—Conowingo Station Operation; High-Flow Conditions

Westmoreland Substation. Westmoreland Substation is the Philadelphia terminus of the 66-kv. lines from Plymouth Meeting Substation.

As may be seen from Figs. 18, 19, and 20, the general scheme of connections and the physical arrangement of equipment are similar to those of the 66-kv. portion of Plymouth Meeting Substation and other Philadelphia electric 66-kv. substations, two 66-kv. buses being provided to which any line or transformer bank may be connected through oil-circuit breakers. At Westmoreland Substation, provision is made for an ultimate of twenty-three 66-kv. sections, five or six of which will be utilized by 66/13.2-kv. transformer banks, and the remainder by transmission lines to Plymouth Meeting Substation and to other 66-kv. substations of the Philadelphia Electric System. Bus sectionalizing, including reactors, if their use is found necessary, is provided for at two points in each bus.

The initial installation is two overhead lines to Plymouth Meeting Substation, two underground lines to Richmond Substation, two underground lines to Schuylkill Substation (together with portions of both buses), and two 66-kv. feeders to two 18,750-kv-a., 66/13.2-kv. transformer banks in a nearby substation. By late summer, two 66/13.2-kv. transformer banks and two synchronous condensers will be installed in the 13.2-kv. section of the station. Ultimately, three and possibly four transformer banks and three condensers will be installed.

The two banks to be installed will be rated at 60,000 kv-a. each with possible provision for increasing this rating by air-jet radiator cooling. The banks will also be provided with ratio changing under load equipment giving a 10 per cent voltage range.

The synchronous condensers will be rated at 30,000 kv-a. each, and will be equipped with high-speed excitation.

Relay protection on the 66-kv. lines consists of *C Z* duplex relays for protection against both line-to-line and line-to-ground faults. Transformer banks and condensers will have the usual differential protection.

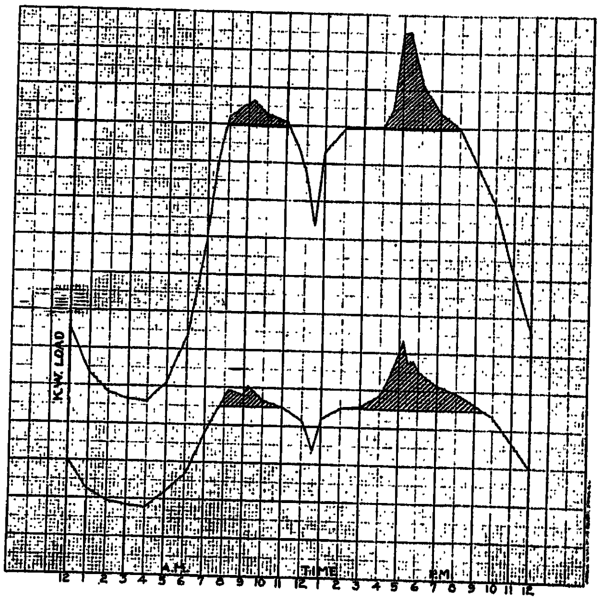


Fig. 22—Conowingo Station Operation; Low-Flow Conditions

The 66-kv., oil-circuit breakers have an interrupting capacity rating of 2,000,000 kv-a.

Relation Between the Conowingo Project and the Interconnection

The Conowingo and Interconnection projects have many favorable reactions upon each other. One of these is the common use of facilities for connecting the 220-kv. lines with the Philadelphia Electric Co.'s existing transmission system. The cost of Plymouth Meeting Substation and of Westmoreland Substation, for either the Conowingo project or the interconnection separately, would have been considerably greater than the proportionate share of the actual combined construction cost.

While two 66-kv. lines from Plymouth Meeting Substation to Westmoreland Substation would have been required for the Conowingo project alone, and one for the initial interconnection step alone, on account of the short period of simultaneous maximum demand, only two lines are being installed for both the first year.

The output of the Conowingo project can be completely absorbed earlier with the interconnection, since the minimum combined load of the three systems is well above the full load capacity of Conowingo Station. Likewise, Conowingo's value in replacement of steam generating capacity at all times of the year is greater with the interconnection.

Figs. 21 and 22 illustrate the more effective utilization of the water available for Conowingo Station, both in high-flow and low-flow periods with the interconnection. In Fig. 21, which represents winter, high-flow conditions, the lower load curve is that of the Philadelphia Electric Co. system alone, and the upper, that of the interconnected companies combined. The cross-hatched area indicates the energy that can be developed from the Conowingo Station, sufficient water being available to run all the turbines in the station at full-load for 24 hr. Operating in conjunction with the Philadelphia Electric Co. system load alone, the area indicated by the double-cross-hatching would be lost on account of the decrease of the system load below the capacity of the station during the night hours. On the other hand, operating with the interconnected systems, the minimum load is well above the capacity of Conowingo Station at all times, and the entire available energy can be utilized.

Fig. 22 illustrates winter, low-flow conditions, in which a very small amount of water per day may be available. In this case the economical method of operation is to utilize all of the available water energy in carrying the peak load, leaving the base load to the team-generating stations, since in this manner the hydro plant replaces the greatest amount of steam-generating capacity. The cross-hatched areas on the two curves are equal, and it will be readily noted that, utilizing exactly the same limited amount of water in either case, the replacement of other generating capacity by Conowingo Station is about 50 per cent greater when operated on the Interconnection load curve, than on that of the Philadelphia Electric system alone.

During the summer months, when the load curves of both the Philadelphia Electric Co. system alone, and of the interconnected systems, are nearly flat for several hours, the low-flow energy applied at the top of the curve will result in a lower replacement of steam capacity by Conowingo Station, possibly no greater on the Interconnection load than on the Philadelphia Electric Co. system load alone. At such times, on account of the short starting period required by hydro generating units, every generating unit not required for the normal load can be utilized as standing, reserve capacity for the entire interconnected system. The amount of expensive, steam-reserve capacity will thus be reduced.

It is hoped that this brief presentation of the largest interconnection yet attempted, embracing three of the leading power supply companies of the United States, and including Conowingo Station (in its ultimate capacity the largest single hydroelectric station), may be of value to all those interested in the general subject, and especially to those who find themselves faced by similar problems.

The author desires to express his thanks to Messrs. F. C. Ralston, R. A. Hentz, and P. H. Chase of the Philadelphia Electric Co., for their helpful assistance in collecting and compiling the material contained in this paper.

Discussion

For discussion of this paper see page 408.

Progress and Problems
From Interconnection in Southeastern States

BY W. E. MITCHELL[1]

Fellow, A. I. E. E.

Synopsis.—Great progress has been made. Interconnection between independent systems is primarily a protective measure. The greatest economic benefits have been realized when the interconnections have been made by subsidiary companies of one holding company. The capacity of tie lines and the amount of power interchanged has increased greatly. The size of generating units has increased, as has the size of power plants, resulting in lowered cost per kw. The problem of satisfactory voltage and power factor control has increased in complexity as has that of system load dispatching. While much improvement has been made in oil circuit breakers, they still leave much to be desired. Interconnection has made possible more economical operation of existing plants and has resulted in the use of a larger proportion of the available water on systems combining steam, storage, and run-of-river hydroelectric plants.

Long-time forecasting of load and rainfall conditions is important in economical system planning. The 110-kv. and 154-kv. line construction is discussed, also the value of ground wires and lightning arresters. The growing importance of carrier current for supervisory control and communication and their application are discussed.

* * * * *

IN 1924 in a paper on Interconnection of Power Systems in the Southeastern States, the author suggested that our great problem for the next 10 years would be to increase the capacity of the interconnecting links between different systems, to develop water power distant from the power market, and to construct the mine mouth or other strategically located (from an economic standpoint) high capacity steam plants, and connect them with the great load centers by means of high-capacity networks. He also suggested that we should plan for at least 10 years in the future.

Less than four years have passed, yet tremendous strides have been taken along these very lines. The economic possibilities of interconnection are being clearly realized. One of the results has been the coordination in a number of instances of the various individual operating companies under one holding company, thus deriving the benefit of massed capital, massed resources, and unified control. Interconnection, group management, low-cost steam generation, and long distance transmission have come as a logical succession of natural economic steps.

Interconnection between independent systems is, first, a measure of protection in emergencies. It cannot be a success unless the executives and operating forces of the respective companies realize it is a give-and-take proposition, and have mutual regard for the interests of both companies in the service of the public.

In the Southeast, what we term an "operating committee," (on which are representatives from Alabama Power Co., Tennessee Electric Power Co., Georgia Power Co., Columbus Electric and Power Co., Augusta-Aiken Railway & Electric Co., Southern Power Co., and Carolina Power and Light Co.) has been functioning for over four years. This committee meets about eight times a year and compares operating data, load, rainfall, storage reservoir conditions, and other problems of mutual interest. This has proved of the greatest value to all concerned. Getting better acquainted and the resultant better understanding, together with a greater knowledge of each others' systems and their points of strength or weakness, have facilitated the prompt handling of the exchange of power in emergencies. Frequently, improved operating economies through interchange have resulted from a greater knowledge of conditions throughout the entire territory.

Unquestionably the greatest benefits are derived, however, (and this applies both to the general public as well as to the individual companies), when the interconnected companies while maintaining their independent corporate identity are subsidiaries of one holding company. It is practically impossible to have a unified development program for five independent companies, but if these are under the same financial, engineering, and management control, then genuine system planning on a large scale will take place and the most economical plants and tie lines will be built first looking only to the maximum results for the group as a whole.

Electrical development in the Southeast has kept pace with the rest of the country as the tabulation on the following page will show.

The accompanying map, Fig. 1, shows the transmission lines of these companies covering a large part of seven of the most progressive states of the new industrial south.

The system of the Southeastern Power and Light Company is operated in parallel practically all of the time, and much of the time it is in parallel with both the Tennessee and Carolina systems. For successful parallel operation and, at the same time, to secure the proper load division between the various generating stations and between the systems themselves, it is essential that the frequency be very closely controlled.

We endeavor to maintain the frequency rigidly at 60 cycles. By the installation of master clocks at the

---

[1] Vice-President and General Manager, Georgia Power Co., Atlanta, Ga.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

TABLE I

		1923	1926	1927
Installed generating capacity	Hydro-kv-a	782,000	1,170,505	1,362,167
	steam-kv-a	353,000	497,978	622,487
Total generation—kw-hr		3,250,000,000	3,822,718,019	4,150,000,000*
Peak load —kw-hr		740,000	800,000	870,000*
Hydro storage —kw-hr		225,000,000	265,700,000	418,000,000
Miles of 110,000-volt lines		....	....	4,497
Miles of interconnecting lines		....	....	538.41
Interchange power between companies—kw-hr		228,500,000	457,000,000	573,000,000*

*Estimated.

larger generating stations, and the use of extremely accurate frequency recorders at load dispatching centers, it has been possible to make material improvement in the control of frequency over the interconnected systems. Fig. 2 is a typical frequency chart taken when the Southeastern Power and Light Company system was operating in parallel with Tennessee Power Company. While this chart probably does not indicate as steady a frequency as exists upon metropolitan systems, it does show the narrow limits within which it is possible to regulate the frequency upon a very extensive system. It should be noted that one cycle is represented by a space of 2⅜ in. on the chart, so that a variation of 0.05 cycles covers about one-eighth inch.

During the wet season the basic load is carried by the run of river hydroelectric plants, and the load fluctuations by steam plants and vice versa during dry seasons. The system supplying the bulk of the power regulates the frequency. Special attention has been given to the adjustment of governors at all generating stations, so that they will be as nearly as possible equally sensitive, in order that all stations may carry their proper share of the load fluctuations.

The problem of voltage control grows more important and more complicated as larger systems are tied together and as additional interconnections are made. On the system of the Southeastern Power and Light Company, there is at present installed a total synchronous condenser capacity of 130,000 kv-a. This capacity is fairly well distributed over the system at important load centers. The condensers are all controlled by automatic voltage regulators. Three 5000-kv-a. condensers and one 10,000-kv-a. condenser are equipped for full automatic control; that is, from the starting to

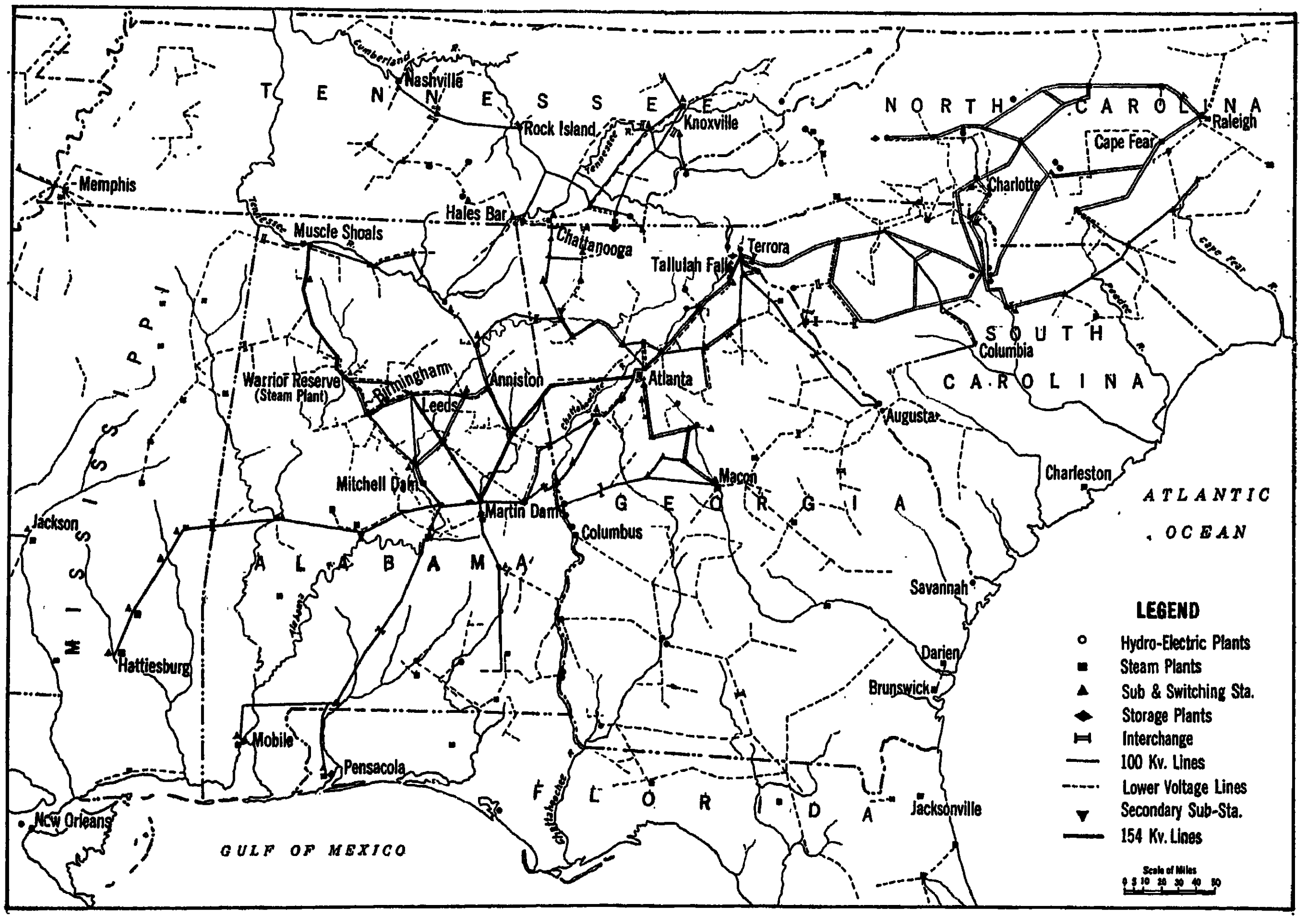


FIG. 1—MAP SHOWING TRANSMISSION SYSTEMS IN SOUTHEASTERN STATES

the stopping operation. Alabama Power Company has installed a 25,000-kv-a. turbo generator at the Gorgas steam plant, with provision for disconnecting the generator and the turbine and operating the generator as a synchronous condenser. During the wet season, when the steam unit is not required for supplying power, the generator is disconnected and operated as a synchronous condenser, supplying approximately 15,000-kv-a. corrective capacity to the system. In addition to this, the hydroelectric units are used for power factor correction, as required during the peak load hours, and when not required for supplying power. Usually, during the dry season of the year, when steam plants are operating on base load, a large portion of the hydroelectric capacity is used during peak-load hours for power factor correction. It is the practise, when these units are operating as synchronous condensers, to close the gates tightly. Vacuum breakers have been installed upon all of the more important units so as to reduce the power input to the generators when they are operated as synchronous condensers. The action of the water wheel when running at full speed is such as to pick up water at its center and discharge it at the perimeter, much the same as a centrifugal pump, whenever the gate openings are so low that the wheel is not fairly well filled with water. It is evident that this increases the power input to the unit. The opening of the vacuum breaker so as to admit air and break the column of water being pumped, therefore, results in considerable saving in losses, with a consequent reduction in gate opening for low ratings and saving in water. In the more modern plants, automatic vacuum breakers have been installed. Naturally the hydroelectric plants are not as beneficial as synchronous condenser stations located at load centers. However, they are in general at opposite ends of the system from the steam plants, and therefore they have been of much use in the correction of voltage. It has been more economical to use them for this purpose than to install the additional synchronous condenser capacity which would be required.

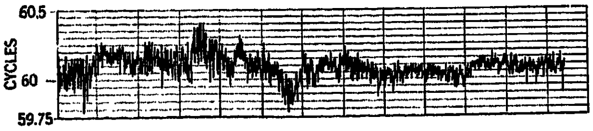


FIG. 2—TYPICAL FREQUENCY CHART OF SOUTHEASTERN POWER & LIGHT CO. SYSTEM

It has not yet been found necessary to install regulating transformers in tie lines to govern the division of load and wattless current, but it is very likely that this or some other method will be required as the number of ties between the systems increases. With the increasing amount of load carried over the lines, the problem of system stability has been encountered. In 1926, when it was necessary to supply abnormal loads over two tie lines between Alabama and Georgia, frequently an outage of one of the tie lines due to lightning or other trouble would result in a separation of the Alabama and Georgia systems by pulling out the other tie lines. Operating experience indicates that with a load of 30,000 kw. on each line, an interruption on one line will result in the separation of the two systems. As a result of this experience, an extensive series of stability tests was made in cooperation with one of the larger manufacturers. A technical paper is now being prepared describing in detail these tests.

On a large network system, it is becoming increasingly difficult, and, in fact, practically impossible, to make the necessary calculations for load division over transmission lines, voltage regulation, etc. Therefore, the practise has been established of at least once or twice each year taking simultaneous readings at all of the most important generating stations and load centers of the interconnected network. Readings of load, power factor, voltage, etc., are taken simultaneously and tabulated and are of considerable assistance in engineering calculations, as well as in the proper operation of the system. In order to analyze troubles and provide remedies, permanent recording equipment, such as the Hall Recorders, or oscillographs, are now being installed at stragetic points upon the system. These instruments, starting automatically and practically instantly when trouble occurs, should prove of considerable value in analyzing trouble and in determining settings for relays, loading of lines, etc.

The problem of relaying is becoming increasingly difficult as single circuit transmission lines are added. A great need is being felt for a system of relaying which, while selective, will instantly disconnect a faulty circuit. At present, the ordinary induction type overload relays are being used on single-circuit lines, naturally having a short time setting to secure selective operation. In a few instances, it has been found that the settings on these relays have been such as to hold the short circuit on the system long enough to affect the system stability, and in one or two instances, has resulted in rather serious interruptions. Upon the double-circuit tower lines, the balanced relays are functioning very satisfactorily. However, during lightning storms, frequently both circuits on the tower lines are interrupted simultaneously.

There are at present three heavy ties between the Alabama and Georgia companies of the Southeastern Power and Light Company system—two direct and one through the Columbus Electric and Power Company. So far, the main flow of power has been from West to East and this will probably continue to be the case for about three more years. However, there are now numerous short periods when surplus run of river power is sent from Georgia to Alabama. So far we have been able with the existing synchronous condenser capacity to handle as much as 90,000 kw. over these tie lines and maintain satisfactory voltage conditions. Within four months a new 154-kv. tie., 145 mi. long, will be completed between Martin Dam and the East Point Substation at Atlanta of the Georgia Power Company. This tie will have a capacity of approximately 60,000 kw. It will be a single-circuit line of special cross-braced H frame steel towers, 60 ft. high with 39-ft. steel crossarms mounted 56 ft. above the ground. Conductor spacing is 19 ft. 6 in. Conductors will be 397,500 cir. mils A. C. S. R. with two No. 2/0 composite aluminum steel ground wires flexibly attached to the towers. Insulation for 154 kv. will consist of 11 units in suspension and 12 units double yoke on strain with grading shields used throughout the line.

On September 18, 1927 a 90-mi. line between Gorgas Steam Plant and Wilson Dam was put in operation at 154,000 volts, this being the highest voltage at present in use in the southeastern states. This line was built during the war and used at 110,000 volts until this changeover. The need of additional capacity caused the change-over. This line now has a capacity of 60,000 kw.

A third 154,000-volt line of the same type as the Martin East Point line is practically complete now between Martin Dam and Leeds substation. For the present and until the construction of Lower Tallassee, this line will be operated at 110,000 volts.

The construction of these lines brings out forcibly the economy of unified control. They permit the postponing of the construction of additional power plants by one company until those under way by other companies under the same control are loaded up.

As the systems grow larger, interconnection between those under one management makes for greater operating efficiency. Full advantage is taken of diversity of time, diversity in rainfall, and in seasonal load. Without the tie lines this would be impossible. To a somewhat lesser degree, these same factors may be taken advantage of by means of ties between independent companies. With the large storage reservoirs practically no water goes to waste during the summer season. If one section has a heavy rain and another none, the tie lines permit sale of the surplus at such a price that the company that has had no rain can afford to buy power and store its own water. Matters of this nature are handled by the load dispatchers or operating engineers and may cover a period of a day, week, or even longer.

With the more careful studies of the curves of rainfall and run-off and expected load, a greater use is made of all hydroelectric plants. The run of river plants which on the isolated system wasted water all night long because it had little or no storage, now that it is linked into a comprehensive system, generates many more kw-hr. than it previously did, thereby probably saving fuel generation.

On a system combining run-of-river and storage hydroelectric plants and steam plants, great savings can be made by accurate knowledge of river flows and available plant capacities. This involves not only studies of all available data as to river flow in the past, but the forecasting of the future flows and the formulating of careful rule curves for the day to day operation of storage plants. Much careful study is being given to this by all large systems and what the subsidiaries of Southeastern Power and Light Company have done is only typical of the other companies. Messrs. J. P. Armero and D. M. Booth have, however, done some very interesting original work on this problem, which they describe as follows:

"In order that future operating conditions can be forecast as intelligently as possible, careful studies have been made of the hydrographs of all rivers involved over a period of not less than 25 years. From these studies a forecast hydrograph is drawn for the ensuing year. This hydrograph is determined in this manner:

"To obtain results with values of any proximity whatsoever to the actual hydrograph, the data of the rivers studied had to be treated as periodograms or curves representing functions which, although depending on cyclic variables, do not by themselves show a definite cyclic character. They are, in fact, cyclic functions whose periods are so long that the observations available cover but a small fraction of a cycle. The object of the periodogram analysis is to determine from the table of observations the equation of the curve, and with this to compute the values to be expected from the functions during the rest of the cycle. The periodogram is formed by the addition of a series of sinusoidal functions, and in some cases contains also lineal functions and constants. Generally, it is impossible to find the actual composition. Unless the observation covers at least one cycle, the problem is indeterminate. The indetermination is removed when a number of components is known, but even in such cases it is necessary for obtaining consistent results that the data be free from any error or perturbation. The results shown here in plotted form have been prepared by using a method whereby each component's period was estimated, and then computing their phase and amplitude. This method of attack is especially useful when observations, although not covering the whole cycle of the resultant, cover at least one cycle of each component.

"Fig. 3 shows the actual massed hydrograph and resultant curve obtained by the addition of the components found by the analysis.

"Fig. 4 shows the forecasted and actual hydrograph for 1926—Coosa River.

"Fig. 5 shows the forecasted and actual hydrograph for 1926—Tallapoosa River.

"This gives an idea of how closely the actual flow has followed the forecasted during previous years. Based upon studies of previous years flow, with consideration of wet, dry, average, or medium years, and with the aid of the study as mentioned above, rule curves are made up, covering the operation of the storage plants.

Figs. 6 and 7 show typical rule curves for the Georgia and Alabama storage reservoirs. The rule curves are followed very closely, and deviations are permitted only within certain limits, and to take care of unusual conditions on stream flow plants. Of course, occasionally there is insufficient rainfall to fill up the reservoirs along the rule curve. By following these curves, the various plants are operated in so far as possible at their most efficient points. Occasionally it is necessary to increase the load beyond the most efficient point to eliminate higher priced power during peak hours. By properly coordinating the operation of all plants on the systems, these occasions are rare.”

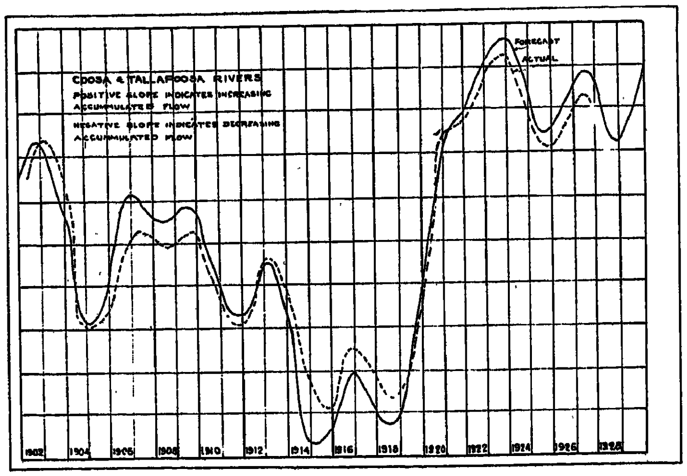


FIG. 3—MASSED HYDROGRAPH AND RESULTANT CURVE—COOSA AND TALLAPOOSA RIVERS

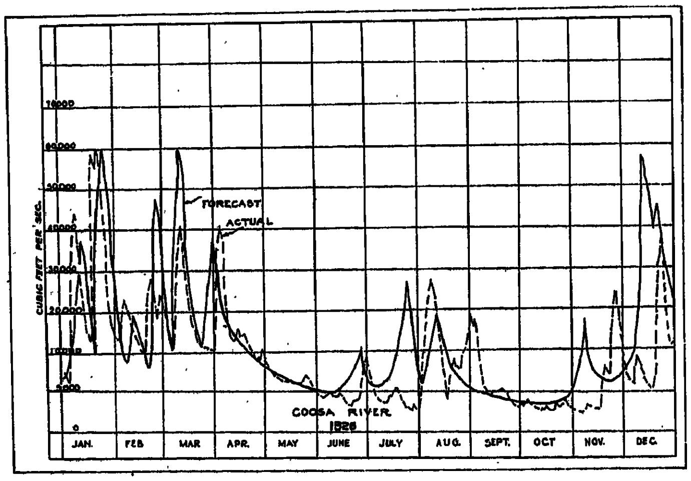


FIG. 4—FORECASTED AND ACTUAL HYDROGRAPHS OF COOSA RIVER—1926

Operating and maintenance practise has also improved with the exchange of operating data and ideas. More careful maintenance schedules for all equipment are followed, thereby reducing maintenance cost and keeping all equipment in better and more efficient condition. This is particularly true of steam plants where all heavy maintenance work is done during the four normally inactive months of December, January, February, and March.

Tables I, II, and III show the effect in a system operating both steam and hydro plants of load growth and rainfall on production and operating costs of typical steam and hydroelectric plants.

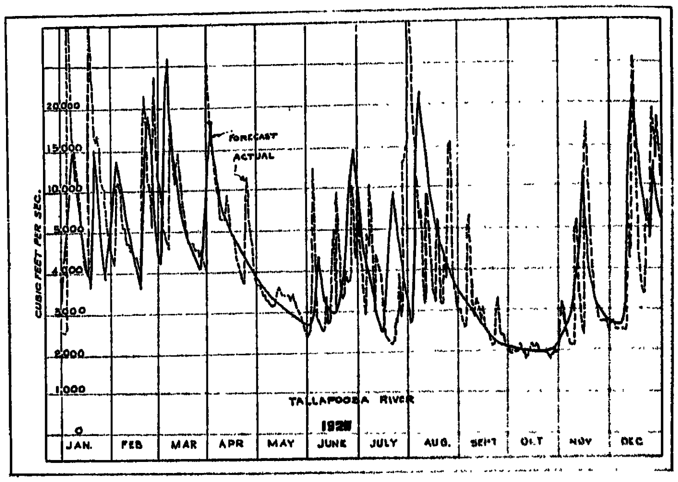


FIG. 5—FORECASTED AND ACTUAL HYDROGRAPHS OF TALLAPOOSA RIVER—1926

At the present time the first 66,667 kv-a. unit of four which will ultimately be installed in a new mine mouth steam plant at Gorgas is now under construction. This plant, together with the existing steam plants and the big storage plants at Martin and on the Tallulah River, make possible the maximum use by both Georgia Power Company and Alabama Power Company of their run of river hydro plants. All combine to develop more kw. of capacity per dollar of investment and to produce more kw-hr. at a lower cost than would be possible under independent management. In a similar manner the Southern Power Company, the Carolina Power and Light Co., and Tennessee Electric Power Co.

TABLE I

PLANT PERFORMANCE—HYDROELECTRIC PLANT OPERATING IN A SYSTEM WITH STEAM PLANT

Year	Output	Load factor	Maximum demand
1923	153,094,600	20.4	69,500
1924	255,520,500	48.5	60,000
1925	227,387,000	44.0	58,500
1926	302,292,000	58.3	59,200
1927	291,880,000	56.7	58,000

OPERATION AND MAINTENANCE COSTS COMPARED

	Unit cost in mills per kw-hr.		
Year	Operation	Maintenance	Total
1923	..	..	0.1553
1924	0.122	0.023	0.1450
1925	0.117	0.073	0.1900
1926	0.080	0.047	0.127
1927	0.074	0.131	0.205

*Maintenance unusually high in 1927 due to cost of welding runners.

TABLE II

COMPARISON OF YEARLY STEAM PLANT PERFORMANCE

Year	Tons coal per year	Output kw-hr.	Load factor in per cent	Maximum demand kw.	Lb. coal per kw-hr.	Unit cost mills per kw-hr.			
						Oper.	Maint.	Fuel	Total
1920	118,159	88,088,100	20	45,000	2.56	..	..	..	6.30
1921	137,763	112,000,600	25.6	55,000	2.46	0.81	0.57	4.92	6.30
1922	188,773	188,414,600	43	55,500	2.00	0.40	0.40	4.00	4.80
1923	145,184	145,250,200	33.1	54,600	2.00	0.52	0.95	4.00	5.47
1924	198,947	207,678,000	34	78,500	1.91	0.38	0.49	3.82	4.59
1925	334,196	350,795,400	54	78,800	1.90	0.32	0.40	3.80	4.52
1926	190,624	198,064,800	29	78,500	1.90	0.50	0.73	3.80	5.03
*1927	263,807	276,553,800	46	75,000	1.90	0.34	0.52	3.80	4.66

*11 months only.

The cost of coal in the above figures is $4.00 per ton at the plant.

TABLE III

SOME MONTHLY PRODUCTION COSTS FOR A LARGE HYDROELECTRIC PLANT OPERATING IN SYSTEM WITH STEAM RESERVE PLANTS IN MONTHS OF DIFFERENT FLOWS

	High flow	Medium flow	Low flow
Gross generation in kw-hr.	38,474,600	16,505,800	9,250,200
Station use kw-hr.	119,600	103,500	115,900
Net generation kw-hr.	38,355,000	16,402,300	9,134,300
Peak kw.	58,200	50,000	36,000
Turbine hours	2,221	1,600	1,297
Operating cost	$ 2,270	$ 1,840	$ 1,745
Labor	$ 1,654	$ 1,524	$ 1,534
Material	$ 617	$ 316	$ 211
Maintenance cost	$ 2,266	$ 5,574	$ 8,919
Labor	$ 1,710	$ 1,670	$ 1,709
Material	$ 556	$ 3,904	$ 7,210
Production cost	$ 4,536	$ 7,414	$ 10,664
Substation cost	$ 921	$ 1,219	$ 946
Total payroll	$ 4,570	$ 4,653	$ 4,714
Average number men	43	46	46

Note: Maintenance during medium and low months includes contract expense shown under material for welding runners.

are working to the same end on their great systems.

The development of all the economical hydroelectric sites is not distant. The material improvement in steam generating efficiencies, together with the lowering plant costs, the reliability of the large steam units and the varying character of our southern streams are all making utility engineers in the south realize that a greater and greater proportion of power will be produced by steam in the future.

The extent and feasibility of interconnection and long distance transmission was thoroughly demonstrated in 1925 when over 600 mi. of line between Muscle Shoals in northwest Alabama and points north of Raleigh, North Carolina were linked up for several weeks and large blocks of power transmitted.

The year 1927, as far as Alabama and Georgia were concerned, was nearly as dry as 1925. Georgia had very little additional generating capacity and a very greatly increased load. Due to coordination of the systems, however, and the additional tie lines, water was conserved in the Burton storage all during the spring and drawn out in the late fall when most needed. Under independent operation the tendency would have been to have taken a chance on getting summer rains (which never came) and so to have drawn upon the stored water that when the final pinch came there would have been none left and then industries would have been forced to curtail because the tie lines could carry no more and the hydro plants would have been greatly reduced in capacity by lack of water. The accompanying load curves, Figs. 8, 9, and 10, and the rule curves, Figs. 6 and 7, demonstrate most clearly the value of unified operation of two large systems.

On December 3, 1927, systems from Chicago to Mobile and Pensacola were interconnected and thus over a thousand miles of transmission line and literally millions of kw. of generating capacity operated in parallel for some fifteen minutes. No power was interchanged but the availability of the interconnections in case of emergency was demonstrated.

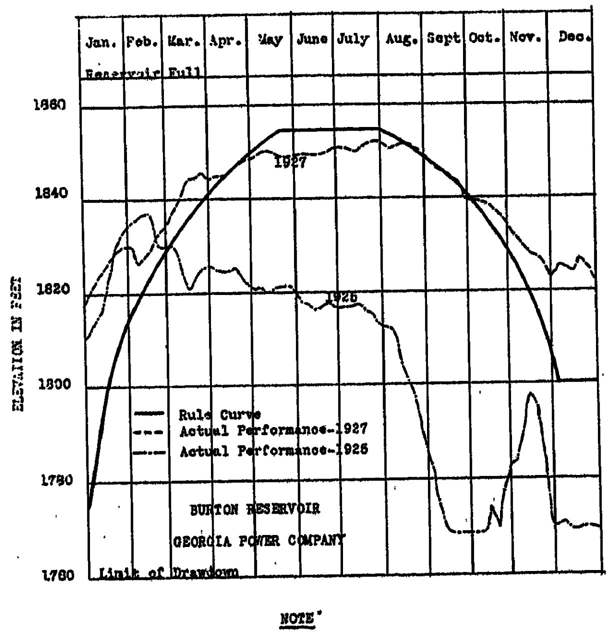


FIG. 6—RULE CURVE BURTON RESERVOIR—GEORGIA POWER COMPANY

In October 1927 a large cotton mill in Georgia had a failure in their isolated plant. Within twenty-four hours from the time notice was received, the Power Company has picked up over 3000 hp. of the mill's load and within seventy-two hours had picked up the entire 10,000 hp. This was only possible due to the capacity of two big interconnected systems, but it meant continued employment for several hundred people who would otherwise have been thrown out of work for two or three months.

In 1927 the Savannah River dropped to a flow nearly as low as in 1925. The local plant near Augusta was entirely inadequate under these conditions to carry the system load so that for several months from 15,000 to 20,000 kw. capacity was taken over the interconnecting 110-kv. line with the Georgia Power Company and thus a curtailment of industry in a large territory entirely avoided.

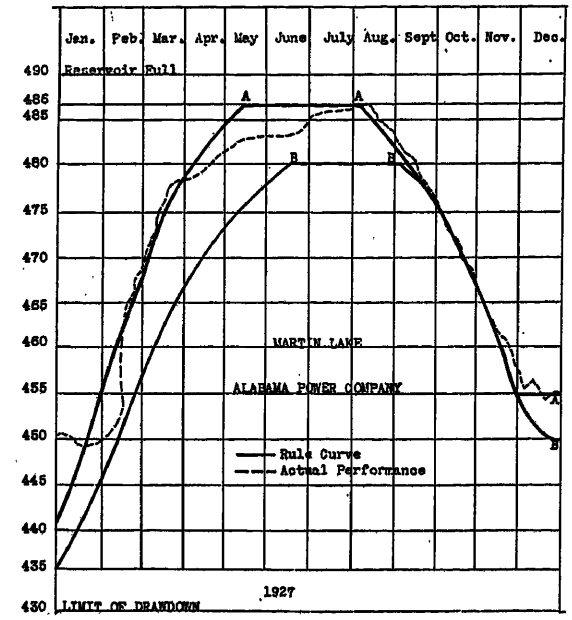


FIG. 7—RULE CURVE, MARTIN LAKE—ALABAMA POWER COMPANY

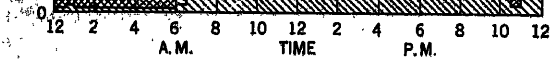


FIG. 8—GEORGIA POWER CO. LOAD CURVE OCT. 8, 1925

Another example of the greater use of available resources: Company A had available 25,000 kw. of transmission capacity but no surplus generating capacity and was interconnected on one side of its system with Company B, which had surplus power, and on the other side with Company C, which was short of power. Transmission facilities were leased from Company A, power purchased from Company B, and Company C's needs were completely taken care of.

The growth of each independent system, but even more the unified control of several systems, has promoted the use of larger power plants and larger generating units. When a system load is of the order of 400,000 kw. the annual growth will be 50,000 kw. at least; therefore, it is logical to install very large units. In the Buck steam plant, a powdered fuel plant, which the Southern Power Company recently put into operation, two turbo generators of 35,000 kv-a. each were installed. In the Martin Dam plant of Alabama Power Company three 37,500-kv-a. units were installed for an average head of 120 ft. At Jordan Dam (Lock 18), now under construction, four 29,000-kv-a. units,

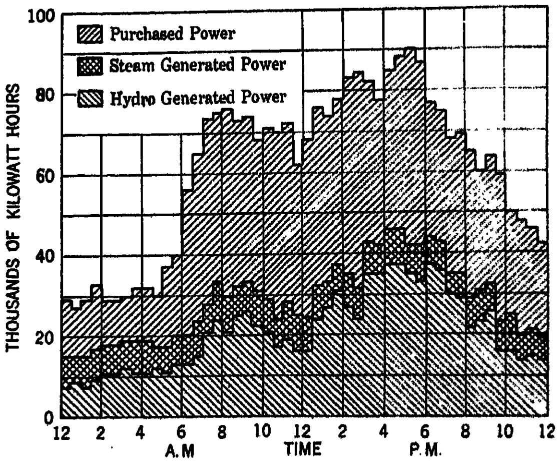


FIG. 9—GEORGIA POWER CO. LOAD CURVE NOV. 2, 1925

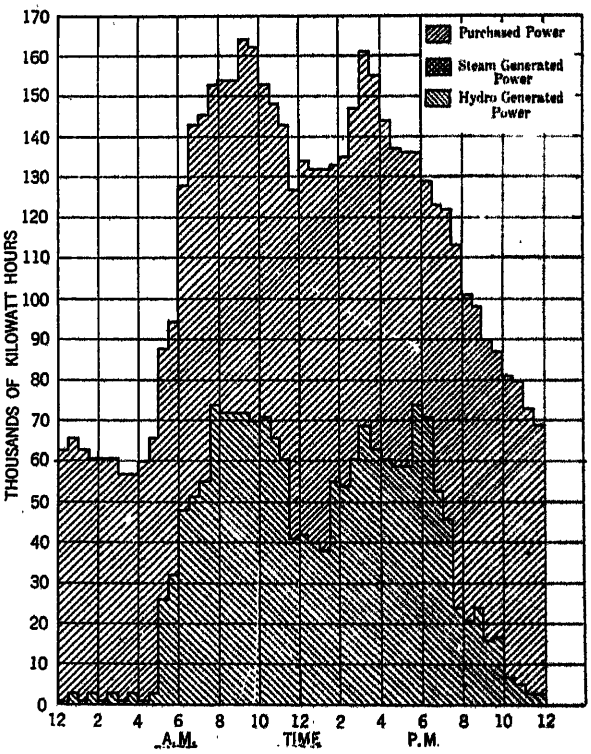


FIG. 10—GEORGIA POWER CO. LOAD CURVE SEPT. 28, 1927

operating at 93-ft. head, are being installed; at Upper Tallassee two 19,000-kv-a. units, operating at 55-ft. head will be in operation early in 1928. The Southern Power Company will have 45,000 kv-a. at its Oxford hydroelectric plant in service early in 1928. The High Rock Plant of the Aluminum Company of America was brought in in 1927 with 40,000 kv-a. The Southern Power Company will receive a large portion of the output of this plant over a new 40-mi. 110-kv. tie line. The Tennessee Power Company revamped their old Nashville steam plant, equipping two boilers for powdered fuel, and installing a 23,500-kv-a. turbine. Carolina Power and Light Co. late in 1926 brought in their Cape Fear Steam Plant with 30,000-kv-a. capacity and are now working on their Norwood hydro plant of 65,000-kv-a. capacity. The generators in this plant will be of the outdoor type. A third 17,500-kv-a. unit is now being added to the Bartletts Ferry Plant of the Columbus Electric & Power Company, bringing its capacity up to 52,500 kv-a.

When the new Gorgas steam plant of the Alabama Power Company was under discussion, very careful studies were made and 66,667 kv-a. units finally decided on as the most economical size for our conditions. This plant when completed will probably be the most economical plant in the Southeastern States. A brief description of this latter steam plant may be of interest.

The turbo generator is a 66,667-kv-a., 90 per cent power factor, 400-lb., 725-deg. fahr. machine arranged for four bleed points. The generator is cooled by four motor-driven blowers having a total capacity of 105,000 cubic feet of air per minute at a static pressure of 10 in. of water. The blowers drive the air through the generator air cooler in a closed system.

The condenser is 70,000 sq. ft., two pass, with 1-in. tubes, two circulating pumps, two condensate pumps, and one bank of air ejectors.

Two 30,000-sq. ft., 475-lb. pressure Babcock & Wilcox boilers and superheaters with Combustion Engineering Company Fin type water walls on three sides of the furnace are provided. Each boiler is guaranteed to produce a maximum of 450,000 lb. of steam per hr. at 475 lb., 725 deg. fahr. total temperature.

The boilers will be fired with pulverized fuel fed from a bin system. Three 15-ton mills discharge to cyclones and thence by conveyors to the pulverized fuel storage bin for each boiler. The single-line diagram, Fig. 11, shows the extreme simplicity of the electrical layout. This plant is expected to make a kw-hr. on the switchboard with 15,000 B. t. us. While many northern plants are more efficient thermally, this figured out the most efficient plant for this territory when the price of coal and the increased capital cost per kw. for greater efficiency were considered.

This plant plainly shows the rapid change in conditions in a few years and indicates the prime importance of careful study of load growth and of consideration of the improvement in efficiency of steam units. Whereas eleven years ago the first large unit installed at Gorgas was 25,000 kv-a., and the plant was built purely as a reserve steam plant to operate at from 10 to 25 per cent annual load factor, the new plant will be a base-load plant operating on a 50 per cent annual load factor at first and probably ultimately on a 70 per cent load factor. Another interesting fact is that while the new plant will be 50 per cent more efficient than the old, its cost per kw. will be but little, if any higher; such is the effect of larger units. Again, units of 66,667 kv-a. would not be the proper size were there not ample reliable tie lines between Alabama and Georgia so that such a unit would be available to both companies.

In present day practise all substations are of the outdoor type with steel girders or steel pole supporting structures. They are simple and sturdy with large clearances. Small control houses are built for the relay, protective and control equipment and telephones with a small storage space for spare parts that must be housed. In older substations the low-voltage buses and switches were put indoors, now for 11,000 volts or above nearly all are placed outdoors.

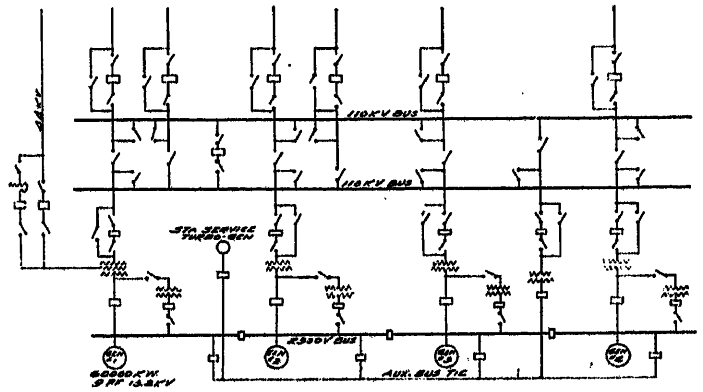


Fig. 11—Single-Line Diagram New 266,668-Kv-a. Gorgas Steam Plant—Alabama Power Co.

Double-circuit 110,000-volt lines and many single-circuit lines are built on steel towers but the majority of single-circuit 110,000-volt lines are of the familiar H-frame wood-pole type with one or two ground wires and with the structures guyed at frequent intervals. Chestnut, juniper, and cypress poles are frequently used, but by far the majority of lines are constructed with creosoted long leaf yellow pine poles. These make a splendid looking and a long lived, low maintenance cost, line.

By far the greater part of the territory served in the Southeastern states is sparsely settled with nothing like the density of load per sq. mi. that is encountered in the northeast or middle west. Therefore, types of construction have been developed which peculiarly suit these conditions. First cost must be kept at a minimum and at the same time operation and maintenance must be kept very low. A brief description of a typical line may be of interest. A single-circuit 110,000-volt line, 225 mi. long, from Mitchell Dam via Lock 18 to Hattiesburg, Mississippi. The total load was only about 12,000 kw., distributed at eight load centers. Prior to inauguration of hydroelectric service the more important of these load centers were supplied from local steam plants, and had good service, to which the hydroelectric service will of course be compared. Due to the small size of the various loads, it follows that the distribution organizations are small, and that there are therefore few men available for emergency patrols on the high tension line and substations. The redeeming feature about this territory was the fact that the prospects for growth in load looked very good. It has actually increased 2000 kw. in the last year.

While first cost had to be low, there had to be considerable spare capacity, at least in the line, to provide for future growth. It was also evident that the operating and maintenance costs must be comparatively low. To get low operating costs, it was evident that the line and connecting substations should be as nearly automatic as possible, requiring a minimum number of men in constant attendance. It was also clear that the line and substations would have to be very reliable, inasmuch as the service requirements were high and there were few men available for putting on the line to run down and repair faults.

In satisfying these requirements, we worked out a system covering the 225 mi., which operates with only three men regularly on duty, looking after the line and substations. This attendance is supplemented by monthly patrols of the line, using men from the local distribution organizations, and by daily inspections of the primary substations, utilizing members of the local supervisory organizations. These inspectors are, of course, supplemented by sending in our regular line and substation crews to do maintenance work when inspections indicate this to be necessary. Based on a comparison of the line design with that of existing lines, we do not expect an average of more than one major interruption per year on this line, and in general expect this to be of not over 7 or 8 hours' duration. There will, of course, be a number of momentary interruptions of one to two minutes' duration during the lightning season. One year's operation seems to be justifying our assumptions.

Considering the line construction, most of the distance the conductors are 397,500-cir. mil steel reinforced aluminum. Structures are of creosoted pine poles, 65 ft. high, with 8-in. tops, each structure being an H-frame consisting of 2 poles and a creosoted pine crossarm (6 in. by 8 in.), 32 ft. long. These structures are spaced on the average of a little over 1000 ft. apart. Insulators consist of 8 and 9 units for suspension and strain service. Poles are set approximately 7 ft. deep, and where soft ground is encountered, all structures are side guyed. In better ground, alternate structures are guyed. Galvanized tension rods are provided for bracing the cross arms, and tying them in with the tops of the poles. All hardware is grounded, there being two 4-lb. copper grounds per structure, one on each pole,—these terminating in ground pipes one in. in diameter and 8 ft. long. Two caps wires are provided which are of ⅝-in. Siemens Martin steel. These wires are insulated on 20,000-volt pin-type insulators, and serve a number of purposes. In the first place they stabilize the line mechanically, serving as head guys on each structure. They are also utilized in connection with a 500-cycle supervisory control system, to control certain substation switches and also motor operated switches which are placed at intervals of 15 mi. along the line, these being operated by the supervisory control system referred to, and power being supplied by small local 12-volt storage batteries. These cap wires are transposed at three points between sectionalizing switches and at sectionalizing switches, and are supplied with drainage units at sectionalizing switches and substations. They can therefore be used as an emergency communication system, and for this purpose are provided with drops at two-mile intervals which extend down the poles and terminate in small protective equipments, so that a patrolman can plug in a portable test set and secure communication with adjacent substations during line interruptions. These cap wires also terminate in telephone equipment at the various primary substations. Finally, being provided with drainage units and grounded through same, these cap wires serve as ground wires to some degree. We are satisfied they have considerable value in this respect, but as yet do not know just how effective they are as compared with the ordinary ground wire.

Considering next the substations, these are as near automatic as they can be made without going to undue complications. A typical substation consists of a tap on the 110 kv. line, with air break switches on each side, coming to an air break switch in the substation. Following this switch, which ordinarily is supervisory controlled, there is a 110-kv. fuse. Following the fuse are the transformers, which are in all cases out of door and self-cooled. It will be noted that no 110-kv. arresters were provided, as our experience indicated that on these small substations it is good economy to omit them. From the transformers, connections go to a double, low-voltage bus, the voltage being 44 kv. in some cases and 11 kv. in others. The various transformer and feeder switches are equipped with various relays, depending upon service required, and wherever possible, they are automatically reclosing. At three of the substations, Lock 18, Demopolis and Meridian, 110-kv. oil circuit breakers are provided in the main line; these, in the case of Lock 18 and Demopolis, being provided with CZ relays and automatic reclosers. At Meridian and Hattiesburg, synchronous condensers are installed which are fully automatic and provided with various protective relays, so that they will take care of themselves during line interruptions. Communication is provided along this line by means of antenna-coupled carrier-current sets, which are installed at all substations. In practically all cases these sets are remote controlled from the nearest city office, which in most cases is several miles away. As mentioned before, emergency communication between substations in case of line interruptions is provided by the cap wires.

The only substations along the 225-mi. of line where men are regularly on duty are at Lock 18, Meridian, and Hattiesburg. At Meridian a man is necessary because this is a dispatching point. His presence is made further advisable by the synchronous condenser installed. The condenser at Hattiesburg makes it advisable to maintain a man at this point also.

Transmission lines still constitute the weakest link in the chain between generating station and customer. The question of ground wires is still a mooted one but the majority of southern operators, after from 15 to 20 years' experience, feel sure that they are justified on important lines and afford material protection against lightning. The new 154-kv. lines being built in Alabama and Georgia are being over-insulated, but the equipment is not. For its protection, a weak link will be put in the lines near the substations by leaving off two or three of the insulator disks. On the question of high-voltage lightning arresters opinion is much divided. On the Southeastern Power and Light Co. system we feel that in the more important stations the transformers should be given the benefit of the doubt and arresters installed. Routine annual testing of transmission line insulators is now carried out by most companies. Some use a megger but the majority use the buzz-stick method. This has proved very satisfactory in eliminating defective insulators. Between the improvements in design and the elimination of defectives by various tests, the number of insulators punctured or destroyed by lightning is very few, nothing like the number we lost 10 years ago. It is still the general practise to patrol lines on foot at regular intervals with emergency patrol after every interruption.

With the growth and interconnection of systems the necessity of uninterrupted communication circuits has grown in importance. Important stations and load dispatching centers are usually coupled by several private land-line circuits as well as by Bell system lines wherever possible. In the last two years extensive use has been made of the carrier-current telephone system, and exceptionally good results have been obtained. On the Southeastern Power and Light Co. system carrier-current sets are now being installed in Atlanta and Birmingham in the load dispatchers' offices to permit immediate communication between load dispatchers on the two systems; also, for the purpose of intercompany commercial calls. They are being used regularly for load dispatching on over 1000 mi. of transmission lines. An accurate check was kept of the calls for 75 days, and 86 per cent of them were completed. The failure to complete most of the remaining calls was due to the operator called being where he could not hear the signals.

Oil circuit breakers continue to be one of the most important factors affecting satisfactory long distance transmission and system interconnection. Here again the increase in capacity of transmission lines and the concentration of power necessitate ever increasing capacity in the oil circuit breaker. Today short circuits of 1,000,000 kv-a. are possible in the Birmingham District substation and of 750,000 kv-a. at Atlanta. The additional tie lines between Alabama and Georgia will cause a complete change in 110-kv. oil switches in all of the Atlanta district oil switches. The general layout of the Southeastern Power and Light Co. system is such that we believe we shall continue to give best service by operating on so-called solid ties as compared to the loose link method, and limiting short circuits by opening it up at certain predetermined points. Later we shall probably add 110-kv. reactors through which normally no current flows. In contrast to this the Southern Power system operates to much better advantage on the loose-link coupling. Because of lack of sufficient data on the performance of high-capacity oil circuit breakers under full rating, a group of five large utilities formed about two years ago an Informal Committee on Cooperative Oil Circuit Breaker Tests. The breakers to be tested were bought by the interested companies and tested under actual system operating conditions. Manufacturers' representatives were invited to be present when their switches were tested. Much important information was gained and we feel that definite improvements in design have resulted directly or indirectly from these tests. As the Alabama Power Company's part, tests were made during 1926 and 1927 on six 44-kv. breakers of four different manufacturers and one 110-kv. breaker.

To summarize: The past four years have witnessed a growth by all utilities in the southeast. Additional and higher capacity interconnections have been made. Individual system operation has been improved as has coordination between systems. More use is being made of interchange facilities. More careful forecasts of load growth are being made and more careful system planning to handle such growth is being done. Advantage is being taken of the lower costs of larger units and particularly of the improvement in efficiency of the modern steam generating plants. Long distance transmission is growing more reliable.

But there are still many problems unsolved or only partially solved. Load dispatching grows more difficult and complicated with the growth of systems. From the standpoint of protection to service proper relaying will continue to require much study. The division of load between parallel circuits, the control of wattless current, system stability, and more reliable oil circuit breakers are problems worthy of the best engineering brains in the industry. As hydroelectric stations decrease in size, more automatic ones will be constructed to keep down the cost per kw-hr. The future will probably see still larger steam plants with larger units located at the mine mouth, when water is available there, and others as near to large load centers as available condensing water will permit. These will, in turn, bring up the problems of still heavier main trunk transmission lines operated at higher voltages than at present generally used in the southeast. The last word has not yet been said regarding lightning protection. Improvement in communication circuits also offers a fertile field for study.

If the record of the industry for continued reduction in cost per kw. and per kw-hr. is to be maintained and, at the same time, the service carried farther and farther afield to thinner and thinner territory, there is no danger of our engineers running out of work.

Discussion

For discussion of this paper see page 408.

Some Aspects of Pacific Coast Interconnections

BY P. M. DOWNING[1]
Member, A. I. E. E.

Synopsis.—This paper tells in general of the reasons for, and the advantages of, interconnection between power supply systems. It enumerates five essential factors which must be considered in successful operation of interconnected systems; namely, (1) dependable communication; (2) centralized authority and control over operations; (3) location and isolation of trouble; (4) voltage regulation and control over the entire system; and (5) frequency control. A description is given of the interconnections along the entire Pacific Coast.

* * * * *

INTERCONNECTION, as that term is used when applied to the electrical industry, means nothing more nor less than the pooling of the production and distribution facilities supplying power within a given area. It differs from transmission in that interconnection implies the transfer of energy from one system or area having a surplus to another system or area having a shortage, whereas transmission implies the transfer of energy from the point of production over a considerable distance to the point of usage.

Electric service was once local in character but with the advent of higher voltages, which made possible the transmission of power over greater distances, communities which theretofore had been supplied from local generating plants found it to their advantage to interconnect with other adjoining systems as a most logical and common sense way of making a more beneficial use of existing facilities.

Interconnection does not necessarily imply capital consolidation of the power supply systems involved. There are many instances where two or more independent systems are operating very successfully under contracts or working agreements that permit of economies not possible under separate operation. Very often, however, all of the advantages to be had from the interconnection are not obtained under contractual arrangements, due to the lack of sufficient flexibility in the contracts or working agreements in effect. To obtain maximum economic and beneficial results from interconnection, control and management should be unified. If power is to be distributed under the most favorable conditions, more than anything else there must be local territorial monopoly. No state, however richly endowed with water power, coal, oil, or other fuel, can make maximum beneficial and economic use of its facilities without interconnection.

At various times we hear of "giant power" and "superpower" systems. While these fascinating phrases probably make a stronger appeal to the imagination of the lay mind than does the more homely and commonplace expression "interconnection," aside from the generally accepted idea that giant power implies state or federal ownership, they mean substantially the same as does interconnection and all seek to accomplish the same end.

Some of the more enthusiastic proponents of giant power have gone so far as to suggest that there should be high-voltage lines of large capacity connecting different power-producing and power-using areas within the radius of practical electrical transmission, thus in effect producing a single pool of power into which all generating stations would deliver power and out of which all distributing systems, irrespective of ownership, would take delivery of such amount as they would need. From an idealistic standpoint such an arrangement would probably be well nigh perfect, but from a practical and economic standpoint the investment in transmission facilities would probably be so great that the purpose sought to be accomplished would be defeated.

Some of the more important advantages to be gained by the interconnection of two or more different power-supply systems are the following:

1. Greater reliability of service and increased economies. In case of a shortage of power due to accidents to equipments, other similar equipment on the interconnected system can supply the shortage until repairs are made or other units put into service. Under normal conditions, the more efficient units can be operated, the less efficient being held in reserve and operated only in emergencies.
2. Increased diversity, thus reducing the power requirements of the combined systems and by improving the load factor, reducing the capital investment in plant facilities.
3. Common use of reserve or spare capacity, thus further reducing the capital investment in plant.
4. In cases where the network is supplied with power from both steam and hydro sources, the hydro stations can be operated continuously at full load when water is available, thus making a maximum beneficial use of water power and a minimum use of steam. On a smaller system there would be much less opportunity of doing so.
5. General availability of power over large areas, thus making it possible for factories to locate where raw material, labor, market conditions, etc., are most favorable. This is particularly important, as it permits manufacturing plants to locate in smaller towns and agricultural areas and use labor normally employed

---

[1] Vice-President Pacific Gas & Electric Co., San Francisco, Calif.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-17, 1928.

during certain seasons only. In this way interconnection becomes an active force in the decentralization of population.

Although there are many arguments in favor of interconnection as a most logical means of effecting economies, the extent to which power systems can be advantageously interconnected has certain limitations beyond which there is no advantage in going. Since the economies of interconnection lie in improved load factor, higher diversity, and greater use of existing facilities that is possible by combining loads of dissimilar character, it naturally follows that no great advantage would be gained by the interconnection of two or more systems unless by so doing these elements could be improved.

Obviously, there would be no advantage, economic or otherwise, in making any considerable investment in facilities necessary to interconnect two or more systems supplying different areas if those systems had similar power supply and load characteristics unless such interconnections would give a combined load of higher diversity, better load factor, improved service or one that would permit of a more general use of existing facilities.

Unfortunately, the wide-spread discussion of this subject that has taken place during recent years has given the public the impression that it would be practicable to interconnect all of the power systems in the country irrespective of the distances at which they might be separated, and, by parallel operation and exchange of power, effect economies to a point where the cost of power to individual consumers might be reduced. Nothing could be further from the fact. In the light of our present knowledge of the art, the advantages of interconnection are regional in character and limited to a radius of 300 to 400 miles.

Interconnection between various sources of electric generation and distribution on the Pacific Coast was the inevitable result of environment and economic pressure and became a reality almost immediately after the possibility of transmitting considerable quantities of power over long distances at high voltages had been fully demonstrated.

In California, where perhaps more has been done in the way of unifying the ownership and operation of electric utilities than has been accomplished elsewhere, topography was an important factor in bringing the systems together. In the eastern part of the State, running north and south for hundreds of miles, are the Sierra Nevada Mountains, having hydro resources amounting to approximately 9,000,000 hp. In the extreme western part of the state lie the cities with markets for large blocks of electric power. Between the two are the wonderfully fertile agricultural valleys which, of necessity, had to be crossed by the transmission lines carrying power from the hydro plants in the mountains to the cities on the coast. The result of such a situation was the construction of a number of transmission lines converging toward the cities as a common point of delivery. Later, by a natural process of consolidation, various systems were united into larger units which were in turn interconnected, thus forming a network of lines which permits of a ready exchange of power between areas supplied from a system under a single ownership and management or among several systems under separate ownerships and managements.

The powerhouses and transmission lines in California as shown on the accompanying map can all be interconnected to form a single network covering the entire state. Ordinarily there are three major connected groups. The first includes: California-Oregon Power Company, Pacific Gas and Electric Company, Snow Mountain Water and Power Company, City and County of San Francisco, Utica Mining Company, Coast Counties Gas and Electric Company, Truckee River Power Company.

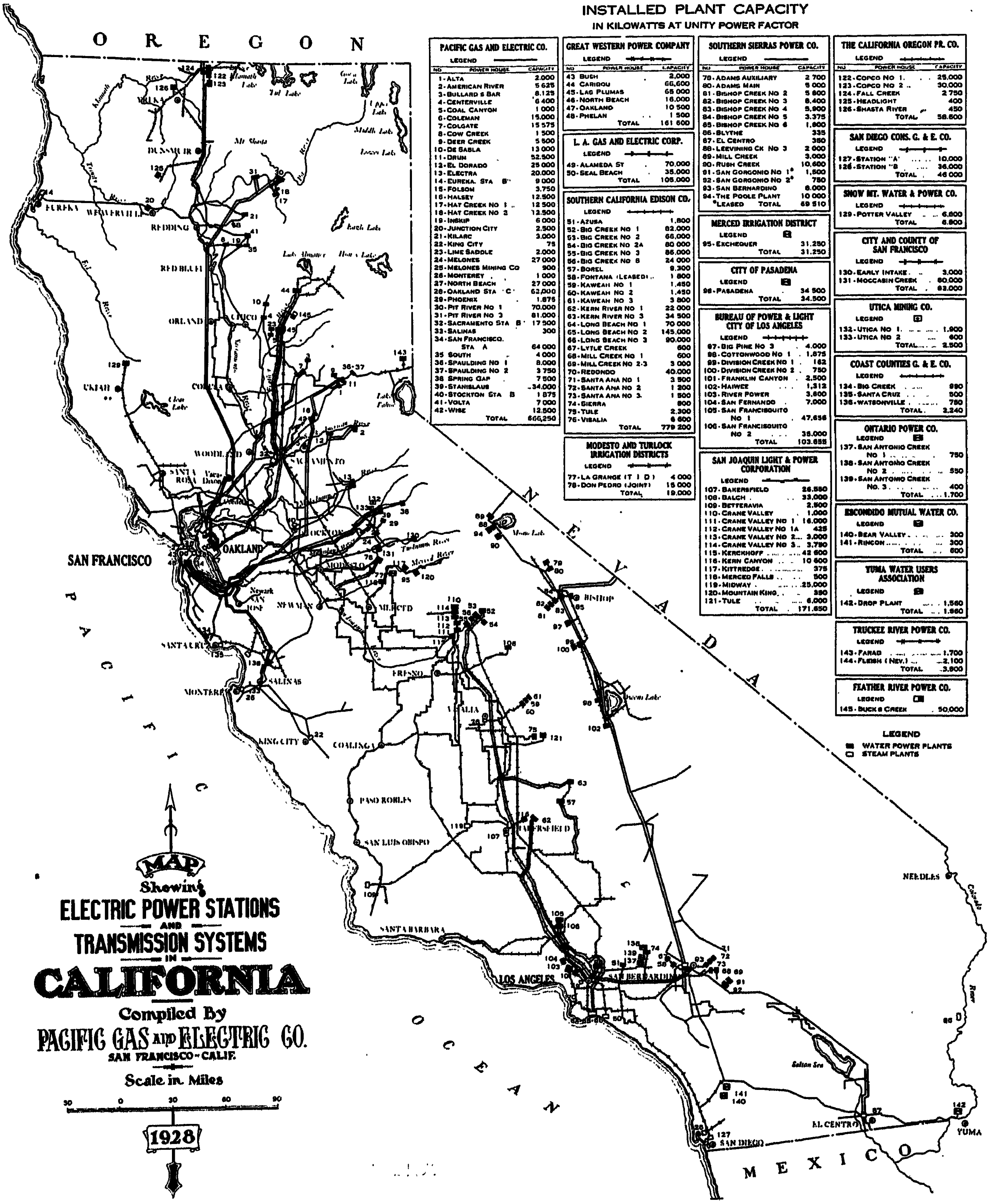
The second: Great Western Power Company of California, San Joaquin Light and Power Corporation, Merced Irrigation District, Feather River Power Company.

The third: Southern California Edison Company, Southern Sierras Power Company, City of Pasadena, Bureau of Power and Light, City of Los Angeles, San Diego Consolidated Gas and Electric Company. (See Fig. 1.)

Along the route of these transmission lines, running from the hydro plants on the western slope of the mountains to the larger cities along or near the coast were many smaller cities, towns, and agricultural communities. Some of these were supplied with power from small isolated plants, while others were without electric service. These small plants were inefficient and the opportunity to shut them down and obtain hydro power from transmission sources at less than it was costing to produce it locally was welcomed by all. At the same time the load thus obtained afforded a ready and quick revenue to the transmission systems. As a result of these mutual advantages, practically all of the isolated plants were abandoned very shortly after power was available from transmission sources. Eventually most of these small plants, together with the distributing systems, were taken over by the larger companies and became a part of a larger and more extensive network.

The agricultural areas were greatly benefited by the extension of these transmission lines. California is essentially an agricultural state, having a wet season and a dry one. Its semi-arid climate necessitates irrigation during the summer months and in certain localities at the lower elevations reclamation pumping is necessary during the winter months. By installing step-down transformers at various points, electric power was made available for both of these uses.

Some indication of the extent to which electric power is being used for agricultural purposes may be obtained



INSTALLED PLANT CAPACITY
IN KILOWATTS AT UNITY POWER FACTOR

PACIFIC GAS AND ELECTRIC CO.

LEGEND ——

NO	POWER HOUSE	CAPACITY
1	ALTA	2.000
2	AMERICAN RIVER	5 625
3	BULLARD S BAR	8.125
4	CENTERVILLE	6 400
5	COAL CANYON	1 000
6	COLEMAN	15.000
7	COLGATE	15 575
8	COW CREEK	1 500
9	DEER CREEK	5 500
10	DE SABLA	13 000
11	DRUM	52.500
12	EL DORADO	25 000
13	ELECTRA	20.000
14	EUREKA. STA "B"	9 000
15	FOLSOM	3.750
16	HALSEY	12.500
17	HAT CREEK NO 1	12 500
18	HAT CREEK NO 2	12.500
19	INSKIP	6 000
20	JUNCTION CITY	2.500
21	KILARC	3.000
22	KING CITY	75
23	LIME SADDLE	2.000
24	MELONES	27 000
25	MELONES MINING CO	900
26	MONTEREY	1 000
27	NORTH BEACH	27 000
28	OAKLAND STA "C"	62,000
29	PHOENIX	1.875
30	PIT RIVER NO 1	70.000
31	PIT RIVER NO 3	81.000
32	SACRAMENTO STA "B"	17 500
33	SALINAS	300
34	SAN FRANCISCO. STA "A"	64 000
35	SOUTH	4 000
36	SPAULDING NO 1	8.000
37	SPAULDING NO 2	3 750
38	SPRING GAP	7 500
39	STANISLAUS	34.000
40	STOCKTON STA "B"	1 875
41	VOLTA	7 000
42	WISE	12.500
	TOTAL	666,250

GREAT WESTERN POWER COMPANY

LEGEND —+—+—+—

NO	POWER HOUSE	CAPACITY
43	BUSH	2,000
44	CARIBOU	66,600
45	LAS PLUMAS	65 000
46	NORTH BEACH	16.000
47	OAKLAND	10 500
48	PHELAN	1 500
	TOTAL	161 600

L. A. GAS AND ELECTRIC CORP.

LEGEND —+—+—+—

49	ALAMEDA ST	70,000
50	SEAL BEACH	35.000
	TOTAL	105.000

SOUTHERN CALIFORNIA EDISON CO.

LEGEND —+—+—+—+—

51	AZUSA	1,800
52	BIG CREEK NO 1	82,000
53	BIG CREEK NO 2	66,000
54	BIG CREEK NO 2A	80 000
55	BIG CREEK NO 3	86,000
56	BIG CREEK NO 8	24 000
57	BOREL	9,300
58	FONTANA (LEASED)	1 800
59	KAWEAH NO 1	1,450
60	KAWEAH NO 2	1,450
61	KAWEAH NO 3	3 800
62	KERN RIVER NO 1	22 000
63	KERN RIVER NO 3	34 500
64	LONG BEACH NO 1	70 000
65	LONG BEACH NO 2	145,000
66	LONG BEACH NO 3	90,000
67	LYTLE CREEK	600
68	MILL CREEK NO 1	600
69	MILL CREEK NO 2-3	3 000
70	REDONDO	40.000
71	SANTA ANA NO 1	3 500
72	SANTA ANA NO 2	1 200
73	SANTA ANA NO 3	1 500
74	SIERRA	800
75	TULE	2,300
76	VISALIA	6 600
	TOTAL	779 200

MODESTO AND TURLOCK IRRIGATION DISTRICTS

LEGEND —+—+—

77	LA GRANGE (T I D)	4 000
78	DON PEDRO (JOINT)	15 000
	TOTAL	19.000

SOUTHERN SIERRAS POWER CO.

LEGEND ——

NO	POWER HOUSE	CAPACITY
79	ADAMS AUXILIARY	2 700
80	ADAMS MAIN	5 000
81	BISHOP CREEK NO 2	5 800
82	BISHOP CREEK NO 3	8.400
83	BISHOP CREEK NO 4	5,900
84	BISHOP CREEK NO 5	3.375
85	BISHOP CREEK NO 6	1,800
86	BLYTHE	335
87	EL CENTRO	350
88	LEEVINING CK NO 3	2 000
89	MILL CREEK	3,000
90	RUSH CREEK	10,600
91	SAN GORGONIO NO 1*	1,500
92	SAN GORGONIO NO 2*	750
93	SAN BERNARDINO	8.000
94	THE POOLE PLANT	10 000
	*LEASED TOTAL	69 510

MERCED IRRIGATION DISTRICT

LEGEND ■

95	EXCHEQUER	31.250
	TOTAL	31.250

CITY OF PASADENA

LEGEND ■

96	PASADENA	34 500
	TOTAL	34.500

BUREAU OF POWER & LIGHT CITY OF LOS ANGELES

LEGEND —+—+—+—+—+—

97	BIG PINE NO 3	4.000
98	COTTONWOOD NO 1	1,875
99	DIVISION CREEK NO 1	162
100	DIVISION CREEK NO 2	750
101	FRANKLIN CANYON	2,500
102	HAIWEE	1,312
103	RIVER POWER	3,800
104	SAN FERNANDO	7,000
105	SAN FRANCISQUITO NO 1	47,656
106	SAN FRANCISQUITO NO 2	35.000
	TOTAL	103.655

SAN JOAQUIN LIGHT & POWER CORPORATION

LEGEND ——

107	BAKERSFIELD	26.550
108	BALCH	33,000
109	BETTERAVIA	2,500
110	CRANE VALLEY	1.000
111	CRANE VALLEY NO 1	16.000
112	CRANE VALLEY NO 1A	425
113	CRANE VALLEY NO 2	3.000
114	CRANE VALLEY NO 3	3,750
115	KERCKHOFF	42 600
116	KERN CANYON	10 600
117	KITTREDGE	375
118	MERCED FALLS	500
119	MIDWAY	25,000
120	MOUNTAIN KING	350
121	TULE	6.000
	TOTAL	171.650

THE CALIFORNIA OREGON PR. CO.

LEGEND —+—+—+—+—

NO	POWER HOUSE	CAPACITY
122	COPCO NO 1.	25,000
123	COPCO NO 2	30,000
124	FALL CREEK	2 750
125	HEADLIGHT	400
126	SHASTA RIVER	450
	TOTAL	58.600

SAN DIEGO CONS. G. & E. CO.

LEGEND —+—+—+—

127	STATION "A"	10,000
128	STATION "B"	36,000
	TOTAL	46 000

SNOW MT. WATER & POWER CO.

LEGEND —+—+—+—

129	POTTER VALLEY	6,800
	TOTAL	6.800

CITY AND COUNTY OF SAN FRANCISCO

LEGEND —+—+—+—

130	EARLY INTAKE	3,000
131	MOCCASIN CREEK	80,000
	TOTAL	83.000

UTICA MINING CO.

LEGEND ▣

132	UTICA NO 1.	1,900
133	UTICA NO 2	600
	TOTAL	2.500

COAST COUNTIES G. & E. CO.

LEGEND —+—+—+—+—

134	BIG CREEK	990
135	SANTA CRUZ	500
136	WATSONVILLE	750
	TOTAL	2,240

ONTARIO POWER CO.

LEGEND ■

137	SAN ANTONIO CREEK NO 1	750
138	SAN ANTONIO CREEK NO 2	550
139	SAN ANTONIO CREEK NO. 3	400
	TOTAL	1.700

ESCONDIDO MUTUAL WATER CO.

LEGEND ■

140	BEAR VALLEY	300
141	RINCON	300
	TOTAL	600

YUMA WATER USERS ASSOCIATION

LEGEND ■

142	DROP PLANT	1,560
	TOTAL	1.560

TRUCKEE RIVER POWER CO.

LEGEND —+—+—+—

143	FARAD	1.700
144	FLEISH (NEV.)	2.100
	TOTAL	3.800

FEATHER RIVER POWER CO.

LEGEND ■

145	BUCKS CREEK	50,000

LEGEND
■ WATER POWER PLANTS
□ STEAM PLANTS

FIG. 1

from the fact that the more than 35,000 irrigation pumping installations in this state represents a greater number than is to be found in all of the other states combined. But pumping is not the only use made of electric power by these agricultural communities. This is evidenced by the fact that of the almost 1,700,000 electric consumers in the state, approximately 300,000 or 20 per cent are outside of incorporated cities and towns. When it is considered that the agricultural business is the least remunerative of any that is served by the supplying companies, it will at once be apparent that the rural communities have benefited greatly by the fortunate fact that nature has placed them between the source of power supply in the eastern part of the state and the principal market in the western part.

While California is generally recognized as having pioneered the long distance transmission of power and pointed out the advantages of maintaining interconnections between systems supplying adjacent territory, the field of interconnection has not been limited to this state. On the contrary, it has been extended until it now includes Oregon, Washington, Nevada, Idaho, and British Columbia. Physical connections between systems operating in these various states are maintained and, if found desirable, they could all be connected into a common network. As a practical matter, however, this has never been attempted. The most extensive interconnection that is continuously in regular operation is that between the California-Oregon Power Company, supplying the central and southern parts of Oregon and the northern part of California, the Pacific Gas and Electric Company, supplying the northern and central parts of California, and the Truckee River Power Company supplying the western part of Nevada. Involved in this interconnection are 4760 mi. of high-voltage lines, 546,000 kw. of hydro generating capacity, and 183,000 kw. of steam generating capacity.

Although the diversity of load found on most large interconnected systems tends to materially improve the load factor of the system, nevertheless, from a strictly economic standpoint, it is still desirable, if best operating results are to be obtained, to maintain a ratio between hydro and steam capacities that will call for base load being carried on hydro and peak and emergency loads on steam. The ideal relationship between these two sources of power supply will depend upon a number of elements, among which the chief are, load factor, relative capital costs of steam and hydro installations, cost of fuel, etc. With a load that is continually growing and the economic advantages that come from the construction of larger generating plants, as compared with those of smaller capacity, the ideal allocation of load between hydro and steam to obtain maximum economy of operation is seldom, if ever, obtained.

From the standpoint of continuity of service, steam plants play a less important part today than they did a few years ago. Several factors have contributed toward bringing about this change. In the first place, as the number of hydro plants feeding into an interconnected network is increased, temporary cessations of operation of one or more generating stations whose output represents a comparatively small percentage of the total have but little effect on the general service of the system as a whole. A similar situation obtains in the case of transmission lines. However, a much more important factor in improving service in so far as transmission is concerned is due to the better quality and design of insulators that are now available and the better and more substantial physical construction used.

Some indication of the dependability of service from hydro sources may be had from the operating records of Pacific Gas and Electric Company for the year 1927. Climatic conditions in California during the past year were such as to afford an abundant water supply at all hydro plants. The steam plants located in the more important cities were floated on the line as a protection against interruptions to service that might occur in case of transmission line trouble. By operating in this manner the load carried on steam was reduced to a minimum. With a peak load of 439,568 kw. and an energy output of 2,380,000,000 kw-hr. less than 0.9 per cent of it was produced by steam.

Some of the more essential features that must be given consideration in connection with the operation of interconnected systems are:

1. Dependable communication,
2. Centralized authority and control over operations,
3. Location and isolation of trouble,
4. Voltage regulation and control over the entire network,
5. Frequency control.

One of the most important adjuncts in connection with the successful operation of any large interconnected network of transmission lines is dependable communication. This is particularly important in the case of the large and more important generating stations and distribution centers. Under normal operating conditions where the distances are not too great, satisfactory communication can be very readily provided by carrying the communication circuits on the same supporting structures that carry the power circuits. Unfortunately, however, with this type of construction, when trouble occurs on the power circuits, the inductive disturbances to the communication circuits are generally sufficient to make them inoperative when they are most needed. The use of such circuits has, therefore, been very largely limited to local and routine service rather than for handling the more important dispatching service.

Much more dependable communication can be had over circuits independent of and located a sufficient distance from the power line to be out of range of inductive interferences. Most of the larger power systems in the West have communication systems of their own independent of the power systems, or lease from the telephone companies, circuits for their own exclusive use.

Recent developments in carrier-current radio equipment give promise of providing a cheaper and more dependable means of communication between different stations on a transmission network than any other now available. This method of communication differs from ordinary broadcasting in many respects. First, the carrier current is superimposed on the power lines at a frequency below that used by regular broadcasting stations, thus avoiding any interference of one with the the other. Second, it can be installed on any voltage power line and is not materially influenced by switching, short circuits, or other transient disturbances on the power circuit. Third, in the case of long lines the first cost of the installation is less than that of a metallic circuit. Fourth, it has all of the flexibility of a metallic system.

The most recent installation of this kind is that on the Pacific Gas and Electric Company's 220- and 110-kv. lines running between the hydro plants on Pit River and the load dispatcher's office in Oakland, a distance of approximately 240 mi. Although couplers have not yet been installed around the transformers used for stepping the voltage down from 220,000 to 110,000 volts, communication between stations at the extreme ends of these lines is better and during the few months the system has been in operation the service has been more dependable than that over an independent metallic circuit running between the two places.

The success attending the operation of large interconnected networks is due in a very large measure to the fact that authority and control over the details of operation are centralized in the load dispatcher. In emergencies his authority is supreme. To successfully handle the important duties of this position calls for good judgment, resourcefulness, a thorough and complete detailed knowledge of the entire systems, but more important than anything else the ability to think quickly and act with decision when trouble occurs. So important is this position that service from an interconnected network may be good or bad depending on whether the dispatching is handled well or otherwise. Allocation of load among the various generating substations and over the different lines in a way that will make a maximum beneficial use of the more efficient plants or equipment conserve water or fuel and at the same time be prepared to meet any operating emergency that might arise, are some of the more important responsibilities of this position. Under normal operations, these conditions can be met without any great difficulty but once the regular operating routine is upset by storms, breakdowns, water shortage, or otherwise, the resourcefulness of the load dispatcher may be taxed to the limit in his efforts to maintain service.

Where generating stations and switching centers are separated by distances sometimes as great as several hundred miles, there is always the liability of partial or complete failure of the communication circuits connecting into the load dispatcher's office. To meet emergencies of this kind, dispatching orders are generally designated by code numbers. By referring to them in this manner, the time required to issue the order is reduced to a minimum. Also the liability of mistakes occurring where orders have to be repeated one or more times before reaching the station to which it is issued, is also minimized.

Localizing and isolating troubles that might be serious enough to interrupt or impair service over a considerable area is another big problem that comes up in connection with the successful operation of an interconnected network. At one time this was more or less a matter of guess work. Practically all switches were manually operated. Such instruments as were available were less reliable than those of today and were not designed to indicate the direction of trouble from where they were installed. Alert attendants, expert at guessing correctly the location of trouble and having the athletic ability to open the right switch in the shortest possible time, were therefore greatly in demand. Present day equipment has materially simplified this situation. Automatic overload, low-voltage, reverse current, time limit, reclosing, and other similar relays have almost entirely superseded manual operation in modern up-to-date installations. The almost uncanny dispatch and precision with which these devices function has brought about a greatly improved service and, by reducing man power, a much more economical service.

As an interconnected network grows and the amount of power delivered into it increases, the necessity of having dependable switches becomes of increasing importance. The construction of high-voltage switches has been materially improved during recent years, but there has been practically no change in the general design. Failure to keep pace with the development of other facilities used in connection with the production and transmission of high voltages has made them the weakest link in a power system.

The first attempt to build switches for operation at voltages as high as 60,000 was made in 1899 by the engineers of the Bay Counties Power Company, now a part of the Pacific Gas & Electric Company's system. The consolidation and interconnection of a number of smaller systems soon made it apparent that switching could no longer be done on the low voltage side of the transformers. The manufacturers had not yet developed switches for this voltage. To meet a situation precedent to the giving of satisfactory service from an interconnected network, it was necessary for the company to design and build its own high-voltage switches. The first attempts were necessarily crude. Entrance bushings suitable for operation at 60,000 volts could not be had. To further increase the electrical factor of safety, the conductors leading through these bushings into the switch contacts were heavily insulated with varnished cambric. Such small porcelain bushings as were available were mounted in well seasoned wood tops and the oil containers were ordinary fiber or pressed paper washtubs. The three legs of the switches were insulated from each other and from the ground.

Improvements in the design and manufacture of bushings have to a very considerable extent eliminated the inflammable material used in the original design and present day switches are much more rugged and dependable than those of 25 years ago. Although these improvements have added to the mechanical and electrical efficiency of the equipment, they have also increased their first cost to a point where, with the usual number of air switches for by-passing and cutting out the oil the high-voltage switch equipment in a modern generating plant represents an investment equal to 50 per cent of that in transformers.

No other piece of electrical equipment offers greater opportunity for improvement than do high-voltage switches. This is becoming more and more apparent as voltages increase. The very satisfactory operation of the two 220,000-volt lines in California has proved beyond question the practicability of going to even higher voltages just so soon as the economics of the situation warrant. What the future may bring forth in the way of improvements in high-voltage switches remains to be seen. Thus far the vacuum type switch has given more promise of relief than any other that has been proposed. Laboratory tests have shown that there is practically no deterioration or damage to the contacts after opening a 15,000-volt circuit carrying 120 amperes more than 500 times. If this type of switch can be developed to a point where it can be used on a commercial basis, there is every reason to believe that it will rapidly supersede the present oil type.

If electric utilities fulfill their obligations to serve the public, they must be able, ready, and willing to supply at proper rates any kind or character of load that may be offered them. Much of the load on a general power supply system will be of low power factor and unless a consumer is penalized because of the low power factor of his load or is offered some inducement in the form of reduced rates or otherwise to warrant his providing the necessary facilities for improving it there will be a considerable amount of wattless current on the system that will have to be taken care of somewhere.

This may be carried on one or more generating stations to suit operating conveniences or a much better plan, and one that becomes absolutely necessary as the length and voltage of the transmission lines increase, is to install synchronous condensers at or near the distribution centers of the system. The number and capacity of such installations will depend on the voltage of the system, the load characteristics, the permissible variation in voltage, etc., but whatever capacity may be needed to give proper service under normal operating conditions, it is very desirable, if satisfactory service is to be given under abnormal conditions, to have a reasonable margin of spare capacity to maintain adequate service in emergencies.

Condenser installations can be fully justified not only as a most satisfactory means of improving voltage regulation but they also have a tremendous economic value in that by improving the power factor, the amount of power that can be carried over a given line is materially increased.

Speed control on an interconnected system is a comparatively simple matter. Irrespective of the number, size, or location of generating stations, one station does the governing. Local conditions determine which one can best perform this function. Ordinarily the station near the load center, where a sufficient governing margin can be maintained at minimum cost, is selected. Where a hydro plant does the governing it is desirable that one having sufficient forebay capacity to supply the varying drafts necessary to meet load fluctuations be selected. Where the network is supplied from both hydro and steam sources, especially where the latter is operated as a standby or reserve source of power supply there is but little, if any, economy in governing with steam. Maximum economy will be obtained by loading all hydro plants except the one doing the governing, thus reducing the steam load to a minimum and keeping hot only enough boilers to pick up the predetermined amount of load that is to be thus protected against interruptions in case of trouble.

To prevent see-sawing of load between different plants, the governors on the governing plant are adjusted sensitive enough to operate on a minimum variation in speed. Those on the other plants are adjusted to operate only on the wider variations that would occur if and when for any reason they might become separated from the governing station.

On a reasonably large interconnected network of lines, carrying a highly diversified load, the instantaneous variations in load or what is ordinarily called the governing margin is comparatively small. In fact it is not at all unusual during short water periods to reduce this to practically zero and follow the gradual variations in load by hand control. On the system of Pacific Gas and Electric Company, with a peak of almost 440,000 kw-hr. supplied to an area larger than any other interconnected network in the West, the normal governing margin is between 1000 and 2000 kw.

Discussion

For discussion of this paper see page 408.

Interconnection and Power Development
in Chicago and the Middle West

BY H. B. GEAR[1]
Fellow A. I. E. E.

***Synopsis.**—The development of power stations in the Chicago District is described, showing how small stations have been replaced, and the energy supply for light, power, and traction consolidated by the use of high-voltage lines.*

The interconnections between stations and the developments of recent years and for the immediate future are outlined. This includes cables at 66,000 and 132,000 volts, with circuit capacities of 60,000 kv-a.

A station in central Illinois, near the coal supply, is being tied in with the Chicago District.

Interchange of energy between companies is conducted under an agreement, which is an important factor in the success of interconnection. Some of the operating problems are outlined briefly.

THE development of power producing equipment for the supply of electrical energy in utility service during the past quarter of a century has been greatly influenced by the development of electrical transmission of energy which has accompanied it.

The ability to send energy economically from a principal point of production to numerous points of distribution was the prime factor in a period of development in which many small pioneer stations were replaced by a few larger power stations, with a considerable gain in economy of construction and operation.

A sketch of the course of power station development in the Chicago District, which is typical of progress in general in this respect, may serve to illustrate some of the phases of this period of growth.

POWER DEVELOPMENT IN CHICAGO

At the time the pioneer turbine-driven units were being planned for installation at the Fisk Street Station in 1901-2 in the Chicago District, there were from 10 to 12 power stations supplying electricity, 12 power stations supplying trolley and elevated train service at 600 volts d-c., a total of 24 power stations generating electricity chiefly for use in a local area. Only two of these stations supplied power to areas outside the radius of direct economical distribution from the point of production, because transmission at higher voltages by cables was yet in its infancy.

The year 1903 marked the inauguration of service at the Fisk Street Station at a generator voltage of 9000, chosen to make it available for transmission to substations, from which local distribution was effected.

The former generating stations being centers of distribution naturally formed the nucleus of this development, and lines to these points formed the first interconnections, since the stations were continued in operation for peak-hour purposes for some years before being abandoned.

The production of energy at Fisk Street Station was attended by a material reduction in the investment per kilowatt of plant capacity, and this opened the way for the economical supply of energy for electric transportation service which followed.

Thus, when rehabilitation of the equipment of the surface car lines followed the granting of new traction franchises in 1907, the additional power required was purchased and delivery was made over transmission lines to converter substations, whence 600-volt energy could be economically distributed.

The former cable car service, which had required 10 separate power houses, was converted to electric cars and these sources of power were abandoned, throwing this load to the electricity supply stations.

Similar unification of power supply took place in the suburban areas.

Within 10 years from the time the first turbine station went into service, the total number of power stations in Chicago and its suburbs had been reduced from 24 electric and 10 cable power stations to not more than 9 or 10 stations in all. Of these, five were producing the major part of the energy, the others being retained for use during peak-hours only.

The merging of traction supply with general light and power load approximately doubled the load on the supply systems and made it possible to use turbo generator units much larger than would have been chosen if the power supply of the district had not been thus unified and consolidated.

This advantage was by no means small, as the total turbo generator capacity installed during the first decade of this development amounted to about 275,000 kw. in three generating stations, whereas the total capacity in the 24 electric stations in service in 1903 did not much exceed 100,000 kw.

Progressive increases in the capacity of units were made as rapidly as the manufacturers could work out the problems involved, and the following tabulation of the maximum sizes of units and the year in which they came into service is of interest:

Year first put into service		Maximum size of turbo generator
1903	Vertical	5,000 kw.
1906	"	8,000 "
1908	"	14,000 "
1912	"	20,000 "
1913	Horizontal	25,000 "
1915	"	30,000 "
1917	"	35,000 "

---

[1] Asst. to Vice-Pres. Commonwealth Edison Co.

Presented at the Winter Convention of the A. I. E. E. New York, N. Y., Feb. 13-17, 1928.

Development was retarded for a few years by the events of the World War, but active growth was resumed in 1919 and in the Chicago District, in the middle western states, and indeed throughout the United States, proceeded at an unprecedented rate.

The accelerated growth from 1919 on necessitated a complete readjustment of power station development programs in all large centers of population, and necessitated the choice of still larger units and station capacities in the Chicago District.

For example, the Calumet Station in South Chicago was planned in 1919, on a pre-war basis, for six 30,000-kw. units, with the expectation that this would care for the requirements of growth in that District for perhaps 10 years. But, as a matter of fact, it had to be developed to full capacity in the second year, after the first unit went into service; that is, the sixth and last unit went into service in about four years, after construction was started on the first unit.

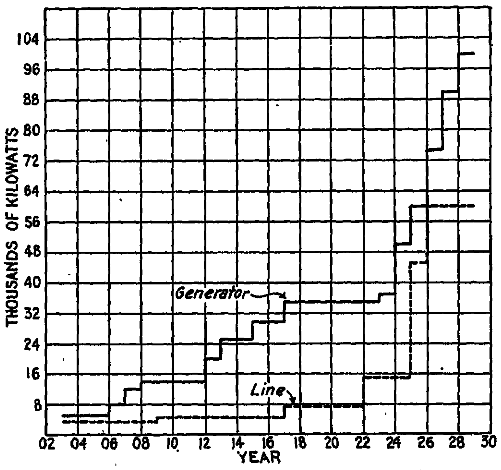


Fig. 1—Largest Unit in Service, Chicago District

The construction of the Crawford Avenue Station followed immediately upon the completion of the Calumet Station, and the first two units, of 50,000- and 60,000-kw. capacity respectively, went into service in the fall and winter of 1924-5.

Subsequent installations in service and under construction at Crawford Avenue Station follow: (Fig. 1)

1925 Unit No. 3 50,000 kw. rated capacity.
1926 Unit No. 4 75,000 kw. rated capacity.
1927 Unit No. 5 90,000 kw. rated capacity.
1928 Unit No. 6 100,000 kw. rated capacity.

The chart, Fig. 2, giving annual increments of maximum load in the City of Chicago, illustrates the demands for service which necessitated such a program of installation of power station capacity.

In the suburban area, outside the limits of the city of Chicago, a similar process of consolidation of generating capacity, and elimination of smaller and less efficient stations was in process from 1912 on.

To supply the northerly suburban area stations were constructed at Waukegan, some 30 mi. north of the Chicago city limits, at Blue Island, about two miles southwest of the city limits, and at Joliet, some 33 mi. southwest of the city. These stations supplied towns in the valley of the Illinois River and elsewhere in that section of the State, at 33,000 volts.

These lines served to interconnect the Blue Island and Joliet Stations, and, in later years, the Joliet Station was interconnected with the station at Peoria.

The Waukegan Station, being located on a large site where water from Lake Michigan is available for condensing purposes, has been developed as one of the larger stations of the interconnected Chicago group. It has a capacity of 110,000 kw., the third unit of 50,000-kw. capacity having been put into service in 1927.

Similar developments took place in other parts of Illinois and in the neighboring states.

Development of Interconnection

The use of interstation tie-lines has been in general practise since power station development reached a point where two stations under the same management were located in such proximity as to make interconnection practicable.

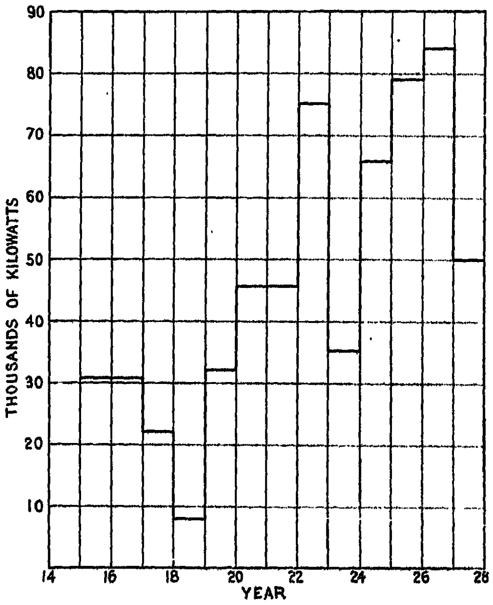


Fig. 2—Annual Increase System Maximum, Chicago

Such interstation lines were operated at the voltage of the generator. Later, the voltage used for transmission of the supply of energy to substations was more often employed.

Interconnections of this type were of relatively small capacity until the size of generator units became such that an excessive number of circuits was required to provide an adequate reserve for the larger units.

In the Chicago District such interconnections were established between the Fisk Street and Northwest

Stations, a distance of about eight miles, at 12,000 volts.

Other interconnections were made between stations in suburban districts at 33,000 volts, with overhead lines forming a part of the general transmission system.

These interstation lines were however, of minor importance when compared with the transmission capacities necessitated by the rapid growth of the past 10 years.

The Calumet Station was located at South Chicago, 13 mi. southeast of the Fisk Street Station, to provide a supply of energy for the great industrial district in that section of Illinois and Indiana. However, the unexpectedly rapid development of load made it necessary to transmit a considerable part of the energy produced there to the Fisk Street zone. This called for a group of lines having a capacity of the order of 100,000 kv-a., and, since these must be underground cables, it was very desirable that the capacity of individual cable units be as great as it was practicable to secure at that time from manufacturers of cables.

This was accomplished by the selection of a 33,000-volt, three-conductor cable unit, having a capacity of 15,000 kv-a., and these cables became the first link in a chain of interstation lines of high capacity, which have been installed since 1921 between all of the principal generating stations in the Chicago District.

Unfortunately the development of "ionization" phenomena interfered with operation of some of these cables at 33,000 volts after about a year, and they were reduced to 22,000 volts, with, of course, a corresponding reduction in capacity.

When the Crawford Avenue Station went into service, cables forming a part of the 22,000-volt system were converted into a tie-line between that station and Fisk Street, and other cables were laid to Northwest Station as well.

From Northwest Station a 33,000-volt cable of foreign manufacture was laid northward to Evanston to interconnect with 33,000-volt overhead lines of the Public Service Company of Northern Illinois. This formed an interconnection between Northwest Station and the Waukegan Station, some 30 mi. north of the Chicago city limits, and this cable has continued to operate at 33,000 volts without serious trouble.

In 1924 it became apparent that the use of a much higher voltage would be necessary for overhead tie-lines to secure the required line capacity, and the first of a system of steel tower lines on private right-of-way, operated at 132,000 volts, was constructed from Calumet Station to the Joliet Station, and to the Gary steel-making district in Northern Indiana. A little later this was extended to Michigan City, some 50 mi. east of the Chicago city limits.

Following the extension of this line to Michigan City, a connection was completed from that point to the Twin Branch Station of the Indiana and Michigan Electric Company, near South Bend, Indiana.

This was the first interconnection from the affiliated group of Chicago systems to one of outside ownership and is, at present, used to receive energy from Twin Branch for use in the Gary-Hammond District. It is also available as reserve to the Indiana and Michigan Electric Company when necessary.

From Twin Branch, other lines extend east to Ohio, and thence to West Virginia and Pennsylvania. It was through this interconnection that the paralleling of a group of systems reaching eastward to Massachusetts was accomplished (for a few minutes) some months ago.

Continuous parallel operation of the Chicago District with the Ohio system is fairly stable when more than 10,000 kv-a. is being transferred from one system to the other.

More recently, parallel operation was carried on for over an hour with the Ohio system, while it was tied in with lines extending through West Virginia, Tennessee, and Georgia to Pensacola, Florida. The fluctuations were not greater at the Chicago end than under normal operation with the Ohio system only.

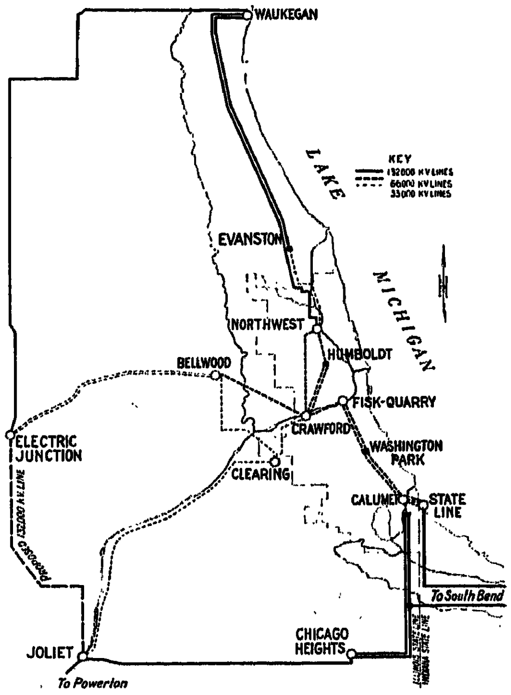


Fig. 3—Interconnection Chicago District, 1920

Sufficient experience has been had to make it apparent that when the weaker links have been increased to be comparable with the stronger, these systems can be operated in parallel successfully for such periods of time as may be necessary to meet the needs of operation during emergency transfers of energy.

Current Developments

In view of the rapid absorption of the ultimate capacity available at the Crawford Avenue site, where six of the 10 ultimate units will have been installed within five years, sites have been acquired at points on the shore of Lake Michigan for future power development (Fig. 3).

The first of these to be developed was Waukegan, 40 mi. north of the center of Chicago, and about midway between Chicago and Milwaukee, where three units, aggregating 110,000 kw., are now in service, and building has been provided for a fourth unit of 50,000 kw. rating.

This station is connected to Chicago by a 132,000-volt line, with a capacity of 60,000 kv-a., which is overhead from Waukegan to the Chicago city limits; thence to Northwest Station it is underground. This six-mile link of 132,000-volt cable is one of two pioneer installations put into service in 1927, the other being in the city of New York.

There is also a 132,000-volt overhead line, from Waukegan to Milwaukee, which acts as a reserve to the systems at either end.

Power developed at this site is used as needed in the neighboring territory and the northern part of the Chicago system, where the load exceeds the capacity of the Northwest Station. This latter station is dependent upon a branch of the Chicago River for condensing water, and its further development is limited by the sluggish flow of this stream when rainfall is below average.

Another site on Lake Michigan is at the Indiana-Illinois state line, on the Indiana side. The city of Chicago adjoins this property on the westerly side.

Most of the land on which the station is to be built has been created by hydraulic dredging operations.

The first unit of this state line station is under way and is to consist of a three-part unit, with high-and low-pressure elements so arranged as to be the operating equivalent of two units, aggregating 208,000-kw. capacity. This site will permit the installation of five units of this capacity, or a total in excess of 1,000,000 kv-a. The first unit is planned to go into service in 1929.

This station will be unique in the Chicago District, in that all of its energy will be transformed to higher voltages and sent to other zones for distribution and utilization.

Approximately half of the output of the first unit will be sent into the city of Chicago, and the remainder into northern Indiana and Illinois, outside of Chicago.

Energy sent into Chicago will be carried by underground cables having a capacity of 60,000 kv-a. per circuit at 66,000 volts.

Energy sent into the outlying territory will be carried by overhead lines at 132,000 volts, and to the neighboring cities of Hammond and East Chicago at 33,000 volts.

Within the city of Chicago, the use of 66,000-volt cables for transfer of large blocks of energy between stations was adopted in 1926, and several such lines have been in satisfactory service for more than a year between Crawford Avenue Station and the Fisk Street and Northwest Stations.

A new type of distribution center, receiving energy at 66,000 volts and distributing it at 12,000 volts in blocks of 4000 to 8000 kv-a., has been developed as a means of decentralizing the amounts of energy concentrated at generating stations on 12,000-volt busses.

These distribution centers will have an initial transformer capacity of 120,000 kv-a., and will distribute ultimately, about 200,000 kv-a. each. Being located midway between generating stations, they will effect economies in 12,000-volt cable which will offset other costs and result in a net saving in investment.

The first of these distribution centers, to be known as Washington Park, will come into service in 1928, and will be located midway between the Fisk Street and Calumet Stations on the south side of the city.

In Chicago and vicinity, the interchange of energy between power stations for the winter of 1928-9 at the peak hour is graphically depicted in Fig. 4.

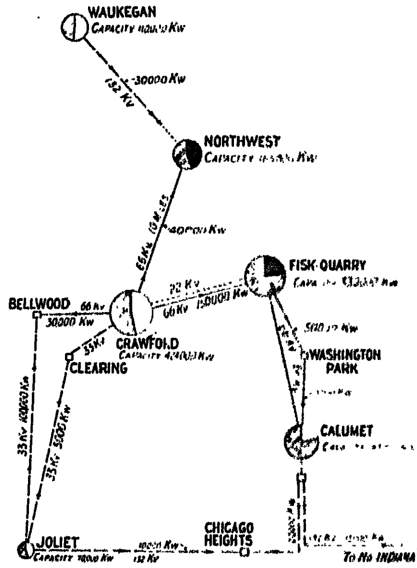


FIG. 4—POWER INTERCHANGE AT PEAK HOURS, CHICAGO DISTRICT, 1928

In this chart, the load in the station zone is indicated by the shaded portions. Where the zone load is greater than the station capacity, the overload is indicated by cross-hatching. The area of the circles is proportionate to the capacity of the respective stations.

Rights-of-way have been secured for overhead lines at 132,000 volts, connecting Joliet with Crawford Avenue Station, and for an outer belt line between Joliet and Waukegan, to be constructed as load conditions may make their installation advantageous.

Much discussion and calculation has centered about the idea that energy should be produced at points near to coal mines, and sent thence by high-voltage lines to the electricity market, instead of sending the fuel to stations located in the districts where the electrical energy is produced.

In the Chicago District, the major part of the fuel supply is brought from mines 200 mi. or more distant, and the problem of delivering hundreds of thousands of kw. of energy by overhead transmission lines at such distances is by no means simple.

Coupled with that problem is the one of finding a place where a supply of water for condensing purposes is present, as well as a supply of coal.

In Central Illinois, a location has been found which approximates this combination.

The Illinois River has a flow which is maintained in dry seasons largely because of the water supplied from Lake Michigan through the Chicago Drainage Canal. This river flows southwest from Joliet and passes to the west of the coal fields in central Illinois.

A site has been chosen on the Illinois River, about 14 mi. southwest of Peoria and some 70 mi. northwest of the coal fields, from which the major supply of coal used in Chicago power stations is taken. A direct railroad connection from mines to this site has been provided.

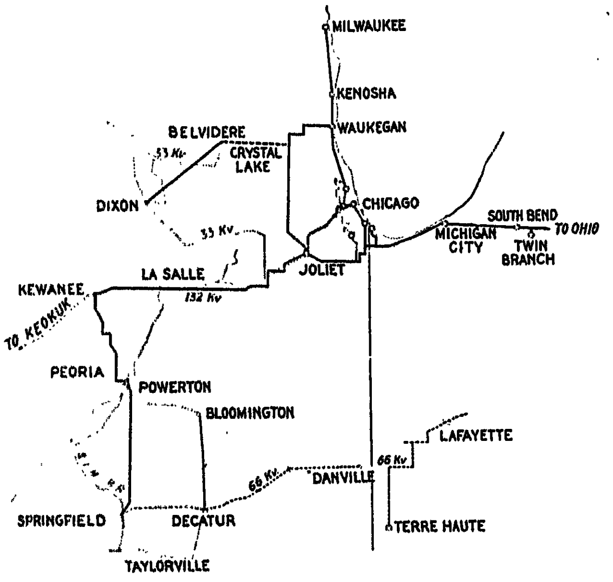


FIG. 5—POWERTON DISTRICT, 1928

This site has been named "Powerton," and is now being developed for two units of 55,000 kw. each, the first of which will go into service in the fall of 1928, (Fig. 5).

This station is to be operated by a separate company, whose business will be the generation of energy, which will be taken by the utilities serving the neighboring cities, such as Peoria, La Salle, Ottawa, Springfield, and Decatur. The remainder will be delivered to the Joliet District of the Public Service Company of Northern Illinois, and perhaps, at times, to the southwestern part of the city of Chicago.

The development of an economical station in this locality not only provides for future growth in this part of Illinois, but will effect large savings in fuel and operation cost, by producing energy now generated in smaller and, for the most part, much less efficient stations. Most of these stations will be shut down, except for the peak-hour shift.

The energy delivered to the cities north and east of Powerton will be transmitted by overhead lines at 132,000 volts.

Nearby cities will be supplied by connecting into the 33,000-volt system of lines now used for bulk supply in that part of the state, and through an overhead, 66,000-volt system to Springfield and cities easterly therefrom.

BASIS OF OPERATION

The use of reserve generating capacity and low-cost energy to the fullest practical extent is facilitated in the Chicago District by the provisions of a three-party interchange energy agreement, which pools reserve capacity and permits the flow of energy to neighboring zones whenever the load in the immediate zone permits; or when the energy so transferred costs less than that produced in the neighboring zone.

The difference between the total capacity in the pool and the coincident maximum load of the three systems is taken as the group reserve (until a higher maximum has been established, or additional capacity put into service).

The percentage of group reserve is taken as the standard and any company having a reserve percentage higher than the group percentage is compensated for the carrying charges on its excess station capacity, by the other company or companies whose percentage of reserve is less than that of the group. This permits the smaller companies to install larger units than would be possible if only the requirements of their individual systems were to be met, and thus gives them capacity for future growth at a lower investment cost, with higher operating efficiency.

This is not accomplished at the expense of the larger companies (except temporarily), as they are able to draw upon the excess capacity thus created and so defer, for a year or more, additions to their own plants.

The program of power station construction for the entire district is coordinated through a committee of two representatives from each company, which studies load increases in the various zones, and recommends to the executives the size and location of units to be added from year to year, their recommendation being made about two years in advance of the dates when the units are to come into service.

This committee also has a subcommittee of load dispatchers, which meets weekly to arrange operating schedules for the transfer of energy from the more efficient stations to other zones at off-peak hr., to arrange a coordinated program of turbine outages for overhauling purposes, and, in emergencies, to confer by telephone and arrange relief where it may be required.

The conditions, as they existed in one of the summer months of 1927, will serve to illustrate the way in which the interchange energy agreement is accomplishing the result contemplated by the parties thereto.

Company	A	B	C	Total kw.
System maximum.......	864,300	139,800	44,700	1,041,000
Reserve capacity, kw.....	101,245	97,980	2,075	201,300
Reserve quota (19%).....	165,875	26,240	8,585	201,300
Excess and *Deficiencies.	*64,630	71,740	*6,510	..

Company "A" paid Company "B" for 64,630 kw. of reserve capacity, and Company "C" paid Company "B" for 6510 kw. of reserve capacity. Thus, Company "B" was reimbursed for an excess of 71,740 kw. in its reserve capacity.

During the same month, Company "A" delivered to Company "B" 9,135,000 kw-hr., Company "B" delivered to Company "A" (in other zones) 16,936,000 kw-hr., and Company "A" delivered to Company "C" 9,172,000 kw-hr.

Of the total of 35,243,000 kw-hr. thus transferred, about 19,000,000 kw-hr. were delivered at off-peak hour, and resulted in a saving in fuel consumed of about $19,000.00 in that month.

The three systems are operated normally in parallel with each other and with the Twin Branch Station at South Bend, most of the time.

Parallel operation with the Milwaukee system is not normal, as this line is maintained, primarily, as a reserve connection for use in emergencies.

Interconnections with companies west and south of the Chicago District are also operated, for the most part, as reserve connections and are paralleled only as operating conditions require it. These lines are of small capacity and are not suitable for other than local reserve service.

SUMMARY OF RESULTS

The interconnection of power stations by lines of relatively large capacity has resulted as follows:

a. the size of generating units is increased, while the total capacity and reserve for a district is decreased;

b. the investment per kw. of capacity in generating stations and the total investment is decreased;

c. the consumption of fuel is reduced by ability to give the more economical units the "base load" of an entire district;

d. the reliability of service is enhanced by increased availability of the capacity and reserve in generating units, during emergencies and routine operating work, such as overhauling and repairing equipment.

PROBLEMS OF INTERCONNECTION

It has been apparent, from the foregoing recital of experience with plants in the Chicago District, that there are some problems which require solution before an interconnected group may realize the advantages sought: These problems relate, chiefly, to the ability to control the division of energy between neighboring stations and to make delivery continuously when occasion requires.

The experience in the Chicago District indicates that these problems lie outside of the generating equipment almost entirely, and relate to the size, voltage, length, and other characteristics of the interconnecting lines.

When line capacity is high, and lengths are short in proportion to the voltage of transmission, the transfer of energy between stations is readily accomplished by governor control of the generating units.

When lines are relatively long and have a capacity which is approached by the normal surges of synchronizing energy, which flow between the two sources of power, there is instability of operation, and lack of continuity is likely to result.

The presence of several successive transformations in a tie-line has the effect of increasing its impedance and is, thus, equivalent to an increase in length. More than two transformations (one each way) are to be avoided in such lines, if possible.

In longer lines and in lines where the direction of flow of energy is at different times in opposite directions, it is often necessary that means of voltage control be provided. These are required to permit the receiving station to operate its bus at a voltage level suited to the requirements of the circuits supplying energy within its own area of distribution.

Transformers with tap changers, adjustable under load, are finding a field of useful application in such situations.

At the higher voltages the use of synchronous condensers for voltage control are often found desirable through the ability to regulate the power factor of the system secured thereby.

These problems have been met and dealt with in those locations where long distances are involved, as in California hydroelectric systems, more than in the industrial centers of the Middle West; but continued development of longer lines in these states suggests the probability that the problems related to line characteristics will be an increasing factor in the developments of the next decade.

CONCLUSION

In reviewing the foregoing discussion, the author finds a predominance of illustrative data taken from experience in the Chicago District and Illinois, which raise in his mind the fear that the scanty reference to the interconnection work done in other important cities of the Middle West has left the picture incomplete. His apology is, therefore, offered with the explanation that had he been able in the time at his disposal to familiarize himself with the experience in such centers as Detroit, St. Louis, and Cincinnati, he would gladly have made use of it to illustrate some of the aspects of the relation of interconnection to power development.

In conclusion, it has been shown that interconnection and power development have already been well coordinated in some of the large centers of population.

The coordination of major points of power production by interconnections in a group of states is under way.

The next decade will see a continuing development of large power stations, joined by lines of high-voltage of great capacity, coordinating the supply of energy to great areas, as has been done in metropolitan centers of population.

Bibliography

INTERCONNECTION OF POWER SYSTEMS

B. VAN NESS, JR.
Associate, A. I. E. E.

1. "A Superpower System for the Region between Boston and Washington." W. S. Murray and others, U. S. Geological Survey *Professional Paper* No. 123, 1921.

2. "Evolution of Interconnected Power Lines and Effect on Utility Regulation." M. H. Aylesworth, *Electrical World*, Vol. 79, Feb. 18, 1922, pp. 325-327.

Growth of interconnection and regulatory commissions reviewed. State-wide, rather than municipal regulation is strongly advocated.

3. "Limitations of Interconnection." E. C. Stone, *Electrical World*, Vol. 77, April 30, 1921, pp. 990-991 (See also reference No. 42).

Rupturing capacity of oil circuit breakers, synchronizing power, and voltage regulation outlined as important considerations in interconnection problems.

4. "Interconnection, Superpower, and Electrification." J. Q. Hayes, *Electric Journal*, Vol. 21, Feb. 1924, pp. 72-77; March 1924, pp. 118-126; April 1924, pp. 166-176; May 1924, pp. 247-254.

The general aspects of the problem are presented together with data on a number of existing interconnections.

5. "The Economics of Interconnection." M. L. Sindeband, *Electrical World*, Vol. 85, January 3, 1925, pp. 25-26.

Careful economic study required before interconnection is attempted. Typical problems for investigation are outlined.

6. "The Interconnected Transmission System of California." W. G. Vincent, Jr., *Journal of Electricity*, Vol. 54, June 15, 1925, pp. 563-575.

Growth of the interconnected system in California is reviewed and outstanding engineering achievements cited.

7. "Operating a Fully Interconnected System." A. M. Perry, *Electrical World*, Vol. 86, Sept. 26, 1925, pp. 657-662; Oct. 3, 1925, pp. 693-696.

Operating methods in force on system of Consumers' Power Company; normally paralleled system is sectionalized during threatening conditions to limit area affected by disturbances.

8. "Loose Linking of Power Sources." Alex. Dow, *Electrical World*, Vol. 87, Jan. 16, 1926, pp. 139-141.

Method adopted by Detroit Edison Company to increase reliability and continuity of service. Interconnection between power areas is limited to minimize system disturbances and lessen oil circuit breaker duty.

9. "Interconnection Saves the South." *Electrical World*, Vol. 87, Jan. 9, 1926, pp. 91-94.

Service maintained during severe water shortage by interconnection of utilities. Attitude of executives toward permanent interconnection.

10. "Electric Power Development in the Chicago District." W. S. Monroe, *Power*, Vol. 63, Feb. 23, 1926, pp. 294-296.

Review of growth of Chicago regional interconnection with outline of proposed future developments.

11. "Handling a Power Exchange." *Electrical World*, Vol. 87, April 10, 1926, pp. 761-762.

Methods adopted by Connecticut Valley Interconnection for economical utilization of exchange power and plant capacity.

12. "Interconnection of Electric Power Systems." *Power Plant Engineer*, Vol. 30, May 15, 1926, pp. 595-601.

Description of major interconnections in the United States giving capacities and system layouts involved. The subject of operating problems is briefly treated.

13. "Interconnections in Virginia and North Carolina." W. C. Bell, *Mechanical Engineering*, Vol. 48, Oct. 1926, pp. 1036-1038.

Types of interconnection power exchange agreements in general use are described and their application to the operating interconnections in North Carolina and Virginia discussed.

14. "Problems Arising from the Expansion and Interconnection of Power Systems." F. C. Hanker, *Electric Journal*, Vol. 24, March 1927, pp. 96-100.

Engineering and operating problems involved in system interconnection. The status and importance of stability studies are discussed in the latter section of the paper.

15. "Interconnection of Power Systems Surrounding the Pittsburgh District." Geo. S. Humphrey, *Electric Journal*, Vol. 24, June 1927, pp. 251-258.

Operating conditions and problems in the Pennsylvania-Ohio interconnection are discussed. The Chicago-Boston interconnection described. Typical interchange agreements between utilities of the Northeastern United States are briefly reviewed.

16. *Equipment for 220 Kv. Systems*. J. P. Jollyman, JOURNAL A. I. E. E., Vol. 46, Sept. 1927, pp. 877-880; Vol. 47, Jan. 1928, p. 54, (Discussion).

Desirable characteristics of prime mover governors, generators, excitation systems, transformers, high voltage oil circuit breakers, transmission lines and substations for 220 kv. systems, also for extensive networks at lower voltages.

17. *Synchronous Condensers*. P. L. Alger, JOURNAL A. I. E. E., Vol. 46, Dec. 1927, pp. 1330-1339; Vol. 47, Jan. 1928, p. 54, (Discussion).

Economic speeds and voltages, ratio of lagging to leading kv-a. output, stability, starting, and improvements in design of synchronous condensers are discussed.

18. *High Voltage Oil Circuit Breakers for Transmission Networks*. R. Wilkins and E. A. Crellin, JOURNAL A. I. E. E., Vol. 46, Dec. 1927, pp. 1340-1348; Vol. 47, Jan. 1928, pp. 57-64, (Discussion).

Importance of speed in switching emphasized. Speed-time characteristics of 110 kv. and 220 kv. breakers illustrated by curves. Electrical and mechanical characteristics required of oil circuit breakers for use on interconnected networks outlined.

19. *The Chicago Regional Power System*. E. C. Williams, TRANS. A. I. E. E., Vol. 47, No. 1, 1928, p. 246.

Description of the interconnecting network in the Chicago region, and the interchange contract between the three largest companies of the interconnection.

20. *The Application of Relays for the Protection of Power System Interconnections*. L. N. Crichton and H. C. Graves, Jr., TRANS. A. I. E. E., Vol. 47, No. 1, 1928, p. 259.

Relay requirements for interconnected systems and new methods for protection against phase to phase and line to ground faults are discussed. A typical relay installation on a 220 kv. interconnection is described.

21. *The Conowingo Hydroelectric Project of the Philadelphia Company's System—With Particular Reference to Interconnection*. W. C. L. Eglin, see p. 372.

Important features of the Conowingo Project described. Relation between the Conowingo Project and the 220 kv. interconnection of the Philadelphia Electric Company, the Pennsylvania Power and Light Company, and Public Service Electric

and Gas Company discussed, particularly with reference to economies involved.

22. *Progress and Problems from Interconnection in the Southeastern States.* W. E. Mitchell, see p. 382.

Growth of utilities in the Southeast described. Importance of unified operating and financial control in economic system planning and operation is stressed. Operating methods and problems are outlined and attention called to particular problems as yet unsolved. Present trend in power plant, line, and substation design and construction is discussed.

23. *Some Aspects of Pacific Coast Interconnections.* P. M. Downing, see p. 393.

Conditions necessary for economical interconnections are outlined. Unified control and management necessary for most economical operation. Major facts to be considered in operation of interconnected systems are described. The Pacific Coast interconnection is used to illustrate these points.

24. *Interconnection and Power Development in Chicago and the Middle West.* H. B. Gear, see p. 399.

Growth of power development and interconnection in the Chicago District is described and current developments outlined. Operation under the three party exchange agreement is discussed. Interconnection problems requiring solution are cited.

Interconnection Through Frequency Changers

25. "Frequency Converter Ties Between Large Power Systems." J. W. Dodge, *General Electric Review*, Vol. 26, June 1923, pp. 435-444.

The characteristics and field of application of the following types of frequency changer sets are discussed; (1) synchronous, (2) induction, (3) speed regulating-induction (Sherbius), (4) induction-synchronous (systems electrically tied through windings of induction end), and (5) synchronous converter.

26. *A 35,000-kw. Induction Frequency Converter.* O. E. Shirley, Trans. A. I. E. E., Vol. 43, 1925, pp. 1011-1024.

Induction-synchronous set for New York System. Power interchange is possible in either direction. In addition to providing mechanical tie the set permits interchange of wattless between systems through windings of induction machine.

27. "35,000 Kw. Synchronous Frequency Changer Set." M. W. Smith, *Electric Journal*, Vol. 21, July 1924, pp. 343-346.

Description of straight synchronous set for Brooklyn Edison Company.

28. *The Use of Frequency Changers for Power System Interconnection.* H. R. Woodrow, Trans. A. I. E. E., Vol. 44, 1925, pp. 369-371.

Considerations determining selection of 35,000 kv-a. synchronous type set for interconnection of 25 and 60 cycle systems of Brooklyn Edison Company. The pull out point (70,000 kv-a.) is sufficiently high for the systems to hold together on loss of largest 25 cycle unit (30,000 kv-a.).

29. "A 49,000 Kv-a. Frequency Converter Set." R. A. McCarty, *Electric Journal*, Vol. 23, July 1926, pp. 339-341.

Large synchronous set for Commonwealth Edison Company.

Interconnection Through Transformers

30. *Voltage and Power Factor Control of 60,000 Volt Transmission Lines Connecting Two Generating Stations.* R. Bailey, Trans. A. I. E. E., Vol. 40, 1921, pp. 995-1015.

Installation of three-phase induction regulators in combination with step-up transformers for voltage and power-factor control of 60 kv. tie lines. Ability of this type of equipment to withstand severe short circuits.

31. *Transformers for Interconnecting High-Voltage Transmission Systems; for Feeding Synchronous Condensers from a Tertiary Winding.* J. F. Peters and M. E. Skinner, Trans. A. I. E. E., Vol. 40, 1921,, pp. 1181-1191.

Discussion of special problems encountered in use of star connected three winding transformers and auto-transformers with tertiaries for interconnection.

32. *Performance of Auto-Transformers with Tertiaries Under Short Circuit Conditions.* J. Mini, Jr., L. J. Moore, and Roy Wilkins, Trans. A. I. E. E., Vol. 42, 1923, pp. 1060-1068.

Tests on transformers with tertiaries described and results discussed. Capacity of tertiary windings is dependent on magnitude and duration of short circuit. The kv-a. capacity and reactance of tertiaries are interdependent.

33. *Theory of Three Circuit Transformers.* A. Boyajian, Trans. A. I. E. E., Vol. 43, 1924, pp. 508-529.

Characteristics of three winding transformers, including auto-transformers with tertiaries. Discussion of general theory, regulation, division of load, impedance for short circuits, behavior in parallel with a two-circuit transformer, and characteristics under unsymmetrical loads.

34. "Three Winding Transformers." J. F. Peters, *Electric Journal*, Vol. 22, Jan. 1925, pp. 12-16; Feb. 1925, pp. 71-77.

Practical discussion of three winding transformer theory, with illustrative calculation of typical problems.

35. *Transformer Tap Changing Under Load.* H. C. Albrecht, Trans. A. I. E. E., Vol. 44, 1925, pp. 581-586.

Comparison of the synchronous condenser, the transformer with load ratio control, and the induction regulator as a means of regulating voltage on interconnecting lines. Discussion of fields of application of the various methods, or combinations of methods.

36. "Tie-in Transformer Equipped with Load Ratio Control." M. H. Bates, *General Electric Review*, Vol. 28, Dec. 1925, pp. 823-826.

36,000-kv-a. three-phase auto-transformer for interconnection tie between Duquesne Light and West Penn Power Companies.

37. "Arrangements for Load Ratio Control." H. R. Wilson, *General Electric Review*, Vol. 29, May 1926, pp. 335-338.

Description of load ratio control arrangements employing parallel windings with ratio adjusters, and combinations of single or parallel windings with auxiliary regulating transformers of different types.

38. "Two 60,000 Kv-a. Load Ratio Control Transformer Banks for Philadelphia Electric Company." A. Palme and H. O. Stevens, *General Electric Review*, Vol. 29, Sept. 1926, pp. 634-638.

Combination of standard power transformers and three-phase regulating transformer equipments. Method is adaptable to existing installations where load ratio control is required.

39. "60,000 Kv-a. Transformer Banks Equipped for Change of Ratio Under Load." W. M. Dunn, *Electric Journal*, Vol. 23, August 1926, pp. 398-402.

Parallel winding type load ratio banks for interconnection between Philadelphia Electric Company, and Public Service Company of New Jersey.

40 (a). *Application and Design of Load Ratio Control Equipment.* A. Palme, Journal A. I. E. E., Vol. 46, Nov. 1927, pp. 1202-1206; 1269-1273, (Discussion).

40 (b). *Transformer Tap Changing Under Load.* L. H. Hill, Journal A. I. E. E., Vol. 46, Nov. 1927, pp. 1212-1221; pp. 1269-1273, (Discussion).

Applications for load ratio transformers are discussed in these two papers. Parallel winding, single winding, and regulating transformer types of load ratio control are described and discussed.

41. *Characteristics of Interconnected Power Systems as Affected by Transformer Load Ratio Control.* L. F. Blume, Trans. A. I. E. E., Vol. 46, 1927, p. 590.

Comparison of voltage control by generator field with that by combination of generator field plus transformer load ratio control. Comparison of synchronous condensers and load ratio control for voltage regulation. Voltage control in transmission line loops.

STABILITY OF POWER SYSTEMS

42. *Some Problems in the Operation of Power Plants in Parallel.* E. C. Stone, TRANS. A. I. E. E., Vol. 38, Part 2, 1919, pp. 1651-1674.

Characteristics and capacity of transmission circuits for interconnecting power systems are discussed. Calculation of synchronizing power, the division of wattless, and means for eliminating excess circulating wattless current are described.

43. *Power Control and Stability of Electric Generating Stations.* C. P. Steinmetz, TRANS. A. I. E. E., Vol. 39, part 2, 1920, pp. 1215-1287; *General Electric Review*, Vol. 23, Aug. 1920, pp. 688-701, Sept. 1920, pp. 756-768.

Unstable condition on system of Commonwealth Edison Company in 1919 is analyzed. Effect of sectionalizing reactors on flow of synchronizing power and stability of the system. Formulas developed for calculation of synchronizing power transfer, low frequency power fluctuations and energy transfer, and armature and field transients in synchronous machines.

44. "Calculation of Synchronizing Power." E. S. Henningsen, *General Electric Review*, Vol. 24, Nov. 1921, pp. 951-956.

Defines synchronizing power and develops formulas for its calculation under following conditions; (1) constant bus potential, (2) constant magnetic flux in generators, and (3) constant field excitation.

45 (a). *Some Theoretical Considerations of Power Transmission.* C. L. Fortescue and C. F. Wagner, TRANS. A. I. E. E., Vol. 43, 1924, pp. 16-23; 71-103.

45 (b). *Power Limitations of Transmission Systems.* R. D. Evans and H. K. Sels, TRANS. A. I. E. E., Vol. 43, 1924, pp. 26-38; 71-103, (Discussion).

45 (c). *Limitations of Output of a Power System Involving Long Transmission Lines.* E. B. Shand, TRANS. A. I. E. E., Vol. 43, 1924, pp. 59-70; 71-103, (Discussion).

The preceding three papers present the results of analytical investigations of the power system stability problem. The applications of general circuit constants and the circle diagram to the problem is outlined and discussed.

46. "Analytical Solution of Networks." R. D. Evans, *Electric Journal*, Vol. 21, April 1925, pp. 149-154; May 1925, pp. 207-213.

Short circuit network calculations including determination of current in branches of power networks outlined. Discussion is paralleled by typical calculations.

47. *Power System Transients.* V. Bush and R. D. Booth, TRANS. A. I. E. E., Vol. 44, 1925, pp. 80-103.

Presentation of an analytical step-by-step method for calculation of power system stability under transient conditions.

48. *Fundamental Considerations of Power Limits of Transmission Systems.* R. E. Doherty and H. H. Dewey, TRANS. A. I. E. E., Vol. 44, 1925, pp. 972-984; 994-1003, (Discussion).

General discussion of the factors entering into the problem of power system stability.

49. *Transmission Stability.* C. L. Fortescue, TRANS. A. I. E. E., Vol. 44, 1925, pp. 984-994; 994-1003, (Discussion).

Behavior of a simple transmission system during steady load conditions, switching operations, sudden increase in load, and short circuits discussed and illustrated by angle diagrams. Presentation of problems of transient and static stability.

50. *Stored Mechanical Energy in Transmission Systems.* J. P. Jollyman, TRANS. A. I. E. E., Vol. 44. 1925, pp. 935-937.

Performance of stored mechanical energy in synchronous machinery connected to a transmission system during sudden changes in system conditions. A tabulation of typical flywheel effects is included.

51. *Calculation of Steady State Stability in Transmission Lines.* Edith Clark, TRANS. A. I. E. E., Vol. 45, 1926, pp. 22-41; 80-94, (Discussion).

Methods of calculating steady state stability by (1) the equivalent circuit, (2) the circle diagram, are presented with formulas and illustrative calculations.

52 (a). *Practical Aspects of System Stability.* Roy Wilkins, TRANS. A. I. E. E., Vol. 45, 1926, pp. 41-50; 80-94 (Discussion) and

52 (b). *Further Studies of Transmission Stability.* R. D. Evans and C. F. Wagner, TRANS. A. I. E. E., Vol. 45, 1926, pp. 50-80; 80-94, (Discussion).

Description of tests on the system of the Pacific Gas and Electric Company, with a comparison of calculated and test values obtained. The second paper presents, in addition, an analytical method for calculation of power system transients.

53. "Calculation of Single Phase Short Circuits by the Method of Symmetrical Components." A. P. Mackerass, *General Electric Review*, Vol. 29, April 1926, pp. 218-231; July 1926, pp. 468-481.

Discussion of method of symmetrical components for calculation of single-phase short circuits on three-phase networks, with illustrative examples.

54 (a). *Experimental Analysis of Stability and Power Limitations.* R. D. Evans and R. C. Bergvall, TRANS. A. I. E. E., Vol. 43, 1924, pp. 39-58; 71-103, (Discussion).

54 (b). *An Investigation of Transmission System Power Limits.* C. A. Nickle and F. L. Lawton, TRANS. A. I. E. E., Vol. 45, 1926, pp. 1-21; 80-94, (Discussion).

Experimental shop tests with commercial machines on miniature networks. Discussion of stability problem together with comparison of calculated and test results.

55. "Stability of Alternating Current Turbine Generators with Fluctuating Loads." John Strasser, *Electric Journal*, Vol. 23, Aug. 1926, pp. 413-420.

Development of a method for the determination of the stability of turbine generators at instantaneous overloads.

56. *Stability Characteristics of Alternators.* O. E. Shirley, TRANS. A. I. E. E., Vol. 45, 1926, pp. 1108-1115.

Alternator characteristics with (1) constant power output, (2) variable impedance, and (3) constant impedance, loads are discussed. A formula and series of curves for specifying limiting short circuit ratios of general purpose alternators are proposed.

57. "A Mechanical Analogy to the Problem of Transmission Stability." S. B. Griscomb, *Electric Journal*, Vol. 23, May 1926, pp. 230-235; see also Westinghouse Electric and Manufacturing Company Reprint 282.

Behavior of a typical power system under stable and unstable conditions as illustrated by a mechanical model.

58. "Review of Papers on Stability of Power Systems." Appendix to Westinghouse Electric & Manufacturing Company's Reprint 282.

Brief statement of scope of each of a selected list of twenty-one papers dealing with stability of power systems together with a very complete bibliography on this subject to January, 1927.

59. *Transients Due to Short Circuits.* R. J. C. Wood, L. F. Hunt, and S. C. Griscomb, TRANS. A. I. E. E., Vol. 46, No. 1, 1927, p. 68.

Results of stability investigation on system of California Edison Company 1925-1927. Majority of short circuits found to be single phase to ground. Factors affecting stability on this system (switch operation, fault reactance, fault resistance, load, etc.) are discussed.

60. *Static Stability Limits and the Intermediate Condenser Station.* C. F. Wagner and R. D. Evans, TRANS. A. I. E. E., Vol. 47, No.1, 1928, p. 94, (Discussion).

A method is developed for determining static stability of transmission systems with three or four synchronous sources. A special application is the stability problem with one or two intermediate synchronous condenser stations.

61 (a). "Stability Characteristics of Machines." R. D.

Evans and C. F. Wagner, *Electrical World*, Vol. 89, Jan. 15, 1927, pp. 141-143.

61 (b). "Quick Response Excitation for Alternating Current Synchronous Machinery." A. C. Powel, *Electric Journal*, Vol. 24, April 1927, pp. 157-162.

61 (c). "High Speed Excitation for Stability." *Electrical World*, Vol. 89, May 21, 1927, pp. 1061-1063.

These three articles discuss advantages of (1) low generator reactance in reducing system phase angle, (2) quick response excitation systems (including special exciters, regulators, and positive sequence networks). The effect of special design characteristics on system stability is treated.

62. "Super Excitation." D. M. Jones, *General Electric Review*, Vol. 30, Dec. 1927, pp. 580-582.

Discussion of a special form of high speed excitation. Applied to synchronous condensers it offers considerable increase to the stability limits of a power system. Comparison of super excitation, quick response excitation, and standard excitation.

63. "Super Excitation to Protect Conowingo System." *Power*, Vol. 67, Jan. 31, 1928, p. 272.

Brief description of special excitation for terminal condenser equipment on Conowingo system.

64. "Electrical Characteristics of Transmission Circuits." William Nesbit, Westinghouse Technical Night School Press.

Very complete reference book of transmission line calculations.

VOLTAGE STANDARDIZATION

65 (a). *Voltage Standardization, From a Consulting Engineer's Point of View.* R. E. Argersinger, TRANS. A. I. E. E., Vol. 46, 1927, p. 172, (Discussion).

65 (b). *Voltage Standardization, Its Relation to the Interconnecting Power Companies of the Southeast.* H. J. Scholz, W. W. Eberhardt, and S. M. Jones, TRANS. A. I. E. E., Vol. 46, 1927, p. 187, (Discussion).

65 (c). *Standardization of Voltage Ratings for Power Systems and Equipment.* A. E. Silver and A. L. Harding, TRANS. A. I. E. E., Vol. 46, 1927, p. 175, (Discussion).

65 (d). *Voltage Standardization of A. C. Systems.* F. C. Hanker and H. R. Summerhayes, TRANS. A. I. E. E., Vol. 46, 1927, p. 161, (Discussion).

The four papers discuss voltage standardization from the viewpoint of the consulting engineer, operating engineer and manufacturer.

Discussion

PAPERS ON INTERCONNECTION AND ITS EFFECT ON POWER DEVELOPMENT

EGLIN, MITCHELL, DOWNING, GEAR

C. L. Edgar: Inasmuch as none of the papers in this symposium touches upon New England, it may, perhaps, be appropriate for me to tell you, in a few words, what the situation is in that part of the United States.

As you will see by the map in the accompanying Fig. 1, interconnection has been carried on in almost every center in the New England States. Owing to the restrictive export laws of the State of Maine, there is no transmission line crossing the Maine-New Hampshire border, but in the other five states there is scarcely a center of distribution which is not connected with some other center, either in the same or in one of the neighboring states.

I have looked over the latest map of the interconnection systems of the United States and have compared the density of these lines in New England with the remainder of the country. This comparison shows that, with the exception of Maine, the density is as great in the other five New England states as in the leading eastern states of New York, New Jersey, and Pennsylvania; perhaps not quite as dense as those centering around Chicago, but considerably more so than in any other part of the United States. Distances, however, are short and interconnection has not attracted as much attention with us because of this fact as in some of the more scattered centers of the rest of the country. So much for the situation in New England. I am very much impressed with the papers in this symposium. They cover reduction in plant capacity, reserve capacity, sizes of generating units, reduction in investments, differences in peak hours, and in fact all the various problems which have to do with deciding whether the time is ripe for interconnections and if so, how they should be brought about.

They give considerable attention to what, in my mind, is one of the most important factors in connection with this whole subject, and that is the necessity of maintaining an adequate control over the operation of an interconnected group in order to secure maximum results. This is, perhaps, merely another way of saying that the load-dispatching system of an interconnected group is the heart of the whole question, and it must be solved not only from the engineering but from the executive standpoint on very broad lines.

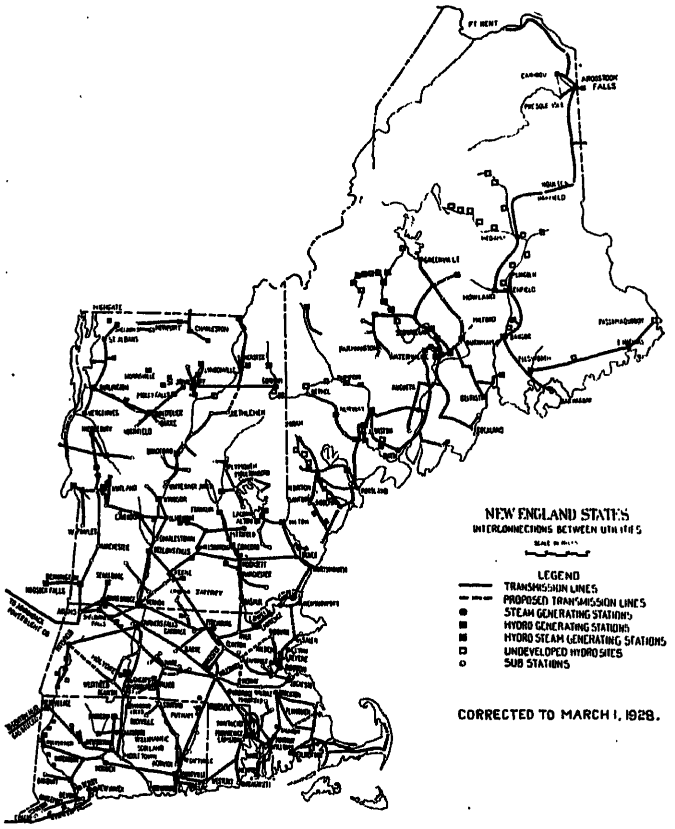

NEW ENGLAND STATES
INTERCONNECTIONS BETWEEN UTILITIES
SCALE IN MILES
LEGEND
TRANSMISSION LINES
PROPOSED TRANSMISSION LINES
STEAM GENERATING STATIONS
HYDRO GENERATING STATIONS
HYDRO STEAM GENERATING STATIONS
UNDEVELOPED HYDRO SITES
SUB STATIONS
CORRECTED TO MARCH 1, 1928.

FIG. 1

They also discuss some of the difficulties arising from improper, inadequate, or unnecessary interconnection. It does not seem to me that they stress sufficiently the inadequacy of some existing interconnecting lines. Some engineers seem to assume that if two systems are connected together, even by what is commonly called a shoestring, they have accomplished their purpose. They are, in fact, going in the right direction, but have only made a poor beginning. The writers especially emphasize the interruptions caused by lightning troubles and improper insulators, and other questions of this same general character.

One, at least, of the papers also refers to the effect which the introduction of high-tension underground cables is likely to have upon the entire interconnection problem. In fact, the writers seem to have covered all phases of the question.

A year or two ago it was my privilege to present the problem of interconnection, especially in New England, before a body not at all connected with engineering or with public utility work. I tried to explain what interconnection really meant in terms or in a language with which they were familiar. It may not be out of place, although I am talking to engineers, to make the same sort of comparisons that I did at that time.

I started out with the statement that interconnection was no new problem. As far back as 1892, the three generating stations of the Boston Edison Company were connected together so that the one which was located on the water front, and therefore had condensing engines, could relieve the other two, which were located on interior streets and were running non-condensing. This was as much true interconnection as that which is shown in Fig. 2 and which has recently been given considerable space in the public press. Electricity was started in Boston and carried through to Chicago on the one hand and down into Florida on the other. I do not mean to imply by this statement that any current was actually carried, but this was the general interpretation that was put on it by the newspaper articles. What actually happened, of course, was that the three centers were connected together electrically, that the systems were all in phase, and that current actually flowed between all these points. The same thing happened when Boston and Chicago were connected together a year or two ago.

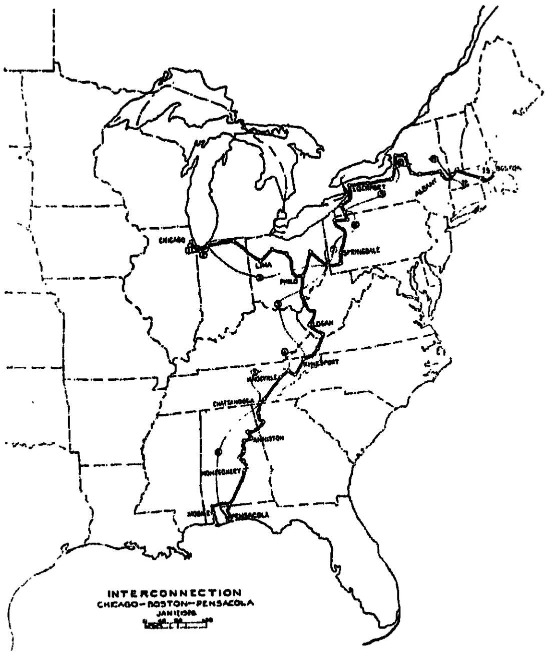


FIG. 2

In trying to bring out the fact that interconnection was no new thing and that it was developing along absolutely normal lines, I used two illustrations of similar developments in our industrial or rather in our transportation life.

The first of these was the federal or state construction of highways, and the second was the development of the privately owned steam railroads of the United States. Using the Lincoln Highway as an illustration of highway development, let us study the methods by which it is being brought about.

Of course the title "Lincoln Highway" has kept this particular development in the public eye. If it had been designed merely as a method of connecting up existing systems of roads so that a main thoroughfare thus would be created between New York and San Francisco, it would, as a matter of fact, have attracted very little attention. The name, however, caught the public eye, and today, or if not today at some time in the near future, one can travel from coast to coast seeing the Lincoln Highway sign throughout the entire distance.

As a matter of fact, the road from New York to Philadelphia was in existence long before the name "Lincoln Highway" was thought of. Some changes have been made, corners have been cut off, roads have been straightened, but these all would have come about in due course in any event. What is true between New York and Philadelphia is true between many other cities on the line of the Lincoln Highway. There were, of course, certain districts that were covered with automobile roads which were separated by many miles from other districts covered in the same way. These districts, where they came close to one another, were connected by a first-class road, and the Lincoln Highway, therefore, came into existence between these centers. Whereas the open gaps were small in the East, they became larger as one traveled westward, and even now some gaps are not yet filled. The Lincoln Highway, when completed, will be nothing more than an interconnection system, enabling the citizens of one city to travel to neighboring cities, and citizens of those, in turn, to travel to other cities, and so on ad infinitum. The problems which have arisen in the development of this transportation system have their exact parallel in the development of our interconnecting systems.

Another illustration: The steam roads of our country, a half century ago, were in a situation similar to that of the automobile roads of fifteen years ago, or to our power line situation as it is today. The interconnection in steam roads has been practically completed, whereas the automobile interconnections are in process, and the power-line interconnections are progressing to almost the same extent.

Referring to the railroad analogy, the Pullman Company, by an arrangement with all the roads, has enabled passengers to go from New York to Chicago, or from Chicago to San Francisco, without change. It is merely an interconnecting system, providing an interchange of passengers from one community to another with ease.

If the Pullman Company did not exist and there were no company doing their sort of business, the effect upon the railroad companies' investments would be tremendous. If the Pennsylvania Railroad, for example, had to own all the parlor, sleeping, and dining cars required to take care of the peak of its business, and every railroad in the country had to do the same thing, the number of cars of this character necessary to be owned by all the railroads would be far in excess of the number now owned by the Pullman Company. If, in order to preclude some of this tremendous investment, an arrangement were made between these companies for borrowing one another's equipment, it would cut down some of this investment, but it would not nearly reach that which now exists as a result of the Pullman Company's existence.

For example, the New Haven Railroad, on the day of the Harvard-Yale race at New London, or of the football game, either at Cambridge or New Haven, has to call upon the Pullman Company for a number of parlor cars far in excess of what the New Haven Company would own if it had to supply its own equipment. Possibly it might borrow a few from the Pennsylvania on the one hand and from the Boston & Maine or the Boston & Albany on the other, but if these cars did not happen to be idle or located at the proper terminal, they could not be depended on with any degree of certainty. These things are well known to all travelers.

Here again the parallel between railroad interconnection and power-line interconnection is very clear. The Pullman Company in the railroad world is equivalent to a reserve in the power industry which can be shifted from one section to another as the needs for it arise. For the same reason that interconnection among the railroads has enabled the products of one district to be easily transported to another and thus to be of advantage to both districts, the construction of these interconnecting power lines is enabling water powers, on the one hand, and steam powers on the other, to work hand-in-hand. Until very recently steam powers could not avail themselves of the advantage of cheap water power for their base load, neither could water powers gain the advantage of steam connections for reliability. This development, therefore, has marked a new era in combining the advantages of steam and water power into one enterprise.

The conclusion that I want to draw from all this is that the highways for automobile traffic and the power lines of the country are all working in the direction of saturation, a saturation which was achieved by the steam railroads many years ago. The electric power companies and electric interconnections are going through exactly the same program that the steam roads went through when they were in process of building.

Interconnection is just as inevitable in electric circuits as it is in the world of transportation. The various steps by which this will be brought about are just as unknown as they were in the railroad world a quarter or a half century ago. All that we can say is that they are bound to come if experience in other lines is any criterion.

If a word of warning is needed, it would be well to bear in mind that there are many cases in existence where railroads have built unwisely and unnecessarily. Possibly when the construction was undertaken, the needs were apparent, but in the light of later years the need has disappeared. This is also true, possibly to a lesser degree, in highway construction.

We must realize that interconnection in itself is not a panacea for all ills. It must be used intelligently, having in mind not only the light of today, but the probabilities of tomorrow. It is the preparation and presentation of papers like these which will do more than any other one thing to prevent the mistakes of the future.

A. C. Marshall: In Mr. Eglin's paper it is pointed out that the advantages of interconnection arise from diversity of one kind or another, which leads to a consideration of the many kinds of diversity which exist, some of which are often overlooked.

There may be diversity as to the shape of the daily load curve, or diversity as to the annual load curve. There may be diversity not only as to the annual load curve but also as to the annual variations in available capacity, such as changes in water-power capacity in wet and dry seasons. There may be diversity due to latitude and longitude, and there should be, of course, the artificially made diversity as to the program of construction in the two systems. There may be diversity in the character of population—in their habits of living, working, and play. There may be diversity in the character of industry which would affect the amount of electricity used and the time in which it is used.

It would be hard to justify the expense of interconnecting two systems that were identical in all these respects. As a matter of fact, no two such systems could exist, because if the systems were large enough to qualify under this discussion there would necessarily be a difference in geographical location, involving at least a small difference in time.

It is not necessary to explain here how these various brands of diversity get in their work. Some of them are inherent in the nature of things, but some of them are more or less under control, A change in the prevailing working hours of industry is to a certain extent in the latter class. It is doubtful if the periodic spring and fall changes of the clock to provide more afternoon daylight have had any appreciable effect on the daily load curves, especially considering the fact that such changes are made in the summer season when the daily peak is not in evidence.

There are, however, districts where permanent changes in the clock have been made which being effective the year round have changed the load characteristics of some public utilities.

When the railroads first adopted standard time in the early eighties the country was divided into zones, the time to be uniform over each zone and the time of each zone to differ from that of the next by one hour. The zones were laid out so that the time for each zone was the correct sun time for the meridian running through approximately the center of such zone. This was a natural arrangement, but in view of our present appreciation of afternoon daylight we would probably do it differently now. As the arrangement was made, the people living along the eastern edge of each zone were deprived of about one-half hour of daylight in the late afternoon. This was not noticed particularly for some time although the movement for daylight saving started some years before the World War. The War gave it a big boost and it has since become very popular.

If the standard time for each zone had been fixed as the sun time of the meridian running through the eastern edge of each zone, no one would have been deprived of any afternoon daylight although some would have had more than others. To illustrate, if the Central Time zone had been made as it is, but if the time had been fixed as that of the 82nd meridian instead of the 90th, and the time of the eastern time zone had been fixed as that of the 68th meridian instead of the 75th, we think now everyone would have been happier. All of which leads up to the situation in Detroit and eastern Michigan.

According to the original standard time layout Detroit was put on Central time, that is, 90th meridian time, being on the extreme eastern edge of that zone. Windsor, across the river in Canada, was running on Eastern, or 75th meridian time. This arrangement deprived Detroit of one-half hour of afternoon daylight, compared with true sun time, but of a full hour of afternoon daylight, compared with Windsor. This was the reason why Detroit people were so reluctant to change and why many of them persisted in keeping their own clock set for the old sun time for a long while.

After some years an agitation began for adopting eastern time for legal time in Detroit. After much discussion and several votes the change was made and Detroit officially and legally adopted Eastern time, although the railroads still published their timetables with Central-time figures. It took another long campaign of education before the Interstate Commerce Commission officially told the railroads to operate in and out of Detroit on Eastern Standard time. The people of Detroit liked the change so much that it soon became the fashion in the State to make similar changes and today almost all of the eastern half of the state runs on Eastern Standard time.

The effect on the daily load curve of the Detroit Edison Company has been marked. The winter afternoon peak has disappeared. All the big industries in Detroit operate on an 8-hr. day and this, together with the change in time, results in the afternoon power load's going off, even in the middle of winter,—before any afternoon lighting, to amount to anything, comes on. The peak load of the year usually occurs in the forenoon and may occur in almost any month of the year. It has occurred in the past few years in June, December, November, September, October, and February. The change may have resulted in the loss of some kw-hr. of lighting in the afternoons, but a small gain in kilowatt-hours no doubt occurs in the morning due to the later sunrise, by the clock. On the whole the saving in capital charges due to the absence of a sharp afternoon peak more than offsets the loss of a few kilowatt-hours.

There has just been completed an interconnection line between the Detroit Edison Company system and that of the Consumers' Power Company. The average day load of the Detroit Edison Company is now about 450,000 kw., while that of the Consumers' Power Company is somewhat less than this. This is an interconnection where a great many of the varieties of diversity are absent. The general load characteristics of the two systems are about the same. Both companies operate in a state formerly agricultural, but rapidly becoming industrial. The daily load curves of the two companies look very much alike. However, the expense of the interconnection was justified in the opinion of both companies by the ability to coordinate the construction programs of the two companies and by the diversity brought in by the large amount of water-power capacity in the Consumers Power Company's field.

This line consists of a single circuit of No. 0000 copper wire on steel towers, spaced about 800 ft. apart. The line operates at 132,000 volts. The line connects with the Consumers' Power Company's 140,000 volt system and with the Detroit Edison Company's lines at Superior substation near Ypsilanti. The line was first put into experimental operation two weeks ago and has already proved of service to both companies. The line is connected to both systems through transformers with oil circuit breakers on each side of each transformer. Complete metering equipment is installed at each end of the line.

Some study has been made of the possible advantage of extending this line west and southwest to reach the northern Indiana district. While it is too early to make many statements regarding this project, it is very evident by a little study that the difference in the daily load curves due to the difference of one hour in time would make such a connection desirable.

On the whole it seems that the engineering of interconnections has been worked out farther and better than the commercial or contract features. A great variety of contracts are in existence but there seems to be room for improvement. Where the two systems to be connected are not owned by the same corporation there is a need for a simple but equitable form of agreement that does not require a Philadelphia lawyer and a Schenectady engineer to interpret.

James Lyman: Mr. Gear has covered more particularly interconnection in the Chicago District. In the country districts of the Middle West the era of interconnection was inaugurated in 1912, with the adoption of an aggressive policy to serve every town in Illinois over a 33-kv. interconnected system from power stations located at economic points. From this beginning the movement has spread with increasing momentum until it has nearly covered most of the states of this region. As the system grew, 33 kv. became inadequate as a primary transmission voltage, and other systems of transmission lines at 66 kv. and 132 kv. have been superimposed on the 33-kv. system.

The accompanying map, Fig. 3, shows the 33-kv. interconnected system covering ten states having 17.3 per cent of the land area of the United States, and 31.8 per cent of the population of the United States in 1920. Outside the urban industrial centers the principal pursuits in this region are agriculture and mining. That the policy of interconnection has had a marked influence on their growth is shown by the following figures:

During 1926 the electric-power companies in these states produced:

41 per cent of the steam electric power produced in the U. S.
14 per cent of the hydro electric power produced in the U. S.
32 per cent of the total electric power produced in the U. S.

This map shows how completely the 33-kv. system now covers the area of this region. Fig. 4 shows on the same map the system of higher-voltage lines, 66-kv. and 110- to 140-kv. connecting the principal centers of this region, and extending far beyond its boundaries. There are only one or two short breaks in these high-tension lines between Minnesota and West Virginia, and between Michigan and Missouri. Wisconsin is actually connected through to Kentucky, Virginia, and states still farther south.

Fig. 5 shows the relative central-station production of steam and hydroelectric power for each of the several states for the years 1920 to 1926. It is of interest to note that the production of power in Indiana, Illinois, and Wisconsin, practically doubled in the six-year period 1920 to 1926. It will be observed that the only states producing appreciable hydroelectric power are Minnesota, Iowa, Wisconsin, and Michigan. All the other states are practically limited to steam-electric power. While the states of Illinois and Ohio represent but 19 per cent of the area, and but 37 per cent of the 1920 population, they produce 46 per cent of all the central-station power produced in this territory.

As power generation on a large scale first grew up in the cities so also did interconnection have its beginning in the cities. As it grew it gradually covered the city areas, then reached out to the suburban areas, and finally spread beyond them to link up with the systems which, growing independently, were covering the country districts. And the final step, which we are now witnessing, is the connecting of these different power centers with high-voltage, high-capacity trunk lines. The Chicago District illustrates this evolution.

Beginning with the epoch-making Fisk Street Power Station, and followed successively by Quarry, Blue Island, Northwest, Joliet, and Calumet, and more recently, by Crawford and Waukegan power stations, this interconnection grew and spread as each successive plant, profiting by new developments, was able to produce energy cheaper than its immediate predecessor. In a measure, each plant in turn functioned as a base-load plant for the interconnected system. Because of this, each of the Chicago plants, in its turn, contained the largest steam-turbine unit available at the time.

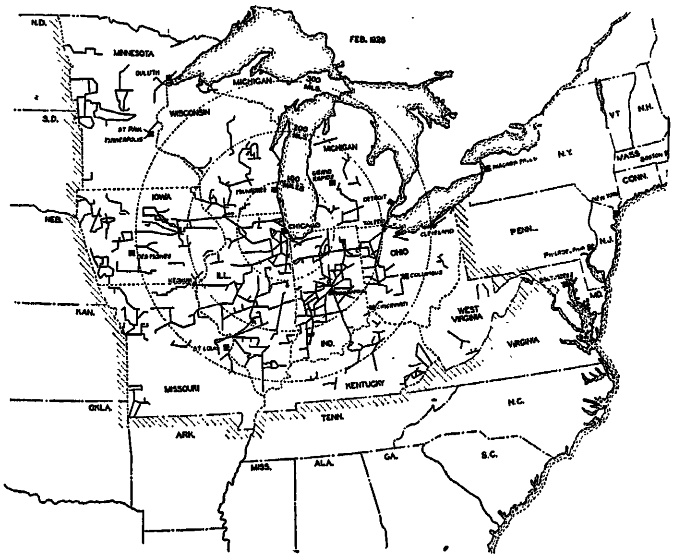


Fig. 3—33-Kv. Lines in Middle West States

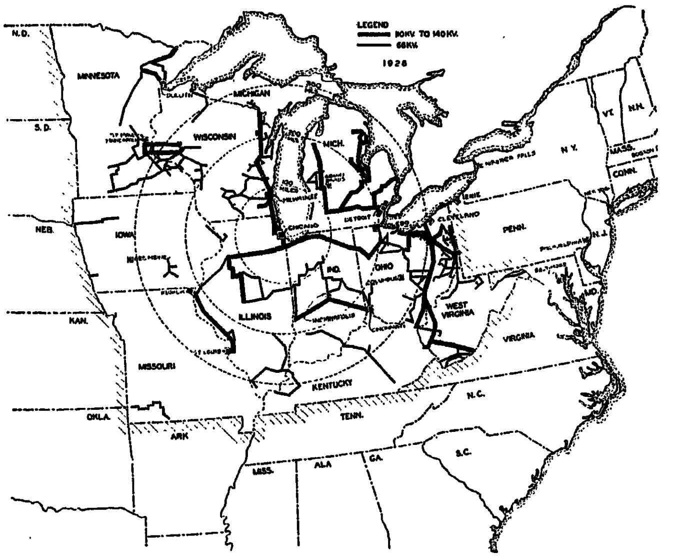


Fig. 4—Superpower Lines in Middle West States

Interconnection of Other Systems in the Middle West. The American Gas & Electric Company transmission system stretches from Southwestern Michigan and Northern Indiana through the states of Ohio and West Virginia to Pennsylvania, Virginia, Kentucky, and Tennessee. It connects with the transmission lines of the Chicago District on the West, the Toledo and Cleveland systems on the North, the systems of the Ohio River Edison Company and West Penn Power Company on the East, and the systems of the Carolina Power & Light Company and Tennessee Public Service Company on the South.

The 140-kv. hydroelectric lines of the Consumers Power Company in Michigan, put into operation in 1912, had at that time the highest commercial voltage in the world. These lines are now interconnected with the transmission system of the Detroit Edison Company.

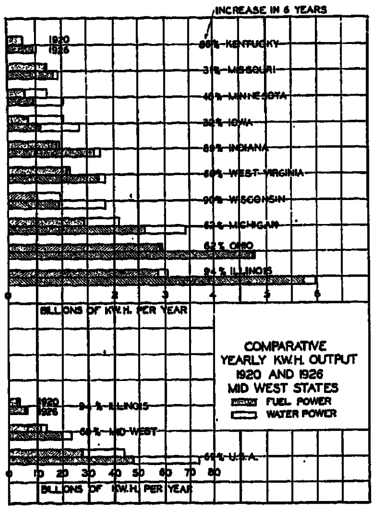


FIG. 5

The systems of the Insull companies also interconnect successively with the Columbia Gas & Electric Company, the Illinois Power & Light Company, and the North American Company on the South, and with the North American Company and Byllesby Company on the North.

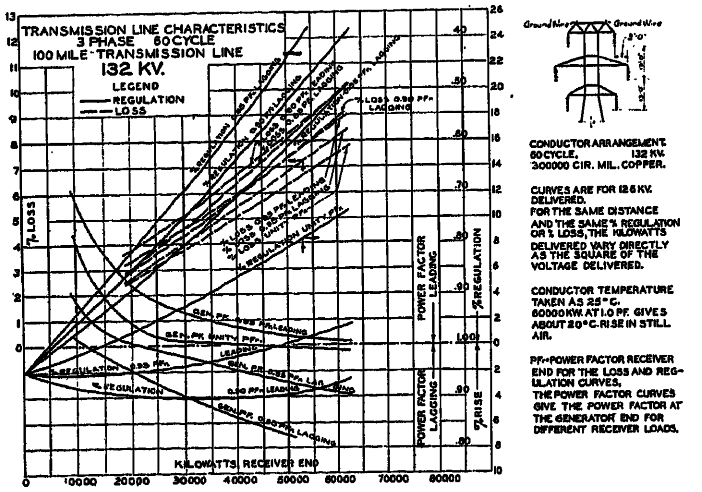


FIG. 6

Transmission Voltage. In the Middle West where large economical steam power stations are often located not more than 100 mi. apart, 132 kv. is a good standard voltage. Power in the order of 50,000 kw. to 60,000 kw. per circuit can be directed to or from these power stations as operating conditions dictate.

A high power factor is required and preferably obtained through the use of synchronous condensers installed at the plants of the largest customers.

Desirable flexibility of control is obtained by the use of equipments for tap changing under load, for the terminal banks of transformers.

Fig. 6 shows characteristic curves for 3-phase, 60-cycle, 100-mi. transmission line of 300,000-cir. mil copper cable, with 126-kv. delivered voltage for various power factors.

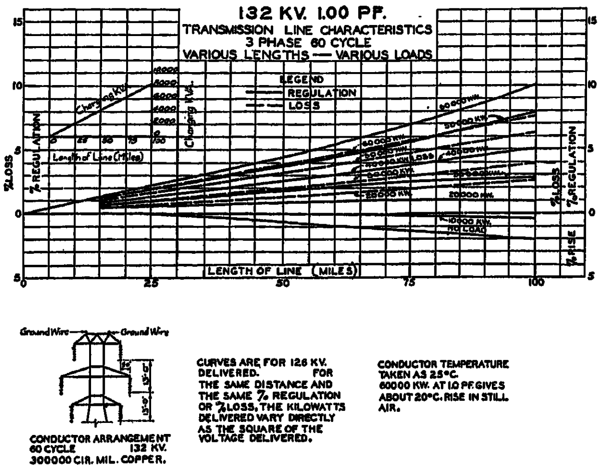


FIG. 7

Figs. 7 and 8 show characteristic curves for unity power factor and transmission lines of 100 mi. and of 300 mi. respectively.

Figs. 9 and 10 show characteristic curves similar to 7 and 8, except at 0.95 power factor lagging.

A study of these curves shows the necessity for high power factor. The increased kilowatts capacity and reduction in $C^2 R$ losses of the interconnected system very much more than pays for the capital and operating costs of the synchronous condensers. These are now being developed in large sizes for out-of-door installation, cooled either by recirculation of air or hydrogen, and with automatic control of voltage.

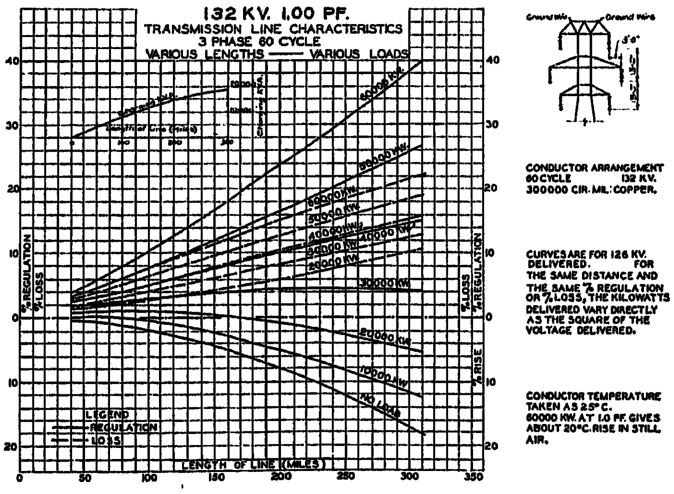
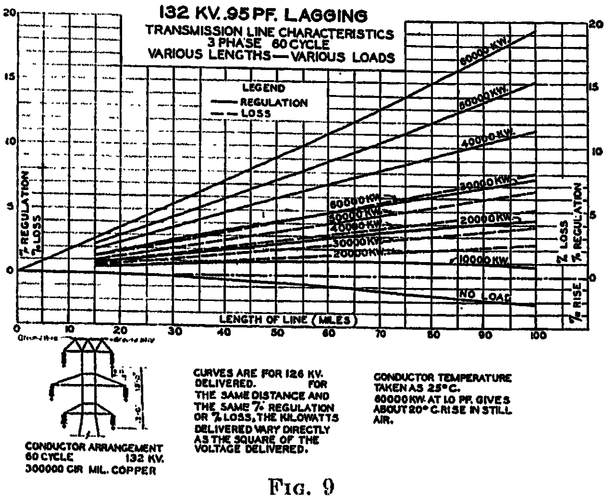


FIG. 8

Fig. 11 shows a steel tower typical of the design used in the Chicago district.

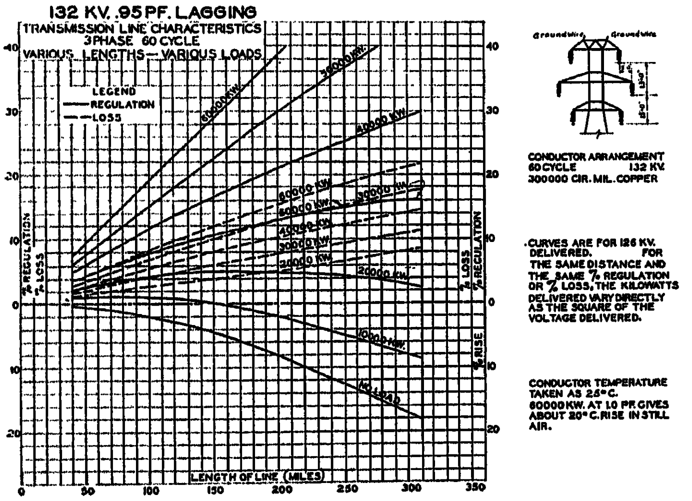
Experience with 150 mi. of such 132-kv. heavy power circuits in the Chicago District indicates that when the circuits are equipped with ground wires and shielding rings the number of interruptions is practically negligible. Where shielding rings are used to protect the insulator strings, in case a flashover does occur, the line can immediately go back into service without inspection or repairs. Transformers and other high-tension apparatus are now made to give equal reliability when properly protected by the most approved lightning arresters.



CURVES ARE FOR 126 KV. DELIVERED. FOR THE SAME DISTANCE AND THE SAME % REGULATION OR % LOSS, THE KILOWATTS DELIVERED VARY DIRECTLY AS THE SQUARE OF THE VOLTAGE DELIVERED.

CONDUCTOR TEMPERATURE TAKEN AS 25° C. 60000 KW. AT 1.0 PF. GIVES ABOUT 20° C. RISE IN STILL AIR.

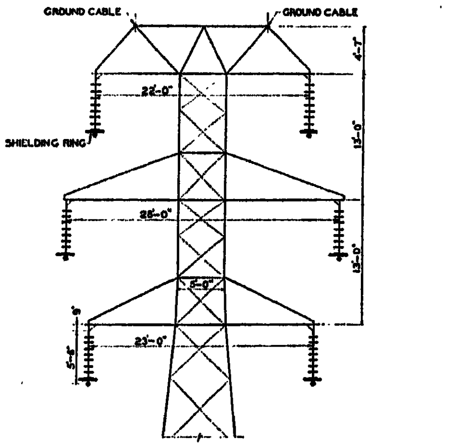
Fig. 9



CURVES ARE FOR 126 KV. DELIVERED. FOR THE SAME DISTANCE AND THE SAME % REGULATION OR % LOSS, THE KILOWATTS DELIVERED VARY DIRECTLY AS THE SQUARE OF THE VOLTAGE DELIVERED.

CONDUCTOR TEMPERATURE TAKEN AS 25° C. 60000 KW. AT 1.0 PF. GIVES ABOUT 20° C. RISE IN STILL AIR.

Fig. 10



TYPICAL 132 KV. TOWER
USED IN CHICAGO DISTRICT

Fig. 11

Advantages of Interconnection. Interconnection makes available throughout a wide area the power from a number of large generating stations. It permits their most favorable location with reference to fuel and water supply. By diversity of the system load a much higher load factor is obtained, thereby reducing in direct proportion the required active generating-station capacity. Further, the reserve generating capacity need only be a fraction of that required were the stations operated independently. The combined saving in generating capacity due to diversity in load and to reduced reserve is in many cases very great.

The entire interconnected territory is served from several sources of reliable power supply so that in case of failure of any generating unit or transmission-line section, the possibility of interruption of power supply is remote. Instances of this character taken from actual experience might be cited if space permitted.

The rapidly increasing power demand throughout the country necessitates frequent installation of new steam power stations. The latest and most economical stations can operate as base-load stations for the system. The most economical generating units can operate as base-load units, and the less economical units or stations can be operated as semi-peak-load and peak-load units or stations; thus the interconnected power companies can continually maintain the highest economies of production for the large bulk of their power supply.

The operators in power stations located at the different centers can advise each other over their carrier systems of communication, of rising storms in advance of their arrival, and thus their power demands may be anticipated. Also, in case of trouble developing in one of the several stations, the others can promptly prepare to take the necessary load.

Frequently systems under different ownership interchange large blocks of power daily to their great mutual advantage.

Interconnection has made economically possible practically the universal supply of electric power, the greatest factor in the continued prosperity of our country.

Farley Osgood: That interconnection is useful seems no longer to be a question. I shall try to generalize on the whole subject and summarize what I have learned from these papers and observed in my work.

In these four papers I think unquestionably we have the best national presentation of the interconnection problem and its possibilities that has ever been given.

I am glad to see that the fantastic names formerly given to high-voltage line interconnections are going out of fashion. I noticed on one chart presented the word "super." In all the discussion on these papers, I think it is the only time that word has appeared. I think it will soon be wrapped up and will appear no more.

There is no such thing as super power, or giant power; that isn't what it is. It is just the everyday work of the electrical fellow connecting together his physical links. That is all there is to it. As Mr. Edgar described the railroad and the Lincoln Highway development, it occurred to me that some day there might be a physical-electrical connection between Boston and San Francisco which we might very appropriately term the Edison Highway. There may be a complete electrical highway from the East to the West Coast and just as the passenger today can get on a train in Boston and go to California (and if he is lucky enough to get in certain of the Pullmans he may not have to change) so too, when we have the Edison Highway a kilowatt can get on the highway at Boston and reach California by being shoved along by the various systems hooked into the interconnection.

That is the picture we should give to the public. We should destroy as fast as possible and whenever we get an opportunity, any thought that California will furnish current to New England or vice versa. It is absurd and it is the duty of the electrical engineers to explain to people why it is impracticable.

We should not make any extravagant statements so that newspaper reporters, politicians, and anybody who may not understand our language will misinterpret it to our detriment before the public.

A word about the financing of these inter-system lines: It isn't any kindergarten problem, this laying out and completing and financing a major interconnection. Engineers must understand that tremendous amounts of money are required and it is their duty to prove to their executives responsible for the financing of their undertakings that a large investment must be made in advance of actual income, because these things take more than two years to do and sometimes take three, five, or ten years before full realization on the capital invested. Financiers who back us in our work should not be afraid to do so, but we should tell them what to expect in the way of return at first, and year by year thereafter, because they depend on us and our statements should be clear and conservative.

The usual possible economies of interconnection have been dwelt on so much that I will not even mention them. We all know what they are.

I should like to emphasize the fact that the strongest construction possible under the economic condition of the work undertaken is an essential, not only because the service is dependent upon these major interconnections but also the growth of the respective systems usually is faster than we expect and an early avoidance of duplication of construction is there if we build large and strong in the first place.

Continuity of service of course also depends upon the sturdiness of the construction, assuming that it is of the proper type for the service.

I notice that there are only two or three large pieces of work going on in the country at the present time of the highest service voltage, namely 220,000 volts. The California fellows developed this voltage as soon as the originating engineers felt it safe to have them do so. This ring here, near New York, connecting electrically Philadelphia, Newark, and the coal fields of Eastern Pennsylvania is also a 220-kv. system.

In the discussions of the Chicago situation I note that 132 kv. has been adopted as the maximum voltage apparently desirable for that district. Far be it from me to criticize because those gentlemen have proved they know where they are heading, but just as a general word of caution, I should like to say: when building, make your voltage as high as the traffic will bear at the time, because you will save yourself a lot of trouble in the future. Even if you do not operate at the highest possible voltage available at the time, have that in mind when your right-of-way is purchased, when the spacing of your conductors and towers, etc., is laid out so that if fortune or big business overtakes you sooner than you dared hope, you can save your executives putting up additional money too soon.

The synchronous condenser is a joy to the operating man in helping him pull up his voltage. It was interesting to hear the last speaker say the best place for the synchronous condenser was on the big customer's property and I might also add, to be bought at his expense.

The customer can be given a favor in the way of rates, if he helps pull up the power voltage of the whole system. It works both ways.

The matter of selective relays for the eliminating of lines and trouble, is a question about which we have had so much discussion, I am afraid to say anything about it, except where you do not have to be tied in solid for a proper reason, if you can eliminate any circuit that is in trouble of course you are going to save slams on your whole system and you are going to add to the safety and protection of your generators and your transformers. I think where the loose-link system can be used, it is healthier for the apparatus.

The Chicago paper was the only one which mentioned the use of high-voltage cable. I believe that is one of the things that the transmission engineers need most for their service at this time.

When we were designing the 132,000-volt domestic ring for New Jersey service, if we could have obtained cable at the time, it would have saved us a great deal of trouble and because we could not get high-voltage cable to go through a congested area we had to go outside into the country with open lines which cost much more than the cable would have cost if it had been available.

In congested areas, think of what Mr. Gear has told us about Chicago. Cable is indispensable. If he can double the voltage of his interconnecting cables from 66 kv. to 132 and to 220 where he has these tremendous loads, think of the money he is going to save and the comfort he will have.

In these installations, it is desirable to have every possible recording instrument and device of every nature to make record of the actual performance of the line and everything connected with it in regular service and particularly at times of failure.

We are just on the threshhold of major interconnection work in America and the only way to learn about it is by absolutely infallible records. We have had to go by guesswork largely up to date but now devices are coming into service which will record for us what takes place normally and under abnormal conditions. Let no argument of cost have them eliminated from your budget. If you do not have anything else, be sure and get all the best possible recording instruments; in fact, you might put that in your budget first and say "If we cannot record what is going to happen to us when we get through, we do not want to build." Mr. Mitchell brought out rather pointedly the fact that interconnected systems could get much better operating results if the several systems are under one ownership, which naturally tends to a closer coordination of personal effort. Mr. Downing cites the excellent results that are obtained in California even though the systems are not under one ownership.

By frankness, fairness, the elimination of individual system selfishness, it is perfectly possible to get, if not the very highest efficiency of operation, at least an entirely satisfactory result. By the formation of a committee composed of the responsible officers of the individual systems, which will make up a set of operating rules, which rules must be thoroughly understood by all operators, and if carried out to the letter by a master operator, unanimously selected and in absolute operating control, efficient and satisfactory operation can be obtained.

In closing, I just want to point out the two things which seem to be the weak links in our chain at the moment and urge the engineers responsible for these two weak links to get all of this discussion in their minds that they may need to help them develop their art so that we may have these links up to the strength of every other link in our electrical systems. The two weak links are high-voltage underground cable, and oil switches.

E. C. Stone: At this time, because of the recent rapid extension of transmission lines and the multiplication of interconnecting links, which make the transmission-line map look something like a railroad map, the public is prone to conclude that there is no limit short of the continental confines of the United States to the transfer of energy; and when the minds of legislators are considering the necessity for federal regulation of power systems which are rapidly over-reaching state boundaries, it is important that engineers particularly, in their speeches and writings, give proper emphasis to the limitations as well as advantages of interconnection so as to be sure that the public does not get a distorted view of the situation.

There is already before us the history of three great public utility services which grew from local beginnings until they covered the entire country and the transfers of the commodities handled by them became nationwide in extent—namely, the railroads, the telephone, and the telegraph. There are inherent reasons, however, why this movement in the electric power industry will not be carried to the same limits.

The articles of commerce carried by the railroads differ widely in character, and the wants and likes of the consumers of the country are sufficiently strong and varied so that they are willing to pay the cost of transferring these articles to points all over the country. The fruits and vegetables of California are consumed throughout the states. The shoes made in Massachusetts are sold in St. Louis, and I have no doubt that St. Louis made shoes are sold in Massachusetts. The messages transmitted by the communication companies are all different regardless of the relative locations of the source or destination; hence the nation-wide movement of such messages. In the power industry, however, all kilowatt-hours are alike so that the consumer will always take advantage of the obvious economy of obtaining his kilowatt-hours from the nearest source of power.

Interstate transmission of electric power is the result of the natural physical relation between the markets for power and the nearest suitable sites for the location of economical power plants. Thus the transmission of power is essentially local in character, and very few instances can be found where any given block of power is transmitted across more than one state boundary. An analysis of the output of eighteen generating plants from which is produced approximately 45 per cent of all of the power transmitted over state lines, shows that the average distance of transmission from plant to point of utilization is less than 75 mi. Probably the average mileage of all interstate transmission would not exceed this figure. Although many plants located in many different states are being electrically connected, the effect of such interconnection is merely to distribute the combined load among the various plants more nearly in proportion to their capacity. It does not to any measurable extent increase the radius of transmission of power from any of the individual plants in the group.

Hence the power situation is again radically different from that in respect to freight handled by the railroads, which in some cases is shipped across a great many state lines. In this connection, Secretary Hoover spoke as follows at the 37th Annual Convention of the National Association of Railroad and Utility Commissioners at Washington, D. C., October 14, 1925:

"The argument is sometimes used that the power situation is parallel with the railroads, where federal regulation has been found necessary. It differs in several profound respects. Power has no such interstate implication as transportation. The grower of oranges shipping from California is affected by the freight rates of every different railway all the way to the Canadian border and the Atlantic seaboard. The same farmer as a buyer of power is interested in only one power company—the one in his immediate neighborhood. What the rates may be of 200 other power companies between him and the borders of the continent do not affect his rates in the remotest degree. Therefore one state is not vitally interested in the rates of another state as in the case of the railways."

As a result of these inherent limitations, only 6,172,000,000 kw-hr., equal to approximately 9 per cent of the total of 68,145,000,000 kw-hr. generated during 1926 by the electric light and power companies in the United States, were transmitted across state lines. This is in striking contrast to conditions with the railroads, of whose business probably 70 per cent to 80 per cent is interstate in character.

It is reasonable to suppose that all of the power required for use within the individual states could be generated within their own boundaries, so that interstate transmission would be unnecessary, but this would result in less use of water power, in higher cost of fuel, in longer transmission distances and, in many instances, in greater first cost of plant on account of utilization of less favorable locations, all of which conditions would increase the cost of power. The margin of profit to be realized by interconnection is narrow, however, and if, as a result of misconception, burdensome restrictions were placed on such interconnection, it is reasonable to assume that public-utility companies, finding themselves hampered by requirements and regulations so burdensome as to offset any advantages to be had, will curtail their interconnections and thereby the consuming public will lose those limited advantages which are now being utilized or being sought.

Therefore, it is to be sincerely hoped that no artificial barriers will be set up which might result in restricting this sort of transmission in such a way as to increase the total cost of the production and delivery of power.

The economies resulting from the interconnection of power systems as disclosed by the papers in this symposium represent all told a very large number of dollars, and are therefore well worth while. Expressed as a percentage of the cost of electric power to the average user, however, the figure will be relatively small, possibly of the order of 5 per cent. Hence the most comprehensive development of system interconnections cannot possibly bring about any radical reduction in the cost of energy to the average user of electric power.

The major engineering problems involved in the interconnection of large power systems will continue to receive careful study. These appear to be as follows:

1. Determination of the total spare generating capacity actually required to render adequate service at all times on systems of large capacity and operating many units. Due regard must be given to the periodic shut-down of machines for maintenance and the probable loss of capacity due to breakdown. Present practise of many systems provides spare capacity equal to the capacity of the largest units. It is yet to be determined whether this will be adequate when adjacent systems are interconnected, so that substantially larger numbers of units are included in one system. More attention will also be given to the most economical size of unit giving full effect to the spare capacity requirements.

2. Control of the amount of energy and reactive kv-a. passing over interconnection points in complicated networks. This will involve not only proper frequency adjustment at the generating stations but in many cases phase and voltage control at strategic points in the transmission networks.

3. System stability both under normal load conditions and on the occurrence of abnormal transient surges. In this connection it is quite probable that it will be found necessary to equip high-tension transmission lines with reactors or their equivalent at the power-house end, following the practises now generally standard on lower voltages of using line reactors which sustain the voltage of the system as a whole as well as reduce rushes of current when the individual line short circuits near the power plant.

4. Design and manufacture of circuit breakers for the voltages and interrupting capacities involved in the interconnection of large power systems.

5. Development of selective relaying schemes such that tie lines will not be interrupted when short circuits develop on one or the other of the interconnected systems. One of the chief operating advantages in interconnection lies in the fact that the loss of a single generating unit will result in a smaller percentage loss in operating capacity. This advantage is lost if tie lines are opened by system short circuits. Furthermore, the amount of power transmitted across a tie line may be greater than the receiving company can handle on its own system if separated from its interconnections so that opening of tie connections will result in interruptions to service.

L. W. W. Morrow: With the rapid growth of light and power systems over extensive areas it is inevitable that energy should be sent across state lines. It is rather astonishing to note, however, that the amount of energy so transmitted was only 9 per cent of the total energy generated in public-utility stations during 1926. According to the Bureau of Business Research of Harvard University the total energy transmitted across state lines in 1926 aggregated 6,171,530,837 kw-hr. while the total generation in that year amounted to 68,145,217,000 kw-hr.

TABLE I

EXPORTS AND IMPORTS ON INTERSTATE POWER BY ELECTRIC LIGHT AND POWER COMPANIES UNITED STATES—1926

	Power Generated* (Kw-hr.)	Interstate Power		Net Interstate Power	
		Exported (Kw-hr.)	Imported (Kw-hr.)	Net Exported (Kw-hr.)	Net Imported (Kw-hr.)
New England:					
Maine	503,105,000	2,803,706	823,946	1,979,760	—
New Hampshire	259,133,000	15,514,336	47,320,485	—	31,806,149
Vermont	254,508,000	234,544,352	21,207,590	213,336,762	—
Massachusetts	2,097,538,000	226,737,240	420,760,736	—	194,023,496
Rhode Island	405,299,000	79,465,342	154,920,753	—	75,455,411
Connecticut	906,941,000	52,106,110	51,219,136	886,974	—
Middle Atlantic:					
New York	9,146,059,000	147,214,876	17,112,855	130,102,021	—
New Jersey	1,722,749,000	19,780,642	202,783,965	—	183,003,323
Pennsylvania	7,220,386,000	614,801,002	477,281,883	137,519,119	—
East North Central:					
Ohio	4,596,945,000	285,558,782	487,588,090	—	202,029,308
Indiana	1,420,528,000	152,210,606	146,616,303	5,594,303	—
Illinois	5,892,130,000	646,354,014	305,976,537	340,377,477	—
Michigan	3,184,591,000	57,715,760	42,943,150	14,772,610	—
Wisconsin	1,779,973,000	225,598,472	126,545,506	99,052,966	—
West North Central:					
Minnesota	843,258,000	50,842,474	195,984,387	—	145,141,913
Iowa	1,308,622,000	632,315,767	76,680,703	555,635,064	—
Missouri	760,910,000	59,690,208	885,509,239	—	825,819,031
North Dakota	45,393,000	0	6,194,161	—	6,194,161
South Dakota	80,342,000	7,786,857	3,187,987	4,598,870	—
Nebraska	358,494,000	19,591,133	1,490,491	18,100,642	—
Kansas	761,429,000	129,960,481	32,631,815	97,328,666	—
South Atlantic:					
Delaware	118,866,000	21,019,362	10,238,314	10,781,048	—
Maryland	586,917,000	78,365,841	438,524,065	—	360,158,224
District of Columbia	350,014,000	42,182,891	0	42,182,891	—
Virginia	920,255,000	10,692,478	109,770,362	—	99,077,884
West Virginia	1,682,968,000	736,930,012	95,349,606	641,580,406	—
North Carolina	1,116,043,000	25,029,500	329,140,958	—	304,111,458
South Carolina	1,023,076,000	318,670,000	67,861,902	250,808,098	—
Georgia	709,705,000	68,094,902	272,418,703	—	204,323,801
Florida	494,745,000	0	1,314,718	—	1,314,718
East South Central:					
Kentucky	457,770,000	73,701,994	182,259,635	—	108,557,641
Tennessee	893,693,000	21,797,033	11,469,426	10,327,607	—
Alabama	1,138,804,000	283,984,216	8,196,946	275,787,270	—
Mississippi	58,667,000	279,300	57,577,615	—	57,298,315
West South Central:					
Arkansas	163,517,000	873,775	61,481,112	—	60,607,337
Louisiana	514,552,000	82,720,682	15,421,073	67,299,609	—
Oklahoma	429,735,000	11,469,200	85,921,944	—	74,452,744
Texas	1,430,356,000	17,470,344	1,356,058	16,114,286	—
Mountain:					
Montana	1,407,783,000	18,432,000	0	18,432,000	—
Idaho	738,876,000	465,743,608	78,554,534	387,189,074	—
Wyoming	42,084,000	0	0	0	0
Colorado	517,405,000	0	0	0	0
New Mexico	25,993,000	1,214,848	2,006,383	—	791,535
Arizona	167,164,000	21,181	4,664,592	—	4,643,411
Utah	347,857,000	0	421,274,000	—	421,274,000
Nevada	36,553,000	0	46,019,035	—	46,019,035
Pacific:					
Washington	1,716,557,000	73,913,249	97,346,669	—	23,433,420
Oregon	806,296,000	91,167,309	68,562,288	22,605,021	—
California	6,700,633,000	67,164,952	21,181	67,143,771	—
United States Total	68,145,217,000	6,171,530,837	6,171,530,837	3,429,536,315	3,429,536,315

*Interpreted from United States Geological Survey figures—Bulletin 68, Bureau of Business Research, Harvard University.

Of the energy transmitted in 1926 across state lines 83.3 per cent went to markets supplied as a normal service and 16.7 per cent went to special or temporary markets. Of the total interstate energy 26.28 per cent involved no change in ownership. The number of interstate lines aggregated 463 but 81 of these carried 61.7 per cent of the total interstate power.

Three general conditions exist which may bring about interstate transmission.

The first type is a regular supply of energy to a market in a territory where generating facilities may not coincide with the location of the load. At Keokuk, Iowa, the nearest large market for hydroelectric power is St. Louis, Mo. and in Illinois; at Cahokia, Ill., a site is most suitable for a fuel-burning plant to supply energy to St. Louis. Thus the city of St. Louis receives a large proportion of its energy from plants controlled by the local utility but located in other states. Out West waterpower is in Idaho but the load is in Utah. Many examples of this type of generating-station location which supply energy to loads across state lines may be found in Ohio, Illinois, Indiana, Pennsylvania, and other states. The stations are operated to supply energy to permanent markets and are owned by the utilities distributing the energy in these markets. This type of interstate business accounts for about 75 per cent of the total energy and is not necessarily involved with major interconnections or transmission systems of large magnitude.

A second condition which brings about energy transmission across state lines is in areas where steam and hydroelectric generating facilities are interconnected for temporary or emergency energy service.

A water shortage on a hydroelectric system may cause energy to be bought from a system in another state; a hydroelectric property may be able to sell or buy energy from a utility using fuel generation; during the construction of a new station energy may be bought so as to load fully the new station when it comes into operation. Many economic combinations in a large area may exist from year to year whereby savings are made by temporary energy interchanges on lines which cross state lines. For example, in the South, Alabama may sell energy to Georgia at one season of the year and may buy energy from Georgia at another period.

The third condition which brings about the transmission of energy across state lines is found in large-capacity interconnections between utilities which operate over an extended area. It is economical on such systems to shift generating-station loads to get the best economic results. For example, the American Gas and Electric system extends from near Chicago to western Virginia. It has generating stations in several states connected by transmission lines. Energy may go to or flow from West Virginia or Ohio depending on load conditions. In the heavily loaded area of eastern Pennsylvania, New Jersey, and New York, stations may be located in several states and energy will be sent across state lines to take advantage of diversity in loads.

A study of interstate energy shows that Pennsylvania, Illinois, Iowa, West Virginia, Vermont, Alabama, South Carolina, and Idaho lead in the export of energy and that energy is imported in large amounts to Massachusetts, Pennsylvania, Ohio, Missouri, Maryland, North Carolina, Georgia, and Utah. A knowledge of generating conditions in these states shows that the exportation and importation of energy involves a few large generating stations and special conditions. For example, South Carolina waterpower goes to the load in North Carolina. Energy from the coal fields of northern Alabama and from Muscle Shoals goes to Georgia. Hydroelectric energy from Idaho goes to the load in Utah. Holtwood waterpower in Pennsylvania goes to the load in Maryland. Hydroelectric and steam power in southern West Virginia goes to Virginia and Ohio. Hydroelectric power from Vermont and New Hampshire supplies the load in western Massachusetts. Keokuk in Iowa and Cahokia in Illinois supply the energy to St. Louis in Missouri. Windsor steam station in northern West Virginia supplies energy to Ohio and Pennsylvania. Thus a few large stations or systems supply most of the energy that crosses state lines. An economic and very practical background exists for all this business.

With the modern trend toward putting loads and generating sources near together for economic reasons and because load density grows rapidly it is evident that long-distance transmission projects involving interstate energy traffic will not become of great importance and that special conditions in the future, as in the past, will control interstate energy traffic. There is no reason to expect interstate energy to grow at a rapid rate or at a rate corresponding to that at which the light and power industry will grow.

W. S. Lee: It would be impossible for me or anyone to enter into any detailed discussion of the problems of interconnection or parallel operation of power systems.

This matter has to be developed in connection with the particular system or section of the country. There are certain waterpower developments, limited in number, and the time will soon come when they will all be developed. There are growing up around these plants and in the territory which they are near, certain demands for power that should be furnished from the nearest point of generation.

Efficiency, economy, and reliability of service has caused various hydroelectric and steam stations to be connected. This interconnection of local hydroelectric and steam stations produces the big central-station system. The central-station system assumes the duty of service to all contiguous territory.

Now the phase in which the engineer today is going to function is to build the water plants at such locations determined by the physical conditions, such as head storage, etc., then intersperse among these plants at the proper place on the load centers, steam stations.

Two important factors in connection with the design of steam stations are load factor and size of plant. The larger the central system the more opportunity the engineer has to build these plants efficiently and economically. The larger the system developed the better opportunity he has of locating these stations so as to shorten the transmission lines and thereby lessen the cost of power to the consumer.

I am delighted to see these papers for if there is any abused subject it is super-power and giant-power. Every engineer is aked the question, "How far can you carry electric power?" You all know it is not economical to carry it very far. The result is, there is an impression that power can be taken in great amounts from one section and dumped into another economically. You can't do it.

Mr. Mitchell's paper gives a very fine description of the Southern Systems. The Duke System tied in with the Eastern Carolina in 1908 and this connection has been found very useful from time to time ever since. About 1911 the Duke System tied in with the Georgia System. Both of these connections have been used for exchanging power and shifting power from one system to the other, but to carry large blocks of power great distances has not been done. I am glad to see that the whole trend of this discussion has not been for the purpose of carrying power in large blocks to great distances but to lay out hydroelectric and steam stations in a connecting system for the purpose of shortening transmission distance.

C. F. Hirshfeld: I have been asked to present, by way of discussion, material on what is commonly designated as loose linkage.

In presenting matters of this sort one is greatly handicapped by the fact that our language grows as required by the development of the art so that the art is always ahead of the language. As a result, it frequently takes a long time for a given expression to acquire a very definite meaning and in the interim there is much danger of misunderstanding. I shall therefore attempt to indicate the meaning of each questionable term that I use.

The term loose linkage, as contrasted with tight linkage, is first used to indicate a system of connections adapted to fall apart comparatively easily in case of trouble, in contrast to one in which connections are intended to be maintained intact as long as it is physically possible to do so.

We all recognize the fact that with present equipment and present methods we must expect troubles of various sorts which result in temporary reduction of voltage and increased flow of current. In a simple, generalized way, these troubles may be divided into two classes, those which correct themselves quickly by being cleared in some way, and those which do not, which "hang on" as we say. These may, for present purposes, be designated as controllable and uncontrollable respectively. With respect to the first class, there is a decided advantage in having connections remain intact in order to give the most immediate return to conditions of normal service after the trouble has been cleared. With respect to the second class, there is a decided advantage in having certain links break easily so as to confine uncontrollable trouble to the smallest practicable area, thereby producing a minimum of disturbance to the system as a whole and no long time effect upon service to the remainder of the territory.

Thus far probably all will agree. It is when we go beyond this rough generalization that we enter the argumentative zone. As I see it, the direction in which the next step leads depends partly upon one's experience, partly upon one's idea of character of service, partly upon one's estimate of the sorts of trouble to be experienced, partly upon one's estimate of the frequency of the different sorts, and partly upon the type of system with which one is dealing.

We may well consider extreme examples to bring out the great differences that are possible. Take as one case a system completely underground and consisting of a perfect network to which energy is furnished from a number of multiplied sources. Take as the other case a system which is entirely overhead, arranged practically radially throughout and in a territory in which lightning storms are prevalent. One would hardly expect the operators of such widely divergent systems to adopt similar ideals of service or similar methods for handling troubles of the type here referred to.

We must then admit that in all probability both loose-linkage and tight-linkage systems have their proper fields of use. At least, I do not desire to be understood, in what follows, as advocating one or the other under all different conditions.

An ideally developed loosely linked system may be assumed to consist of a collection of areas so arranged and so served that each can readily become self-contained and self-sustaining in case of uncontrollable trouble in one or more other areas. For want of a better term I shall call each of these a power area.

In the ideal loosely linked system, each power area normally operates in synchronism with all the others during all periods of normal operation. However, a power area subject for the time being to uncontrollable trouble cuts loose completely and automatically from all other power areas. It then shifts for itself as best it can until those in charge have had an opportunity to clear the situation by any means that may be available, after which it may once more be synchronized with the remainder of the system when, as, and if that becomes desirable.

These requirements necessitate certain characteristics which may be listed as follows:

(a) Each power area must be capable of becoming self-sufficient practically instantaneously.

(b) All power areas must be so tied together under normal conditions that the necessary synchronizing currents may flow.

(c) All power areas must be so tied together that any one developing uncontrollable trouble will automatically cut itself loose from the others before service in them is seriously menaced.

Requirement (a) indicates immediately that each power area must have sufficient running reserve, or the equivalent, to care for the total existing load within its boundaries if it happens to be isolated because of trouble in one or more other power areas. This does not mean that each power area must at all times be supplying all the power consumed within that area. The distinction should be carefully noted. Power may flow across power-area boundaries as may be required to affect most efficient or most convenient plant operation. The only requirement is that each area thus supplied from outside its boundaries shall have sufficient running reserve to care for itself if this supply is suddenly interrupted. The requirement is easily met through the reserve capacity of turbo generators above their best points when the sizes of units in the system are properly coordinated and when the boundaries of power areas are properly chosen.

Self sufficiency with respect to power supply may be obtained by the use of one or more generating stations per power area or by the use of one or more major switching stations supplied from several sources over different routes, both sources and routes being so chosen as to minimize the danger of complete loss of supply. In this connection, it should be noted that the boundaries of power areas need not be fixed and, in general, will not be fixed. This statement applies to different hours within one day as well as to different days, different weeks, months, seasons, and years. It is only necessary that a boundary is known to exist at a certain place at a certain time and that the ties crossing that boundary at that time are properly relayed to prevent them from transmitting a major disturbance.

It will probably be granted that at least under the majority of conditions existing in this country, the loosely linked system offers the greatest security against a major catastrophe to the system as a whole. Undoubtedly, operation on this basis requires a very high order of engineering in planning and constructing and extending the system. Undoubtedly also it calls for very intelligent cooperation between system operator and plant operator if the maximum of results of all sorts is to be obtained. It is probable also that the load dispatcher or other individual who handles system operation must give more thought to the details of his job than he must with a tightly linked system, if the best results of which each system is possible are to be obtained. There is admittedly room for argument as to whether the results obtained from loose linkage justify these costs in human effort. Some executives have demonstrated that they believe the results obtainable do justify such costs.

We may say that there is really a radically different point of view back of the two systems of linkage. In the one, the necessary occurrence of uncontrollable difficulties is recognized and the maximum of security to service is provided by arranging power areas capable of instantaneously becoming self-sufficient. In the other it is assumed that such major uncontrollable occurrences will be of such comparatively rare occurrence that the cost of protecting the service by loose linkage is unnecessarily great and not justified by the hazard.

As we develop extensive interconnection we are automatically driven toward loose linkage whether we like it or not. Overhead transmission lines will cut out under present-day conditions and with present-day equipment. The circumstances in each case must determine the extent to which running reserves will be operated and, therefore, the extent to which ideal loose linkage will be achieved.

F. A. Allner: Attention has been called in Mr. Downing's paper to the limitation or lessened desirability of interconnection when certain essential conditions favoring interconnection are absent. It should be recognized, to start with, that each case presents a problem controlled by the local conditions and that no generalization can be readily made. But it can be pointed out by an academic example that recent trends in power generation are working in favor of interconnection, even though certain obviously favorable factors are lacking, such as substantial diversity in load or in seasonal availability of power resources, or a pronounced differential in generating costs.

We may assume two systems, not interconnected heretofore, which have identical shapes of load curves, *i. e.*, no diversity of load, either in time of the year or hour of the day. We may also assume that the steam plants of the two systems are of nearly the same over-all economy. Under these conditions we should probably expect the least incentive for incurring a substantial investment for interconnection. However, the tendency in recent years to install large-size steam generating units, the capacity of which is equal to a large percentage of system load and considerably in excess of the annual increase of load demand, introduced several new elements in favor of interconnection, such as the staggering of investment steps between the two adjoining systems, savings in carrying charges on reserve equipment and more economical generation due to keeping on the line sufficient generating capacity only to compensate for the loss of one large unit instead of two. An interconnection of this sort has to be a solid tie (*i. e.*, not of the type referred to as loose linkage) which would insure that the two systems stay in parallel at times of transfer of maximum power coincident with short circuit on one tie line or with other system disturbances.

Mr. Gear's paper furnishes a practical example of the advantages incident to the staggering of investment steps involving three companies. It may be of interest to hear from him some further details as to the approximate capacity cost per kw. for which the three parties are billing each other.

In every case of interconnection particular stress should be laid on the quality of service, which feature has been brought out prominently in Dr. Hirshfeld's contribution. Quality of service should not suffer as a result of interconnection, allowing, of course, for the initial period of ironing our minor difficulties incident to the newness of the operating methods. The indirect consequences of impaired service may easily counteract any calculated economies that were originally the underlying reasons for interconnection. Most of us are familiar with the disastrous situation that may develop in a few minutes if two large systems or groups of a single system fall out of step due to major distribution trouble or other causes. As the frequency drops rapidly on that part of the system that has insufficient generating capacity connected to it, restoration of service usually involves the cutting off of a large part of the load and a considerable delay in resynchronizing.

In connection with this problem it may not be amiss to point out a possible by-product of large-scale interconnection under certain conditions when a hydroelectric plant, possessing seasonal storage or pondage, can be tied into an interconnected system. Particularly in the case of low-head hydro-plants, units can be started up from standstill and synchronized in a small fraction of the time required for a steam unit, the total time for this operation being of the order of approximately one minute. As the customary types of waterwheel governors permit synchronizing adjustment down to a low range of speed, restoration of service can be accomplished without cutting off commercial load when such hydroelectric units are available for standby purposes. This quick starting ability of hydroelectric units has been referred to by the late Dr. Eglin as "the hydro a-c. battery service," comparable to a certain extent to the storage-battery service provided on large d-c. distribution systems.

Referring to Mr. Mitchell's paper, a question in connection with the forecasting of river flow may invite a statement of general interest regarding the success that Mr. Mitchell's engineers had in this line of work. Previous attempts of analyzing flow records on large streams over long periods, of the order of thirty to forty years, were not very successful, my recollection being that the cyclic equation was a different one for each period. It would be rather interesting to hear from Mr. Mitchell whether his forecasting has been found as satisfactory in other years than those shown on the chart.

Philip Sporn: (communicated after adjournment) The American Gas and Electric Company has operated an interconnected system for many years, in fact, interconnection on the American Gas and Electric Company system has grown up along with the system. In 1907, at Scranton, the company found itself with a shortage of capacity and the D. L. & W. Railroad, which was operating a plant in Scranton, found itself in the same predicament. The railway plant was furnishing power to a group of coal mines. An interconnection was worked out between them calling for interchange of capacity when and if available. Since the peak of the railway load was a morning peak, this interconnection made available considerable capacity for the lighting company to enable it to carry its evening lighting peak, and it, in turn, was able to help out the railway company on its morning peak.

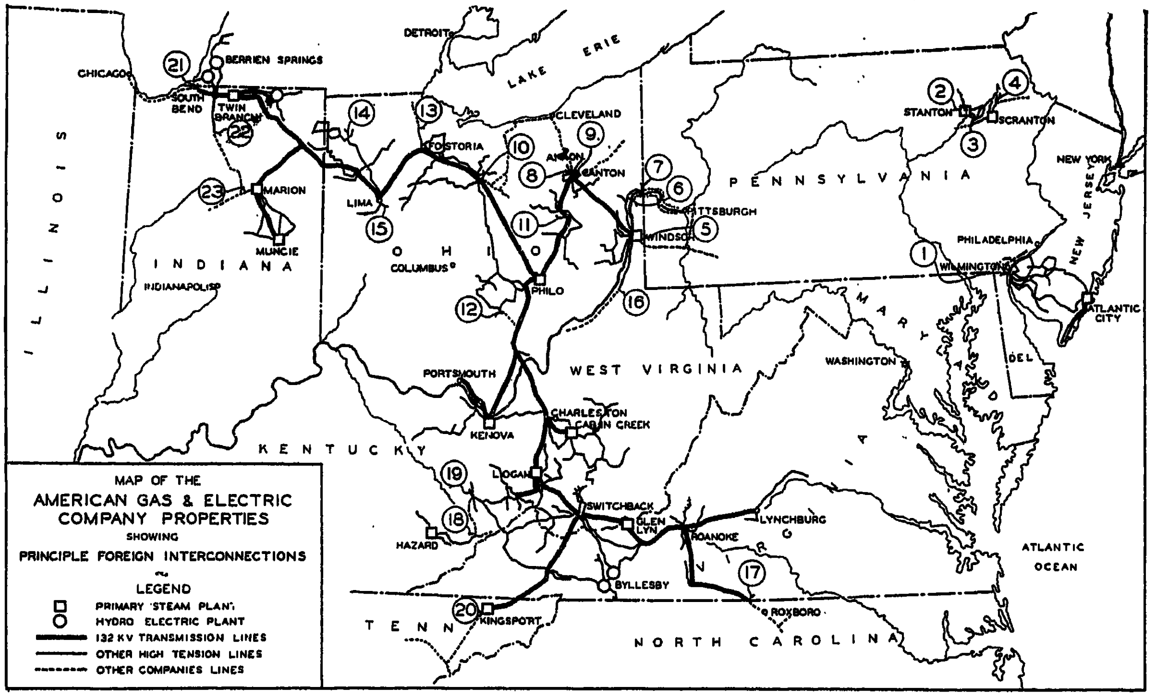


FIG. 12—AMERICAN GAS & ELECTRIC COMPANY'S INTERCONNECTED SYSTEM
The points of interconnections and the capacity of interconnections are shown in Table I.

Since then the American Gas and Electric Company's system has grown tremendously and as already stated, interconnection has grown with the system, in fact, whenever and wherever it was at all economically feasible to make an interconnection, it was made. Very often it was found that interconnections could be made at a rather small expense, but the benefits, based on advance analysis, were not quite apparent. The connections were made, nevertheless, on general principle in such cases and subsequent operation has always brought forth benefits far greater than were anticipated, so that we who have been working on interconnection, have come to believe that where an occasion for an interconnection presents itself, it is at least as rational, and probably more rational, to prove the case against interconnection rather than the other way around.

Fig. 12 herewith, shows the American Gas and Electric Company interconnected system as it is, including that portion of it that is actually under construction.

In Fig. 13 there is shown the Island of Great Britain, Scotland,

and Wales superimposed over the American Gas and Electric Company system.

In Fig. 14 there is shown a map of the British Isles with the central electric-supply scheme as recently outlined in the British Technical Press.[1] It will be seen from Figs. 13 and 14 that taking into account the width of the American Gas 132-kv. network, it is more extensive than the proposed network for Great Britain. A rough check of the distances involved in the network in the two cases may also be of interest. According to the best available data, the total length of lines in the proposed British network will be approximately 2500 mi. The total length of the American Gas and Electric Company system in miles is approximately 1227 and in circuit miles is approximately 1870. No data as to the total capacity that will be connected to the British network have been found available at the present time but it is doubtful that it will approach the capacity already connected to the American Gas 132-kv. grid, if the capacity of the interconnections is included.

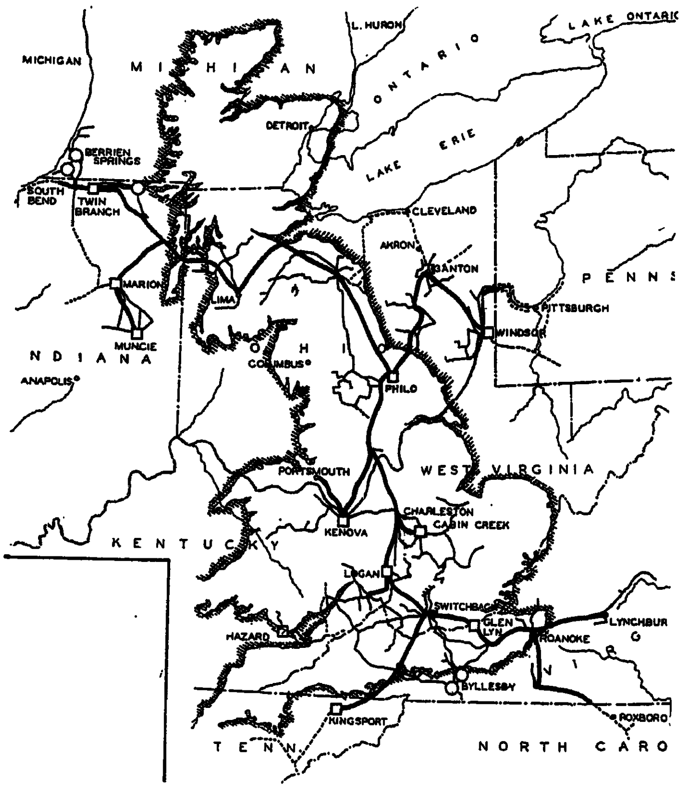


FIG. 13—OUTLINE OF ENGLAND SUPERIMPOSED ON TERRITORY OF AMERICAN GAS & ELECTRIC COMPANY

□ Primary steam plant
○ Hydro electric plant
━━━━━ 132 kv. transmission lines
――――― Other high tension lines
- - - - - - - - Other companies lines

In Fig. 12 the extent to which interconnection has been carried out between various parts of the American Gas and Electric system and foreign companies is shown. It will be noted that including the system in New Jersey and Pennsylvania, there are 23 interconnections in actual operation, and the majority of them are 24 hours a day and 7 days a week interconnections. In other words, they are in parallel all the time.

The approximate capacity of these interconnections is 340,000 kw. The combined system peak of the interconnected American Gas and Electric companies at the present time is approximately 490,000 kw. The total primary generator capacity is approximately 543,000 kw. and the capacity, taking into account steam limitations, is approximately 503,000 kw. Including the stand-by capacity, that is steam capacity that is no longer used for daily carrying of load, the above figures are approximately 690,000 kw. and 565,000 kw. respectively.

It will at once be noted that this does not allow very much leeway in primary capacity for one or two big units being down but this is exactly where interconnection comes in. As illustrative of that, we may take an emergency condition that arose on the system last November when unit No. 1 at Philo (40,000 kw.) developed internal trouble and was taken out of service for inspection and repairs. Two weeks later, before unit No. 1 was replaced on the line, unit No. 2 (40,000 kw.) developed internal trouble and was removed from service, completely shutting down the Philo Plant and reducing the capacity of the system by 80,000 kw. It was not expected that unit No. 1 would be restored to service until the following week. This was on a Saturday, so that the system did not feel it at once but when the Monday load came on the interconnections were immediately pressed into service and the following arrangements made:

1. The connection with the West Penn Power Company (Connection No. 5, Fig. 12) was called upon for an additional 15,000 kw.

2. The connection with the Ohio Public Service Company (Connection No. 9, Fig. 12) was called upon for the maximum possible load.

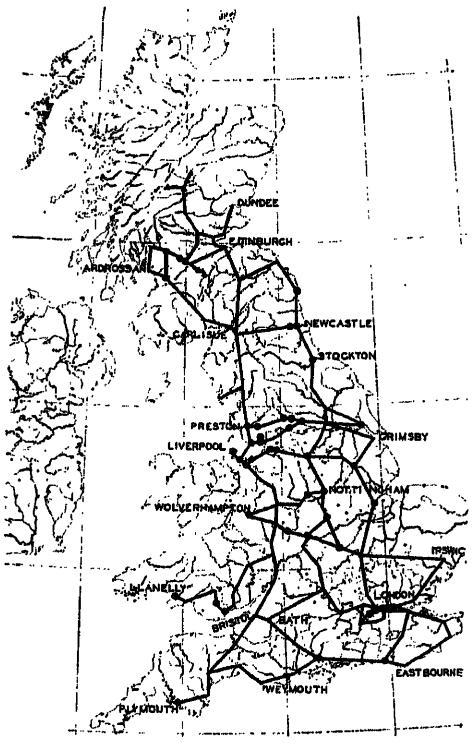


FIG. 14—SIMPLIFIED MAP OF PROPOSED ENGLISH 132-KV. GRID INDICATED IN "THE ELECTRICIAN" VOL. 99, NO. 2578, PAGE 546

● Primary Steam stations ▲ Hydro stations ━━━ 132 kv. lines

3. The Cleveland Electric Illuminating Company (Connection No. 8, Fig. 12) was called upon and Cleveland responded with a promise of all that possibly could be carried on the 66-kv. cables between Lake Shore and Oak Street Substations, allowing approximately 50,000 kw. to feed in to Akron or approximately 30,000 above the normal amount.

4. The interconnection between the Southern Ohio Power Company and The Ohio Power Company at Haydenville (Connection No. 12, Fig. 12) was called upon for 5000 kw.

---

[1] See *Electrician*, Vol. 99, No. 2578, pages 526 seq. and page 546.

5. The system south of the Ohio River was advised that not having been crippled it would have to take care of itself without any help from the north, calling upon its own interconnections if any help were needed.

6. Some of the minor interconnections were called upon for their quota.

The result of all this was that the interconnected system load of the American Gas and Electric Company subsidiaries north of the Ohio River was carried without the slightest disturbance on November 29 and until noon of November 30, when one of the Philo units was placed on to the line, and the load continued to be carried with a lesser call upon the interconnections until December 4 when the second unit came on the line. An important fact in connection with this is that no large quantities of energy were transmitted any long distance but demand was taken either directly into the American Gas and Electric system of a number of near points, or as in one or two cases the relief was obtained by a substitution process. For example, whereas the system normally supplies Akron from 20,000 to 30,000 kw. and Cleveland does the same, the coming in of Cleveland with an extra amount equal to their normal or approximately a total of 50,000 kw., made it unnecessary for the American Gas and Electric system to feed the usual amount to Akron.

On the question of diversity, it may be interesting to offer some figures. As stated, the system peak is approximately 490,000 kw. During the peak, the diversity between the interconnected companies, sub-dividing them into only four groups,[2] is approximately 15,000 kw. or 3 per cent. The diversity on the other hand, between the American Gas and Electric system and the Chicago system, due to the different characters of the loads and due to one hour difference in time, is far greater and is in the order of 60,000 kw. In other words, 60,000 kw. under the most unfavorable conditions could be sent by[3] American Gas and Electric interconnected system into the Chicago District, and Chicago could send a considerable amount in excess of that in a reverse direction.

FOREIGN INTERCONNECTIONS

No.	Foreign Company	Point of Interconnection	Kw. Capacity
	Atlantic City Electric Co.		
1	Delaware Electric Power Co...	Wilmington	8000
	Scranton Electric Co.		
2	Penn. Power & Light Co......	Stanton	50,000
3	Penn. Power & Light Co......	Pittston	4,000
4	Penn. Power & Light Co......	Peckville	15,000
	The Ohio Power Co.		
5	West Penn. Power Co..........	Windsor	90,000
6	West Penn. Power Co..........	E. Liverpool	2,000
7	Duquesne Light Co...........	South Ferry	2,000
8	Northern Ohio Power & Light Co........................	Canton	30,000
9	Ohio Public Service Co........	Canton	15,000
10	Ohio Public Service Co........	Shelby	10,000
11	Northern Ohio Traction Co....	New Phila.	3,000
12	Southern Ohio Power Co......	Haydenville	5,000
13	Toledo Edison Co............	Pemberville	10,000
14	Defiance Gas & Electric Co....	Auglaize	3,000
15	Western Ohio Railway Co.....	Lima	3,000
	Wheeling Electric Co.		
16	West Penn. Power Co..........	St. Marys	6,000
	Appalachian Electric Power Co.		
17	Carolina Power & Light Co....	Danville	15,000
	Kentucky and W. Va. Power Co.		
18	Consolidation Coal Co........	Fleming	3,000
19	Consolidation Coal Co........	Martin	5,000
	Kingsport Utilities, Inc.		
20	Tennessee Public Service Co...	Kingsport	15,000
	Indiana & Michigan Electric Co.		
21	Northern Indiana Public Service Co....................	New Carlisle	40,000
22	Northern Indiana Power Co....	Plymouth	3,000
	Indiana General Service Co.		
23	Northern Indiana Power Co....	Sweetser	4,000

In conclusion, we can say that it is our belief as a result of experience with an interconnected system and with interconnections with foreign companies extending over many years, that in addition to the many benefits of interconnection that have been so well stated in the papers under discussion, there is a number of points that are generally not brought out and these are:

1. Interconnection is a logical outgrowth of the development of the power industry. In most cases where the possibility of interconnection presents itself it is at least as rational to investigate the reasons against the interconnection as to assume that the interconnection has to be proved.

2. The actual benefits of interconnection will in the long run surpass the anticipated benefits.

3. As a general rule it will be found that the maximum benefit of an interconnection can be obtained under those conditions where power can flow back and forth or where a multiplicity of interconnections is involved, where an opportunity presents itself for substitution of energy from one point to the next and not by a direct transmission from any one extreme point of the system to the other.

P. Junkersfeld: (communicated after adjournment) Interconnection in the electric power industry was used as a part of the earliest networks, and later, beginning about 1890, to connect together d-c. networks adjacent or very near to each other for mutual assistance in maintaining approximately proper voltage at and near such points. Soon after, larger mains and then special feeders or trunk lines were used, and as soon as alternating systems and prime movers were sufficiently developed to be kept continuously and satisfactorily in synchronism under service conditions, interchange of energy took place in much larger quantities between generating stations directly, or indirectly by shifting of substation loads from one transmission line to another. It soon became necessary to have a single continuous operating agency to coordinate electrical operation, and in most instances the individuals have been called load dispatchers, and were first referred to before the A. I. E. E. incidentally in a paper presented by the writer in 1901.

Interconnections as generally understood today are maintained between different large elements of any one public-utility system, and between adjacent systems under different ownership or management to a greater and greater extent, both as to quantity of energy and as to distance. Most of the many advantages have been referred to in the various papers and in the discussions. These interconnections are usually a great operating convenience, and result in better service.

There seems to be a tendency sometimes to install such interconnections where the distance and other conditions affecting cost are such that the expenditure may not be justified by the noncoincidence of the peak loads or by the difference in generating costs on the two systems to be interconnected, or by the reduction in reserve generating capacity or by other factors. The interchange of too large a proportion of the energy required on one of two systems to be interconnected may not be profitable, or be less profitable solely as a matter of reducing generating costs than the interchange of a smaller proportion at the much higher load factor that may thus become available.

Instances like the foregoing have, however, heretofore been the exception. In the majority of interconnections the expenditure is amply justified by the many investment and operating advantages already referred to in some of the various papers and discussions. One additional illustration is the interconnection between the San Joaquin and Great Western systems in California, where a 100-mi., 165-kv. interconnection released about 20,000 kw. because of the diversity between these two systems. The sum of the two peaks on these systems was 267,000 kw. in 1926.

---

[2] Appalachian Electric Power Company and subsidiaries; Ohio Power Company and Wheeling Electric Company; Indiana & Michigan Electric Company; Indiana General Service Company.

[3] Upon the completion of the Philo-Fostoria line now under construction.

The trend to operate the most economical sources of steam or hydroelectric power at larger and larger annual load factors and better continuity of service seems to indicate an increasing number of interconnections between systems, and with properly developed load dispatching frequently affords opportunity to reduce total generating costs and also to improve the service rendered to the public.

W. A. Naudain: (communicated after adjournment) Mr. Mitchell states that on a system combining run-of-river and storage hydroelectric plants and steam plants, great savings can be made by accurate knowledge of river flows. Also careful study is being given to this by all large systems, and what the subsidiaries of Southeastern Power and Light Company have done, is typical of the other companies.

If the United States Geological Survey which is primarily interested in the making of maps, could be impressed with the value of this information to the utility companies, they might be induced to include this—furnishing these data in connection with their map data, with very little additional work to themselves.

Also the Corps of Engineers of the several geographic divisions, who are to such a large extent connected with and interested in river flows might in connection with their regular work, supply these data for public utility benefit. This would result in making river data available to the utility companies without the resultant expense, which is a product of such a survey.

N. E. Funk: I should like to say that I agree thoroughly with Mr. Lee. I think the shorter our transmission lines are, the closer we can build our generating stations to our load, the better off we will be.

I say that because with this 220-kv. interconnection, it may look as though we are following the diametrically opposite course.

The 220-kv. interconnection is rather unusual in that there is a very decided diversity in the loads concerned. Starting with the first section of the interconnection, the line between Siegfried and Plymouth Meeting, where about 50,000 kw. can be used, only as emergency because of the fact that there is only one line, the load will build up over 200,000 kw. interchange between morning and evening. A case like this is not so often found, and occurs only in this case because of the entirely differently shaped loads in the territories connected.

W. E. Mitchell: I will answer Mr. Allner's question first, as to how far back the forecasting goes. We only worked it out three years ago, therefore we had only three years to check it in. We have made our forecast for the next three years.

I may say, we think the cycle is too short. If we could get a cycle of two hundred years or two thousand years, we might know something about the rainfall and prognosticate about its cyclic variation. As it is, we are making a good guess and the method of arriving at the guess is simple and very interesting.

Now in the matter of interconnection, I am glad we are getting out of the intellectual and technical field. We are getting down to plain common sense. As Mr. Edgar has said, it is a matter that is inevitable but it isn't going to cure all sorts of troubles. Mr. Lee has hit the nail on the head. We are interconnecting where it is economically wise and we are trying to get the load and the place where the power is made closer together. After that, interconnection pretty nearly takes care of itself in the matter of improvement to service, and the reduction in idle plant capacity. Undoubtedly there is entirely too much of that all over the country today, capacity that isn't earning a dollar for any of us, and yet which with isolated properties was essential for the protection of service. After all, it is service we are selling. We can reduce that idle capacity and therefore reduce the cost of that service through interconnection wisely developed.

H. B. Gear: Replying to Mr. Allner's request for a statement of the basis of charge between the companies in the Chicago District, the amount paid consists of three parts,

(a) the capacity charge,
(b) the regular energy charge, and
(c) the surplus energy charge.

The capacity charge is paid by any company deficient in capacity or reserve, and consists of fixed charges on generating-station capacity, step-up transformers, and interconnecting lines.

The regular energy charge consists of the cost of producing and delivering the kilowatt-hours taken by any company under its deficiency requirements.

The surplus energy charge consists of charges directly proportional to kilowatt-hours (chiefly fuel) for energy taken at off-peak hours, for the purpose of realizing the advantage of lower production costs in the neighboring station than can be realized at stations in the zone where the energy taken is utilized.

It is the intent that the most economical units will be working at their highest practicable load at all hours.

Saturation Permeameter

BY S. L. GOKHALE[1]
Member, A. I. E. E.

Synopsis.—*The saturation permeameter is a device for speedy and accurate measurement of saturation value of magnetic material; that is, for measurement of the limiting value of intrinsic induction* (β) *characteristic of that material. The permeameter in its present form is of the well-known bar-yoke type, with a magnetizing coil capable of producing a magnetizing force of* $H = 4500$ *g. without excessive heating. The permeameter is equipped with a compensating coil whose function is to compensate automatically for the spatial flux enclosed by the potential coil, so that the ballistic deflections indicate the saturation value without further computation or correction. The permeameter is available for accurate measurement of saturation value of such material as is saturated well within the working range of the permeameter, i. e., with a magnetizing force of* $H = 3000$ *g., or less, which includes all magnetic material used for electrical engineering purposes except those for permanent magnets and cobalt. The saturation permeameter is also available for approximately correct measurement of B or* β*, for range of H below the saturation point as far down as* $H = 200$ *g. In speed of testing and accuracy of the measured saturation value, this permeameter surpasses all devices for saturation measurement developed heretofore.*

THE following symbols are used in this paper with the corresponding unit stated in brackets:

B	Total induction or flux density (gauss).
β	Intrinsic induction ($B - H$) (gauss).
H	Magnetizing force (gilbert per cm.; briefly g.).
H	Spatial induction (gauss).
λ	Interlinkage (Maxwell).
I	Magnetizing current (ampere).
S	Saturation value; *i. e.*, limiting value of β.
A	Cross-section of space enclosed by potential coil.
a	Cross-section of material under magnetic test.
n	Number of turns of potential coil.
M_c	Mutual inductance of the compensating inductor.
H/I	H-factor of the permeameter.
H_s	Saturation point; *i. e.*, the value of H at which β reaches the constant value S.
μ_i	Intrinsic permeability, $(B - H)/H$, or $\mu - 1$.

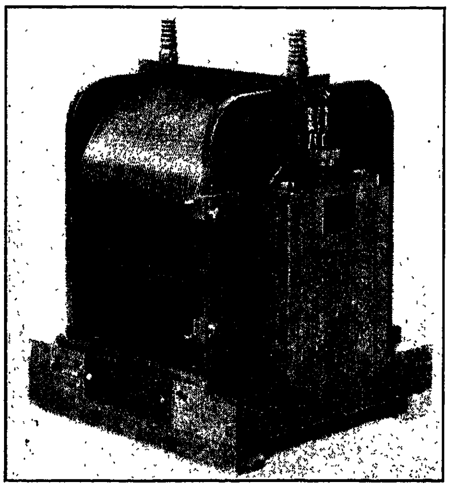


FIG. 1—SATURATION PERMEAMETER

The symbol H is ambiguous, but the ambiguity causes no inconvenience and is therefore left uncorrected.

METHOD OF TEST

For scheme of wiring see Fig. 2. The compensating inductor which is a variable mutual inductor M_c is adjusted to the required value approximately to start with, and exactly by trial as the test progresses. The permeameter is then treated like the ordinary bar-yoke permeameter, using magnetizing currents ranging from 5 to 30 amperes. If the upper part of the magnetization curve thus obtained appears to be straight over a considerable range, after making a reasonable allowance for stray points, it is assumed that the sample is really saturated for that range. The compensator is then adjusted by trial until the deflection for the supposed saturation range appears to be strictly constant. The constant value of β thus indicated is accepted as the true saturation value. The lower part of the magnetization curve is theoretically not reliable, but it has been observed that for magnetizing forces as low as $H = 200$ g., the βH curve by the saturation permeameter is in very close agreement with the corresponding part of that curve by the Bureau of Standards compensation method (Fig. 3). This value of magnetizing force ($H = 200$ g.) is therefore accepted as the lower limit of reliability of the saturation

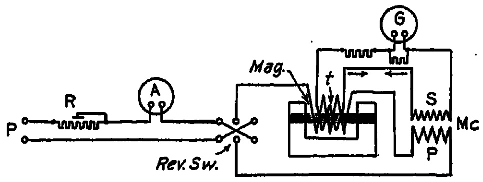


FIG. 2—WIRING DIAGRAM FOR DETERMINATION OF SATURATION

This illustrates the compensation method. It is also the connection used with slight modification in testing the range of reliability of the instrument

P = Source of power, direct current
R = Control rheostat
A = Ammeter
Mag. = Magnetizing coil of saturation permeameter
t = Potential coil for measurement of β
H' = Coil for measurement of H' located in the space between the test sample and the inner surface of the magnetizing coil (not shown in this diagram; used for test in paragraph 5, Fig. 4)
$M\,c$ = Mutual inductor—variable for compensation
P—Primary coil
S—Secondary coil
G = Galvanometer

---

[1] General Engineering Lab., General Electric Co., Schenectady, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

permeameter, although in some cases the agreement may reach down to lower magnetizing forces (Fig. 6).

DETERMINATION OF *H*-FACTOR

The function of the compensating inductor is to compensate for the spatial flux enclosed by the potential coil of the permeameter. When the compensation is exact, as shown by the constancy of the indicated value of β, the interlinkage of the secondary coil of the inductor must be equal to the spatial interlinkage of the permeameter. Expressed analytically, we have the equation

$$\lambda s = \lambda c$$

or,

$$H n A = M_c I/10$$

where λ_s is the spatial interlinkage of the potential coil of the permeameter,

λ_c is the interlinkage of the secondary coil of the compensator.

Then, the *H*-factor of the permeameter is

$$H/I = M_c/10\, n\, A$$

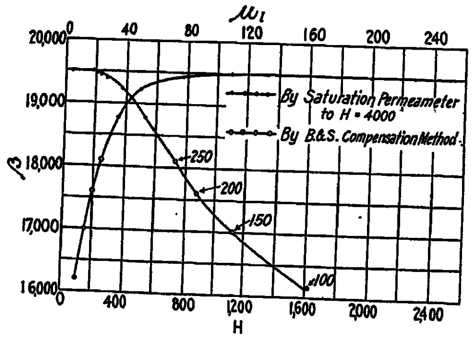


FIG. 3—CURVES OF 2½ PER CENT SILICON-STEEL SHEET

Tests on sample No. D-7. βH curve and $\beta \mu_i$ curve are shown. The numbers on the $\beta \mu$ curve indicate values of H

CONDITIONS OF RELIABILITY; BASIC ASSUMPTIONS

When the compensation is exact, as indicated by the constancy of the observed ballistic deflection, the interlinkage under measurement is constant; hence we have the equation

$$\beta n a + H n A - M I/10 = \lambda_s = \text{constant}$$

where λ_s is the value of the total interlinkage measured,

$\beta n a$, interlinkage of intrinsic flux,

$H n A$, interlinkage of spatial flux,

$M I/10$, interlinkage of compensating inductor.

By differentiation with relation to I, we get

$$n a \cdot \frac{d\beta}{dH} \cdot \frac{dH}{dI} + n A \cdot \frac{dH}{dI} - M/10 = 0.$$

$$d\beta/dH = (M/10 - n A \,.\, dH/dI)/(n a \,.\, dH/dI)$$

Assumption I. Assuming that the *H*-factor of the permeameter is constant for the range under consideration, we have the equation

$$H/I = C$$
$$dH/dI = C$$

Substituting this value in the above equation, we get

$$d\beta/dH = (M/10 - n A C)/n a C$$
$$= \text{constant}$$

Assumption II. At this point, a further assumption is made, *viz.*, that if $d\beta/dH$ is constant, the value of the constant is zero. Graphically, it is assumed that if any extensive part of the βH curve be straight, it is at the same time also parallel to the axis of H. In other words, it is assumed that no large part of the βH curve is straight without being parallel to the axis of H. With this assumption, the above equation is reduced to the two equations,

$$d\beta/dH = 0; \text{and } n A C = M/10.$$

From the first equation we get

$$\beta = \text{constant}$$
$$= S; \text{(saturation value)}$$

From the second equation we get

$$H\text{-factor} = C = M/10\, n\, A$$

The reliability of these two measurements depends upon the validity of the assumptions (1) constancy of the *H*-factor for the required test range; and (2) absence of straightness of the βH curve except when saturation is reached.

The constancy of the interlinkage by itself does not carry with it unconditionally the assurance of saturation, or the accuracy of the measurement of S, or of the *H*-factor. The equation

$$d\beta/dH = (M/10 - n A \,.\, dH/dI)/(n a \,.\, dH/dI)$$

might hold true when neither dH/dI nor $d\beta/dH$ are constant, because it may be that the variations in β due to changes in H might be balanced by changes in H due to variations of *H*-factor. It can be shown, however, that in the case of the saturation permeameter under discussion, the constancy of the observed deflection is not brought about by balanced variations of this kind. Space limitations prevent the discussion of the proof of this point.

Validity of Assumption I. There has been no method discovered for direct measurement of H at a point in the test sample, and not even for indirect measurement, except under certain favorable conditions which are not present in this case. But the value of H at a point inside of the magnetizing coil and in close proximity to the sample can be easily measured by ballistic determination of spatial flux near that point. (For test data see Fig. 4.) A study of the curves shows that the *H*-factor for the proximate space is practically constant, the deviation from constancy $(\Delta\,\theta/\theta)$ being undetectable for $I = 5$ to 20 amperes and less than one part in 1000 for the range $I = 3$ to 29 amperes. On the basis of this observation it seems reasonable to assume that the *H*-factor in the body of the sample is also constant within the same limits, because the causes that tend to produce deviation in the one case are also the only possible causes operative in the other case. It is therefore assumed that in the case of this saturation permeameter, the H-factor is strictly constant for values of I between the limits 5 to 20 amperes (H = 750 to 3000 g.) and almost constant for I = 3 to 29 amperes (H = 500 to 4000 g.).

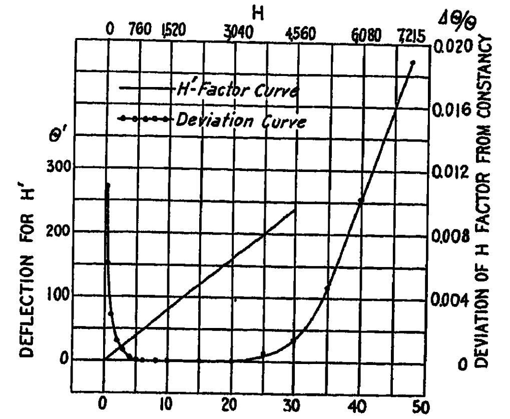


FIG. 4—H-FACTOR CURVE AND PRECISION LIMITS

Validity of Assumption II. The second assumption underlying the compensation method is, that no part of the βH curve is straight without being parallel to the axis of H at the same time. Stated analytically the assumption is that $d\beta/dH$ is not constant but varies with H, except when it (*i. e.*, $d\beta/dH$) becomes zero. The assumption is supported by numerous tests on magnetic material of various kinds, tested in the form of toroid rings to preclude uncertainty due to possibilities of error inherent in other methods of test. It was found in all these cases that the βH curve was not straight until the saturation point was reached, after which point the curve was straight and parallel to the axis of H. In no case was a large part of the curve straight without being parallel to the axis of H.[2] In order to submit the matter to a more severe test, one toroid ring was tested by the differential test. (See Fig. 5.) In this case the value of $\Delta\beta/\Delta H$ is not constant until it is reduced to zero. All reliable data thus far are in support of the Assumption II, and there are none to contradict it.

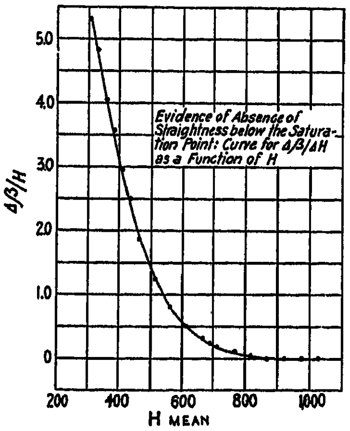


FIG. 5—CURVATURE OF βH CURVE BELOW SATURATION

The curve is for $\Delta\beta/\Delta H$ as a function of H. Standard sheet-steel toroid rings used with compensation for spatial flux

Examples of apparent exceptions are given in paragraph on Form of Intrinsic Permeability Curve (see Incidental Observations).

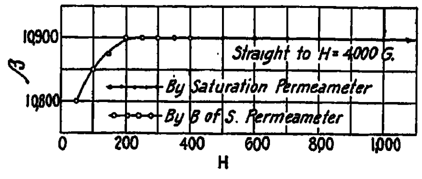


FIG. 6—βH CURVES FOR PERMALLOY ROD

Limit of Error, ± 2½ gausses

Experimental Evidence of Reliability. Fig. 6 represents the βH curve for a sample of permalloy by the saturation permeameter, this sample being selected for the reason that it can be saturated by a magnetizing force of H = 200 g., which permits its saturation value to be determined by the Bureau of Standards compensation method, whose reliability has been accepted as indisputable. The close agreement of the two βH curves by the two methods constitutes a very persuasive argument as to the reliability of the saturation permeameter. This experiment disproves the balanced compensation referred to in paragraph on Conditions of Reliability and confirms the validity of the two basic assumptions. Full discussion cannot be given in this paper.

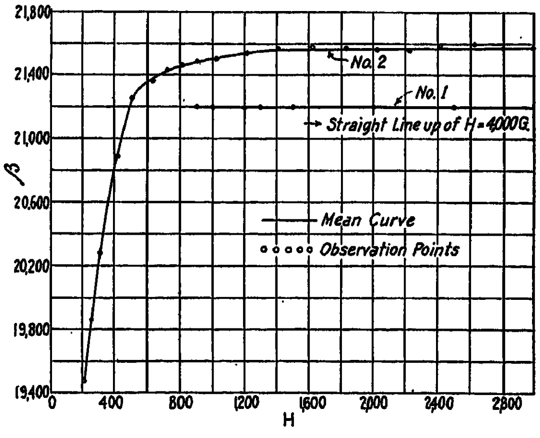


FIG. 7—SATURATION OF PURE IRON

Curve 1 was taken by the saturation permeameter; curve 2, by the Bureau of Standards isthmus permeameter. Curve 1 shows the absence of stray points by the saturation test

Internal Compensation. For further simplification of test procedure a small coil of sufficient interlinkage-coefficient, located in the proximate space, may be used in place of the compensating inductor. This method of compensation has the advantage of simplifying the test procedure, as the adjustment is made once for all and needs no further consideration.

---

[2] Gokhale, Sept. 1926, p. 853, et. seq.

INCIDENTAL OBSERVATIONS

Absence of Stray Points. The saturation part of the βH curve by the method outlined herein is free of stray points. This is due to the fact that the ballistic deflections for this part are not dependent on the exact value of the magnetizing current; any error in reading the ammeter has therefore no effect on the deflection; all the observation points for this part of the curve fall on a straight line parallel to the axis of H. In this respect this method of test surpasses all other methods. (For comparison, see Fig. 7.)

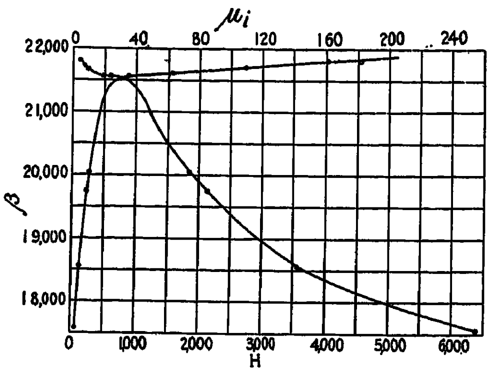


FIG. 8-1—$\beta\mu$ CURVE BY THE LONG-SOLENOID METHOD

This is the method of B. O. Peirce, Academy of Arts and Science, (Vol. 49-2, p. 139, Table 6)

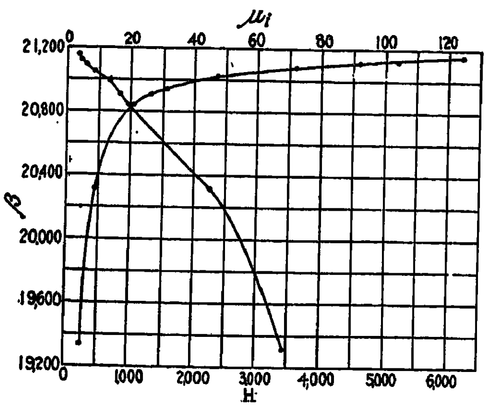


FIG. 8-2—$\beta\mu$ CURVE BY YOKE ISTHMUS METHOD

By Gumlich, *Arch. f. Elec.*, Vol. II, p. 471

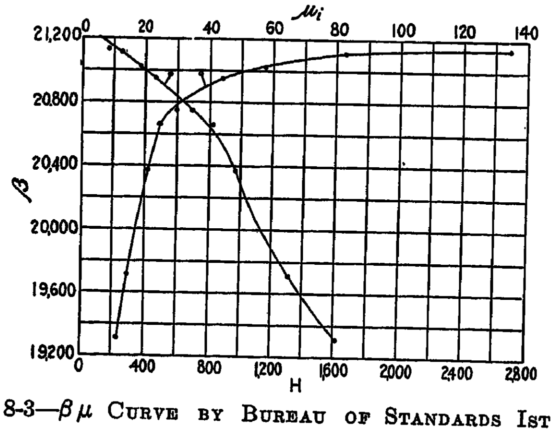


FIG. 8-3—$\beta\mu$ CURVE BY BUREAU OF STANDARDS ISTHMUS PERMEAMETER

By Cheney, Bureau of Standards. Paper No. 361, p. 633

Speed of Test. Since the accuracy of measurement of S does not depend on the accuracy of the ammeter, it is needless to waste time in accurate adjustment or reading of the current. The only reason for reading the ammeter at all is to assure that the observation points which should be sufficient in number and reasonably well distributed are actually so: the operator is instructed to waste no time and effort over accuracy of reading of the ammeter; this contributes to speed of test.

Freedom from Excessive Heating. The rapidity of test prevents excessive heating and thereby adds to the accuracy of the test. It is well known that variation of temperature constitutes one serious source of error in measurement of saturation value.

Form of Intrinsic Permeability Curve. The form of the $\beta\mu_i$ curve obtained by the saturation permeameter (Fig. 3) possesses all the characteristic peculiarities of the curve, by the compensated toroid ring method described in the *Law of Magnetization.*[3] The corresponding curves by other methods of test do not generally possess that form which indicates the unreliability of those methods (see Fig. 8). In such cases only, the βH curve is straight over a considerable range, without being at the same time parallel to the axis of H. These curves constitute the apparent exceptions to the *Assumption* II referred to in *validity of Assumption* I.

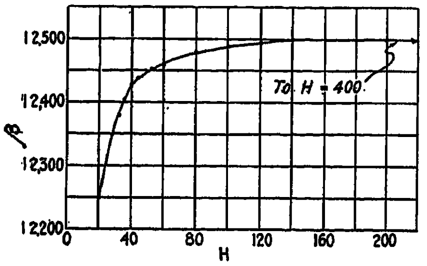


FIG. 9—SATURATION OF NICKEL-IRON ALLOY RING BY INCREMENTAL METHOD

Limit of Error, ± 2½ gausses

Demonstration of Saturation. Incidentally, we have now an experimental evidence of the phenomenon of saturation by a large number of tests on toroid rings, and therefore free from any uncertainty inherent in any type of permeameter whose reliability is unproved. In one case (see Fig. 9), the ring was tested by the method of increment which reduces the limit of observation error to ±2.5 gausses in β. In this case the variation of the curve from straightness—if there be any variation at all—does not exceed one part in 5000. This experiment therefore constitutes a convincing evidence of the phenomenon of saturation, and therefore also a confirmation of Weber's theory of molecular orientation.

Limits to the Use of Saturation Permeameter. The theory of the saturation permeameter implies two limitations to its use. First, the test sample must be capable of saturation well within the upper precision limit of the permeameter, *viz.*, $H = 4000$ g. (See Fig. 4.) For practical reasons $H = 3000$ is regarded as the upper limit. Magnetic material which does not reach saturation at this point, such as cobalt, cannot be successfully tested by this permeameter, for determination of saturation value. Second, the saturation permeameter is not theoretically available for accurate test much below the lower precision limit, (*viz.*, $H = 750$ g., see Fig. 4). But it has been observed in all cases that the curves obtained by this permeameter agree very closely with corresponding curves by the Bureau of Standards method, down to $H = 200$ g. (See Figs. 3 and 6.) This value of H is therefore considered as the lower limit of practical reliability.

---

[3] A. I. E. E., TRANS. 1926 Vol. 45, p. 1023 and Disc. p 1033, Figs. 4-0, 6-0.

CONCLUSION

From the preceding arguments, data, and curves, it follows that the saturation permeameter is speedier and more reliable than all other types of apparatus used until now for determination of saturation value.

Discussion

W. B. Kouwenhoven: I should like to know if the straight line relation between deflections for H and magnetizing current I for the range of 5 to 20 amperes of Fig. 4 of the paper will hold for all specimens tested. It would also be helpful if Mr. Gokhale would give additional data as to the size of specimens which the permeameter will hold, the number of turns, the size of wire, the resistance of the winding, and the dimensions of the device. These data will serve to make the paper more valuable.

The means for holding specimens in a permeameter are of importance as different results may be obtained by various clamping arrangements. The temperature of the specimen will also affect its magnetic characteristics and at high densities there is usually considerable heating. I should like to have Mr. Gokhale tell us something about the methods used for clamping specimens and about the temperature rise of specimens in his permeameter.

Hans Lippelt: The writer gives in his paper a quantity H_s as representing the saturation point, *e. g.*, that value of H at which β reaches the constant value S. As is well known, this value of H is, theoretically, infinitely large.

For practical purposes the distinction of being the saturation point may be assigned to a certain finite value H_s, with which, however, must be associated the fact that the coordinate value β_s is only very near to the actual saturation S of the material.

The definition of H_s should then be qualified by the statement of a conventional "tolerance" $\frac{S - \beta_s}{S}$.

Furthermore, when determining the saturation point by the saturation permeameter, its sensitiveness and that of the galvanometer should be made known. In other words, it should be stated (a) either whether the new permeameter and other parts of the outfit are sensitive enough to record changes of β at the magnetic limit set by the above tolerance, or else (b) what other tolerance determines the upper limit of the permeameter's ability.

In connection with the avoidance of excessive heating, of which the author speaks, it seems that the permeameter could be improved by using wire made of copper having super-conductivity. I understand the General Electric Company has developed such copper of superconductivity.

S. L. Gokhale: The straight-line relation of H' to I, over a wide range of H, as indicated by the zero deviation over the range of I from 5 to 20 amperes, or to express it more precisely, the constancy of the H-factor over that range, does not hold true for all material. We have found by measurement that it does not hold true for magnetic material in the hardened state, even when the sample is apparently saturated within the recognized upper working limit of the permeameter, *viz.*: $H = 3000$ to 4000 gausses. For example, Fig. 1 herewith is the apparent β H curve for two bars of magnet steel indicating a saturation value of $\beta = 17{,}000$ and 16,900 gausses respectively, and a saturation point of $H_s = 2500$ *g*. But the corresponding deviation curve, Fig. 2 herewith, is not straight which makes the saturation measurement unreliable. As far as we know until now, the failure of the saturation permeameter is limited to material for permament magnets. The deviation curve is straight over a wide range for all other material used for electrical engineering purposes in our work. These facts have been stated clearly but very briefly in the synopsis.

With reference to permanent magnets, we have found moreover, that by using a sample of smaller cross section we can obtain the corresponding deviation curve with lesser curvature. Further work in this direction is in progress. From the above observation it follows that if a sample of any material falls within the class for which the permeameter is reliable, a smaller sample of the same material will also fall in the same class. This conclusion has been verified by observation by testing two samples of the same material of very unequal cross sections.

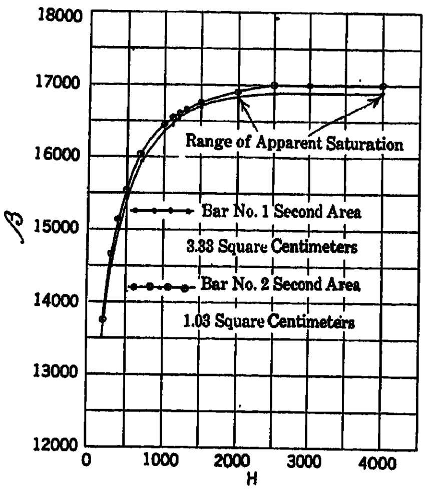


FIG. 1—TUNGSTEN MAGNET STEEL, MAGNETIZATION CURVES BY SATURATION PERMEAMETER

With reference to size of sample, this permeameter was first designed for testing bundles of sheet steel, 3 cm. broad, 25 cm. long, and about ½ cm. thick, (16 strips of 0.014 in. sheet steel). It was subsequently used for testing fewer strips down to single strips and has proved satisfactory in every case. The permeameter has been used for testing flat bars and has proved satisfactory for this purpose also except, of course, in the case of permanent-magnet material.

By using auxiliary pole shoes, the permeameter has been adapted for round rods of 6 mm. diameter, and has proved quite reliable for this purpose. A second permeameter has been constructed for testing round rods of ½-in. diameter; it is also reliable for all material except permanent-magnet steel.

With reference to clamping, it has been observed that variation due to faulty clamping is very slight, and consists of error in the indicated value of the saturation point H_s; it does not affect the saturation value. In one case on record the insertion of one thickness of tissue paper did not affect either of the two readings;

the insertion of a thicker paper apparently increased the value of H_s, but not of S. It follows from this that slight variations in clamping such as will occur in every-day work without exceptional carefulness, lead to no error at all in the measurement of S, and practically no error in the β H curve below saturation.

With reference to heating, I should have called attention to what looks like a pair of horns on the top of the permeameter (see Fig. 1 in the paper). The permeameter is provided with a cooling chamber into which is forced a blast of cold air through one of these horns, the other being used as an outlet. This method of cooling combined with the speed of testing possible with this permeameter, permits an accurate determination of saturation value with a rise of temperature of the sample not exceeding 1.6 deg. cent. The following is a sample of a complete test:

Standard Sheet Steel 0.014 in.
Tested with Saturation Permeameter

H	β	Temp.
4000	20,600	28.5 deg. cent.
3000	20,600	
2500	20,600	
2000	20,600	
1500	20,600	
1300	20,550	
1200	20,525	
1000	20,500	
900	20,470	
800	20,425	
700	20,400	
600	20,300	
500	20,150	
400	19,800	
300	19,200	
200	18,300	
100	17,050	30.1

Rise 1.6 deg. cent.

In attempting to reach higher values of H with the permeameter, the real difficulty lies not in producing the higher values of H, but in the accurate measurement of the values so produced. A reference to Fig. 4 of the paper will show that in that particular case we have actually reached a value of H = 7200 $g.$, that is, 75 per cent above the regular upper limit, but at this point the deviation of the H-factor from constancy is nearly 2 per cent, which is fatal to the reliability of the permeameter. Until this difficulty is surmounted, it is no use trying to force H to higher values by any means.

The desirability of more data is indisputable; in fact they were included in the paper when it was first prepared, but were subsequently omitted for convenience of publication. Such information will be sent to any one who chooses to go deeper into the subject.

With reference to Mr. Lippelt's question, it is not clear to me what theory he has in mind. Personally, I know of no well established theory which calls for a slow and gradual approach of the β H curve to the saturation limits. On the contrary, I have a strong bias in favor of the view that as saturation approaches, the β H curve deviates from whatever law it follows up to then, and rapidly reaches the saturation limit and then by an abrupt change follows the saturation line. I confess frankly that it is merely a bias at present, but it seems to be a very reasonable hypothesis.

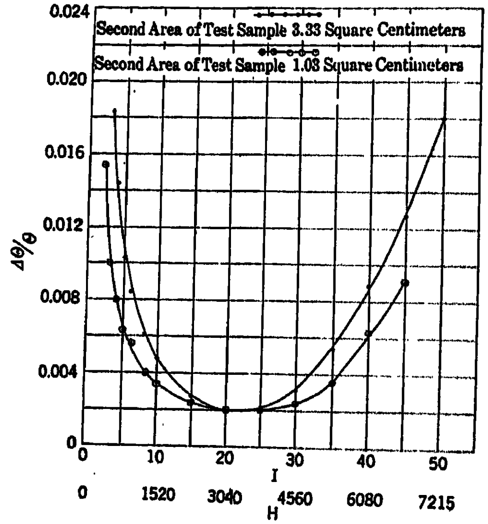


FIG. 2—DEVIATION CURVE FOR TUNGSTEN MAGNET STEEL

Leaving aside the theoretical aspect of the question and dealing with the subject in its practical aspect, the point H_s is defined as that value of H, for which the corresponding value of latent induction γ, that is, $S - \beta$, is less than the least detectable value under the condition of test. For example, in Fig. 6 the value of error is only 2½ gausses. In every case the limit is determined by the sensitivity of the galvanometer used for the particular occasion.

Compressed Powdered Permalloy
Manufacture and Magnetic Properties

BY W. J. SHACKELTON[1]
Associate, A. I. E. E.

and

I. G. BARBER[2]
Non-member

Synopsis.—*The paper gives a brief description of the manufacture of magnetic cores of compressed permalloy powder followed by information covering their magnetic properties with particular reference to their use in loading coils. Production of the powder, and its insulation, pressing, and annealing are discussed. Under magnetic properties, permeability, core loss, and modulation are treated. Curves are given illustrating the characteristics of interest in connection with the design and application of loading coils; and comparisons to corresponding characteristics of compressed powdered iron are made throughout.*

INTRODUCTION

It is now somewhat over ten years since commercial use was first made of loading coils having cores of compressed powdered iron. During this period all but a relatively small percentage of the loading coils produced for the Bell System have been of this type. The general construction of these coils, the practises employed in their application to the telephone plant, and the theory underlying these practises have been thoroughly discussed in previous papers.[3,4,5] The distinguishing feature of these coils that is of interest in connection with the present subject is the type of magnetic material used in their cores, namely, compressed powdered iron. The iron used for this purpose is electrolytically deposited in such a manner as to be readily reducible to a fine powder. This powder, after its particles have been coated with a suitable insulating film, is formed, under high pressure, into rings of approximately rectangular cross-section. One or more of these rings may be used to form a complete toroidal core, in the manner shown in Fig. 1. The method of manufacture and the properties of such compressed, powdered iron cores were described by Buckner Speed and G. W. Elmen[6] in a paper presented before the Institute in 1921.

Some years ago a nickel-iron alloy, particularly valuable for its magnetic properties, was developed.[7] This alloy, known as permalloy, was described by Arnold and Elmen[8] in 1923. Permalloy is especially adapted to many uses in the realm of communication, on account of the fact that its permeability at low fields is many times higher than for any other known, magnetic material. Until recently this material has, however, been fabricated only in the form of ductile sheets that could be blanked to various shapes for use in such telephone apparatus as relays and transformers, or in the form of wire or tape that could be applied to the continuous loading of submarine telegraph cables.

On account of the low hysteresis loss that is characteristic of permalloy, its application to the cores of loading coils seemed desirable. In the sheet or tape form, the material is not particularly adapted to this purpose. As was discussed in the Speed-Elmen paper,[6] such cores must meet the following rather severe electrical and magnetic requirements:

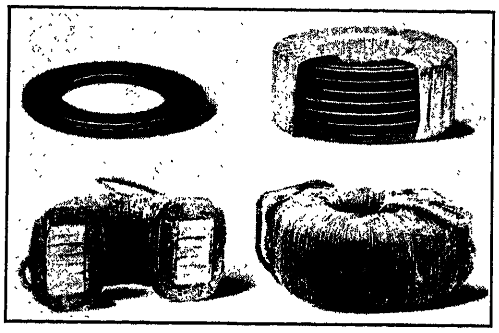


Fig. 1—Compressed Powdered Iron Core Loading Coil

1. The permeability must remain constant over the range of magnetic induction corresponding to the normal range of telephone currents, and it must always return to very nearly its original value after the application of large magnetizing forces.
2. Modulation effects caused by hysteresis must be small.
3. The total of the hysteresis and eddy-current losses must be low.

To satisfy these requirements it is necessary to have the cores operate at low magnetic inductions, a condition which is most readily fulfilled by the use of compressed powdered materials. This type of core affords, in addition, an economical means of obtaining the fine subdivision required to keep eddy-current losses at a minimum. Early attention was directed

---

[1] Bell Telephone Laboratories, Inc.

[2] Western Electric Company, Inc.

[3] *Wave Transmission over Non-Uniform Cables and Long Distance Air Lines*, M. I. Pupin, TRANS. A. I. E. E., Vol. 17, 1900, p. 445.

[4] *Commercial Loading of Telephone Circuits in the Bell System*, B. Gherardi, TRANS. A. I. E. E., Vol. 30, 1911, p. 1743.

[5] *Development and Application of Loading for Telephone Circuits*, Thomas Shaw and William Fondiller, TRANS. A.I.E.E., Vol. 45, 1926, p. 268.

[6] *Magnetic Properties of Compressed Powdered Iron*, Buckner Speed and G. W. Elmen, TRANS. A. I. E. E., Vol. 40, 1921, p. 596.

[7] U. S. Patent 1586884, June 1, 1926, G. W. Elmen.

[8] "Permalloy, An Alloy of Remarkable Magnetic Properties," H. D. Arnold and G. W. Elmen, *Jl. of the Franklin Institute*, Vol. 195, No. 5, 1923.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

toward the possibility of fabricating permalloy into a powder, and of insulating and forming this powder, under high pressure, into core parts.[9] Many difficult problems have been encountered in this work; and it was only after a large amount of experimental development that the method now in use for commercially attaining this end was evolved. The present paper describes this method and discusses the important, characteristic properties of such compressed, powdered permalloy cores.

MANUFACTURE

Manufacture of Permalloy Powder. The first part of the problem, to develop a method for economically manufacturing on a large scale the fine alloy powder, was undertaken and solved by the engineers of the Western Electric Co. Experiments were directed to the production of a brittle, grindable alloy with particular reference to the effects of the melting practise and the subsequent working of the cast alloy ingot. This work, due largely to C. P. Beath and H. M. Heinicke, has lead to the establishment of a melting and rolling practise by means of which the permalloy can be made so brittle that strips ¼ in. thick can be easily broken by hand. Such brittle material is easily reducible to a fine powder by means of a hammer mill, a ball mill, or attrition mill. Fig. 2 shows a photomicrograph of a properly rolled slab. The fine crystals and the fracture following the crystal boundaries are to be noted.

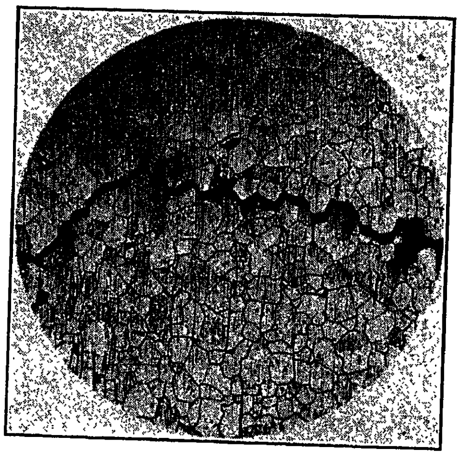


FIG. 2—PHOTOMICROGRAPH OF HOT-ROLLED BRITTLE PERMALLOY. MAGNIFICATION APPROXIMATELY 66

This method of producing permalloy powder has been applied to nickel-iron alloys ranging from 70 per cent to 90 per cent nickel. The permalloy, used for the cores discussed in this paper, contains approximately 80 per cent of nickel, as with such a composition the low hysteresis loss and high magnetic stability desired for loading coil cores are best obtained simultaneously.

Manufacture of Compressed Powdered Permalloy. Cores made from uninsulated permalloy powder have high eddy-current losses, thus making it necessary to insulate the individual particles before pressing. Early attempts at doing this were unsuccessful, partly due to the fact that the particles, hardened by the mechanical working in the ball mill, sheared through the thin film of enveloping insulation during the pressing operation. In order to reduce this shearing action it was found advantageous to soften the powder by annealing it before insulating. This annealing also has a desirable effect upon the hysteresis loss. During the annealing operation, the powder sinters together into a hard cake. This cake, however, is easily reducible to powder again by passing it through a rotary crusher and an attrition mill. The resulting annealed powder is of a silvery white color and contains a wide variety of sizes of particles. Fig. 3 shows a characteristic distribution of particle sizes in annealed permalloy powder.

The next part of the problem encountered, that of insulating the permalloy powder, was decidedly more complex than for compressed, powdered iron cores, as insulated powdered permalloy cores must withstand a heat treating operation in order to develop their characteristic properties. This is largely necessitated because of the detrimental effects produced by the high pressure used in forming the core parts. As the temperature of this heat-treating operation is above that at which the ingredients that would ordinarily be used for this purpose lose their insulating properties, it was necessary to develop a radically new kind of insulator. The fineness of the powder also required that this insulator be of extreme thinness in order to obtain in the compressed aggregate a specific gravity comparable to that of the solid metal; a condition necessary for the attainment of useful permeability values.

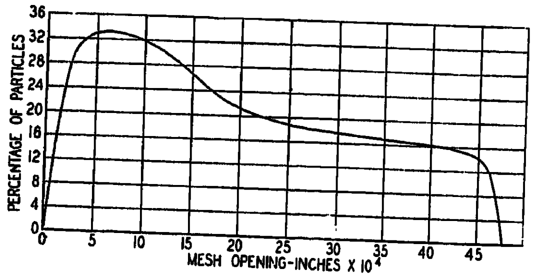


FIG. 3—DISTRIBUTION OF PARTICLE SIZES IN ANNEALED POWDERED PERMALLOY. CURVE SHOWS PROPORTION BY WEIGHT OF PARTICLE SIZES GRADED ACCORDING TO MESH OPENINGS DIFFERING BY 0.001 IN. STEPS

The method employed makes use of several refractory materials, some of which are mixed dry, with the permalloy powder, and others then added in a water solution. The amount of insulating material added is regulated so as to control within limits the permeability of the pressed cores. The mixing is done in a rotating barrel, shown in Fig. 4. This barrel is heated while it revolves, and in the course of mixing the volatile ingredients are evaporated; and finally a brownish, insulated powder is obtained. In the development of methods of insulating the powder, and making cores suitable for loading coils, the work of J. W. Andrews, R. Gillis, and A. F. Bandur, of the Western Electric Co., may be particularly mentioned.

---

[9] U. S. Patent 1523109, Jan. 13, 1925, G. W. Elmen.

The insulated, permalloy powder is formed into cores by pressing it in steel dies at approximately 200,000 lb. per sq. in. Fig. 5 shows the effect of forming pressure upon permeability. For performing this operation the improved press shown in Fig. 6 has been developed.

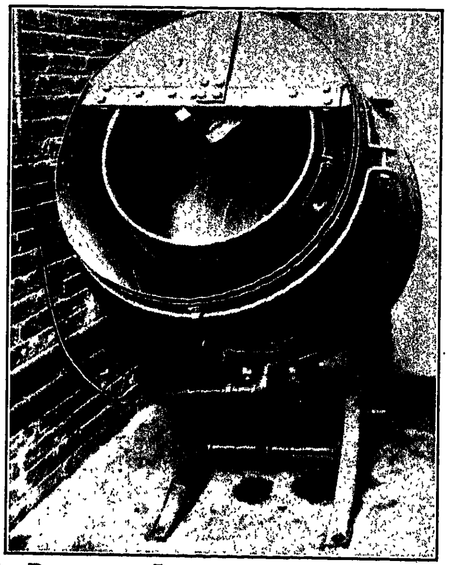


Fig. 4—Drum for Insulating Permalloy Powder

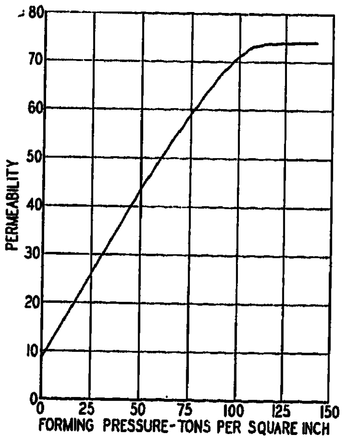


Fig. 5—Effect of Forming Pressure on Permeability of Compressed, Powdered Permalloy

This press is equipped with a pneumatic-cushion base so that, when the pressure builds up to the desired value, it is automatically relieved. On account of this construction the press is more rapid and economical in operation than the steam-hydraulic press mentioned in the Speed-Elmen paper, and gives a more uniform product. The various pressures required by different sizes of cores are attained by the proper setting of a release valve. The press can be calibrated by the use of a Crosby Indicator that records the pressure throughout the whole pressing cycle, as an ordinary gage is not able to follow this fast cyclic change.

It has been mentioned that it is necessary to anneal the compressed powdered permalloy cores. This heat-treating operation also serves to increase the mechanical strength of the pressed cores. It is necessary to control the time and temperature of this anneal rather closely, as annealing at too high a temperature or for too long a time will break down the insulator; and annealing at too low a temperature will not develop the desired low hysteresis loss. This requires that the core parts be brought up to annealing temperature at a rapid rate, and when large charges are annealed the temperature must be uniform. In present commercial practise this annealing is conducted by inserting the core rings in the type of furnace shown in Fig. 7.

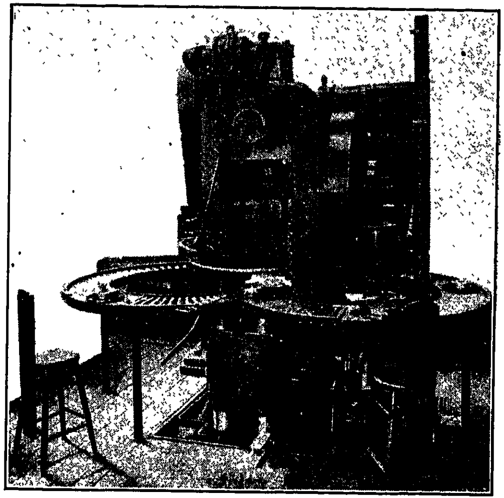


Fig. 6—High-Speed Automatic Release Press

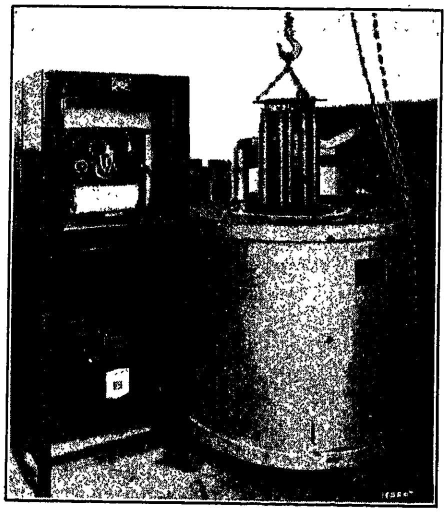


Fig. 7—Annealing Furnace

The heat-treated, compressed powdered permalloy cores contain a small amount of water-soluble matter which is accordingly removed by an extraction process.

After this operation, the cores are vacuum-dried and covered with a thin coating of shellac. The vacuum-drying operation is performed in the same tank that is used for extracting the soluble matter, in order to avoid an extra handling operation. Fig. 8 shows the equipment used for this purpose.

From the above general description it can be seen that while the process of manufacture of compressed, powdered permalloy cores is in some respects similar to the method of manufacture of compressed, powdered iron cores, in many respects the processes are very different. In order to ensure a satisfactory product it is necessary to control closely the manufacturing variables at all stages of the process. The brittleness of the rolled alloy and grain-size of the powder depend upon the melting and hot-rolling conditions that are used, and this grain-size, in turn, has an effect upon the eddy-current losses. The amount of insulator used and the purity of the insulating ingredients also greatly affect the permeability of the core. The hysteresis loss is vitally dependent upon the annealing cycle applied to the formed core parts. The effects of manufacturing variables have, however, been extensively explored. As a consequence, it is possible, by the exercise of careful control, to manufacture a product having well-defined characteristic properties, such as described below.

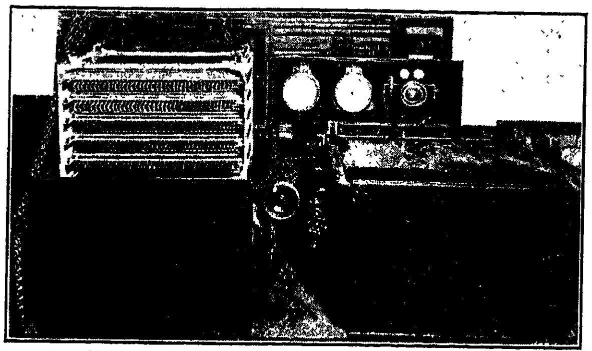


Fig. 8—Washing and Drying Tanks

Properties of Compressed, Powdered Permalloy

Specific Gravity. As noted, the relative proportion of powdered alloy and of insulating material is varied in the control of the volume permeability. Within the limits generally found necessary for this purpose, the specific gravity varies from 7.8 to 8.3. These values may be compared with the value of 8.6 for 80-20 Ni-Fe alloy in sheet form.

Tensile Strength. As is the case with powdered-iron core rings, the compressed-powdered permalloy rings are quite strong enough to satisfy all manufacturing process requirements, relative to handling and to machine winding. Tests to rupture on individual rings give an average breaking point strength of 265 lb. per sq. in. in tension.

Resistivity. The resistivity, as measured by d-c. methods directly on the rings, may vary over a considerable range without having significant effect on the magnetic properties. Typical values are from 1 to 20 ohm-cent. It is found that a resistivity below this range is an indication of incomplete insulation of the particles, and of higher than normal eddy-current losses.

Permeability. This property is of prime importance in a material for use in loading-coil cores. The function of the loading coil is to add a specific amount of inductance to the telephone circuit at each of the periodic points at which these coils are inserted serially into the line. In the ideal case, the inductance, so added, would be perfectly constant under all conditions of service, and the coils used for the purpose would introduce no additional losses into the circuit. In practise, of course, these ideal characteristics are only approximately realized. Each coil necessarily adds the d-c. resistance of its windings to that of the circuit. Power losses in the magnetic material of the core contribute an added resistance factor. The constancy of the core permeability under the conditions of normal operation and of service hazards limits the inductance constancy of the coil. At a given cost for the loading, the magnitude of the permeability defines the lowest winding resistance that can be obtained and, likewise, but more indirectly, the core-loss resistance. From the standpoint of obtaining coil windings of low d-c. resistance alone, the permeability should be as large as possible. However, it is a matter of experience, with magnetic materials now available, that high permeabilities are not associated with high degrees of constancy nor with the lowest power losses. Consequently, a compromise giving the best balance between the conflicting factors must be sought. This usually leads to permeabilities of lower value than can be advantageously employed in other types of electrical apparatus. Fig. 9 shows a typical permeability-induction characteristic for both powdered permalloy and for Grade B powdered iron. The significant differences in favor of the new material are (1) its much larger initial value, being over twice the value for the powdered iron (2) the wider

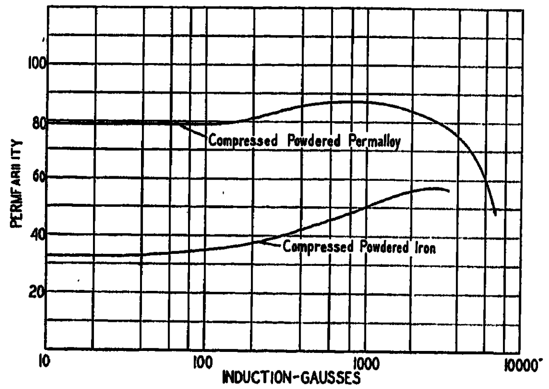


Fig. 9—Permeability-Induction Characteristics of Compressed Powdered Permalloy and Grade B Compressed Powdered Iron

range of induction over which it is substantially constant, from 0 to 100 gausses as compared with 0 to 30 gausses, (3) the smaller percentage change over a rather wide range of induction. For example, the maximum change from the initial value up to an induction of 4000 gausses is about 10 per cent as compared to 75 per cent for the powdered iron. The values exhibited by these curves are for typical samples. Differences of initial permeability between individual cores must be allowed for commercial manufacture. For the compressed powdered permalloy all cores have values between 69 to 81, allowing a maximum variation from the mean of about ± 8 per cent. Considering the varied nature of the processes through which the material for the cores, and finally the cores themselves, have to pass, in the course of which the permalloy is subjected to severe mechanical working and to large stresses (and the well-known sensitivity of permalloy in these respects), this tolerance is seen to be extremely close. In comparison, it may be noted that in other magnetic materials as ordinarily manufactured, variations of ± 30 per cent in permeability are very common.

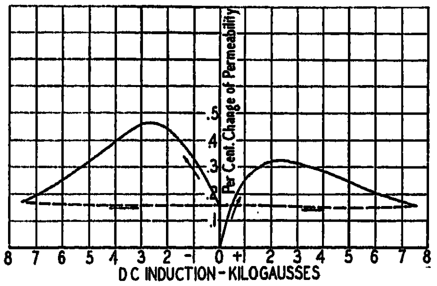


FIG. 10—RESIDUAL EFFECT OF D-C. INDUCTION ON THE INITIAL PERMEABILITY OF COMPRESSED POWDERED PERMALLOY

In the manufacture of the coils their inductances are still further controlled by adjustment of the windings.

Loading coils are designed so that over the normal range of telephone-current values the induction in the coil cores due to these currents does not exceed the initial nearly horizontal portion of the permeability-induction curve of Fig. 9. Currents of much larger value may, however, result from induction or accidental connection to sources of d-c. or low frequency power; and it is a requirement that these service hazards shall not seriously alter the coil inductance. In this respect the portion of the curve at inductions above 600 gausses is of interest. An effect of more importance, however, than that shown by this curve is indicated in Fig. 10, which shows typical changes in initial permeability due to the residual effect of previous large inductions, both positive and negative. It will be noted that the change has a maximum value in the vicinity of an induction of 2500 gausses. The changes are, however, markedly small, the maximum departure from the initial point being less than 0.5 per cent. Having once been heavily magnetized the material acquires a new mean permeability, about 0.2 per cent higher than it had initially; and departures from this value do not normally exceed 0.3 per cent.

Besides remaining constant with respect to the normally transmitted current, as well as to the effects of larger accidental currents, the loading coil must remain quite uniform in inductance over a wide range of frequencies. Voice-frequency circuits, having the highest cut-off frequency used at present for commercial service, require efficient operation up to about 6000 cycles.[10] Variations of inductance with frequency may result from the screening effect of eddy-currents in the core. Hysteresis losses, if large enough, will also cause the inductance (equivalent series value) to vary with frequency. The losses in the powdered permalloy cores are so small, however, that variations from these sources can be entirely neglected over the above range of frequency. In fact it is not until frequencies of the order of 100 kilocycles are reached that these effects begin to be significant.

Core Losses. In the application of the material to specific designs it is necessary to know the relation between the core losses, frequency, and value of the operating induction, or flux density. For the purpose of studying these relations the equation:

$$W = \eta\, v\, f\, B^{x} + \gamma\, v\, f^{2} B^{2} \qquad (1)$$

is a useful starting point. However, as the matter of final interest is the equivalent series resistance of the coil, it is convenient to reduce the above equation to the following form:

$$\frac{Rt}{L} = 8\,\pi\,\eta\, f\,\mu\, B^{x-2} + 8\,\pi\,\mu\,\gamma\, f^{2}. \qquad (2)$$ [11]

The symbols in these equations have the following significance:

W = power loss in ergs per sec.
η = hysteresis coefficient.
v = core volume in cm.^{3}
f = frequency in cycles.
B = maximum flux density.
γ = eddy-current coefficient.
Rt = equivalent series resistance due to core loss.
L = inductance of coil in henrys.
μ = permeability
x = hysteresis exponent.

The first terms of the right-hand members of the two equations give, respectively, the hysteresis loss and the equivalent series resistance caused by it, and the second, the eddy-current loss and its equivalent series resistance. Considering the latter it is evident that the effective resistance due to eddy-current losses is independent of the operating induction; that is, a coil of given inductance having a constant permeability core will have the same resistance due to eddy-current loss, regardless of whether it is made small or large. This, especially for the higher frequencies where the eddy-current losses may predominate, is a serious limitation to the design of high-efficiency coils. The factor $8 \pi \mu \gamma$ is a constant for the material that defines its quality from the eddy-current loss standpoint. To insure low eddy-current losses, it is necessary to make this factor relatively small. In the case of the powdered permalloy this is accomplished not only by reducing the eddy-current coefficient γ by the use of insulated finely ground particles, but also by limiting the working permeability of the compressed core. Table I-*A* lists averages values of eddy-current coefficients and the product $\mu \gamma$ for the powdered permalloy and for grade B powdered iron. The coefficient for the permalloy material is nearly 50 per cent less than that of the powdered iron, in consequence of which the product $\mu \gamma$ is only 38 per cent larger—even though the permeability is more than doubled.

---

[10] Shaw-Fondiller. *Loc. cit.*

[11] For the derivation of this equation reference is made to Speed-Elmen, *Loc. cit.*

TABLE I
MAGNETIC PROPERTIES OF COMPRESSED-POWDER CORES AT LOW INDUCTIONS

A. Permeability and Eddy-Current Coefficient

	Permalloy	Grade B iron
Permeability—μ (mean).......	75	35
Eddy-current coefficient—γ.....	0.0021×10^{-6}	0.0035×10^{-6}
Product—$\mu . \gamma$..................	0.16×10^{-6}	0.123×10^{-6}

B. Hysteresis Loss

Induction (B_{max})	Loss, ergs per cm.³ per cycle Permalloy	Iron (see note)
1 Gauss......................	0.017×10^{-4}	0.064×10^{-4}
2 "	0.08	0.33
5 "	0.64	3.5
0 "	3.7	23.8
15 "	10.7	75.5
20 "	23.5	172
30 "	72	

Note: Hysteresis loss values for the powdered iron are about 8 per cent lower than those given by Speed and Elmen; this reduction being due to a small improvement made in the Grade B material subsequent to the publication of the Speed-Elmen paper.

Table I-*B* gives data showing the hysteresis loss in ergs per cu. cm., per cycle, for a typical range of flux densities. These data are plotted in Fig. 11. On account of the large ratios represented, logarithmic scales are used so as to open up the plot at the lower values. The curves show a slight departure from linearity, indicating that the hysteresis exponent X is not a constant even for the small inductions represented. The curves for the two materials are, however, nearly parallel showing them to have at each point about the same exponent. The outstanding difference is the large displacement between them. For a given hysteresis loss powdered permalloy is capable of operation at much higher densities.

The factor ηB^x of the hysteresis term of the loss equation evidently is the loss per cubic centimeter, per cycle. Hence, dividing the latter by B^2 there is obtained the variable part of the corresponding term of the equivalent resistance equation, that is ηB^{x-2}. Such values multiplied by the mean permeability are plotted in Fig. 12. The ordinates of these curves multiplied by the factor $8 \pi f$ give the hysteresis loss resistance in ohms per henry for the frequency and flux density desired. Here the outstanding superiority of the powdered permalloy is more clearly evident, the ratio of resistances due to hysteresis losses mounting very rapidly with increasing density.

These curves show at a glance the density at which hysteresis loss resistances are equal. Of course, it is often more important to have the total core loss resistances equal at a particular frequency. Since the eddy-current resistances are, as previously mentioned, constant with respect to flux density, it follows that their difference is also constant. Hence, a curve drawn parallel to, and displaced from, the hysteresis loss curve along the loss axis to an amount proportional to this

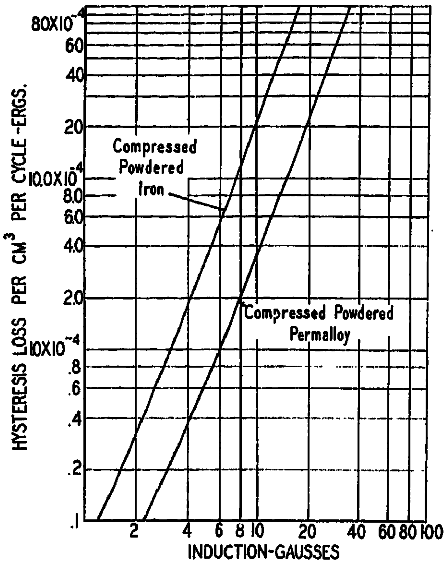


FIG. 11—HYSTERESIS LOSS CHARACTERISTICS OF COMPRESSED POWDERED PERMALLOY AND GRADE B COMPRESSED POWDERED IRON

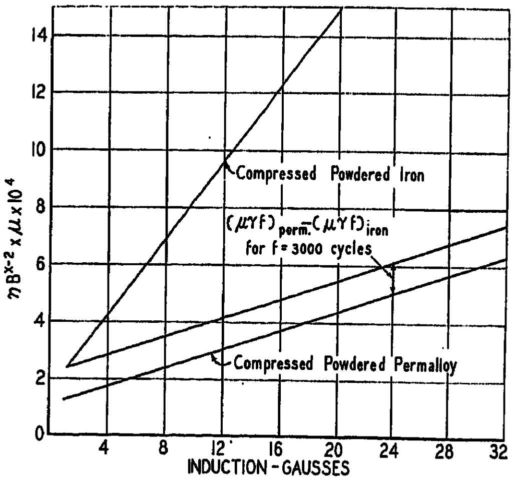


FIG. 12—RELATION BETWEEN A-C. INDUCTION, B_{max}, AND HYSTERESIS LOSS RESISTANCE FACTOR, $\eta B^{x-2} \times \mu$, FOR COMPRESSED POWDERED PERMALLOY AND COMPRESSED POWDERED IRON

difference gives the densities at which total core-loss resistances are equal. Such a curve is included in Fig. 12 for a frequency of 3000 cycles, this being slightly above the maximum frequency which certain powdered-iron core loading coils were designed to transmit efficiently.[12]

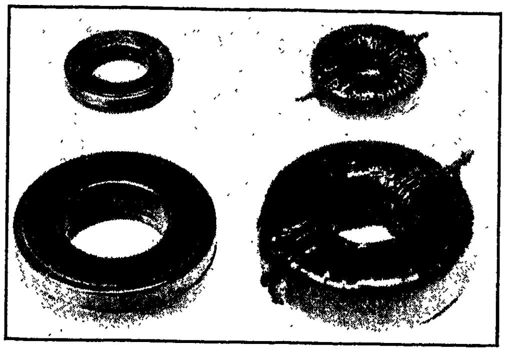


Fig. 13—Right—Compressed, Powdered Permalloy and Compressed, Powdered Iron Core Loading Coils of Equal Efficiencies. Left—Cores Used in these Coils

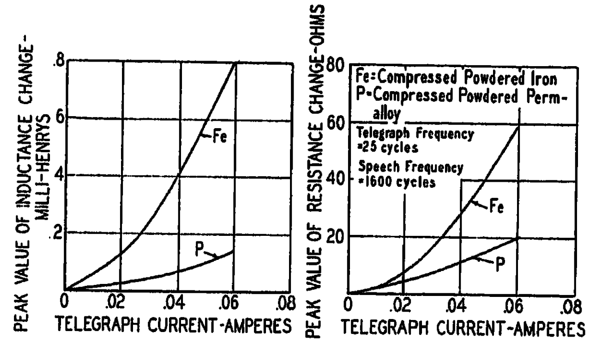


Fig. 14—Relation between Current and Third Harmonic Modulation Product for Coils having Cores of Compressed Powdered Permalloy and Compressed Powdered Iron

It is interesting to note here the reduction in core volume that can be effected by the substitution of the powdered permalloy for the previously standard powdered iron, equal or higher efficiency as to core loss resistance being obtained. Assume a toroidal core of uniform cross-sectional area A and of mean circumference l, wound with N turns of wire.

$$\text{Since } L = \frac{4\pi N^2 A \mu}{10\, l},\ N^2 = \frac{10\, l\, L}{4\pi A \mu} \text{ approx.} \tag{3}$$

$$B = \frac{4\pi N I \mu}{10\, l} \tag{4}$$

$$\text{Combining (3) and (4) } B^2 = \frac{4\pi \mu L I^2}{10\, l A}$$

$$\text{Substituting } V \text{ for } l A,\ B^2 = \frac{4\pi \mu L I^2}{10\, V} = \frac{K\ \mu}{V}$$

$$\text{And} \qquad V = \frac{K\ \mu}{B^2}$$

That is, for a given inductance, core volume is proportional to permeability and inversely proportional to the flux density squared.

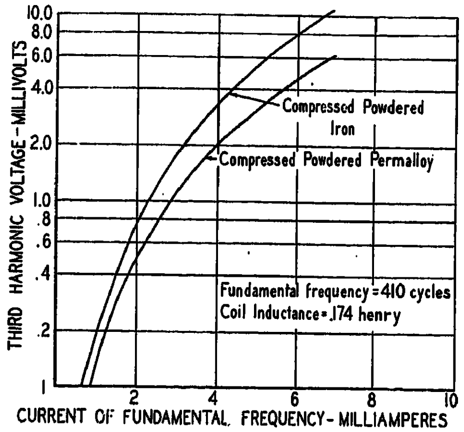


Fig. 15—Telegraph Flutter Effects in Coils Having Cores of Compressed Powdered Permalloy and Compressed Powdered Iron

A largely used toll cable circuit loading coil of the compressed powdered iron core type operates at an average flux density of 5.6 and the cut-off frequency of the circuit is about 3000 cycles. From Fig. 12, the flux density for a permalloy-core coil to have the same core loss resistance at this frequency would be 19; hence the core volume of the latter would be $\frac{75}{35}\times\left(\frac{5.6}{19}\right)^2 = 0.187$ times that of the former, a ratio of about 1 to 5. On the other hand, the lower hysteresis loss characteristics of the permalloy material may, without increase in cost, be utilized to give coils of higher quality not only in respect to effective resistance, but also in respect to their tendency to produce "flutter"[13] when used in circuits having joint telephone and telegraph service, and to introduce inter-channel modulation in multi-frequency carrier telegraph systems. For the initial application of this material to commercial use, an intermediate course is being followed; that is, coils both lower in cost and better in quality in all of these respects have been developed. For the two main types of facility required, core volumes have been decreased by 70 per cent in one case and by 85 per cent in the other. Fig. 13 illustrates the larger of these reductions in core and coil sizes. Combined with these reductions in size, and consequent savings in cost, there are appreciable improvements in effective resistance and also, as noted below, in flutter and modulation.

Modulation Effects. Under this heading are included the effects mentioned above as telegraph flutter and also what is more commonly recognized as a modulation effect, namely the production of harmonic electromotive forces by a varying flux of fundamental frequency. As these effects are closely associated with hysteresis characteristics it would be expected, from the general data previously given, that the permalloy material would be decidedly superior to the powdered-iron material. In view of the dependence of hysteresis loss on the working flux density, as shown by Fig. 11, it is clear that the specific results obtained can be controlled by design. For a more definite comparison therefore Fig. 14 is shown, giving the relation between the line current of fundamental frequency and the induced, third harmonic voltage for two standard designs of loading coils, one having a powdered-iron and the other a powdered-permalloy core. A reduction of from 30 to 40 per cent in the third harmonic product is obtained by the use of the permalloy core even though it has only 30 per cent of the volume of the iron core and is operated at about 3 times the flux density. Fig. 15 shows the relation between telegraph current and peak values of inductance and effective resistance increments occurring in the speech-frequency circuit in each cycle of the telegraph current for the same coils. Again marked improvement is effected through the use of the permalloy material.

---

[12] See Table X, Shaw-Fondiller, Loc. cit.

[13] *Hysteresis Effects with Varying Superposed Magnetizing Forces*, W. Fondiller and W. H. Martin, Trans. A. I. E. E., Vol. 40, 1921, p. 443.

SUMMARY

An improved, magnetic material eminently suited for use at audio frequencies, for example, in the cores of loading coils, has been described. Its permeability at low inductions is of sufficiently high value to permit the design of small coils with relatively low-resistance windings, and is constant to a marked degree under all conditions of service. Its eddy-current and hysteresis losses are much smaller than those of any other magnetic material of comparable constancy of permeability. These characteristics have made possible the development of new loading coils for exchange area and toll cable loading of about 1/5 and 1/3 the size respectively of former coils, not only with no sacrifice as to quality but, actually, with very appreciable improvements in transmission efficiency, inductance stability, and modulation.

Despite the high cost of nickel as compared with iron and the relatively large proportion of it used in the permalloy, the reduction in coil sizes results in large savings in manufacturing and installation costs. It would not be possible, of course, to effect these savings were it not for the fact that the net cost of manufacturing the cores is sufficiently low. A method that accomplishes this result has been described. In the development of this method difficult technical problems involved in powdering the permalloy, and in insulating the powder, so that subsequent high-pressure forming and high-temperature heat-treating operations could be applied have been solved. The processes are adaptable to economical, large-scale production and at the same time can be controlled so as to insure a highly uniform product meeting the stringent requirements for a loading-coil core material imposed by recent long-distance telephone circuit developments.

Discussion

F. B. Jewett: The commercial use of permalloy-dust-core loading coils as successors to iron-dust loading coils, which in turn were improvements over and successors to the earlier types of iron-wire-core coils, has brought about a number of very important improvements which have not been particularly stressed in the paper, which is primarily concerned with a description of the manufacturing features and physical characteristics of the new core material. I should like to draw attention to two of these important service features, one of which affects the local distributing plant in our larger cities, and the other our very long loaded cables such as those connecting New York, Chicago, and St. Louis.

In the local distributing plant, one of the principal advantages of permalloy-dust loading coils is in their very small size, which permits much smaller cases to be used for potting a given number of coils and allows a much larger number of coils to be potted in the largest cases, thus resulting in a great conservation of space in the underground loading vaults. It is a formidable job to tear up city streets and to put in new structures. A large part of the underground construction in a city like New York is more or less permanently placed. There is a duct system, which was placed in past years, to amplify which would require the opening of the streets and the laying of new ducts.

Since loaded cables are used in our cities, there are, in addition to the ducts, man-holes, called loading vaults, designed to house the pots containing the loading coils. In many places, a few years ago, we were potentially confronted with an early necessity for materially enlarging these housings, either by the building of more and still larger vaults under the city streets or by going off the city streets onto private property. With the coming of the permalloy coil and its small dimensions, and the smaller potting space required per coil, we find it possible to place a considerably greater quantity of coils in the existing structures, and so not only obviate the cost of making changes but also the great inconvenience to street traffic. In the large metropolitan areas, therefore, with their great congestion of circuits, running up to tens of thousands or more along the main routes, we have an advantage which is even more important than the cost saving of the coils themselves.

The other feature which concerns long-distance service is interesting because it is part of a progressive change in the type of line which we use.

In the early days, the telephone plant was on the single-wire, ground-return basis. That had to be abandoned in favor of the all-metallic or two-wire circuit, partly for telephone reasons and partly because of inductive interference troubles which followed the great growth of power and light circuits. For many years our whole telephone engineering was based on the assumption of a two-wire form of communication line. With the coming of amplifying devices, the so-called telephone repeaters, and the extension of these long circuits to very great distances, there was introduced a form of line known as the four-wire circuit. We had gone from one to two wires, for a circuit, and we were proposing to go from two to four, in which one pair of wires served to carry, say, the west-bound conversation, and the other pair the east-bound conversation.

It is not the time and place here to go into the technical discussion of why that was the desirable thing to do, beyond the statement that it resulted in a more flexible and more efficient plant to separate the ordinary two-wire local circuit at one end into a four-wire toll circuit, and at the distant end to recombine it again into a two-wire circuit. This made possible the use in each of the four-wire circuits of one-way repeaters having a very high amplification. With the four-wire circuit, roughly speaking, the same amount of copper for a given distance is involved in a telephone cable as with the two-wire circuit. For instance, over a given distance with a two-wire circuit we use 16-gage copper, while on the four-wire circuit we use 19-gage copper wire—exactly the same amount but drawn into four instead of two wires. Since, with cable, the cost is largely made up of material values, the copper and paper and the lead encasing sheath, the inherent cost of the four-wire against the two-wire circuit is nearly the same, but when you begin to add loading coils on these very long circuits you are adding something to the material cost of the cable itself, which is not an inconsiderable part of the total cost. So with the higher priced loading coils, despite the other advantages of the four-wire circuit, there was a tendency to restrict its use to conserve cost, because of the loading coils themselves. But with the introduction of the permalloy-dust-core coils, which are not only much smaller but considerably cheaper, there is introduced a factor which makes it possible for us on long telephone cable circuits to go further in using the four-wire type of structure.

Bancroft Gherardi: In round figures, today, we have in our plant 3,000,000 loading coils. That is to say, for every 6 telephone instruments connected with the Bell System, there is one loading coil, and we are adding to that number at the rate of about 700,000 a year, the same rate with which we are adding telephone stations connected with the system. So it is evident that the loading coil, not only in the results which it produces but in the numbers used, is a factor of major importance in our plant.

B. A. Behrend: It will be interesting to recall the first suggestion of the use of powdered iron for increasing the inductance of a telephone or telegraph circuit without materially increasing the resistance. In his "Electrical Papers," vol. II, p. 123, 1902, Oliver Heaviside makes this statement: "One way is with my non-conducting iron . . . . . an insulator impregnated with plenty of iron dust." Further references occur on p. 275 in the same work where Heaviside states that he "rolls the resulting compound, when in a slightly yielding state, under considerable pressure, into the form of solid round cylinders." In "Electromagnetic Theory" vol. I, p. 443, he suggests again "loading the dielectric itself with finely-divided iron" and he calls this his "ironic" circuit. This reference occurs within a page of the famous suggestion of using "inductance in isolated lumps" for loading cables or circuits though there appears no suggestion that the cores of these loading coils should be made of this powdered iron. With the increasing appreciation of the theoretical work of Heaviside's this historical reference may be of interest.

Wm. Fondiller: There are one or two points that are mentioned in the paper that I want to emphasize, because I think that they are rather important.

First is the matter of the higher permeability of the powdered-permalloy cores as compared with the powdered-iron cores. You can get high permeability, by suitable heat treatment of any magnetic material, but unless at the same time you can keep down the losses in the case of loading coils that increase in permeability is of no avail. The fact that it has been possible with powdered permalloy to more than double the permeability that was available with powdered iron, marks a really important accomplishment, particularly when it is coupled with extremely high magnetic stability. The magnetic stability is evidenced by the fact that up to an induction of $B = 100$, the permeability is almost absolutely constant, a performance not attained with any other magnetic material. It is particularly striking in view of the seemingly anomalous fact that in sheet permalloy you get enormous changes in reduction with small changes in value of H. The flat characteristic of the powdered-permalloy cores is due to the beneficent effect of numerous air gaps introduced by the core design.

It is also significant that with a core with 80 per cent nickel in it, a material which costs over ten times that of iron, in ingot form, it has been possible to produce loading coils having better properties at a lower cost, by virtue of a great reduction in the amount of core material and copper windings.

I want to mention one other thing of interest. The powdered permalloy core has practically the composition, that is, between 80 and 81 per cent nickel, that was pointed out by Dr. McKeehan to be the alloy which would have zero magneto striction. To this fact Dr. McKeehan attributed the extremely low hysteresis loss of permalloy.

Eugene Herzog: Can this same type of material be extended to uses in other electrical apparatus? I see from Fig. 9 that the permeability of this material has an induction of about 2000 and from there drops off very rapidly, and as in most power apparatus a much higher induction is used, I should like to ask the authors whether they have made any attempt to develop a material which could be used successfully at a higher induction.

W. J. Shackelton: In response to the last question, I will say that we have not made any particular attempt as yet to apply this material to other fields than the telephone field. The fact that its permeability over a wide range of induction is low as compared to the values ordinarily used in power apparatus would lead me to believe that its chance for application in this field would be rather small. Where its permeability is not a limitation, however, it can be used even at fairly high inductions.

The Effect of Humidity on the Dry Flashover Potential of Pin-Type Insulators

BY J. T. LITTLETON, JR.* and W. W. SHAVER*
Associate, A. I. E. E. Non-member

Synopsis.—This paper shows that humidity affects considerably the flashover voltages of porcelain and Pyrex insulators. The flashover potential rises as the humidity is increased. The tests were made on pin-type insulators and on rods. The results show that this rising characteristic is a surface effect which varies with the absolute water content of the surrounding atmosphere.

INTRODUCTION

IT is probably generally recognized that the humidity or water content of the atmosphere has some effect on the flashover potential of insulators. The A. I. E. E. specifications for testing insulators[1] state that relative humidity measurements should be taken at the time of the test and recorded as part of the data. No correction factor for this condition has been worked out, however, and little seems to be known as to the quantitative effect of humidity changes. It seems to be a general belief that an increase in humidity leads to a decrease in flashover. Peek[2] has stated that humidity does not affect the flashover potential of sphere-gaps. Schwaiger[3] has given data on porcelain and glass rods in uniform tangential fields, showing a lowering of flashover potential with increase of humidity, and curves showing an increase in flashover potential on large insulators and constant flashover potential on small insulators with increase of humidity. Accordingly, it seemed worth while to determine the quantitative relationship between humidity and flashover potential on different types of insulators and insulators made of different materials, and, if possible, to work out a correction for this condition.

While the work on this problem has not been completed, inasmuch as no correction factor has been completed, it is believed that the measurements obtained are of sufficient value and interest to justify their presentation.

EXPERIMENTS

The electrical equipment consisted of a 150-kv-a., center-grounded, 60-cycle transformer with maximum voltage of 300 kv., controlled by an induction regulator. Flashover voltages were measured in either of two ways; (1) by a voltmeter previously calibrated with a sphere-gap, or (2), directly by the sphere-gap in parallel with the insulator with suitable high resistances on each side of the gap to prevent errors due to oscillations from the insulator under test.

The insulators were mounted and the tests conducted as nearly as possible according to the A. I. E. E. specifications for testing insulators.[4] The tests were made in a wooden box 6 ft. square and 5 ft. high with a hinged cover, all lined with waterproof paper. These dimensions gave less than the standard clearance for the insulator mounting so that in this respect the tests were not made according to the standard specifications. This condition did not noticeably affect the results, as measurements made in the box agreed with those taken in the room for comparison. The iron pipe holding the insulator pin ran diagonally through the box and projected through the walls so that the one lead was connected to it on the outside. The other lead to the half-inch brass rod tied in the line groove was insulated from the chamber wall by a large bell shaped pyrex insulator.

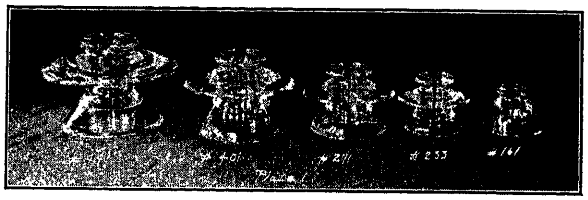


FIG. 1—PYREX INSULATORS USED IN TESTS

FIG. 2—PORCELAIN INSULATORS TESTED

The humidity in the test chamber was controlled by the introduction of dry steam circulated throughout the box by a fan. Because of small leaks it was found necessary to introduce steam continuously in order to maintain a given humidity. Humidity measurements were made by means of a dry and wet bulb hygrometer suspended in the current of air from the fan, just inside a window through which the readings were taken. These results were converted to absolute humidity by means of the data given in the Psychrometric and Smithsonian tables.[5]

The experimental procedure was as follows; the insulator was mounted on the pin in the test chamber

---

*Both with the Corning Glass Works, Corning, N. Y.

1. For all references see bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-17, 1928.

and a standard dry flashover test made with the natural humidity of the atmosphere. Steam was then introduced in the chamber and the humidity thereby increased and maintained at a constant value for 15 min. A second flashover test was then made and the cycle repeated. If it were desired to decrease the humidity after it had reached a point near saturation the cover of the box was raised and fresh air admitted. So far as possible, a given series of tests was made at a constant temperature, but on account of the steam used in the test, the temperature slowly increased, in an extreme case changing by as much as 4.5 deg. cent. However, temperature and pressure as well as relative humidity were recorded for each test and a correction applied to reduce the values to those for 25 deg. cent. and 760-mm. pressure.[6]

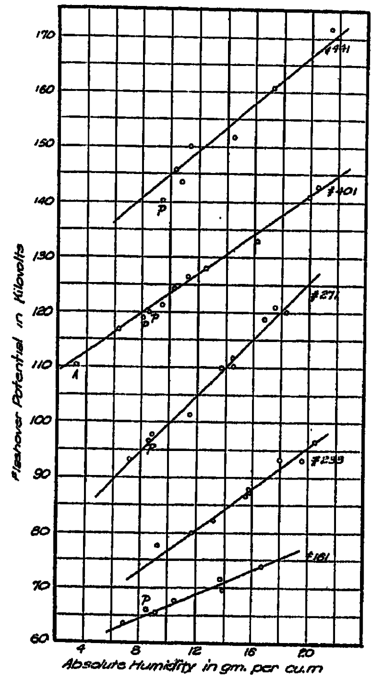


FIG. 3—VARIATION OF FLASHOVER POTENTIAL WITH HUMIDITY, ON PYREX INSULATORS

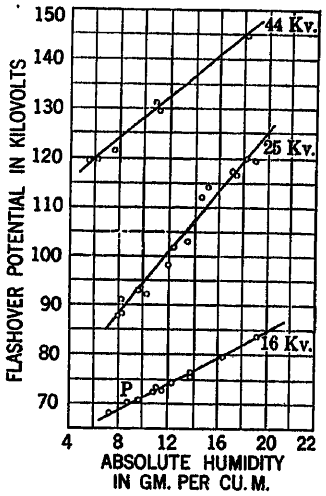


FIG. 4—VARIATION OF FLASHOVER POTENTIAL WITH HUMIDITY, ON PORCELAIN INSULATORS

Measurements were made upon different types of Pyrex pin-type insulators (see Fig. 1) and upon porcelain pin-type insulators, (Fig. 2), varying from 16- to 44-kv. rating. Tests were also made on the effect of humidity on the flashover potentials between No. 6 copper wire electrodes wrapped around Pyrex rods of various diameters, and at various spacings. In addition, in order to determine whether or not the phenomenon was merely due to the humidity effect on the electrodes, measurements were made on the flashover potentials between a one-inch steel bar bent in the shape of a hook, and a loop of No. 8 copper wire, as well as between two loops of No. 6 copper wire.

RESULTS

Figs. 3 and 4 show the results of a series of tests on the different Pyrex insulators and porcelain insulators, respectively. In every case there was a marked increase in the flashover potential when the humidity was increased. These curves do not show the maximum variations that have been observed, as considerably greater variations are possible when the experiments are made at a higher temperature, the maximum possible water vapor content in that case being much greater. In general it was found possible to increase the flashover potential of both Pyrex insulators and porcelain insulators about 30 per cent over the value at a low humidity.

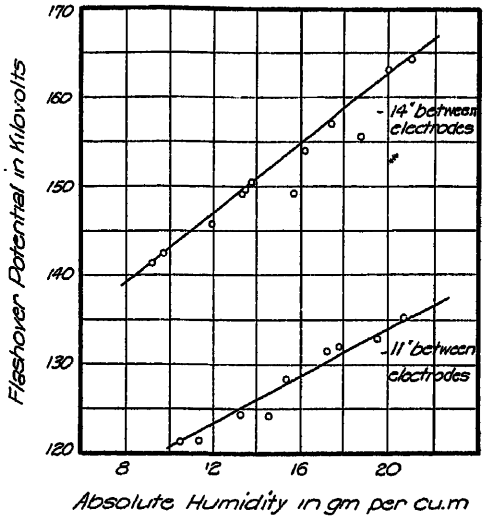


FIG. 5—VARIATION OF FLASHOVER POTENTIAL WITH HUMIDITY, ON PYREX ROD, 0.9 IN. IN DIAMETER

The results of some of the experiments on Pyrex rods are shown graphically in Fig. 5 and again, increased humidity caused increased flashover potential. The results shown were obtained with a rod 0.9 in. in diameter and with the spacings between the electrodes as indicated on the graphs.

Fig. 6 shows the results of the measurements on the flashover potentials between the loop of No. 8 copper wire and the one-inch steel rod bent in the form of a hook and between the two loops of No. 6 wire. In the first case, they were spaced so as to give a flashover potential about the same as that of a 44-kv. insulator; and in the case of the loops of No. 6 wire, the spacing was the same as when they were wrapped around a pyrex rod. From the slopes of these curves, it is quite evident that the humidity effect on the flashover potential of an insulator and of a Pyrex rod cannot be attributed to any effect on the electrodes.

DISCUSSION AND CONCLUSIONS

As mentioned above, the curves show humidity values expressed in absolute units, since such units bring the measurements taken under different temperature and pressure conditions more nearly in agreement with each other than do those of relative humidity. Schwaiger[7] shows data plotted on a relative humidity basis which also come out as a straight line relation if temperature and pressure are kept constant, but in order to correlate data made at different temperatures, or to work out a correction factor, it is necessary to use absolute humidities. In addition he shows that in the neighborhood of 100 per cent relative humidity the flashover potential falls very rapidly, this effect beginning at even 80 per cent humidity on certain insulators. This decrease in flashover potential was noted in our experiments at humidities above 90 per cent, but it is believed that this was due to condensation on the insulator surface, which could be avoided by having the insulator at the proper temperature. Schwaiger's results indicate no humidity effect on flashover potential with an insulator of approximately the same dry flashover potential as the Pyrex insulator No. 161 and the 16-kv. porcelain insulator. Our observations made on the latter insulators indicate a very definite effect and one which is also of considerable magnitude. Weicher[8] investigated the effect of humidity on the flashover potential between points and 2-cm. spheres so spaced as to give a flashover potential approximately the same as indicated in Fig. 6, and found that the flashover potential increased with increasing relative humidity. This does not agree with our observations on the flashover potentials between the two loops of copper wire and between the copper wire and the pin in which no humidity effect was observed.

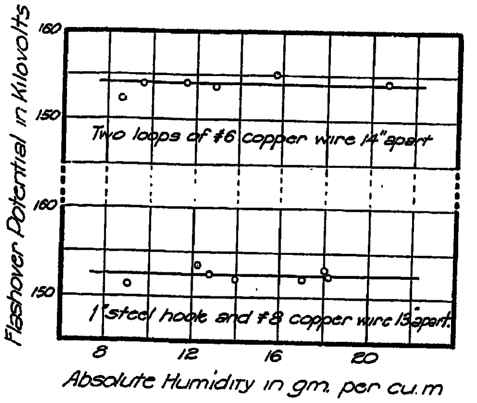


FIG. 6—FLASHOVER POTENTIAL BETWEEN METAL WIRES AND BETWEEN HOOK AND WIRE

Above, two loops of No. 6 copper wire, 14 in. apart.
Below, a 1-in. steel loop and a No. 8 copper wire, 13 in. apart

The question as to whether the curves shown in Figs. 3 and 4, which were obtained under more or less artificial conditions, actually represent the phenomena when observed under standard conditions, is perhaps best answered by a reference to the point A on the curve in Fig. 3 for the 40-kv. Pyrex insulator. This point was obtained from the average dry flashover values of five 40-kv. insulators of the same type as used in this test. The tests on the five insulators were made in the standard way outside the test box under low, absolute humidity conditions some months previous to the time when the other results shown were obtained. When correction is made for temperature and pressure, however, this value shows good agreement with the other results obtained in the special test box. As a further proof of this, the points marked P on the curves represent measurements taken under natural humidity conditions at Purdue University Electrical Engineering Laboratory.

It is not proposed to advance at this time any theoretical explanation of the results obtained, as the data at hand are not sufficient to warrant the development of any very definite theory. The results given on the accompanying graphs show that the effect is a surface one which varies with the absolute water content of the surrounding atmosphere. Since this is true, it is not surprising that the results are similar for both Pyrex insulators and porcelain insulators, as in both cases the surface is of glass. It is perhaps worthy of note that the slopes for porcelain insulators are slightly steeper than those for Pyrex insulators having approximately the same flashover potential. This may be accounted for by the high chemical resistivity surface of Pyrex insulators.

Regardless of the cause of the phenomenon, experiments have shown that humidity has a marked effect on the dry flashover potentials of pin-type insulators. It is further apparent that the large change occasioned by humidity has a direct effect on the rating of an insulator. Extrapolation from the curve, together with temperature and pressure corrections, shows that a No. 441 Pyrex insulator at 760-mm. pressure and 35 deg. cent. temperature would flash over at 128 kv. with a 10 per cent relative humidity and at 193 kv. with a 90 per cent relative humidity.

Moreover, the apparent straight line relation between flashover potential and absolute humidity makes it possible to adopt a correction factor for the humidity effect on a given insulator, and to catalogue the dry flashover potential at a standard humidity as well as at a standard temperature and pressure. Until a satisfactory theoretical correction factor or formula has been determined so that humidity corrections can thereby be made it is believed that curves such as those given should be used as a measure of the standard dry flashover value of an insulator rather than a measurement at one known humidity.

Bibliography

1. A. I. E. E. Standards, Insulator Test Specifications, July, 1925.

2. "Dielectric Phenomena in High-Voltage Engineering," Peek, 1920, p. 88.

3. *Elektrotechnische Zeitschrift*, Vol. 26, June 29, 1922, p. 875. *Elektrische Festigkeitslehre*, 1925, p. 424, published by Julius Springer.

4. Loc. cit.

5. Psychrometric Tables by C. F. Marvin, U. S. Dept. of Agriculture. Smithsonian Tables, 1927 edition, p. 185.

6 Peek, loc. cit., page 111. The maximum possible correction as given by the formula $\frac{3.92 \times \text{Bar. pressure in cm.}}{\text{Absolute temperature}}$ was used in all cases.

7. Loc. cit

8. *Elektrotechnische Zeitschrift*, May 4, pp. 436-440, May 11,pp. 460-464, 1911.

Discussion

W. L. Lloyd, Jr.: (by letter) It has long been recognized that humidity has an effect upon the sparkover voltage of certain gaps and insulating members. The effect varies with the type of gap or the type of insulator or bushing. The effect of humidity on the needle-gap was shown in a paper before the A. I. E. E. by F. W. Peek, Jr.[1] The spark-over voltage increases with the humidity. The sphere-gap on the other hand is apparently not affected by humidity; at least in the papers referred to above, tests indicated that the effect, if any, is within the ordinary limits of accuracy for the sphere-gap so that no connection is possible.

For an insulator or bushing of the usual design an increase in humidity causes an increase in sparkover voltage. The effect, however, is ordinarily not of sufficient magnitude to be a factor in the insulation of a line or the number of flashovers on the line. The variations due to humidity are small compared with the difference between the flashover voltage and the operating voltage. Moreover, where the laboratory tests are made in a room that is not too damp, the variation from the laboratory test results will be in the direction of safety, *i. e.*, towards a higher sparkover voltage. In tests, to meet specifications, however, where the guaranteed sparkover is close to the actual sparkover of the insulator or bushing a few kv. may be a matter of acceptance or rejection of the product The effect of humidity then becomes of importance and it is necessary to know the humidity correction to be made.

I agree with the statement made on the third page of the paper in that the correction curves are similar "for both Pyrex insulators and porcelain insulators as the surface is a glass in both cases," but I am perplexed at the next two sentences. I have plotted the data given in Fig. 3 for porcelain on top of the data given in Fig. 2 for Pyrex and I cannot notice any difference in the average slope of those curves. If any difference in the slopes were to be found I would attribute the difference to a difference in the design of the insulators rather than to a difference in the materials or the chemical properties of the materials since it is a fact, and a fact admitted by the authors, that the surface in both cases is a glass.

E. D. Eby: This subject of the effect of humidity upon the arc-over potential of insulators, and bushings, and gaps, is one of great importance to us from the standpoint of testing, because the variations encountered are such as to require a correction factor. And it seems strange that up to this time no serious attempt, apparently, has been made to collect sufficient data from which could be derived a correction factor which could be used for different classes of insulators. We need such correction factor to harmonize the results of our tests, and also to provide a basis for the rating of insulators and bushings. As the authors have pointed out, the great variation in the arc-over potential is something which immediately affects the rating of the insulator.

It isn't so important that this matter should be investigated from the standpoint of operation of the insulators, because the 60-cycle tests over the range of humidities which have been observed, do not seem to fall below the wet flashover of the insulator; that is, these so-called dry tests at different humidities are always above the wet-flashover value. Since the wet-flashover value is determined according to the rating of the insulator, the variation in dry-flashover voltage due to humidity would not affect the choice of the insulator when that choice is made by the wet-flashover value.

On the other hand, humidity does not affect the operation of the insulator from the standpoint of impulse potential because the impulse flashover is essentially the same whether the insulator is dry or wet; at least that is true generally, not only of so-called insulators but of apparatus bushings as well.

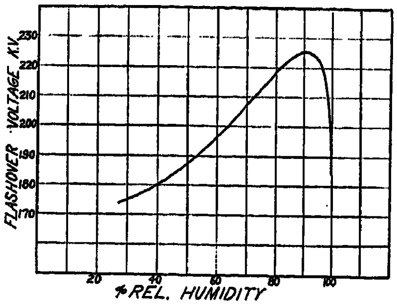


FIG. 1—VARIATION OF DRY FLASHOVER VOLTAGE OF HIGH VOLTAGE BUSHING WITH CHANGES IN RELATIVE HUMIDITY

We have long been aware that different gaps were affected differently by relative humidity in the atmosphere, and probably each different design of insulator, bushing, or gap has a characteristic of its own; but also those designs which are similar in nature probably could have assigned to them a correction factor which would be approximately correct for the group. And so it would be a practical thing to classify insulators and bushings and gaps, and assign to each class some correction factor which would be approximate for the group but nevertheless practical.

Our observations in the past of the great variations in the flashover voltages of apparatus bushings, led us about two years ago to make some tests on apparatus bushings to discover what the effect of humidity might be. We built a box, similar to that of the authors, in which we were able to control the humidity. We did not, however, convert our results into absolute humidity but interpreted them in terms of relative humidity. We varied the relative humidity approximately from 40 to 100 per cent, and with a given design of bushing we found that in that range of relative humidity its flashover potential varied from 180 kv. at 40 per cent to 225 kv. maximum. We noticed that the maximum, in this series of tests, occurred between 90 and 95 per cent relative humidity.

Fig. 1 herewith shows the shape of that curve, which is somewhat more complete than that given by the authors in their paper. These tests were made on a number of different units, and were later confirmed by a large amount of data collected over the succeeding two years on specimen units which have been tested almost every day. This has been done not only at Pittsfield, but also at our Philadelphia works, and the two sets of data are remarkably in agreement. In fact they vary but little from each other.

---

[1] See also F. W. Peek, Jr., *Dielectric Phenomena in High Voltage Engineering*, pp. 87-88, Fig. 93.

We found over this period of time that the range of relative humidities encountered in our testing departments at these two plants varied from about 30 per cent to 70 per cent, corresponding to the left-hand portion of the curve in Fig. 1, which is not quite a straight line, but without the rest of the curve might easily have been thought to be a straight line. We found also in this range of humidities that the variation of voltage was about ± 9 per cent from the average, taking the average at 50 per cent humidity, which was the average of the humidities encountered.

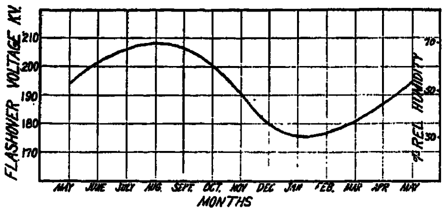


FIG. 2—SEASONAL VARIATION OF RELATIVE HUMIDITY AND FLASHOVER VOLTAGE OF HIGH-VOLTAGE BUSHINGS

We found also a very interesting seasonal change in flashover voltage of these specimen units, following almost identically the shape of the curve of the change of humidity with the seasons. Fig. 2 herewith is such a curve, representing both humidity and flashover voltage of units. We observed that both the humidity and flashover were maximum in August, or approximately that season, from July to September. The minimum humidities, and likewise the minimum arc-overs, were encountered in the month of January, or the December to February period.

F. M. Clark: Concerning the effect of humidity on the arc-over values for pin-type insulators, the work which we have done in Pittsfield appears to be in agreement with the results described by the authors; that is, as the humidity increases—we speak of it in terms of relative humidity—we get an increase in arc-over value. In the case which I have at hand this increase amounts to about 20 per cent, a maximum arc-over value being obtained with about 90 per cent relative humidity.

I should like to offer a possible explanation of the phenomena involved. This the authors have avoided. In *La Verre* for 1924, page 54, P. Woog described experiments which he had carried out having for their purpose the detection of changes which occur under certain definite conditions in the surface of glass. Woog set up an apparatus which might be roughly described as a spherical electrode mounted inside of a cylindrical glass vessel which, in turn, was placed on a metallic conductor. The cylindrical glass vessel was filled with oil. When the voltage was slowly increased a discharge was finally obtained from the spherical electrode through the oil to the surface of the cylindrical glass vessel and over this surface to the outside plate electrode.

Working at 25 cycles, with dry oil, Woog found that as the temperature passed from 11 deg. cent. to 100 deg. cent. the voltage to discharge dropped from 67 to 54 kv. There is nothing in the way of experimental error to explain the result.

Using wet oil Woog found that as the temperature was changed from 18 to 12 deg. cent. the arc-over voltage dropped from 65 to 27 kv. approximately. As the temperature was brought back from 12 to 18 and 20 deg. cent. the arc-over voltage again increased to about its original value. In other words, speaking entirely of wet oil, as the temperature increased the arc-over voltage rose.

The author's explanation was somewhat as follows: On the surface of the glass were silicates which were hydrolyzed to give bases with the final formation of a film of sodium carbonate. This material can exist with water of hydration. As the humidity increases this hydration increases until a maximum value is obtained where the ratio between carbonate to water molecules is 1 to 10. What happened, therefore, in Woog's experiments, according to his own explanation, was that the voltage to flashover varied as the author increased or decreased the amount of water in the sodium-carbonate film on the glass. The important thing is that he found that with 10 molecules of water, we will say, in the sodium carbonate, the arc-over value dropped as compared to what it was with the sodium carbonate in the dry state. The effect of temperature tends to increase the arc-over value of moist oil because the decahydrate of sodium carbonate which is formed at low temperature possesses a greater vapor pressure than that characteristic of moist oil with the result that sodium carbonate in a lower stage of hydration is formed.

The present authors have found, as illustrated in Fig. 5, that as the humidity rises the arc-over characteristics also increase. As I have stated, this result is in agreement with the work in our laboratory but it is in direct contradiction to the idea that Mr. Woog has expressed. As Mr. Peek has already demonstrated, and as illustrated in Fig. 6, the dielectric strength of air is not involved. The authors have therefore concluded that the increase in arc-over value with humidity is a surface effect. This they have demonstrated by taking a glass rod and wrapping electrode wires around it, taking the arc-over values between carefully spaced wires. We have often made creepage tests in our work at Pittsfield on all sorts of materials and we find that if we place electrodes as noted by the authors we get an arc-over which is not strictly accounted for by the surface condition of the insulation. Rather we get a result which depends upon the ratio distribution of stress between the insulating surface and the medium in which it is immersed. We have found that we must use some method of holding the voltage stress down into the surface which we are testing.' I would suggest to the authors that they accomplish this effect by taking a glass rod, drilling out one end, and filling the well so formed with mercury. They should then wrap a wire around the glass rod directly over the bottom of the mercury well and spaced a definite distance away from a similar wire electrode wrapped about the glass away from the mercury well. The mercury well and its superimposed wire electrode are connected electrically and the arc-over value is taken in the usual manner. With this method the authors may find that they will obtain an arc-over value which is in agreement with the carefully obtained research results described by Woog.

C. F. Harding: In view of the fact that the authors of this paper have quoted some of the test values that are closely associated with the curves shown in the paper, these tests having been made in the high-voltage testing laboratory at Purdue University, it may be of interest to have these confirmed in slightly greater detail.

These tests were made upon a wide variety of sizes, not only of Pyrex insulators but also of several makes of porcelain insulators of the pin type, covering the same voltage range.

RESULTS OF SIXTY-CYCLE DRY-FLASHOVER TESTS ON PIN-TYPE INSULATORS WITH VARYING HUMIDITY
PYREX GLASS INSULATORS

(Average of three readings upon each of three insulators of six different sizes)

Insulator No.	Humidity per cent	Flashover voltage kv.	Humidity per cent	Flashover voltage kv.	Increase humidity per cent	Increase flashover voltage per cent
1	48	140.0	59	161.1	23	15
2	40	137.0	60	152.0	50	11
3	43	95.4	61	101.0	42	6.3
4	40	65.2	59	66.8	48	2.4
5	40	50.0	59	51.5	48	3.0
6	40	51.3	59	51.8	48	1.0

PORCELAIN INSULATORS

(Average of three readings upon each of three insulators of ten different sizes of two different manufacturers)

7	46	152	62	165.3	34.8	8.8
8	46	127	61.6	145	34	14.2
9	48.3	119	61	133	27	11.7
10	43	105.5	60	125	39.5	19.4
11	43	105	60	127	39.5	21
12	43.3	91.2	63	93.1	45.5	2.1
13	42	69	59	70.3	40.5	1.9
14	43	68.7	58	70.0	35.0	1.9
15	40	63.6	59	65.4	47.5	2.8
16	40	48.7	59	49.6	47.5	1.9

In general, the average of 3 readings under A. I. E. E. 60-cycle standard testing conditions, on three samples of large types of insulators would increase about 15 per cent in flashover, with approximately 23 per cent increase in humidity, that is, from the neighborhood of 40 or 45 per cent to 60 or 65 per cent humidity. This was also true within a smaller range with a similar voltage gradient for the porcelain insulators. With the lower voltage insulators, the per cent increase, of course, was naturally less.

In addition to what has been said about standard or uniform specifications, it would seem desirable to have these A. I. E. E. specifications include the humidity correction, or some humidity measurement plus a correction. If we are to depend at all upon the dry flashover test at 60 cycles, and the Institute has standardized such a test, and if in normal laboratory testing such humidity variations exist, we certainly should have a humidity correction.

Similar tests with steep wave fronts are now being made, in which it is probable that such variations will not exist because, as has been pointed out in the paper and in the discussion, the change in humidity does not affect the sphere-gap and the steep-wave-front measurements in air. So we have the condition that we are testing insulators at 60 cycles, referring our dry-flashover voltages to a sphere-gap which does not change with humidity changes, and yet, from one laboratory to another, unless we do hold humidity constant, we have a 14 to 15 per cent variation in the dry-flashover voltage of such insulators.

D. H. Rowland: (by letter) No pains should be spared in studying these phenomena until sufficient data are obtained so that some means may be devised for intelligently taking account of humidity in the A. I. E. E. specifications for testing insulators.

As the authors state it has for a long time been known that humidity affects insulator flashovers. R. H. Marvin pointed this out some years ago.

It has been found in our laboratory that two porcelain insulators having exactly the same flashover values at a certain definite absolute humidity but differing somewhat in corona point, are affected entirely differently by a change in humidity. Thus the curves of flashover against moisture content of the air for two such specimens would not have the same slopes. It is believed, therefore, that the shape of the pieces rather than "chemical resistivity" of the surface is the factor that accounts for the variation of flashover value with humidity for any given insulator. Certainly the curves in the paper under discussion are not definite enough to draw conclusions as to the effects of the surface of the materials under consideration, *i. e.*, glass and porcelain.

Suppose for the sake of argument that there were specimens made up of glass and porcelain of identical shapes for test purposes, there would still be an inherent difference in the dielectric constants of the materials which would affect the electrostatic field and the initial corona point, and as pointed out in the case of the two porcelain insulators, this factor seems to have a bearing on the situation. It would be very difficult to study the surface conditions alone.

Several investigators have found changes in the flashover values obtained between needles with varying humidities. If these data are true it points to the fact that the conditions observed by Dr. Littleton and Mr. Shaver are not as they maintain, entirely a surface effect. More work should be done in this connection.

W. A. Lewis: (communicated after adjournment) One characteristic of the experimental conditions, which appears to have an important bearing on the effects obtained, is the form of the electric field surrounding the conductors. As an example, we may compare the results obtained on glass rods by the authors of the paper and by Schwaiger, those of the latter appearing to be checked by our own investigations. The author found that flashover voltage increased with increasing humidity when loops of wire were used as electrodes, whereas Schwaiger found the opposite effect when the glass rods were placed between parallel plate electrodes. The essential difference appears to be in the form of the field, as in the first case the field is non-uniform, the gradient being very high near the electrodes, and in the second case the field is uniform and the gradient constant along the surface of the insulators. Consequently, in the first case initial breakdown occurs near the electrodes and corona forms, but in the other case initial air breakdown and complete flashover are simultaneous.

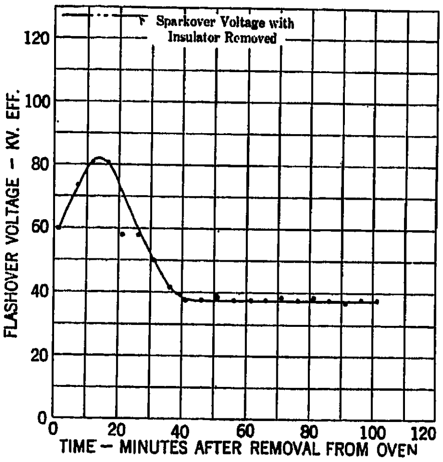


Fig. 3—Flashover Characteristic of Dried Glass Rod, Diameter 214 Cm., Length 7.62 Cm., Air Temperature 21 Deg Cent., Air Density 0.985, Relative to 76 Cm. and 25 Deg. Cent.

Peek has shown that the sparkover voltage of needle gaps, with which marked corona is present, increases with increasing humidity(Diel. Phen., p. 88), and so it does not seem unreasonable for the same phenomena to appear here, when the corona formation is sufficient. The data obtained from flashover between the two wire loops without glass present are insufficient to determine whether this phenomenon is actually present or not, since the field form is considerably altered by the introduction of the glass.

If there is present another phenomenon, independent of corona, which tends to decrease flashover with increasing humidity, it is readily seen that flashover voltage may increase, decrease, or remain constant with increasing humidity, depending upon the relative magnitudes of the two opposite effects. In this way the divergent experimental results may be reconciled.

During the past year experiments have been conducted at the California Institute of Technology, using cylinders of glass between parallel plates. As no test chamber was available in which the humidity could be varied, the expedient was adopted of drying the cylinders in an oven at 90 deg. cent. for several days, and then testing in the plate gap under natural humidity conditions. Flashover voltage was plotted as a function of time, and curves obtained, of which Fig. 3 herewith is a good example.

All curves exhibited the property of starting at a low value, rising to a maximum and falling again to a constant value. The time required to reach the constant value was in no case less than 30 min., which suggests that great care should be exercised to make certain of equilibrium before taking readings, when the controlled humidity method is used.

We explained the shape of the curves in the following way. The low initial flashover voltage was due to the presence of a film of heated air around the insulator, and the flashover voltage rose as the air cooled. At the same time the surface was collecting moisture from the atmosphere, tending to reduce the flashover voltage. When the latter effect overbalanced the former, the curves began to fall and continued until equilibrium was reached. If this explanation is correct, our results would certainly seem to check Schwaiger's, since the flashover voltage decreases with increasing surface moisture, corresponding to increased humidity. The range of atmospheric humidity was insufficient to obtain reliable data by comparison of the equilibrium values of various curves.

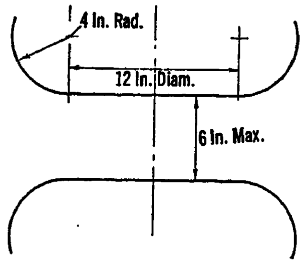


FIG. 4—SECTION OF CIRCULAR, PARALLEL PLATE ELECTRODES

Although a knowledge of flashover characteristics of practical insulators is the ultimate aim, it would appear from the above that much fundamental research must be carried out before we can reach a sound rational basis for a suitable correction factor.

R. W. Sorensen: (communicated after adjournment) It will be noted that the results of the authors and those found by Mr. Lewis each taken alone are such as to cause exactly opposite conclusions. That is, the authors of the paper under discussion tell us that for their particular test condition flashover voltage increases with increase in humidity. On the other hand, the results presented by Mr. Lewis indicate beyond doubt that the flashover voltage for the glass rods he used in test becomes lower as the surface of the rod becomes moist. In both cases the flashover is an air breakdown.

Peek and others have shown that humidity does not affect the flashover voltage for sphere gaps and the authors of the paper have shown by their Fig. 6 that the flashover between wire loops is also not influenced by humidity.

Lewis' tests were made in a very large open room. The authors of the paper made their tests in a large box in order to be able to control all conditions of temperature and humidity. Lewis has just completed a large box of somewhat different dimensions and will soon have some further results made therein.

The authors made their tests with pins and wires or wires as electrodes. Lewis' tests were made with parallel plates in which edge effects were avoided by the use of large radii of curvature as shown in Fig. 4 herewith.

Schwaiger, whose work has been mentioned, used electrodes similar to those used by Lewis and obtained results in the same direction, *viz.*, indicating a decrease of flashover voltage with increased humidity.

The distinction between the results found by both Schwaiger and Lewis and those obtained by Littleton and Shaver seems then to be rather one of field form than one of humidity, or at least is certainly a function both of humidity and field form.

A possible explanation of this difference is one based on the shielding effect of corona. That is in the case of wires as used by the authors of the paper, corona voltage comes before arc-over. Under the conditions of test the added humidity or moisture content of the air permits a more uniform and a more abundant corona glow on the electrodes or even at edges of the insulator. This glow has the effect of readjusting the electric field so as to reduce the gradient, by a shielding effect thus increasing flashover voltage as the moisture increases.

When uniform fields are used as was done by Lewis and by Schwaiger there is no effect of this kind because flashover occurs at a voltage less than required for corona. Hence an increased moisture content of the air means an increased moisture deposit on the insulator with a corresponding decrease in flashover voltage.

Because of these facts it seems inadvisable to introduce in the A. I. E. E. rules for testing insulators any general correction constant for making humidity corrections because it seems apparent that humidity correction constants can at present be made only by the establishing of a correction factor for each type, and possibly even for each size of insulator; that is, with our present knowledge of the fundamentals involved we certainly are not in position to establish correction factors for humidity which will be applicable to insulator testing.

J. T. Littleton and W. W. Shaver: Several points have been raised in the discussion that require some explanation.

Mr. Lloyd referred to the slopes of the curves as being determined by the insulator designs and not dependent on a property of the surface of the material. This might very well be the case. However, our statement that the difference between the slopes of the curves for the Pyrex insulators and for the porcelain insulators of comparable size might be due to the difference in surface resistance of the material, has as we believe some foundation. It is very evident that the slope of the curve for the Pyrex No. 161 insulator is somewhat less than the slope of the curve for the porcelain 16-kv. insulator. These two insulators are almost identical in form and size. Also the porcelain 44-kv. insulator is quite comparable to the Pyrex No. 401 insulator and the slopes of the curves for these insulators are noticeably different. In this case, however, there is more difference in insulator shape than in the former case.

While the surface of both the Pyrex insulator and the porcelain insulator is a glass, these two glasses are very decidedly different in composition and undoubtedly have large differences in properties. It is well known that the surface resistance of all insulators depends upon the humidity. Even a material such as fused quartz shows a very large humidity effect, so that a difference in the surface resistance of porcelain and Pyrex insulators would be expected. However, it is not believed that in either case any chemical reaction takes place between the water vapor and the material.

Mr. Eby called attention to the fact that even under extremely dry conditions the flashover potential was not reduced to a value below the wet-flashover value and he concluded therefore that humidity curves were desirable only for the purposes of rating the insulators and really had no bearing in actual service. However, there is another point that must be considered and that is the allowable safety factor for a puncture given by the ratio of puncture voltage to dry-flashover voltage. For, instance, an insulator having a dry-flashover potential of 125 kv., tested in air to that point might under extreme conditions have a flashover value of 180 kv. in air which might conceivably be very near the puncture voltage; at least the factor of safety so far as puncture is concerned would be greatly reduced.

We do not understand exactly what Mr. Clark has in mind when he refers to a test made on insulating surfaces under oils of various moisture contents and we do not see in just what manner such tests may be applied to our data. Flashover tests made on a paraffin-coated insulator under various humidity conditions showed the same results as the uncoated insulator tending to prove that there is no chemical combination between the insulating material and the water which gives rise to the curves obtained. While the explanation of this phenomenon is undoubtedly of interest, that feature has been given but slight consideration by us. We believe that the additional conductivity of the surface of the material caused by the film of adhering water molecules has redistributed the field to a certain extent, tending to prevent an overstress of the air around the tie wire and along the surface of the material. Mr. Clark, however, is mistaken regarding the relative value of the dielectric constant of Pyrex insulators and porcelain. The dielectric constant of Pyrex insulators for 60-cycle frequency is about 4.8 which is decidedly lower than any figure which we have seen published for porcelain.

Attention again should be called to the fact that in studies of this type absolute humidities and not relative humidities should be considered. Measurements taken at a given temperature and given relative humidity will be quite different when taken at the same relative humidity and at a different temperature. There is almost twice the amount of water present in air at 25 deg. cent. as in air at 14 deg. cent. with the same relative humidity.

Mr. W. A. Lewis referred to the discrepancy between our results on rods and those of Schwaiger. We did not discuss Schwaiger in more detail inasmuch as his conditions did not duplicate ours, he dealing with uniform tangential fields and we with conditions as described. The fact that the results differ proves the conditions of the two experiments to be different. We trust that Mr. Lewis will make measurements on vacuum-dried rods, eliminating the temperature effect to which he has called attention.

There is, however, a doubt in many minds that the gradient in a field between parallel plates is uniform for there is some evidence that the drop in the neighborhood of the plates is very much greater than in the center of the region between them.

In our paper we state that a constant humidity was maintained for 15 min. before measurements were taken. It was our experience that this was ample. Mr. Lewis' curve further indicates that to be so, for a certain portion of his 40-min. interval is required for cooling. We did not call attention in our paper to a number of tests made on oven-dried insulators carefully cleaned with absolute alcohol, for the results after a sufficient length of time had been allowed for cooling, were identical with regular stock.

Professor Sorensen calls attention to the difficulty in arriving at a correction factor for humidity effect. His conclusions are identical with ours. The fact that our family of curves showed no consistency in change of slope proves that the humidity effect is dependent upon something beyond the absolute flashover potential. It probably depends upon the air path and the distribution of potential over the surface leakage path, in other words, the shape, and the surface resistance coefficient of the material. We believe that these factors can be separated and evaluated but the problem is by no means a simple one. For that reason we suggest a humidity curve as being by far the more simple way of evaluating this effect for any given type of insulator.

The Cathode Oscillograph as Used in the Study of Lightning and Other Surges on Transmission Lines

BY HARALD NORINDER[1]

Synopsis.—Visible records of the effects of lightning surges and transients on transmission lines are described in this paper, which explains the operation of a specially constructed cathode-ray oscillograph. The outstanding feature of this oscillograph is its ability to record phenomena of a microsecond in duration, while at the same time the instrument is maintained in an exposed position for as much as a million times as long as the occurrence. Methods of eliminating load frequency are discussed. Different types of lightning surges are reproduced. The use of the oscillograph for making graphic records of switching surges on transmission lines is also demonstrated.

* * * * * *

I. RESEARCH STUDIES OF LIGHTNING SURGES ON TRANSMISSION LINES

EVER since electric energy has been transmitted by long transmission lines extended over great areas, research into the nature of lightning surges has constituted a problem of great importance. The difficulties of a solution to the problem have been in many ways almost insurmountable.

The limitations of a purely theoretical solution to the problem of lightning surges. Several attempts have been made to solve the problem of surges on transmission lines in a purely theoretical manner. Such a solution has been a necessity for the formal consideration of the effects of surges running in a transmission network with given line constants. The limitations of such theoretical calculations reside particularly in the fact that assumptions have to be made as to the nature of the surges in their initial moments. Evidently it is of vital importance to know the amplitude and the sloping of the surge front, while a knowledge of the front characteristic is of fundamental importance when, for instance, the reactions of a surge into a transformer must be taken into consideration.

Practical value of a direct study of lightning surges. It is, however, not a purely theoretical point of view that has stimulated a direct study of lightning surges on transmission lines. The problem is of more interest in relation to the practical engineering demands which have stimulated research work on lightning surges. On a transmission system, it may, for instance, be quite necessary to make a decision as to whether or not it is necessary to use lightning arresters. Or it may be necessary to take up a comparative study of the protective value of different lightning arresters. In such a situation, it is evidently quite important to be acquainted with the nature and the different forms of the lightning surges. The same holds true for the construction of lightning arresters.

Some special difficulties in a direct study of lightning surges. It is not a lack of interest that is responsible for our shortcomings in understanding lightning surges. Our failure has another explanation. There are many experimental difficulties in direct research work on transmission lines. This is especially the case when we try to secure direct evidence of the nature of surges, etc., under full load conditions on a line. In many earlier research studies we have attempted to eliminate these difficulties by indirect methods. For instance, the damage wrought by lightning surges on transformers, wall bushings, etc., has been observed and from such observations we have derived some knowledge of the frontal characteristics of the surges, their oscillatory character, and their amplitudes. It is a well-known fact that in most cases a flashover is a chance occurrence, and thus it is not easy to derive valuable conclusions from such an empirical consideration of the surge phenomena.

A valuable solution can be arrived at only by direct research where the natural conditions of the surges are observed under full load conditions. From this demand for research under load conditions, some special difficulties arise, as we shall see.

Need for a suitable instrument in studying surges on a line under load. When research study of lightning surges is limited to an observation of their amplitudes and their character of polarity, good results have been gained by the klydonograph.[2] But when it is necessary to give an exact analysis of the wave shape with regard to time, the only possible instrument now developed is a suitably constructed cathode-ray oscillograph with an electron jet directly bombarding a photographic sensitive layer situated within an evacuated vessel.

Difficulties with earlier instruments. The following demands must be met by the specially constructed cathode oscillograph:

1. As the oscillograph must record surges which are generated without the control of the operator, it must be so arranged that a record is possible for the shortest times of passage 10^{-6} - 10^{-8} sec. with a single passage of the phenomena.
2. As the time of arrival of surge phenomena cannot be known by the operator, the instrument must be so arranged that it can be in a continually exposed position for time periods a million times as long as the time necessary for the recording of the surge.

---

[1] Royal Board of Waterfalls, Stockholm, Sweden.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

[2] J. H. Cox, P. H. McAuley, L. G. Huggins, TRANS. A. I. E. E., Vol. XLVI, 1927, p. 315.

3. The instrument must be so arranged in construction and manipulation that its electron emission is steady and sufficient. Instrumental and accidental disturbances must be eliminated.

A cathode-ray oscillograph with a photographic sensitive layer in vacuum was first constructed by Dufour. His excellent instrument has given very good results in the study of such transients and oscillations where the time of arrival was determined by the operator. The oscillograph has thus been used for study of the time lag of sparks and for proper oscillations of transformers.

In the study of lightning flashes and the surges produced by them on transmission lines, however, it has not been possible for us to use an oscillograph of the Dufour pattern. The total time of exposure in such an oscillograph is restricted to a few times the total time of duration of the surges because of the excessive blackening of the film which would otherwise occur. Since it is impossible to foretell the exact time a lightning surge will occur, it is necessary to have a time of exposure of sometimes half an hour or more in order to record a surge with a duration of fifty microseconds; it can easily be seen that it was quite necessary to solve the research problem by elimination of the blackening effects of the electron jet.

The oscillograph especially constructed for transients and surges on transmission lines. In the beginning of our research work on lightning surges in the laboratory of the Swedish Royal Board of Waterfalls, we were soon persuaded that the conditions given above could be met only by an instrument of special pattern. If the occurrence of lightning flashes and of their surges on transmission lines had been very frequent in Sweden, we should probably have been able to obtain results earlier. During some seasons, however, we have only one or two occasions to secure records, which is not sufficient, considering the experimental difficulties that must always be related to the problem.

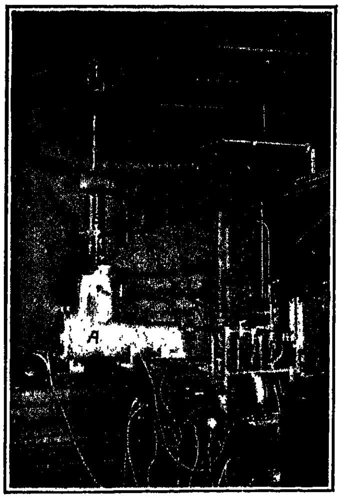


Fig. 1—The Cathode Ray Oscillograph of New Construction

The instrument which we devised in the research station of the Royal Board of Waterfalls near Uppsala is illustrated in Fig. 1.

The instrument is, as may be seen, very robust in its construction. It consists mainly of metal, using only a straight bit of glass between the cathode and the anode. A suitable vacuum pressure can be regulated by the aid of a specially constructed regulation valve. By this arrangement, it has been possible to establish a very steady electron emission from the cathode.

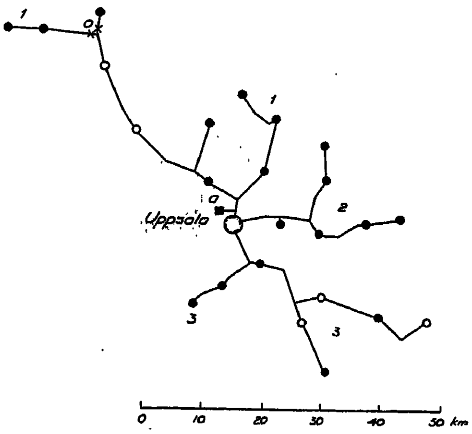


Fig. 2—Map of 20-Kv. Transmission System Used in Recording Lightning Surges

All arrangements have been made to secure a continuous operation with the instrument. The necessary time for evacuation has been considerably reduced. Valves and vacuum tubes have been constructed of metal and the pump connections to the oscillograph have been so arranged that it is not necessary to stop the pumps when passing air into the oscillograph vessel for changing films. The pump arrangement is so effective that it is possible to gain an effective vacuum within 8 to 10 min. with a film surface of 600 sq. cm.

Special arrangements make it possible to reduce the secondary stray radiation from the electron jet, and thus it is possible to work with the oscillograph in an exposed position for one or two hours. During this time of exposure, the instrument still has the property of recording short transient phenomena with a total duration of some millionths of a second. It is quite possible to attain still shorter times by a regulation of the intensity of the electron emission.

In the following will be demonstrated the results gained with the instrument in recording either lightning surges on a transmission line or switching surges under full load conditions The records were taken with the cathode oscillograph connected to a 20 kv. transmission line from a transformer station of the Royal Board of Waterfalls in the vicinity of Uppsala, Sweden. The short connection line from the experimental station to the transmission line is marked *a* in Fig. 2.

The connection of the oscillograph to the transmission line under load. The records of lightning surges were taken mainly from the transmission line group marked No. 1 in Fig. 2. Sometimes surges coming from lines 2 and 3 were also recorded. By the connection of several end points of lines to the record system, it sometimes has been difficult to identify the reflected surges.

Fig. 3—The Experimental Station of the Swedish Royal Board of Waterfalls for the Study of Surges

For recording purposes, the cathode-ray oscillograph was placed in a special building. (See Fig. 3.) The distance between this building and the line was 50 meters, and from the connection point to the transformer of the line was 150 meters. The records thus can be referred to one of the end points of the line.

The oscillograph was connected through condensers to the line, the connection consisting of two chain insulators for each phase. Their capacity was 20 micro-micro-farads and thus could be neglected in comparison with the line capacity. It was easily possible to disconnect the line voltage from the system by the aid of an outdoor switch. (See Fig. 3.)

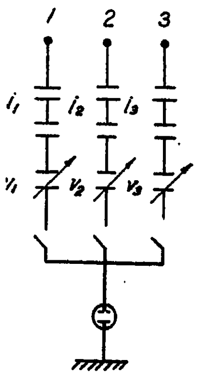


Fig. 4—Method of Three-Phase Connection of the Oscillograph to the Transmission Line

Methods of eliminating load frequency. The measurements were performed under full load voltage and thus it was necessary to make use of an elimination method for the 50-cycle voltage. For this purpose we used different methods.

1. The three-phase voltages from the line were reduced by the capacities of the identical surge insulators, i_1, i_2, i_3, and the small variable air condensers v_1, v_2, v_3. (See Fig. 4.) A small distortion of the line voltages caused by load variations in the transmission lines was easily eliminated by regulation of the variable condensers v_1, v_2, v_3. When the system was in full phase symmetry, the sum of the voltages was zero. And then, if for instance the symmetry were disturbed by an atmospheric surge, a deviation of the electron jet took place within the instrument. (See Fig. 5-a.) This method has a disadvantage. If the surge should cause a breakdown to ground on one of the three phases, the load voltage from the other phases would cause a blackening of the film.

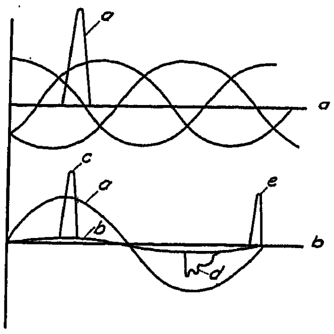


Fig. 5—Two Different Methods of Diminishing the Effect of Load Frequency

2. The oscillograph was connected in single phase to the line, for instance, above the variable condenser v_1, from line 1. (See Fig. 4.) The two other lines, 2 and 3, were disconnected from the oscillograph. By regulation of the small air condenser v_1, it was easy to give such a value to the line voltage that it was covered by the "zero losses" of the oscillograph. (By zero losses is meant that amount of voltage that must be imposed on the instrument before any record takes place.) The line voltage was in such a case reduced from the initial value a as in Fig. 5-b to the reduced value b in the same figure. Surges c, d, or e exceeding the line voltage could in such a case be recorded. The advantage of this method lies in the great deviation possibilities of the surge above the line voltage and an easy control of the latter, while the normal load voltage varies only within the fixed zero losses of the instrument, which may be between five to ten per cent of the whole possible deviation of the instrument.

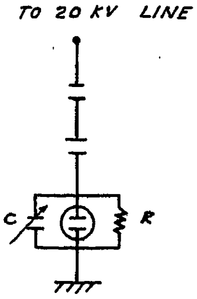


Fig. 6—Method of Diminishing the Effect of Load Frequency by Means of Leakage Resistance

3. The oscillograph is single-phase-connected to the line. (See Fig. 6.) Parallel to the deviation system of the oscillograph is placed a leakage resistance R and a small variable condenser C of such a value in relation to the capacity of the system that it affords an effective leakage for the load frequency, but stops higher frequencies, for instance, those of 1000 periods or above. The leakage of the resistance diminishes with the increasing of the frequency. For an effective regulation, it is still necessary to hold the reactance of the condenser small in comparison with the leakage resistance. If a set of impulses is observed and their time of duration is shifting, difficulties may arise in using this method. The reactance of the condenser system varies with ω, and it is thus quite impossible to get a value for the leakage that can be considered as suitable for a very much wider range of frequency. This may be seen by the relation:

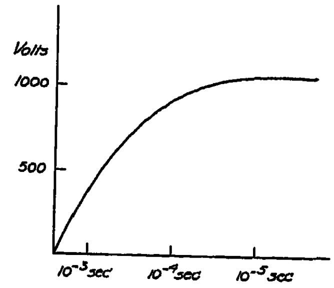


Fig. 7—Voltage Correction Curve for Different Frequencies

$$V_R = V e^{-\frac{2\pi}{R \omega C}} \qquad (1)$$

where V is the voltage between the condenser plates without leakage, and V_R the corresponding voltage with leakage. The leakage resistance is R, the effective capacity C. When recording with variable frequencies, it thus must be noted that the functional correlation between the leakage and the frequency can be represented by a curve. (See Fig. 7.) The modification by the leakage is illustrated by Fig. 8, where curve I represents a real curve and curve II the modification of amplitude that results from the leakage of the resistance R. The regular variation of the voltage with the frequency makes it possible to establish corrections for a defined frequency. It is always possible to give such a value to R and C for a given frequency that the influence of the leakage can be reduced.

In order to reduce the influence of and errors caused by unknown variation with high frequency of the leakage resistances used, it was necessary to test the leakage circuit by superimposing on the system at the high voltage end, voltages of known frequency and amplitude from a separate impulse generator.

Results of the measurements. In the following will be revealed the results of studies of lightning surges made at the above-mentioned experimental station at Uppsala. The general direction of these results has been clear to us for the past four years. By visual observations of the fluorescence spot in the cathode-ray oscillograph connected to transmission lines, we have obtained a first-hand practical experience of the frequency and the variation forms of lightning surges on transmission lines. From this experience we had to decide the velocity to be used in recording lightning surges. In this first research work, we did not prefer an abnormally high velocity for making our records, because it was clear to us that many of the surges had their fronts not quite straight, but sloping. The probable reflections from different points of the transmission system were another reason for our making our records with a velocity that was not too high. In such a case, too high a velocity would cause a confusion of the curves. Our intention with these preliminary research studies was to get a general understanding of lightning surges on transmission lines. With this general understanding, the next step evidently was a more detailed study of the surges.

A feature which we noticed in the beginning of our observations was the following: Sometimes severe thunderstorms developed in the vicinity of the transmission lines without any appreciable occurrence of lightning surges on the transmission lines. The explanation of this circumstance should be plain enough. The general distribution of the field intensity in the thunder cloud was such that either no field lines that were broken down by the lightning flashes ended on the transmission lines, or the breaking down of the lines was performed with such a slow velocity that no lightning surges of any appreciable amplitude could be developed. On another occasion, a similar thunderstorm might develop plenty of surges following one after another on the same transmission line. In such a case, the general field distribution was of a peculiar character, with many field force lines ending on the line, and the lightning flashes developed in as short a time as was necessary to produce surges. Sometimes we could observe surges following one after the other where all were of the same shape. This might be explained by the fact that the lightning strokes were developed during similar physical conditions within a restricted sphere of the thunderstorm cloud.

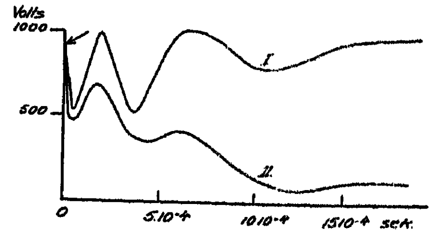


Fig. 8—Appearance of Surge When Using Leakage Resistance Method

It must be noted that our observations of the lightning surges have been hindered by the infrequent occurrence of thunderstorms in Sweden. During the season of 1926, we could get only a few surges on our transmission lines which were under observation. Thus we were forced to economize with the few opportunities for observation which were available, and it was necessary to develop our instruments to a high degree of operating safety. Especially were we obliged to concentrate our efforts on the point of eliminating the secondary stray radiation in order to get the longest possible time of exposure.

The first lightning surges recorded with the specially constructed cathode oscillograph. The first records of lightning surges were taken during the summer season of 1925. A specimen of such a record is given in Fig. 9. The normal 50-cycle load in this case varied within the limits of the bright band. We can observe from the curve surge values of about three times the top values of the load voltage. When these records were made, the cathode ray oscillograph was not so highly developed with regard to elimination of secondary stray radiation and intensity of the electron jet. Therefore in making the record in Fig. 9, it was necessary to hold a relatively small electron emission, which reduced to some degree the sharpness of the curves. For reproducing this oscillogram in a printed paper, it was necessary to intensify to some extent the curves of the original record.

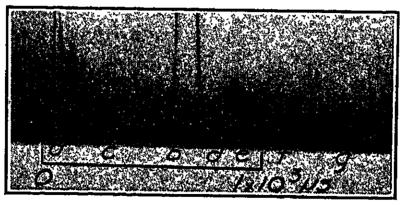


Fig. 9—First Oscillograph Record of Lightning Surge on a Transmission Line

We can see in *a* and *b* of Fig. 9 two different surges with their fronts quite straight. The reflections of these surges are also visible, for the first surge *a* at *c*, and for the second surge *b* at *e, f*. The curves with small amplitudes at *d, f*, and *g* must have their origin in other surges that have been highly damped before reaching the point of observation. The reflected surges have undergone, as we can see, a very pronounced damping. The initial top value of the surge was more than three times the normal line voltage to earth. In spite of these high surge values, the protective relays did not cause any disconnection of the transmission line. We can, in such cases, suppose that flashovers occurred on some of the insulators of the transmission lines where the surge had its highest value, but the paths of flashovers were too weak to produce a connection strong enough to complete a connection to earth for the normal load voltage of the line. Similar situations have been observed several times.

Various lightning surges which were recorded on transmission lines during the thunderstorm season of 1927. Improvements in the cathode-ray oscillograph made it possible to record many surges on transmission lines during the summer season of 1927. The oscillograph was connected through condensers to the line either in a single-phase connection or sometimes a three-phase one. The records of the latter connection were sometimes disturbed by a current to earth from some of the connected phases. The recorded surges were in such a case covered by the 50-cycle load voltage variations and it was sometimes troublesome to identify the surges in the primary records.

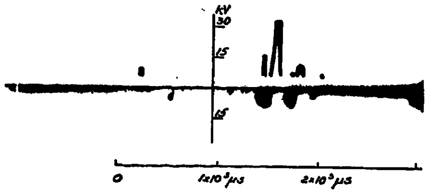


Fig. 10

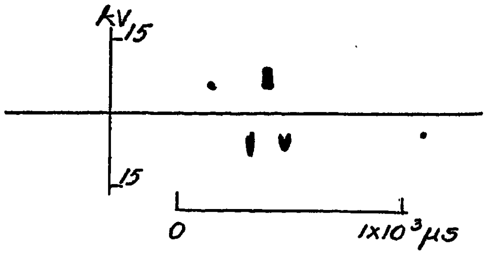


Fig. 11

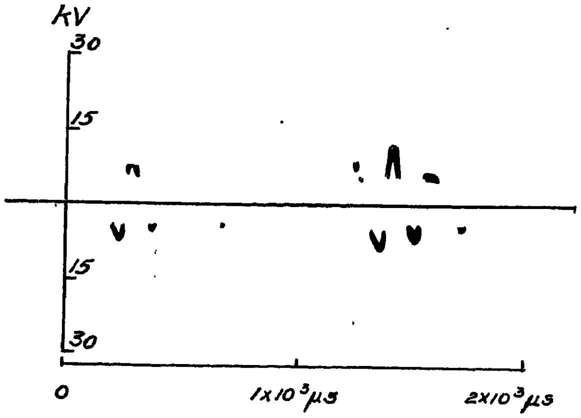


Fig. 12

Figs. 10, 11, 12—More Recent Records of Lightning Surges, Taken With an Improved Instrument

As may be seen by the following reproduced records, we sometimes have records of surges with a small amplitude. The reason for this is that since relatively few thunderstorms produced surges on the transmission line, it was necessary to work with thunderstorms often far away from our observation station. A lightning surge with an initially high amplitude at, for instance, 30 km. distance from our observation point thus reached the oscillograph in a damped state.

In Figs. 10, 11, and 12 are given some examples of surges recorded on the 20-kv. line mentioned above. These figures are direct reproductions of records with the film in an exposed state in the oscillograph's vessel during an hour. In spite of the long exposure time, we can not find any trace of severe blackening by secondary stray radiation.

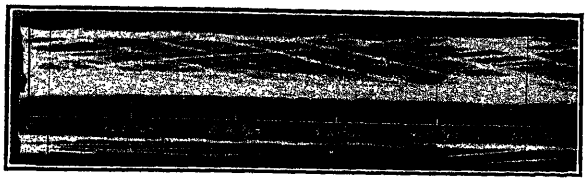


FIG. 13—OSCILLOGRAM OF LIGHTNING SURGES, OBTAINED BY USING THREE-PHASE CONNECTION OF OSCILLOGRAPH

In Fig. 10 there are several surges superimposed in the same period of record while the surges in Figs. 11 and 12 are separated. The surges of Fig. 12 seem to have an oscillatory character caused by special connections in the transmission line system.

The method of a three-phase connection has also been used and in Fig. 13 is a record reproduced where the oscillograph was connected to the three phases of the line through suitable condensers. The earth current in this case gave rise to a dissymmetry of the phase voltage, which to some extent disturbed the record. In other original films it is possible to find several surge types of interest. In Figs. 14 to 16 some of them are given, drawn from the original records taken with a three-phase coupling. The earth current in these records caused a dissymmetry in the load voltage and in such a case a direct reproduction would not give a clear view of the recorded surges.

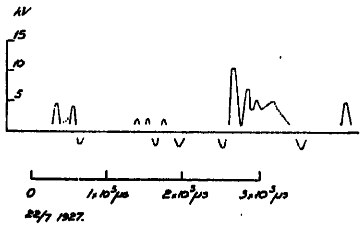


FIG. 14

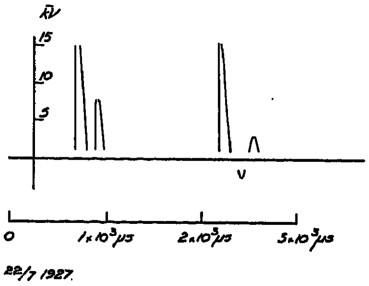


FIG. 15

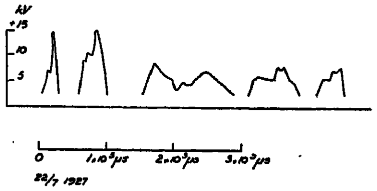


FIG. 16

FIGS. 14, 15, 16—REPRODUCTION OF SEVERAL INTERESTING SURGE TYPES RECORDED

The different types of lightning surges and the need for more extensive research in the future. The different types of lightning surges that we have recorded are demonstrated in Fig. 17. About one-fifth of them have, as in No. 1, a straight front where the highest voltage is reached within a few microseconds or part of a microsecond. We hold it as possible that some surges of this type are generated by direct lightning strokes on the lines. At the very moment some of them were produced, we have observed lightning strokes in the vicinity of the transmission lines.

The other surge types 2 to 5 of Fig. 17 reach their highest value in a time varying from 10 to 150 microseconds. Surges of a highly damped or oscillatory character, types 3 and 4, we have observed only a few times when the line was disconnected by the protective relays. Type 5 has been observed during similar conditions.

A marked variation in wave shape and frontal characteristics of the lightning surges recorded has been observed. This variation is fully explained by the unstable conditions which sometimes are characteristic of the electrophysical development within a thunderstorm cloud. The discharging processes within a thundercloud take place often only between charged cloud volumes. In such a case, lightning surges may be produced where the front is sloping. On the other hand such discharges between cloud volume and the earth may produce surges where the fronts are quite steep.

Pronounced positive surges such as 1 and 2 in Fig. 17 are the most frequent type. Sometimes such surges were followed by earth current in the line. We have often held our film in an exposed position a long time in order to get as many surges as possible; thus it is clear that earth currents from some of the heaviest surges could not be avoided. Very often the repeated surges were of type 5, having a similar amplitude value; therefore, they cannot be regarded as reflected phenomena from an original surge. In Fig. 9 on the other hand, there can be noted a fixed time difference between the original surge and its reflections. That the time between an original surge and its reflections may vary from case to case, can be explained easily. The thunderstorms

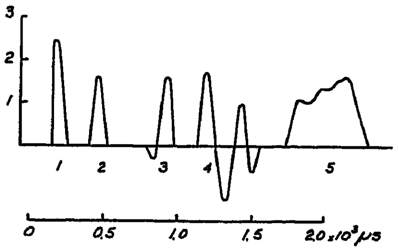


FIG. 17—DIFFERENT TYPES OF LIGHTNING SURGES RECORDED WITH THE CATHODE RAY OSCILLOGRAPH

considered as a generating region of the surges may, of course, change their distances from the transmission line, and sometimes we must calculate with two thunderstorms at a distance along the same transmission line. Therefore we can explain the differences in time from the reflected transients. Characteristic of the reflections is the pronounced damping.

The amplitudes of the surges have varied between 5 to 30 kv. to earth, values from ½ to 3 times the normal load voltage to earth. A number of our recorded surges has a remarkably low amplitude. This may be explained by our special precautions in these researches. We have tried to get as many surges as possible, and we have not restricted our efforts to such surges as have an amplitude surpassing the line voltage.

We have already pointed out the marked damping of many surges. In some cases, we have found a damping to 1/10 of the initial value on a distance of 60 to 80 kilometers. Surges that are developed at a distance of 30 to 40 kilometers from our observation point must, when they are passing this point, already be damped to some extent.

A surge exceeding three to four times the line voltage to earth does not necessarily produce a flashover path of such concentration that the line voltage will follow in the same path. We have, during a thunderstorm, observed lightning strokes quite near the transmission line at a distance of 12 kilometers from our observation point. On this occasion, flashovers were observed on insulators with an insulation twice as high as the normal insulation of the line insulators. It was quite possible that the line insulators at the same moment flashed over. Any disconnection of the lines by the protective relays at that moment was not observed.

An object to be desired in future research studies is to work with a set of oscillographs at different distances along the line, and at the same time put up one or two oscillographs especially adapted for recording the discharge forms of the lightning.

It is of importance with regard to the future to observe the real location of the lightning channels and regions. In other words, their distances to the transmission line on which surges are recorded must be observed in order to get quantitative results. This may be done in a manner which we have earlier used in our thunderstorm investigations in Sweden, by observing the time difference between the lightning and the thunder. In such a case, one can use two or three observation stations suitably located with regard to the transmission lines.

While our studies of lightning surges on transmission lines have fully confirmed our earlier results that were drawn indirectly from the nature of lightning discharges, we cannot yet draw positive conclusions about the character of surges on transmission lines. The primary set of phenomena that controls the generation of surges on transmission lines is quite variable. It is therefore necessary to do an extended piece of work before we are ready to deprive the surges of all their mystical character. From the results given above, it is evident that it is now possible to undertake an exact piece of research on the lightning surges on transmission lines in an entirely different manner than has heretofore been feasible.

II. STUDIES OF SWITCHING SURGES ON TRANSMISSION LINES

For some time it has been desired to record such transients as are produced by switching on long transmission lines. The difficulty of such studies has always been the lack of suitable instruments.

The limitations of purely theoretical calculations. By theoretical methods it has been possible to precalculate the general characteristics of transients. Such a calculation, however, has its limitations. By calculations, only, we can get no answer to the question as to what happens in reality on a transmission line during a switch operation under full load conditions. Several experiments have been made with artificial lines to escape this difficulty. The line constants have in such a case been chosen so that they correspond in their oscillations to the proper oscillations of oscillographs of the usual pattern where the vibrating element is a mechanical one. Such studies have served to confirm the theories of transients, but they cannot give any answer as to what happens on a line during switching with full load conditions.

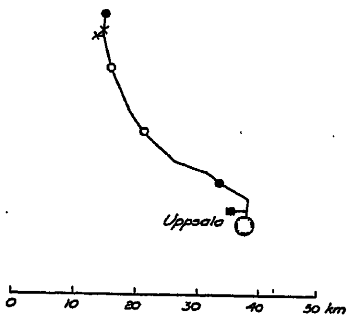


Fig. 18—Diagram of 20-Kv. Line Used in Taking Oscillograms of Switching Surges

Switching transients graphically recorded by suitable cathode ray oscillograph. The uses of the cathode ray oscillograph in recording lightning flashes and surges having been made clear in the first part of this paper, it will be evident that the cathode ray oscillograph constitutes a suitable instrument also for recording switching surges on transmission lines under full load conditions.

The transmission line used for switching surges. For the switching operations, we have used a 20-kv. line of which a diagram is given in Fig. 18. At one of its end points, the line has been connected to a commercial transformer of a ratio of 70/20 kilovolts.

The neutral point of the transformer has been earthed directly without any ground resistance. The line has been set free from all inductive loads in the form of transformers and the other end of the line has been open. In order to get as pure conditions as possible, all side branches of the line have been disconnected. Sometimes two of the three-phase lines have been connected in one of the line ends in order to get a double length of the line. For the switching operations, we have used commercial oil switch-breakers which, in general, have not been equipped with discharge resistances.

Methods of eliminating the normal load frequency. The measurements have been performed either with a single-phase or a three-phase connection to the line, and we have in these investigations used the methods of elimination that were described in the first part of this paper. The latter type of connection has sometimes been of great value, because it is possible by this connection to eliminate the ordinary 50-cycle load voltage in a simple way. If the system has its full symmetry, the resulting voltage must be zero on the deviation system of the oscillograph connected through condensers to the transmission line. In such a case, when the symmetry is disturbed, we can get a voltage on the deviation system and thereby get a record. We have found this method very simple and convenient for the elimination of the ordinary load voltage.

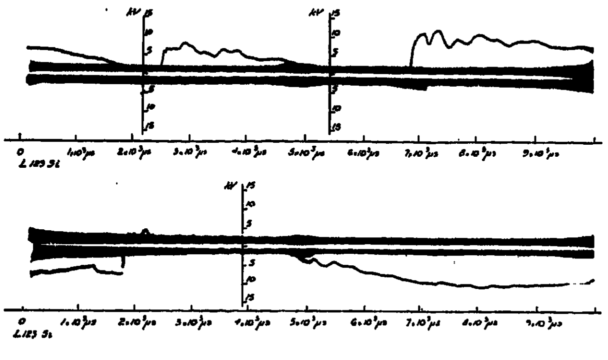


FIG. 19

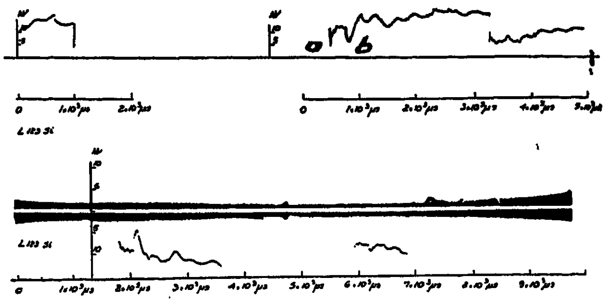


FIG. 20

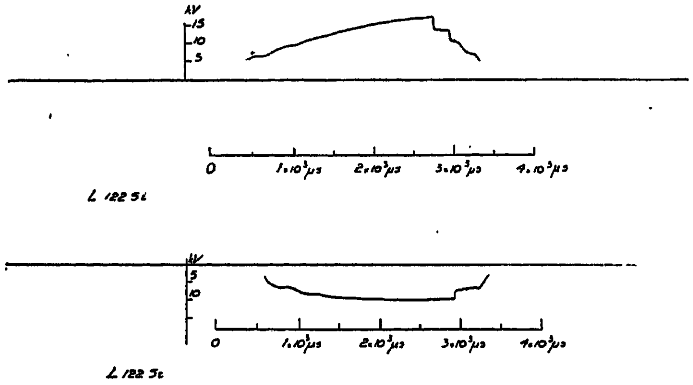


FIG. 21

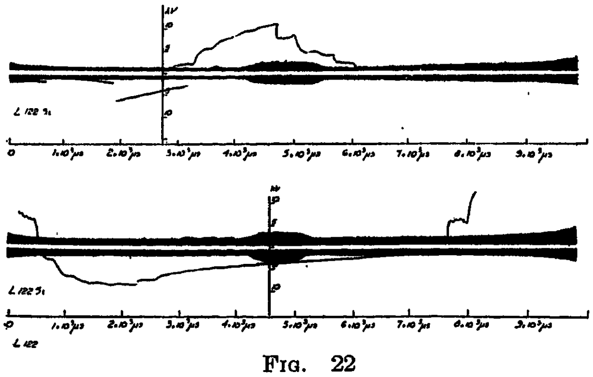


FIG. 22

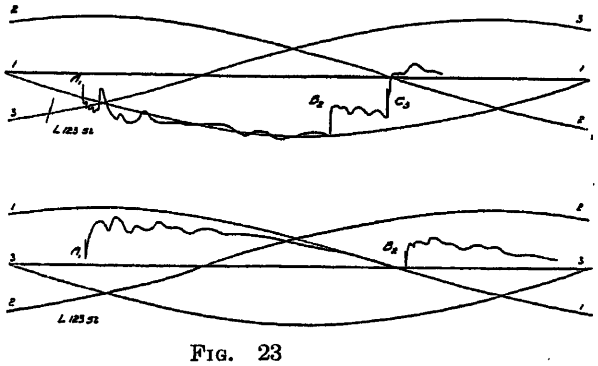


FIG. 23

FIGS. 19, 20, 21, 22, 23—OSCILLOGRAMS OF SWITCHING TRANSIENTS, USING THREE-PHASE COUPLING METHOD

Results gained on commercial lines under load conditions. Our experimental studies have been executed on commercial lines under full load conditions. This has been a circumstance both of advantage and of difficulty. By switching with a commercial breaker on a particular line, we have gained phenomena of similar character, and it has been impossible to vary the operations with regard to the amplitude and the front sloping of the transients. The method has its definite advantages. It has been possible by this method to gain such transients as really occur on transmission lines under natural load conditions. Some variations and irregularities have been observed. These are caused by irregularities lying in the arcs and sparks of the oil switch. For instance, in closing a circuit breaker, the contact at a certain moment will be separated sufficiently to cause a spark, which is followed by an arc. From our researches, we can conclude that the time for this spark is a very short one—the process is developed within a part of a microsecond, or within a few microseconds. In the spark path is produced an arc with changing characteristics. From this originate voltage fluctuations with varying periods and amplitudes. The process can have a duration up to 1/500 of a second.

Switching surges with a three-phase connection. Switching transients, where we have used a three-phase coupling method, are produced in Figs. 19 to 22. From Figs. 19 and 20 may be seen a rapid increase in the voltage when the line energized. Such rapid voltage variations can be observed in all our records.

This straight front wave passes along the line and can very often be observed when it is reflected. In Fig. 20 we can observe the time for the reflected wave marked *b*. The time difference between *a* and *b* is as measured on the original records 280 microseconds, which altogether corresponds to the time necessary for a transient wave to pass and return on the 42.1-km. long line. The marked increase of the resulting voltage at *b* can easily be explained as a summation of the existing voltage and the reflected voltage at the observation point.

A comparison between oil circuit breakers provided with and without discharge resistances. If we compare Figs. 19 and 20 on the one hand, and Figs. 21 and 22 on the other, we can observe a pronounced damping effect of the discharge resistances. If oil switches are used without discharge resistance (see Figs. 19 and 20), we can still observe that the produced voltage variations are not severe. This is in accordance with the practise in the United States. An examination of all our records of transmission transients shows that breakers not provided with discharge resistances produce more rapid voltage variations in comparison with those having resistances mounted. It seems that the difference is not great enough to give any reason for the complications and costs that always follow the existence of the discharge resistances. In their initial moments, voltage transients are developed with straight fronts. The amplitudes of these voltage surges do not exceed the variation of the load voltage.

The advantages of using a three-phase coupling method. The method of using a three-phase connection through condensers has some advantages over the method of a single-phase connection. The former method can give a more general view of the switching phenomena recorded. This can be seen by the Figs. 23 and 24. The time covered by the record in these figures extends over half a cycle of the normal load frequency. In order to explain the records of Figs. 19 and 20, we have drawn the three-phase half periods together with the switching transients with marked initial times. It is easy to observe, by comparison, how the moments of contact are recorded for each of the three-phase voltages. In the upper curve in Fig. 23, we can see that the voltage of curve 1 has gone over at A_1, for curve 2 at B_2, and for curve 3 at C_3. In the lower curve in Fig. 23, the voltage has gone over at A_1 for curve 1, at B_2 for curve 2, and for curve 3 when this curve has passed through zero. The time for the contact voltages is determined by the position of each contact in combination with the suppressed load voltage and the insulation resistance between the contacts.

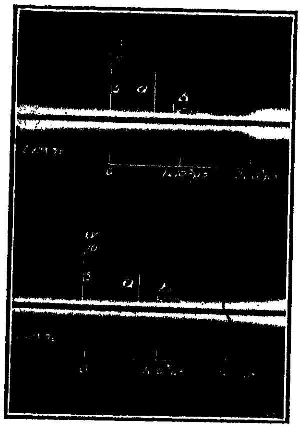


FIG. 24—OSCILLOGRAM OF SWITCHING TRANSIENTS, USING SINGLE-PHASE CONNECTION AND A LOW LEAKAGE RESISTANCE

The characteristic method of the system is as follows. When the three contacts have gained their full load voltage, and the voltage system is symmetrical, the deviation system of the oscillograph comes to zero potential.

The amplitude of the voltage does not need to come up to its full crest value when the contact takes place. The contact value is determined both by the voltage value of the contact and its position with regard to the insulation resistance of the oil.

Results of switchings where the single-phase method has been used. The other method used was a single-phase connection. In this case, the other two phases were either grounded, or in some instances, insulated. Some definite difficulties arose in the use of the single-phase connection method. It thus was necessary to eliminate the normal 50-cycle load voltage. For this purpose, we have the method with a leakage resistance already described. We must remember that this method gives different voltage scale values for different frequencies.

We have not, in these researches, found it necessary to introduce a variable voltage scale. Our scale given in the figures thus refers only to the voltage fluctuations of a voltage variation where full voltage values are gained within about a hundred microseconds.

Some researches with a single-phase connection are reproduced in Fig. 24. In these records we have used a very low resistance and the leakage thus has been appreciable in order to record only the rapid voltage variations. The length of the line was 42.1 km. The straight voltage fronts, that are clearly visible on the original records, are marked *a* in Fig. 24. The voltage reaches its full value within a few microseconds and these curves are then marked *b* after reflection from the other end of the line. The time difference between the original and the reflected curve is 280 microseconds, a value in full accordance with the normal speed of transients on the line.

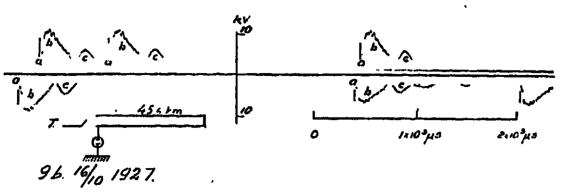


FIG. 25

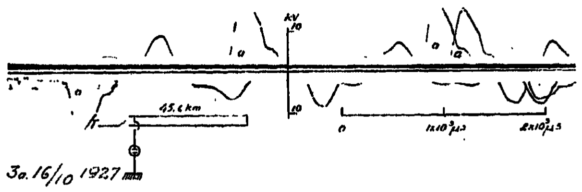


FIG. 26

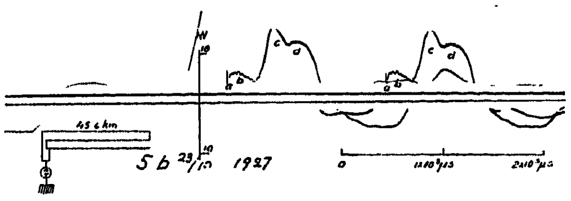


FIG. 27

FIGS. 25, 26, 27—OSCILLOGRAMS OF SWITCHING SURGES, USING SINGLE-PHASE CONNECTION AND LARGER LEAKAGE RESISTANCE

Some studies have also been made where a larger leakage resistance is chosen than in the case first mentioned. By such an arrangement, it has been possible to record slower voltage variations. The line length has been doubled in this measurement.

Some results of such measurements are given in Figs. 25, 26, and 27. The line length and connection diagrams are given in these figures. The straight initial voltage front that is developed when the line was set under load is marked *a* in some of the Figs. 25 to 27. This straight front varies with its full value within a few microseconds and must, with the time scale used in the records, be marked as a vertical line going up from the zero line. These straight fronts are very quickly damped.

In Fig. 25 are reproduced a set of repeated connections of the line to the transformer. It must be noted that these different connections produce similar voltage variations marked *b* in Fig. 25. In connection with this slow variation, we observe a high frequency variation that is quickly damped. The reflected transient coming from the other end of the line is regular and marked *c* in the figures.

The voltage variations in the open end of the line have been recorded and are reproduced in Fig. 26. The initial voltage variations in this figure are marked *a*. As the line is open, we have recorded a doubling of the voltage amplitude. The high frequency which is very marked in Fig. 25 has quite disappeared at the open end of the line.

Voltage variations of a more complicated structure are reproduced in Fig. 27. In this record the voltage variations are composed of the initial, the passing, and the reflected voltage variations.

Conclusions. This article covers the first studies of lightning and switching phenomena with a cathode-ray oscillograph of new construction, ever made on transmission lines during load conditions. Naturally it is only a beginning describing some practical schemes that can be employed in making such investigations, and the author hopes that further work to be done in the near future will bring out many more valuable contributions to the science of lightning and switching phenomena.

Discussion

D. W. Roper: In all of the calculations of the effects of lightning heretofore, it has been necessary to assume the shape of the wave of the transient voltage due to lightning as a basis for calculations. Similar assumptions must also be made in connection with the tests on lightning arresters with the aid of the so-called lightning machines, in order to obtain information regarding the relative protective value of various types of arrester. Now for the first time Dr. Norinder shows, not some assumed values, but some actual curves obtained by his modified form of cathode-ray oscillograph. These records show very plainly that there is a wide variety in the forms of transient voltage waves, and this means that it is not sufficient to assume one particular form of wave in making comparative tests on lightning arresters. Instead, the proper procedure would be to arrange the lightning-machine circuits so that a variety of transient voltage waves would be applied to the lightning arresters under test, and the average of the results obtained in this manner utilized instead of the results from one particular form of wave as has been the practise heretofore. In all probability, such a procedure will account for a part of the discrepancies which have been noted between tests on lightning arresters in the laboratory and the records obtained from arresters in service.

Recent studies of the transformer failures due to lightning in Chicago indicate beyond any reasonable doubt that about 30 per cent of such failures are due to transient high voltages which enter the transformer on the low-voltage leads. The failures from this cause, which are of the order of 0.1 per cent per year, are so small that it is quite impossible to secure the data by the statistical method that was followed in the papers heretofore presented before this Institute, and, accordingly, arrangements are being made to secure this information by means of a cathode-ray oscillograph similar to the one used by Dr. Norinder. It is hoped that, in addition to furnishing information as to how the transformer failures due to lightning entering via the secondary leads may be prevented or reduced under the conditions in Chicago, the studies which are planned will, in addition, furnish information regarding lightning protection which can be applied to higher-voltage circuits.

K. B. McEachron: Those who have attempted to solve the problem of applying the cathode-ray oscillograph to the study of lightning in the field appreciate the difficulties involved.

In the United States we have been operating oscillographs of the Dufour type over a period of three or four years, and have been quite successful in recording the performance of devices on transients produced in the laboratory. The difficulty comes about chiefly in the matter of getting the oscillograph under way when the transient comes along. Needless to say we have no premonition of its happening, and as a rule we are not able to beat the transient to the oscillograph, which means that we must have some means of automatically setting off the oscillograph, or else keeping it in operation all the time. Dr. Norinder seems to have solved that problem in the second manner. In that connection there is one question I should like to ask him. I presume he is using a cold-cathode-ray oscillograph, and I wonder how much difficulty he has with holding a good spot on the film. I assume he has no difficulty because his photographic records are satisfactory. We have had difficulty in that particular respect, in holding a satisfactory spot over a long period of time when using high cathode-anode accelerating potentials.

This problem can perhaps be attacked from the standpoint of the hot cathode instead of the cold. There are certain inherent difficulties with that method, however. In the Lightning Arrester Laboratory in Pittsfield we have worked out a scheme whereby the transient will actuate the oscillograph although the device itself was not previously in operation. By that I mean that there is no cathode stream at the moment the lightning discharge reaches the oscillograph, but it has been possible to set up the circuit so as to start the oscillograph at the moment when the transient appears. It is necessary, of course, to find some means of delaying the transient for a short period of time, as in the method Gabor used in his experiments in Berlin where he delays the transient by passing it through a cable of more or less known characteristics, and then measures the emerging transient. In the meantime the oscillograph has had a microsecond or two to get into operation.

We have come to feel that this problem is an extremely important one. Some valuable work has been done in this country with the Lichtenberg figures. However, they tell us nothing about the duration or tail of the transient and therefore we are unable, very often, to decipher accurately the results which we get from the surge-recorder or klydonograph studies. This is probably due very often to differences in the form of the waves, and therefore our solution at the present time is rather indeterminate.

I feel that we must have on our transmission lines in our country several of these cathode-ray oscillographs so that we can determine the crest values, wave fronts, and character of the tails of waves which appear on our transmission circuits. The effect of the station equipment in modifying and reflecting these waves forms should be also determined.

Harald Norinder: This question of the regulation of the spot in a cold-cathode-ray oscillograph raised by Mr. McEachron has not caused us any difficulties at all. To have a steady spot is only a question of regulation. With respect to an oscillograph where a hot cathode is used it would be of interest if such a type could be developed, but I am not sure that all troubles could be easily avoided. It is for instance very hard to get a suitable concentration through the anode with the hot-cathode type. There have been some trials to solve the problem of the hot-cathode type but I have not seen any publications giving good practical results.

Synchronous Machines—IV

BY R. E. DOHERTY*
Member, A. I. E. E.
and
C. A. NICKLE*
Associate, A. I. E. E.

***Synopsis.**—The special case of cylindrical rotor machines has been treated previously by Boucherot and others. The present paper solves the general case, including salient pole machines. The cylindrical rotor type thus becomes merely a limiting case. The principal assumption which distinguishes the present theory from the extensively studied cylindrical rotor theory is that the total armature self-inductance is here taken as variable with respect to rotor position, whereas the previous theory of short circuits, as represented by Boucherot, for instance, assumes this inductance to be constant—in other words, that the air-gap is uniform.*

The four basic concepts underlying the improvement in theory, both as applied here and in the authors' previous work on Synchronous Machines, are:

1. Characterization of the machine by four reactance coefficients, two corresponding to the main pole axis, i. e., direct axis, and two to the interpolar axis, i. e., quadrature axis. These are: x_D, x_D', x_Q, x_Q'. *(See notation.) Thus the theory has been referred to, more or less aptly, as the "Four-Constant Theory."*

2. Resolution of flux and m. m. f. waves traveling with respect to the rotor into stationary, pulsating components in line with the direct and quadrature axes. The theory involves also, of course, the usual Blondel resolution of the stationary fundamental waves.

3. That the variable component of armature inductance varies between the direct axis value and the quadrature axis value as a second harmonic function of the electrical space angle. This applies to all variable components of inductance which, with armature current of fundamental frequency, produce fundamental voltage.

4. The complete use of a fractional system of notation throughout the equations, thus expressing each quantity as a decimal fraction of some definite, convenient value, such as "normal" value. This is, of course, an outgrowth of the familiar practise of expressing such quantities as a per cent of normal value, but the latter involves carrying along the "100" in calculations, which is both cumbersome and conducive to numerical error. Experience has established a very definite preference for the former—the only difference between them, of course, being the position of the decimal point. Thus, with unity as the base, instead of "100," it is proposed to denote the former by "per unit" system, instead of "percentage" system.

Turning to the results of the paper, expressions are derived for the short-circuit currents in the armature and field, for voltage induced in the open phase, and for voltages across external reactances in the armature and field circuits. Comparison of calculated and test curves for these quantities are shown in Figs. 5 to 8, for a 7000-kv-a. 375 rev. per min. salient pole alternator, and in Figs. 11 to 25 for a 20-hp. 1800-rev. per min. induction machine (cylindrical rotor). These show satisfactory agreement.

The form of the results is interesting and instructive. The short-circuit currents are expressed, as in equation **(9)**, *as the sum of two series, one of odd, the other of even, harmonics, in which the magnitudes of the harmonics, in ascending order, are in geometric ratio. The value of that ratio is*

$$b = \frac{\sqrt{x_Q'} - \sqrt{x_D'}}{\sqrt{x_Q'} + \sqrt{x_D'}}$$

where x_D *and* x_Q *are transient reactances in the direct and quadrature axes.*

The effect of salient poles in reducing the relative magnitude of the harmonics with respect to the fundamental in the short-circuit current is clearly shown by the equations. This is important both in connection with the calculation of mechanical forces, and of the voltage induced in the open phase during single-phase short circuit. It is shown that although this voltage may be 15 or 20 times normal in a laminated, cylindrical rotor machine without amortisseur circuits, it may be only 2 to 4 times normal in a salient pole machine. An amortisseur winding greatly reduces it in either case.

The fundamental component of this voltage across the open phase is shown to be approximately equal to (1 + b) times the voltage before short circuit.

The mathematical work and definitions are in Appendixes as follows:

A Summary of equations.
B Armature current.
C Field current.
D Voltage across external reactance in armature circuit.
E Voltage across external reactance in field circuit.
F Voltage across the open phase.
G Variation of armature inductance with position.
H Decrement factors.
I Reactances, definitions, and relations.

HISTORICAL

At the White Sulphur Springs Convention in 1926 the authors presented the first of a series of papers covering their investigations of the Synchronous Machines. Part I was an extension of fundamental theory; Part II, a treatment of torque-angle characteristics under steady state conditions; and Part III, presented at the 1927 Winter Convention, a study of torque-angle characteristics under transient conditions. In the present paper, an analysis is made of single-phase short circuits. In the future, the treatments of other characteristics, including polyphase short circuits and the torque existing under short-circuit conditions, may be given.

Certain special cases of short circuits have been treated previously. Following the early work by Steinmetz[1] and Berg, Boucherot[2] presented his classical paper in 1912, dealing with alternators of the laminated, cylindrical rotor type, with and without amortisseur windings. In 1915 Diamant[4] reviewed the work of Berg and Boucherot, and, among other mathematical extensions, gave expressions for the envelope of the short-circuit current. In 1918 he presented an analysis of sustained short circuits, dealing principally with the nature of flux distribution under that condition. In the same year[9a] one of the authors proposed the use of the *Constant Linkage Theorem* in dealing with short circuits, and in 1921[9b] and 1923[9c] illustrated its use. Also in 1923 Franklin[5] applied this method to a large number of cases of single-phase and three-phase short circuits. Working from the same fundamental premises, Laffoon[7] solved additional cases in 1924. The following year Karapetoff[6] analyzed, in the main, the same cases as those of Franklin and Laffoon; but he started from a somewhat different theoretical basis, *i. e.*, from Kirchoff's Laws. But he, like Franklin and Laffoon, neglected resistance, which made the fundamental premises the same—since the Constant Linkage Theorem is merely a corollary of Kirchoff's Second Law, for the special case of negligible resistance. That is, his equations involved voltages, instead of magnetic linkages, and were therefore the first derivatives of the corresponding equations in Franklin's work. Involving the same assumptions regarding the circuits, the results naturally also agreed. In 1923, Lyon[13] outlined a method according to which "certain low-frequency transient conditions in electric machinery may be analyzed by a vector method." From the vector point of view, first proposed by Dreyfus[14,15,16] in 1912, the transient voltage or current may be represented by a series of vectors which shrink exponentially as they rotate. The thorough, mathematical treatments of cylindrical rotor machines both by Shimidzu and Ito[8] and by Biermanns,[3] in which the effect of resistance is approximately taken into account, are especially noteworthy. All of the foregoing investigations have dealt with cylindrical rotor machines; and, granting the simplifying assumptions which have appeared to be practically necessary, such as those made by Shimidzu and Ito, and Biermanns, relating to resistance and saturation, it may be said that a practical solution of short circuits for the cylindrical rotor type has been attained.*

---

*Both of the General Electric Co.

1. For references see bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

SCOPE

These previous analyses, however, have not solved the important case of salient pole machines. The present paper treats the general case, including both types. Proceeding from the same basic point of view from which the authors' previous work on Synchronous Machines[9d,9c] has been developed, the present treatment takes up single-phase short circuits and partial short circuits, *i. e.*, with external reactance in the armature and field circuits, developing expressions for armature current, field current, armature voltage across the open phases, voltage across reactance in the armature circuit, and the voltage across reactance in the field circuit. These expressions apply not only to salient pole machines, but also to the cylindrical rotor type, since under the present theory, the latter type merely becomes a special case of the former. Thus, the scope includes the development of expressions, applicable alike to salient poles and cylindrical rotors, for the voltage and current phenomena under single-phase short circuit.

Although the single-phase phenomena of *three-phase* machines only are considered here, the theory is, of course, applicable also to two-phase machines.

DISCUSSION OF THEORY

The four basic concepts underlying the improvement in theory, both as applied here and in the authors' previous work, are: (1) characterization of the machine by four reactance coefficients, two corresponding to the main pole axis, *i. e.*, *direct axis*, and two to the interpolar axis, *i. e.*, *quadrature axis*. These are x_D, x_D', x_Q, x_Q'. (See notation.) Thus the theory has been referred to, more or less aptly, as the "Four-Constant Theory.†

2. Resolution of flux and m. m. f. waves traveling with respect to the rotor into stationary, pulsating components in line with the *direct* and *quadrature* axes. The theory involves also, of course, the usual Blondel resolution of the *stationary* fundamental waves.

3. That the variable component of armature inductance varies between the *direct* axis value and the *quadrature* axis value as a second harmonic function of the electrical space angle. This applies to all variable components of inductance which, with armature current of fundamental frequency, produce fundamental voltage.

The general aspects of this premise should be carefully considered. In a salient pole machine, if the axis of the armature fundamental m. m. f. is not in line with either the pole axis or the interpolar axis, the resulting fundamental flux wave, of course, will not be in space phase with the m. m. f. wave; and therefore calculations involving the total waves would be very complicated. *By symmetry*, the flux wave will be in space phase with the m. m. f. wave producing it, if the latter is in line with either of these axes. The purpose of resolving the fundamental wave of m. m. f. along these two particular axes is thus to obtain component flux waves in space phase with the respective m. m. f. components producing them.

---

*After submitting the present manuscript, a copy of S. Bekku's treatment of "Sudden Short Circuit of Alternator," Bibliography 12, has been received. This treats the case of the cylindrical rotor machine with a three-phase winding on the rotor, each rotor circuit being closed either directly or through the exciter. He employs "Heaviside" in the mathematical work.

†There are, of course, more than four. The four constants which characterize the machine under single-phase operation are not the same as the four which characterize it under polyphase operation. They are, however, of the same general nature, thus corresponding each to each, the chief difference being due to the mutual inductance between phases. Nor are the four constants the same for a line-to-neutral, as for a line-to-line, short circuit. Thus, there are twelve constants in this particular connection, and even some of these may be split up under special conditions; for instance, where saturation exists, or where there are short-circuited windings on the rotor in addition to the main field winding, etc. Moreover, in certain network calculations, it is desirable to use an additional constant, namely, the *zero phase-sequence reactance.* The *positive phase* and *negative phase* values are also used in the latter connection, but these are expressed in terms of the "four constants." And, if one should include also the calculation of all the possible phenomena due to harmonics, the number of necessary constants could be extended almost indefinitely. Thus while it is correct to say that the machine, under single-phase or polyphase operation, is characterized under the premises of this paper by four constants, there are, all told, many more than four values of reactance to be considered in general.

Now it is a mere incident of machine design that there is symmetry about the two axes mentioned, and that these axes happen to be 90 electrical degrees apart. But these facts underly the possibility of expressing the variation of armature inductance as a simple second harmonic, as in equation (3). For derivation of this, see Appendix G.

Such a variation is shown to apply not only in the case where the field winding is open circuited, *i. e.*, to the case of pure reluctance symmetry of the rotor magnetic circuit, but also to the case of artificial reluctance symmetry as produced by short-circuited rotor windings, such as the field winding. A short-circuited secondary on the rotor merely increases the reluctance of the path of the armature flux; and in machines as ordinarily constructed, the artificial reluctance symmetry is about the same two axes as when the secondary windings are open-circuited, since the axes of the latter windings coincide with the direct and quadrature axes. See Fig. 1.

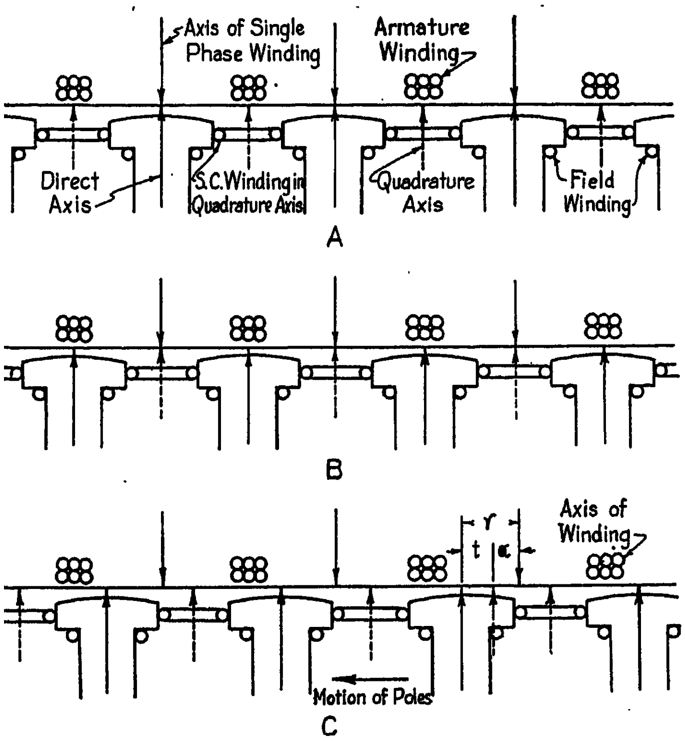


FIG. 1

There is also a relatively small component of *armature leakage inductance*, which varies as a second harmonic. Although this component is due to space harmonics of flux, it nevertheless produces a reactive voltage of the same frequency as the armature current. This has been considered by Karapetoff[6a] and Arnold,[11] its significance is fully discussed in *Synchronous Machines,—I*,[9d] and its magnitude is given by Alger.†

Thus, considering the variable components of inductance, it is permissible, under the premises, to regard the variation as a second harmonic of the electrical space angle, regardless of whether the variation is produced by the shape of the iron magnetic circuit, by short-circuited secondary windings on the rotor, or by both; and regardless of whether the variation affects the fundamental or the harmonic flux waves, so long as the reactive voltages produced by such waves are of fundamental frequency. Fig. 2 shows the variation of the transient inductance of a 20-hp. machine, on which many of the tests described herein were taken. Curve *A* was taken under the condition of no saturation, and is practically a sine wave of double frequency, whereas curve *B* was taken under the condition of partial saturation, and departs considerably from a sine wave—thus containing significant even harmonics. Fig. 3 shows the same curve for a four-pole, 15-k-va., 1800-rev. per min. salient-pole machine without saturation. Fig. 4 shows the curve for a 7000-k-va. machine. This

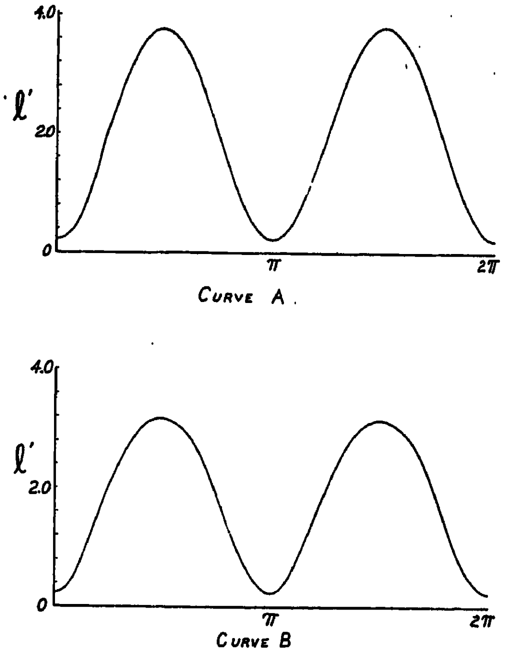


FIG. 2—VARIATION IN TRANSIENT INDUCTANCE, 20-HP. MACHINE

Curve A—No saturation
Curve B—With saturation

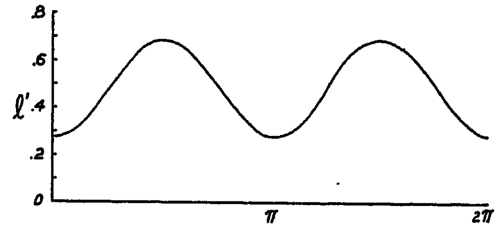


FIG. 3—VARIATION IN TRANSIENT INDUCTANCE—4-POLE, 15-KV-A., 1800-REV. PER MIN., SALIENT-POLE MACHINE

was obtained from an oscillographic record taken in connection with the "slip method."‡

---

† "Calculation of Reactance of Synchronous Machines," by P. A. Alger, p. 493.

4. The complete use of a fractional system of notation throughout the equations, thus expressing each quantity as a *decimal* fraction of some definite, convenient value, such as "normal" value. This is, of course, an outgrowth of the familiar practise of expressing such quantities as a percentage of normal value, but the latter involves carrying along the "100" in calculations, which is both cumbersome and conducive to numerical error. Experience has established a very definite preference for the former, the only difference between them, of course, being the position of the decimal point. Thus with *unity* as the base, instead of "100," it is proposed to denote the former by "per-unit" system, instead of "percentage" system.

The use of the "per-unit" system has distinct advantages. In the first place, the results are in forms which have a definite significance in giving at once an idea of relative values; for instance, a current of 0.5 means 50 per cent of normal current. Another advantage is that cumbersome conversion factors are thus eliminated. For instance, as *per-unit* quantities, current is taken as numerically equal to the m. m. f. of armature reaction, and voltage, at normal frequency, as numerically equal to the magnetic linkages which produce it, and in many cases, as noted below, reactance as numerically equal to inductance. It should be mentioned that in equating *per-unit* current to *per-unit* armature reaction, or *per-unit* linkages to *per-unit* voltage or field current, dimensional relations are not disturbed, because all values are ratios; *i. e.*, all values are numerics. Conversion factors thus become unnecessary.

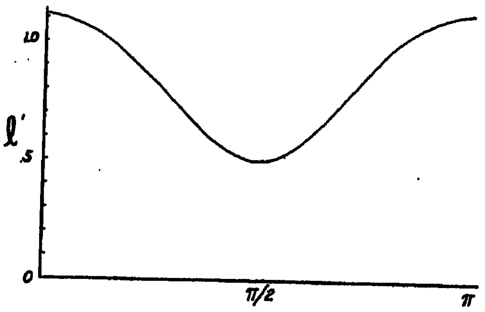


FIG. 4—VARIATION IN TRANSIENT INDUCTANCE—7000-KV-A. MACHINE

At some risk of confusion, attention is also called to the fact that, according to the definitions, reactance and inductance, *as per-unit quantities*, become identical numerically in many special cases where the reactance is based on fundamental frequency. Reactance is still reactance, and inductance is still inductance; but with each expressed as a ratio, *i. e.*, as a numeric, they become, by definition, numerically equal under the special condition referred to. The definitions are: *per-unit inductance for the nth harmonic is the ratio of the nth harmonic linkages, produced by an nth harmonic current of normal value, to normal linkages.* And, *per-unit reactances for the nth harmonic is the ratio of the nth harmonic reactive voltage due to an nth harmonic current of normal value, to normal voltage.* Thus, if l is the circuit inductance, $i_n\, l$ gives the *per-unit* linkages due to the nth harmonic current i_n, and $i_n\, x_n$ gives the corresponding *per-unit* reactive voltage. For the fundamental component of current i,

$$i\,x = i\,l$$

provided all of the linkages due to i produce voltage of fundamental frequency; and if so, $x = l$. But this is not always so. If the inductance l is constant, the linkages must be proportional to the current (under the premises), and the fundamental current could not produce any but fundamental linkages; but if l is a function of time, as it is during rotation, a current of one frequency may produce linkages of another frequency. For instance, the fundamental component of current in the armature may produce 5th harmonic (in time) linkages in the armature. But the reactive coefficient x for the fundamental would not, by definition, include these linkages; it would include only those of fundamental frequency. Thus, as defined, the *per-unit* reactance for the fundamental, and the *per-unit* inductance become numerically equal when l is constant, and the voltage across the inductance, regardless of the character of the current, is given by

$$e = l\,\frac{d\,i}{d\,t} = x\,\frac{d\,i}{d\,t}$$

However, for rotation, the voltage is given only by

$$e = \frac{d}{d\,t}\,(l\,i)$$

In other words, the only difference between *per-unit* x for the fundamental, and *per-unit* l, is the difference between the total linkages produced by the fundamental current i, and that component of the total which is of the same frequency as i. Hence when there is no difference, that is, when the linkages are at all instants proportional to the component of current under consideration, x and l may be taken as numerically identical.

So, considering the system of notation as a whole, although it may possibly be a bit confusing at first, it is nevertheless sound, has very great advantages in simplifying equations, and in giving an added significance to the results, and is time-tried; *i. e.*, has proved its value over years of day-to-day practical use in the solution of complicated problems.

The four basic concepts outlined in the foregoing paragraphs, afford the basis for symmetrical, relatively simple, and very interesting expressions for the voltage and currents under single-phase operation. It has been possible in this way to take into account, without undue mathematical complication, the *predominating factors* of the problem, and therefore to obtain a very satisfactory agreement between calculated and test results, as may be seen by reference to Figs. 11 and others which follow.

---

‡See R. H. Park & B. L. Robertson, p. 514.

These extensions in theory have required not only additional characteristic machine constants, as already mentioned in a footnote, but also have made it necessary, in order to avoid confusion and misunderstanding, to classify and more carefully define them. In the early days, a synchronous machine had *a* reactance; now it has many. Moreover, our ideas about some of the well established terms, for instance, *armature leakage reactance*, and *field leakage reactance*, have needed some revision. As a rough illustration in connection with the general situation; in running a polygonal boundary, an error in the length or bearing of one of the sides is not so apparent, unless it becomes necessary to "close" the polygon. The error is merely passed on the next side, letting the end whip about as occasion may require. In synchronous machines, it was formerly necessary to "run" only one or two sides; now, with the necessity of calculating practically all of the electrical characteristics, we must "close the polygon." So, an orderly classification, description and careful definition of terms is quite in order, as is also the presentation of methods of calculating some of these factors. The former is given in a companion paper by R. H. Park and B. L. Robertson; and the latter, by P. L. Alger.

Returning to the general aspects of the theory, consider now the distinguishing differences between the present treatment and those already referred to, which have preceded it. Boucherot's work is the outstanding early contribution on the subject. Others have attacked essentially the same problem in different and perhaps more convenient ways, and have made noteworthy extensions. But they, like Boucherot, have all treated *cylindrical rotor machines only*, whereas the present investigation has dealt with the general case including salient pole machines, the cylindrical rotor type thus becoming merely a limiting case. It may be of interest, therefore, to draw some specific comparison relating to the differences in fundamental premises, and hence also in the resulting equations.

There are two fundamental assumptions which distinguish the general treatment from that of cylindrical rotors. The first relates to the character of the *total* armature self-inductance; Boucherot's L_2, the authors' $\mathcal{L}_D$ and $\mathcal{L}_Q$. In the cylindrical rotor theory, this inductance is assumed to be constant with respect to position—in other words, that the air-gap is uniform; in the present theory, it is assumed to be variable,* as it actually is in salient pole machines. In the limit, when there is no variation, that is, when $\mathcal{L}_D$ and $\mathcal{L}_Q$ are equal, the equations then apply to cylindrical rotor machines.

What difference does it make in the results if that inductance is assumed to be variable? It makes a large difference in the relative magnitudes of the fundamental and harmonics, and thus also in the ratio of the amplitude of the fundamental to the peak value, both in the short circuit current, and in the open phase voltage. One important result of the theory is that the *peak value* is the same for salient poles as for cylindrical rotors, when the transient reactance, direct axis, is the same. Thus equations (**18b**) and (**19b**) show, as indicated graphically in Fig. 9, that the peak is independent of x_Q'. The magnitudes of the harmonics and of the fundamental are very important, both in determining mechanical stresses and angular vibrations, and also in determining the voltage in the open phase and across external reactance. These are greatly affected by salient poles, and can be predetermined with practical accuracy from the present theory. But they obviously cannot be correctly calculated from cylindrical rotor theory based on the assumption of constant total inductance with respect to rotor position.

The second distinguishing premise is that there are two rotor windings, one in each of the conventional axes, and having different constants. It is thus possible to take into account, at least in an approximate way, the effect of any short-circuited winding which may exist in the quadrature axis, such, for instance, as a non-uniform squirrel-cage winding, as used in laminated cylindrical rotors, or in salient-pole machines, and also such as massive steel rotors in which the transient reactances x_D' and x_Q' are different. And if there are actually only two distinct rotor circuits in the two axes, even though they have different constants, the theory is completely applicable. Moreover, for a special case of cylindrical rotor in which there are two rotor windings at 90 deg., *i. e.*, one in each axis, and also in which $x_D' > x_Q'$, the characteristics can not be calculated, so far as the authors are aware, by the previous cylindrical theory. But they are easily calculated and analyzed by the present theory. This is illustrated by reference to Figs. 13, 14, and 15, which apply to such a case. These are explained under "Results." Thus, the second distinguishing assumption broadens the application of the theory over the previous theory, even in the case of cylindrical rotors.

ASSUMPTIONS

1. *Negligible saturation.* This is, of course, not true practically. However, as mentioned in *Synchronous Machines—III*, this does not prevent the use of the results in cases where saturation is present, by judicious shading of the constants, or by shading the calculated results with respect to the degree and distribution of the saturation; remembering, however, that the method thus applied is an approximation.

2. *The open-circuit voltage is a sine wave.* This is

---

*The total variable component comprises not only the variable term of the inductance of armature reaction, but also the small variable term of armature leakage inductance.

practically true of most modern alternators. The combined effect of coil pitch, distribution of the winding, Y-connection, and perhaps "fractional slot" winding, practically eliminates all harmonics in the voltage wave. So this assumption is reasonably correct for most synchronous machines. The only significant practical departure might be in the case of the line-to-neutral voltage of a machine with a relatively small number of full pitch coils per pole, in which case there might be a significant third, and possibly other odd harmonics.

3. *The machine has a short-circuited rotor winding in the quadrature axis, as well as the main field winding in the direct axis.* (See Fig. 1.)

4. *Resistance of the armature and field circuits is neglected in calculating the magnitude of the initial short-circuit current; and the armature resistance, in calculating the magnitude of the sustained current. But both resistances are taken into account in the decrements.* The effect of resistance in the impedance is usually insignificant, but its effect on the decrement is, of course, very pronounced. Indeed, there would be no decrement without resistance.

5. *The machine is short-circuited at no-load.* A suggested extension of the theory is to take into account the relatively small effect of load current.

FUNDAMENTAL EQUATIONS

The general plan of attack and the general considerations leading up to the fundamental equations will be discussed here, and the final equations will be derived in the Appendix.

Superposition is used in obtaining the initial values of armature and field current. The plan is first to find a general expression for the magnetic linkages of the armature winding, due to any transient armature current, and with no field current except that induced by the armature current; and then to determine the armature linkages due to any other current in the field winding, and superpose these to find the resultant value of armature linkages. Under short circuit, and neglecting resistance, the latter value of linkages must be equal to the known value existing at the instant of short circuit.* These relations are sufficient to solve for the *initial* armature current i.

The *initial* value of field current is obtained by adding the component induced by the armature winding, to the steady value existing the instant before short circuit.

There are certain components of the *initial* current which are known to be transient. These, as a function of time, are obtained by multiplying the individual transient components by the appropriate decrement factor.

The sustained values, of course, comprise those components which exist after the transients have disappeared, and are thus obtained by setting the transient terms equal to zero.

The decrement factors are obtained by an approximation. It seems to be well nigh hopeless, as a practical matter, to take into account rigorously the armature and field resistances in the general case, but it happens fortunately that it is possible to obtain results which are sufficiently accurate for any practical purpose, by making simplifying assumptions.

Before stating these specifically, the physics of the transient currents will be briefly considered. When the armature winding is short-circuited, the magnetic flux linked with the closed field circuit can not change significantly in the first moment, yet, since it can not now enter the short-circuited armature winding, it is forced by the demagnetizing action of the armature currents, to pass through paths (the leakage paths) of greater reluctance; that is, the short-circuited armature is equivalent to an increased magnetic reluctance. Hence, an additional current must appear in the field circuit in order to sustain the flux in those new paths, and this spontaneous additional direct current, being unsupported by the exciter voltage, is, of course, transient in character; that is, the voltage which supports it through the resistance of the field circuit, is generated by the decay of the flux through that circuit. Considered thus, the problem of determining the transient becomes practically identical with that of determining it for the simple inductive circuit, namely, to obtain the simple decrement factor $\frac{r}{L}$, in which r is the resistance of the circuit through which the transient flux is decaying, and L is the total circuit inductance defined by the ratio of the total linkages of the circuit, to the current required to sustain them, this decrement factor being applied to the transient component of current. This general view of decrements was presented by one of the authors[9a] in 1918, and is essentially the same as that used by Biermanns.

How does armature resistance affect the field decrement factor? From the limit of zero armature resistance, on the one hand, for which *all* flux linkages are forced into the high reluctance leakage paths, to infinite resistance on the other, that is open circuit, under which condition none of the flux is forced out of the normal magnetic path of relatively low reluctance, there are, as far as the field transient is concerned, all possible intervening values of effective reluctance and corresponding inductance. Thus, the effect of armature resistance on the field transient may be taken into account in the field decrement factor by appropriate modification of the inductance. This is done when that resistance is large enough to affect significantly the value of the *direct axis* component of armature current. It is rarely necessary to do this, but when it is, it is a much easier task than to attempt a rigorous solution, which, to the author's knowledge, has been obtained only in the special and limited cases treated by Biermanns,[8] Shimidzu and Ito,[8] and Bekku.[12]

---

*Constant Linkage Theorem. See Bibliography 9c.

The same general argument may be applied to the effect of field resistance upon the armature decrement factor, that is, with respect to the transient flux linkages which are "trapped" in the short circuit armature winding.

Thus, the approximation is made that, unless the armature resistance is large enough to become significant in the impedance, *i. e.*, large enough to affect the demagnetizing current in the direct axis, it is neglected in the field decrement factor; and when it is taken into account, this is done by an appropriate modification of the inductance. And under similar limitations, the field resistance is neglected in the armature decrement factor.

The field decrement factor is applied to the induced direct component of current in the field, and to all other phenomena which depend on that current in direct proportion, such as the transient alternating component of current in the armature, and the armature circuit voltages which are due to the latter. And likewise the armature decrement factor is applied to the d-c. component in the armature and to all other phenomena which thus depend upon it, such as the transient a-c. component in the field circuit, and the voltages which are due to the latter. For the derivation of decrement factors see Appendix H.

The armature decrement factor is,

$$\sigma_a = \frac{r}{\sqrt{x_D'\, x_Q'}} \tag{1}$$

and the field decrement,

$$\sigma_f = \frac{x_D + \sqrt{x_D'\, x_Q'}}{x_D' + \sqrt{x_D'\, x_Q'}}\, \sigma_0 \tag{2}$$

where σ_0 is the decrement factor of the field corresponding to *total* field self-inductance.

Another item for consideration is the voltage across an external inductance in either the armature or field circuit. This is found by differentiating the particular current with respect to time, and taking the product of this by the external inductance coefficient. These voltages are derived in Appendixes D and E.

Still another is the voltage across the open phase of a three-phase machine during short circuit of one of the phases. This is determined by first obtaining an expression for the linkages with the open phase in question, and differentiating this with respect to time. This is given in Appendix F.

Turning now to the details of the fundamental equations, refer to Fig. 1A. If a transient current i is caused to flow in the armature winding in the position shown, that is, with the direct axis and the armature winding axis coinciding, a current will, of course, appear in the field winding which is short-circuited either at the terminals or through an external inductance. Thus, fluxes will be produced in the leakage paths of both the armature winding and the short-circuited field winding. Corresponding to these fluxes there will be a *direct static transient* inductance $\mathcal{L}_D'$ and corresponding reactance x_D'; similarly, when the quadrature axis is lined up with the axis of the armature winding, as in 1B, there will be a *quadrature static transient* inductance l_Q and corresponding reactance x_Q'. For any other position, as in 1C, the inductance will vary, as previously shown, between x_D' and x_Q' as a second harmonic function of the angle γ between the armature winding axis and the direct axis. Thus

$$l' = \frac{1}{2}\,[\,(l_D' + l_Q') + (l_D' - l_Q')\cos 2\,\gamma\,]$$

Or, since x_D' and x_Q' are "static" values of *per-unit* reactance, they may be taken, as already explained, as equal to the corresponding inductance. Hence,

$$l' = \frac{1}{2}\,[(x_D' + x_Q') + (x_D' - x_Q')\cos 2\,\gamma] \tag{3}$$

Let the transient armature current be due to a single-phase short circuit occurring during rotation at uniform speed, and at the instant when the direct axis is displaced from the armature axis by the angle α. Counting time from this instant, the angle γ becomes a function of time.

$$\gamma = t + \alpha \tag{4}$$

Therefore, any transient current, i, which exists at any time t, will produce the following flux linkages with the armature winding,

$$\psi_i = l'\,i = \frac{1}{2}\,i\,[(x_D' + x_Q') + (x_D' - x_Q')\cos 2\,(t + \alpha)] \tag{5}$$

The current existing in the field winding before short circuit, would produce in the open circuited armature winding at any time t the alternating mutual flux linkages,

$$\psi_m = k\,e_0 \cos\,(t + \alpha) \tag{6}$$

where e_0 represents the peak voltage, line-to-neutral, generated before short circuit, and therefore also *the flux linkages;* and since on open circuit, terminal voltage is also nominal voltage, e_0 also represents the corresponding field current. k is a factor whose value depends upon the nature of the short circuit, *i. e.*, whether line-to-neutral or line-to-line. Thus, $k = 1.0$ for line-to-neutral, and 1.73 for line-to-line.

The sum of (5) and (6), must equal the linkages existing at the instant before short circuit, namely,

$$\psi_m = k\,e_0 \cos \alpha \tag{7}$$

The above relations determine the total armature current, as derived in Appendix B.

The total field currents are given by

$$\left.\begin{aligned} I_d &= e_d = e_0 + \Delta\, e_d \\ \text{and} \qquad I_q &= e_q = \Delta\, e_q \end{aligned}\right\} \tag{8}$$

where e_d represents *direct* nominal voltage, and therefore also the total field current in the direct axis winding, and Δe_d the increment induced by the transient armature current. Similarly, Δe_q represents the increment induced in the *quadrature* axis winding. It is assumed that the only quadrature component is the induced current. The field current is derived in Appendix C.

RESULTS

The foremost result is the method of analysis which makes possible, in a practical way, the calculation of transient phenomena in any synchronous machine, regardless of whether it is of the salient pole or the cylindrical rotor type. The equations thus derived have made it possible to calculate complicated transient characteristics which check, in minute details, with the oscillographic records of tests; and these equations have revealed a number of important and very interesting facts relating to the form of the current waves, the resulting possibility of relatively simple analysis of the waves, and the induced voltage in the open phase during single-phase short circuit. A summary of equations is given in Appendix A.

Referring to the short-circuit current waves, the current is expressed as the sum of two harmonic series, one of odd, the other of even, harmonics; and the *magnitudes of the harmonics in each series, in ascending order*, are in *geometric ratio*. Thus, the single-phase short-circuit current of the armature is

$$i = \frac{k\,e_0 \cos\alpha}{\sqrt{x_D'\,x_Q'}}\,[1 + 2\,b \cos 2\,(t + \alpha)$$
$$+ 2\,b^2 \cos 4\,(t + \alpha) + 2\,b^3 \cos 6\,(t + \alpha) + \,.\,.\,.\,.\,]$$
$$- \frac{2\,k\,e_0}{x_D' + \sqrt{x_D'\,x_Q'}}\,[\cos(t + \alpha) + b \cos 3\,(t + \alpha)$$
$$+ b^2 \cos 5\,(t + \alpha) + b^3 \cos 7\,(t + \alpha) + \,.\,.\,] \qquad (9)$$

The geometric ratio b is given by

$$b = \frac{\sqrt{x_Q'} - \sqrt{x_D'}}{\sqrt{x_Q'} + \sqrt{x_D'}}$$

where x_Q' is the transient reactance in the quadrature axis, and x_D' the corresponding quantity in the direct axis.

For instance, a 7000-k-va., 375-rev. per min., 25-cycle, 2200-volt salient-pole machine of usual construction has the following constants.*

$$x_D' = 0.4$$
$$x_Q' = 1.0$$
$$b \cong 0.23$$

In this machine, the 3rd harmonic in the short-circuit current would be 23 per cent of the fundamental, the 5th 23 per cent of the 3rd, the 7th 23 per cent of the 5th, and so on. Likewise for the even harmonics. The total wave, both calculated and tested, is shown in Fig. 5 for armature current, and Fig. 6 for field current.

As an extreme case,

$$x_D' = 0.21$$
$$x_Q' = 3.77$$
$$b = 0.618$$

These constants apply to an actual induction machine with a coil wound secondary.† The armature is three-phase, the rotor two-phase; the latter, in this particular case, being connected two phases in series with direct-current excitation. The machine is rated 20-hp., 1800-rev. per min., 60-cycle, 110-volt. For convenience, the *per-unit* quantities are based on 10-kv-a., three-phase, 57.7-volt line-to-neutral.

With the rotor winding thus connected, the harmonics are relatively 2.67 times as large as in the previous case of the 7000-kv-a. salient-pole machine. The waves of current, both calculated and test, are shown in Figs. 11-13, for the armature current, Figs. 16-18 for the field current.

It is interesting to note that if $b = 0$, that is if $x_D' = x_Q'$, all harmonics disappear, and the current wave changes from the peaked wave in Fig. 13 to approximately a sine wave, as in Fig. 14. The values of x_D' and x_Q' were made practically equal by short circuiting one of the two rotor phases, and exciting the other, keeping the external impedance of the latter as low as possible.

Going a step further, x_D' can be made greater than x_Q' by placing external inductance in the field winding which is excited, *i. e.*, the direct axis winding. In this case the sign of the harmonics is reversed, *i. e.*, b becomes negative, thus giving a dip in the wave. This is shown in Fig. 15.

Referring again to equation (9), it is interesting to consider how limiting values of b modify the shape of the current waves. The first series, *i. e.*, even harmonics, has the general form shown in Fig. 9A. The second series has the general form shown in Fig. 9B. The total current, the sum of the two series, has the general form of Fig. 9C.

Now if x_D' and x_Q' are equal, b becomes zero, and all

---

*It should be remembered that these constants represent the *static transient linkages* (or, at normal frequency, the corresponding reactive voltage) as a fraction of normal linkages, *line-to neutral*, (or, at normal frequency, the *line-to-neutral* normal voltage). The values given look high compared with the usual three-phase values, because they are *line-to-line* constants. If the line-to-line voltages were taken as the per-unit base, instead of line-to-neutral, the values would be divided by $\sqrt{3}$. Also, the corresponding line-to-neutral, three-phase values would be just one-half the values given. See companion paper by Park and Robertson regarding the relations between three-phase and single-phase constants.

†This machine was chosen with the two points in mind; one was that our premises should be closely approximated; the other, that it was desirable, in the interest of an exacting test of the theory, to employ extreme limits in reactance coefficients thus involving harmonics of large magnitude, which harmonics, nevertheless, would be capable of control by different connections of the windings, and by variation of constants in the external circuits.

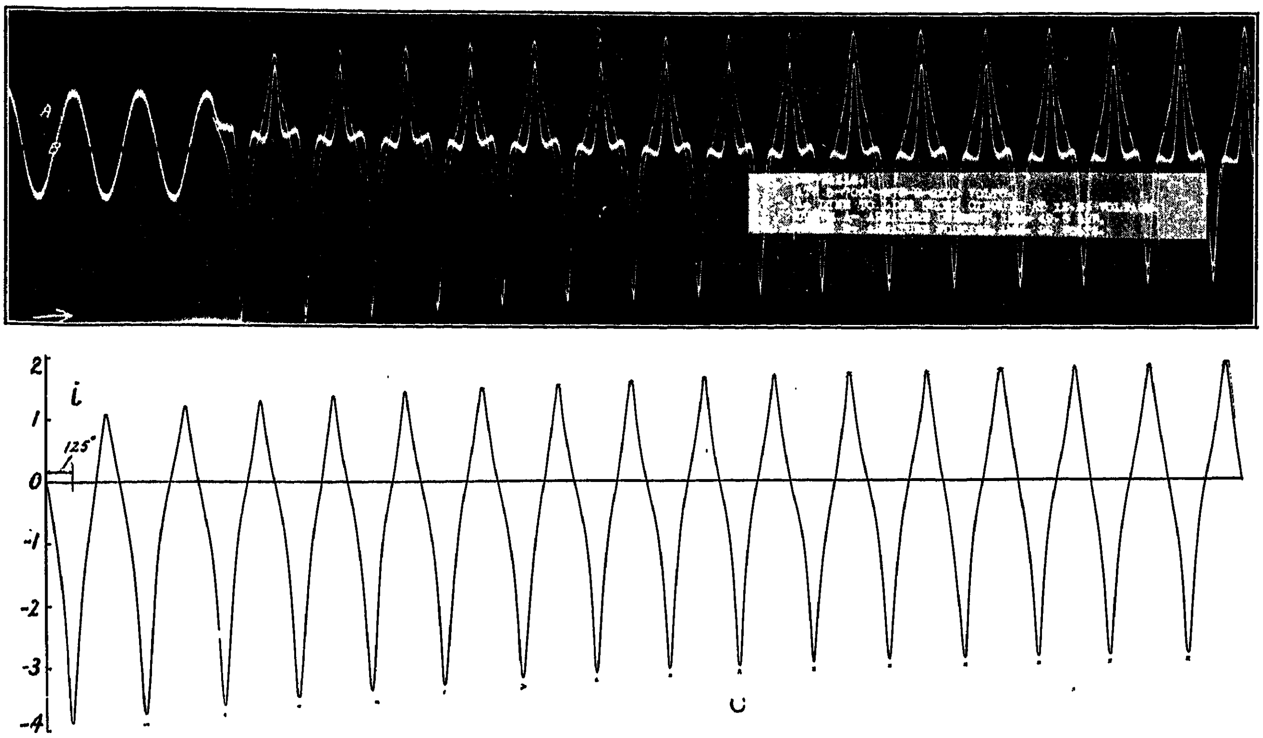


FIG. 5—SINGLE-PHASE LINE TO LINE SHORT CIRCUIT AT $\alpha = 55$ DEG., 7000-KV-A. MACHINE

A—Test curve of armature current

B—Test curve of open-phase voltage

C—Calculated curve of armature current with test points indicating the peaks

Equation:—

$$i = \frac{0.59}{1.4 - 0.6 \cos (2\,t + 110°)} \epsilon^{-0.01168\,t} - \frac{0.358 \cos (t + 55°)}{1.4 - 0.6 \cos (2\,t + 110°)} - \frac{0.641 \cos (t + 55°)}{1.4 - 0.6 \cos (2\,t + 110°)} \epsilon^{-0.00272\,t}$$

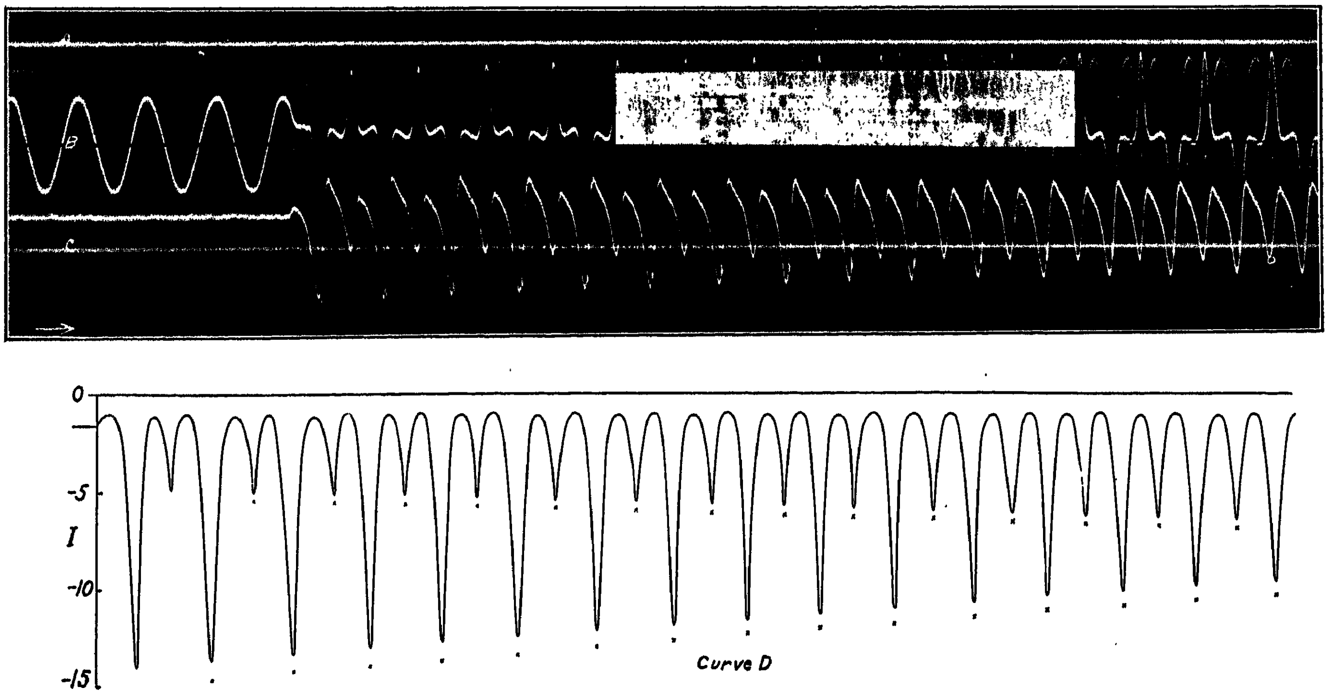


FIG. 6—SINGLE-PHASE LINE-TO-LINE SHORT CIRCUIT AT $\alpha = 55$ DEG., 7000-KV-A. MACHINE

A—Test curve of field current

B—Test curve of open-phase voltage

C—Test curve of voltage across external reactor in field circuit

D—Calculated curve of field current with test points indicating the peaks

Equation:—

$$I = -\frac{0.364 \cos (t + 55°)}{1.4 - 0.6 \cos (2\,t + 110°)} \epsilon^{-0.01168\,t} + \frac{0.624\,[3.24 + 1.24 \cos (2\,t + 110°)]}{1.4 - 0.6 \cos (2\,t + 110°)} + \frac{1.109\,[3.24 + 1.24 \cos (2\,t + 110°)]}{1.4 - 0.6 \cos (2\,t + 110°)} \epsilon^{-0.00272\,t}$$

harmonics vanish, as already indicated. Taking $\alpha = 0$, the first series then reduces to only a d- c. component of an amplitude

$$i_0 = \frac{k\, e_0}{\sqrt{x_D'\, x_Q'}} \tag{10}$$

And the second series then contains only a fundamental term of amplitude

$$i_1 = \frac{2\, k\, e_0}{x_D' + \sqrt{x_D'\, x_Q'}} \tag{11}$$

However, if x_Q' is increased until it is very large compared with x_D', b will approach unity—that is, all harmonics will approach equality—and the three waves shown in Fig. 9, will approach respectively those shown in Fig. 10. The value of b for the 20-hp. induction machine is very high compared with usual machines, but its value of 0.62 is considerably below unity. Yet, the current wave in Fig. 11 indicates a definite approach to Fig. 10C. In the first four cycles, that is, up to $t = 8\,\pi$, the peaks of the component corresponding to Fig. 10A slightly predominate over those of the component in Fig. 10B. But the former, being subject to a more rapid decrement, become less after the fourth cycle, and the little *resultant* "tip" reverses direction. Note that the calculated wave in Fig. 11 checks the test perfectly in this detail.

Another interesting and useful feature of the expressions for short-circuit currents is that they open up new possibilities of analyzing the current waves. In addition to the relations shown in Fig. 9C and Fig. 10C, and the expression for the fundamental component of current in equation (**10**), it is shown in Appendix B that the r. m. s. value of the first series, *i. e.*, even harmonics, of equation (**9**), is

$$i = \frac{k\, e_0 \cos \alpha}{\sqrt{x_D'\, x_Q'}} \sqrt{\frac{1 + b^2}{1 - b^2}} \tag{12}$$

and of the odd harmonics

$$i_b = \frac{\sqrt{2}\, k\, e_0}{x_D' + \sqrt{x_D'\, x_Q'}} \; \frac{1}{\sqrt{1 - b^2}} \tag{13}$$

Since equation (**11**) gives the amplitude of the fundamental term, it follows that the first fraction in (**13**) is the r. m. s. of the fundamental. Hence the r. m. s. of the whole odd harmonic series is different from the r. m. s. of the fundamental by the factor

$$\frac{1}{\sqrt{1 - b^2}}$$

This is usually not significantly different from unity, and hence an ammeter reading will be approximately the fundamental. For instance, $b = 0.5$ would be an extremely high value for any commercial synchronous machine. The corresponding value of the factor is 1.15. However, a representative value of b might be 0.2, for which the corresponding ratio is 1.02; that is, the ammeter would read 2 per cent more than the fundamental. Hence this might be a useful approximation.

Another possibility of convenient analysis of the oscillograph current wave is afforded by the fact that, with the harmonic ratio b, if one harmonic in each series is determined, all of them are known. The convenient relations exist that the peak value of the odd harmonic series is $1/(1 - b)$ times the peak value of the fundamental component, and the peak value of the even harmonic series is $(1 + b)/(1 - b)$ times the direct current component. It is shown in Appendix B that since the three quantities—fundamental current i_1, direct-current component i_0, and the harmonic ratio b—completely determine the wave, it can be analyzed by scaling off three current values from the oscillogram at $(t + \alpha)$ = 45 deg., 90 deg., and 180 deg.

So much for the results regarding short-circuit current. Turning now to the next major point in the results, the equations show that under single-phase short circuit the peak voltage across the open phase may be extremely high; many times normal in cylindrical laminated rotor machines without amortisseur winding. This voltage depends upon a very simple relation, namely the ratio of x_Q' to x_D'. Thus, for a short circuit at $\alpha = 0$—that is, at maximum flux enclosure or maximum flux linkages—the ratio of the voltage across the open phase after short circuit (initial value) to the peak voltage before short circuit is

$$\rho_o = \left(2\frac{x_Q'}{x_D'} - 1 \right) \sin \beta \tag{14}$$

and for short circuit at zero flux linkages, *i. e.*, $\alpha = \pi/2$.

$$\rho\, \pi/2 = \frac{x_Q'}{x_D'} \sin \beta \tag{15}$$

where β is the angle between the axis of the short-circuited winding, and the axis of the open-circuited winding considered.

Thus, as an extreme case, consider the 20-hp. machine. Here the short circuit was line-to-line, and the voltage recorded was across the open phase line-to-neutral. Thus β = 90 deg. The ratio for maximum flux linkages ($\alpha = 0$), therefore, is,

$$\rho = 2\frac{3.77}{0.21} - 1 = 34.9$$

Reference to Fig. 19 will show that this *initial* value, of over 34 times the voltage before short circuit, was obtained. The transient, of course, reduced the value of the first peak to obtain 28 times the value existing before short circuit, which value is also shown on the calculated wave.

When $x_D' = x_Q'$, the maximum voltage for the case considered is unity.

Turning to the test on the 7000-kv-a. salient-pole machine, the short circuit was line-to-line, and the open-phase voltage recorded was also line-to-line. Thus β = 60 deg., and from (**14**),

$$\rho = 3.46 \text{ for } \alpha = 0$$

and

$$\rho = 2.16 \text{ for } \alpha = \pi/2$$

These are limiting values. In the test, the short circuit occurred at $\alpha = 2.2$ (*i. e.*, about 125 deg.) and therefore the peak should fall between those indicated by the above values of ρ. Using $\alpha = 125$ deg. (or, for convenience, the corresponding value, 55 deg.) in the equation of Appendix F, the calculated wave Fig. 7 was obtained. The calculated first peak is 2.73 times the peak voltage before short circuit, and the corresponding test value, 2.97. The calculated value is thus less than the test by 8 per cent, which is a reasonable check, considering the probable error in scaling the angle from the oscillogram, and that due to the simplifying assumption made in Appendix F. And it will be noted that the shape of the curve checks satisfactorily. Thus, the voltage across the open phase during single-phase short circuit may rise to a peak value many times the value existing before short circuit. It varies almost directly with the ratio x_Q' to x_D'. Hence, it would be extremely high—possibly 15 or 20 times normal voltage—on a cylindrical, laminated rotor machine without a squirrel-cage or metal wedges, serving the same purpose; but is practically no greater than normal when $x_Q' = x'_D$, as in a machine which has an amortisseur winding or massive steel rotor. Most of the cylindrical rotors in usual practise have one or the other of these features. In salient-pole machines, the tendency toward this high voltage is less, because x_D' is relatively higher, and x_Q', on account of salient poles, is relatively lower than in cylindrical rotor machines; but as shown in the foregoing actual illustration, the peak voltage, even in this case, may be from 2 to 3.5 times normal. A very important point to observe in this connection is that this high voltage across the open

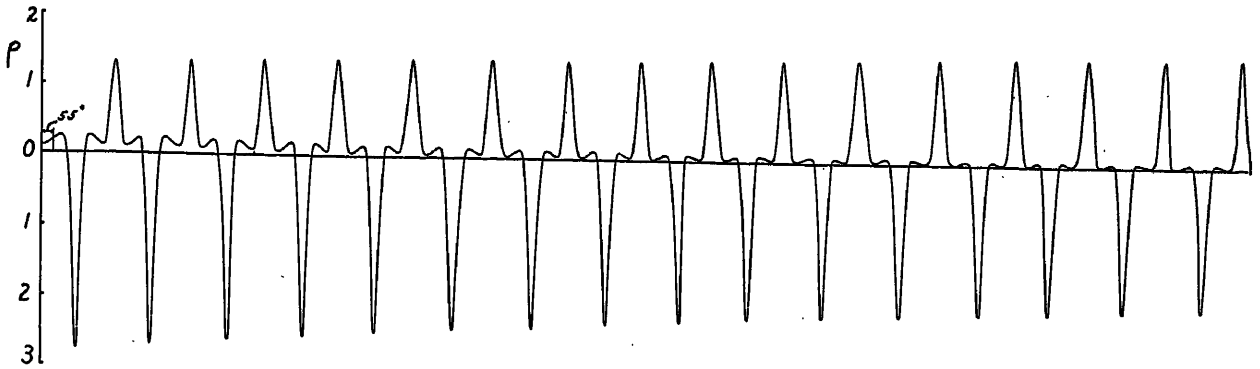


FIG. 7—CALCULATED CURVE OF TRANSIENT OPEN-PHASE VOLTAGE, SINGLE-PHASE. LINE-TO-LINE, SHORT CIRCUIT AT $\alpha = 55$ DEG., 7000-KV-A. MACHINE. (FOR TEST RESULTS REFER TO FIG. 5B.)

Equation:—

$$e = 2.38 \left[\frac{[0.35 - 0.3 \cos (2t + 170°)] \sin (2t + 110°)}{1.4 - 0.6 \cos (2t + 110°)^2} - \frac{1.194 \sin (2t + 170°)}{1.4 - 0.6 \cos (2t + 110°)} \right] \epsilon^{-0.01168t} + (1 + 1.77\, \epsilon^{-0.00272t})$$

$$\left\{ 0.624 \sin (t + 115°) + \frac{0.746 \sin (2t + 170) \cos (t + 55)}{1.4 - 0.6 \cos (2t + 110°)} - 1.24\, \frac{[0.35 - 0.3 \cos (2t + 170°)] \sin (t + 55°)}{1.4 - 0.6 \cos (2t + 110°)} - \frac{1.49 [0.35 - 0.3 \cos (2t + 170°)] \sin (2t + 110°) \cos (t + 55°)}{[1.4 - 0.6 \cos (2t + 110°)]^2} \right\}$$

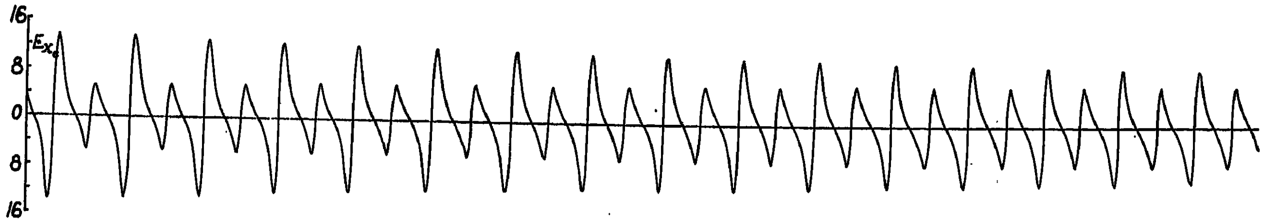


FIG. 8—TRANSIENT VOLTAGE ACROSS AN EXTERNAL REACTOR IN THE FIELD CIRCUIT, SINGLE-PHASE LINE-TO-LINE SHORT CIRCUIT AT $\alpha = 55$ DEG., 7000-KV-A. MACHINE. (FOR TEST RESULTS REFER TO FIG. 6C.)

Equation:—

$$e = \left\{ \frac{0.0425 \cos (t + 55°) + 3.64 \sin (t + 55°)}{1.4 - 0.6 \cos (2t + 110°)} + \frac{4.37 \cos (t + 55°) \sin (2t + 110°)}{[1.4 - 0.6 \cos (2t + 110°)]^2} \right\} \epsilon^{-.01168t}$$

$$- \left\{ \frac{1.55 \sin (2t + 110°)}{1.4 - 0.6 \cos (2t + 110°)} + \frac{0.75 \sin (2t + 110°) [3.24 + 1.24 \cos (2t + 110°)]}{[1.4 - 0.6 \cos (2t + 110°)]^2} \right\}$$

$$- \left\{ \frac{2.75 \sin (2t + 110°)}{1.4 - 0.6 \cos (2t + 110°)} + \frac{1.33 \sin (2t + 110°) [3.24 + 1.24 \cos (2t + 110°)]}{[1.4 - 0.6 \cos (2t + 110°)]^2} \right\} \epsilon^{-0.00272t}$$

phase is due, not to high flux density, but rather to a high rate of change, and hence saturation would not limit it, unless by tending to equalize x_D' and x_Q'.

Boucherot[2] called attention to the possibility of high voltage across the open phase, and to the favorable effect of an amortisseur winding in reducing it. The value of the present treatment in this connection is that the equations, in addition to being generally applicable, show the very significant extent to which salient poles reduce this voltage.

It may be of practical importance to know the value of the fundamental component of the open phase voltage. The ratio of this to the voltage existing across the open phase before short circuit is given approximately by the simple form (see Appendix F),

$$\rho_1 = 1 + b$$

where

$$b = \frac{\sqrt{x_Q'} - \sqrt{x_D'}}{\sqrt{x_Q'} + \sqrt{x_D'}}$$

For instance, in the case of the 20-hp. machine, Fig. 19, ρ_1 would be 1.628; and in the salient-pole machine, Fig. 7, for corresponding conditions, 1.23.

COMPARISON OF CALCULATED AND TEST RESULTS

Calculations and tests were made on the two machines already referred to, namely, a 20-hp., 1800-rev. per min., 60-cycle, 110-volt induction machine, and a 7000-kv-a., 375-rev. per min., 25-cycle, 2200-volt salient-pole alternator. The four major *per-unit* coefficients (measured) for the 20-hp. machine, on the basis of 10 kv-a., three-phase, 57.7 volts line-to-neutral, are:

$$x_D = 3.77, \quad x_D' = 0.21, \quad x_Q = 3.77, \quad x_Q' = 3.77$$

The auxiliary coefficients are:

$$x_{AD} = 3.64, \quad x_{AD}' = 0.076, \quad x_{AQ} = 3.64, \quad x_{AQ}' = 3.64$$

$$\sigma_f = 0.132 \quad \sigma_a = 0.0474$$

For the 7000-kv-a. alternator the corresponding *per-unit* coefficients based on 7000 kv-a., three-phase, 1270 volts line-to-neutral, are:

$$x_D = 2.24, \quad x_D' = 0.40, \quad x_Q = 1.00, \quad x_Q' = 1.00$$

$$x_{AD} = 2.09, \quad x_{AD}' = 0.25, \quad x_{AQ} = 0.85, \quad x_{AQ}' = 0.85$$

$$\sigma_f = 0.00272 \quad \sigma_a = 0.0117$$

A comparison of calculated and test results is shown in Figs. 11 to 25, inclusive, for the 20-hp. machine, and in Figs. 5 to 8 for the 7000-kv-a. machine. Each illustration shows,

a. Calculated curve, from equations in Appendix A.
b. Oscillograph record of test.
c. Equation, with numerical values substituted,* from which the calculated curve was derived.

The reason for making extensive tests on the induction machine, excited by direct current as a synchronous machine, has been stated in the second footnote under "Results."

A careful inspection of each of the figures will clearly show a satisfactory agreement between the calculated and test curves, even with respect to the minute details. It will be noted that the decrements agree well.

The ordinates of the curves, it will be remembered, are scaled as the ratio of the actual value of the quantity to the value taken as the *per-unit* base. Ordinarily, normal value is chosen as that base. In the present case, however, the base for the 20-hp. machine was taken as 10 kv-a., 57.7 volts line-to-neutral, thus making the per-unit value of reactance numerically the same as the value in ohms—merely as a matter of convenience. The normal values were taken as the basis for the 7000-kv-a. machine. These points should be kept in mind in reading the ordinates.

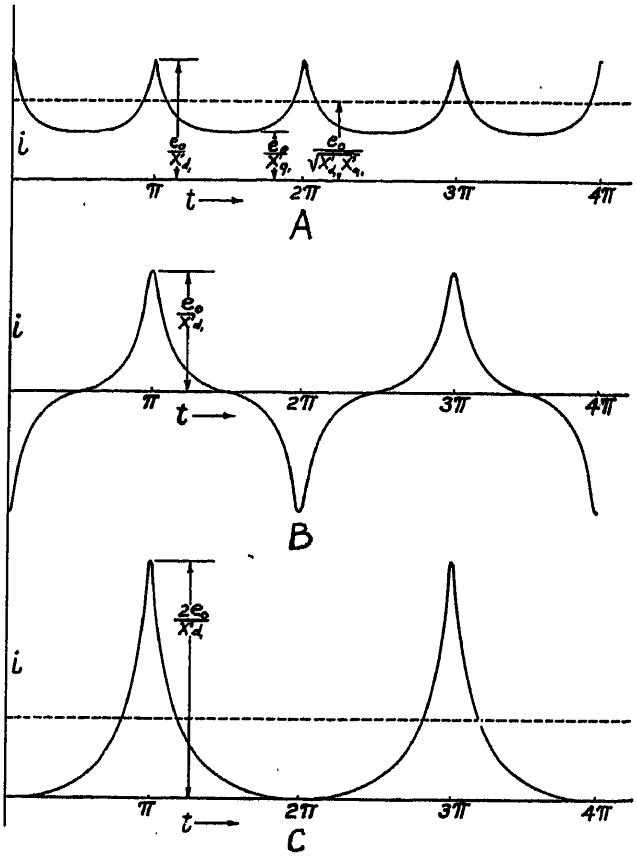


FIG. 9

In the case of the open-phase voltage during short circuit, Figs. 7 and 19, the ordinate gives the ratio of the voltage before short circuit to the voltage after short circuit.

With reference to the 7000-kv-a. alternator, this machine has no closed circuits on the rotor except the main field winding and the massive steel of the rotor spider. The effect of the latter is probably negligible. The poles themselves are, of course, laminated. The tests were taken at a relatively low voltage—at 300 volts; hence the premises were pretty closely approximated, and consequently the calculated results agree closely with the tests.

---

*These equations, being reduced for numerical calculation, are in a somewhat different form from those in the summary of equations in Appendix A.

In the case of the voltage across the field terminals, *i. e.*, across the external impedance, the value of this impedance was not definitely known. Hence with the estimated value, the general shape of the calculated curve is all that may be expected to agree with the test. The calculated curve for this is shown in Fig. 8, the test curve in Fig. 6c.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the valuable assistance of Mr. B. L. Robertson in the preparation of the paper, of Mr. C. C. Herskind in tests and calculations, and of Messrs. W. G. Scarborough and R. Hammar in the preparation of data and illustrations. They also wish to acknowledge the helpful suggestions of Professor W. V. Lyon of M. I. T., and of Messrs. R. H. Park and P. L. Alger.

Appendix A

SUMMARY OF EQUATIONS

In the first part of the paper, under "Scope," it was stated that the present treatment develops expression for,

1. Armature Current—Appendix B.
2. Field Current—Appendix C.

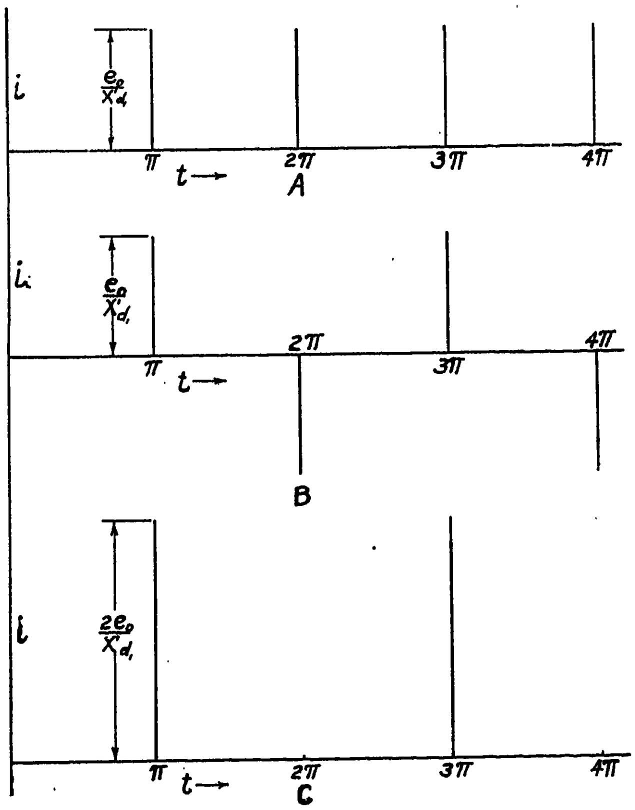


FIG. 10

3. Voltage Across Reactance in Armature Circuit—Appendix D.
4. Voltage Across Reactance in Field Circuit—Appendix E.
5. Voltage Across Open Phase—Appendix F.

These expressions are derived in the appendixes mentioned before, but for ease of reference, they are tabulated here. The order is as stated: First, the general equation for the quantity is given. Equations for special conditions follow readily from this. Thus, for the condition of maximum flux linkages at the instant of short circuit, $\alpha = 0$; while for minimum flux linkages $\alpha = \pi/2$ and the armature decrement is absent. Steady state conditions are obtained by placing $\alpha = \pi/2$, and omitting the armature and field decrements.

Since the equations are somewhat long it is found advisable to put them in a simpler form. By inspection, it is observed that they contain small groups which are more or less common to all of the expressions. These few groups, therefore, may be represented by a single symbol, as follows:

$$A = x_D' + x_Q'$$

$$B = x_D' - x_Q'$$

$$C = x_D + x_Q'$$

$$F = x_D - x_Q'$$

$$G = x_D - x_D'$$

$$\begin{aligned} H &= (x_D' + x_Q') + (x_D' - x_Q') \cos 2\,(t + \alpha) \\ &= A + B \cos 2\,(t + \alpha) \end{aligned}$$

$$\begin{aligned} J &= (x_D' + x_Q') - (x_D' - x_Q') \cos 2\,t \\ &= A - B \cos 2\,t \end{aligned}$$

$$K = \frac{x_D' + \sqrt{x_D'\,x_Q'}}{x_D + \sqrt{x_D'\,x_Q'}}$$

$$N = \sqrt{\frac{x_D'}{x_Q'}}$$

$$P = \frac{x_D - x_D'}{x_D + \sqrt{x_D'\,x_Q'}}$$

$$\sigma_a = \frac{r}{\sqrt{x_D'\,x_Q'}}$$

$$\sigma_f = \frac{\sigma_0}{K} \text{ (general)}$$

$$\sigma_f = \sigma_0 \sqrt{\frac{x_Q'}{x_D'}} \text{ (for cylindrical, laminated rotor only)}$$

$$\sigma_0 = \frac{R}{X}$$ (If R and X are in ohms, σ_0 will be a numeric, in proper terms to use in the above expressions).

1. ARMATURE CURRENT

Transient.

$$i = \frac{2\,k\,e_0}{H}\left[\epsilon^{-\sigma_a t} \cos \alpha - K \cos (t+\alpha) - P\,\epsilon^{-\sigma_f t} \cos (t+\alpha)\right]$$

For maximum flux linkages $\alpha = 0$ in the above equation, but for minimum flux linkages $\alpha = \pi/2$ and the armature decrement does not appear. Thus, for the latter case,

$$i = \frac{2\,k\,e_0}{J} \sin t\,[K + P\,\epsilon^{-\sigma_f t}]$$

Sustained. $$i = \frac{2\,k\,e_0\,K \sin t}{J}$$

2. Field Current

Transient.

$$I = \frac{e_0}{H}\,\{\,[C + F\cos 2\,(t + \alpha)\,]\;[K + P\,\epsilon^{-\sigma_f t}]$$
$$-\;2\,G\cos\,(t + \alpha)\cos\alpha\;\epsilon^{-\sigma_a t}\,\}$$

For maximum flux linkages $\alpha = 0$; and for minimum linkages,

$$I = \frac{e_0}{J}\,\{\,[C - F\cos 2\,t]\;[K + P\,\epsilon^{-\sigma_f t}]\,\}$$

Sustained. $$I = \frac{e_0\,K}{J}\,(C - F\cos 2\,t)$$

3. Voltage Across Reactor in Armature Circuit

$$e_{x_e} = \frac{2\,k\,e_0\,x_e}{H^2}\,[2\,B\cos\alpha\sin 2\,(t + \alpha)\;\epsilon^{-\sigma_a t}$$
$$+ \sin\,(t + \alpha)\;[H - 4\,B\cos^2\,(t + \alpha)]\;(K + P\,\epsilon^{-\sigma_f t})\,]$$

For maximum linkages $\alpha = 0$; and for minimum linkages,

$$e_{x_e} = \frac{2\,k\,e_0\,x_e}{J}\,[\cos t\;(J - 4\,B\sin^2 t)\;(K + P\,\epsilon^{-\sigma_f t})\,]$$

Sustained. $$e_{x_e} = \frac{2\,k\,e_0\,x_e\,K}{J}\,\cos t\;(J - 4\,B\sin^2 t)$$

4. Voltage Across Reactance in Field Circuit

$$Ex_e = \frac{2\,G\,E_0\,X_e\cos\alpha\sin\,(t+\alpha)}{H^2}\,[H - 4\,B\cos^2\,(t+\alpha)]\;\epsilon^{-\sigma_a t}$$
$$+\,\frac{2\,P\,E_0\,X_e\sin 2\,(t + \alpha)}{H^2}\,(B\,C - A\,F)\;\epsilon^{-\sigma_f t}$$
$$+\,\frac{2\,K\,E_0\,X_e\sin 2\,(t + \alpha)}{H^2}\,(B\,C - A\,F)$$

For maximum flux linkages $\omega = 0$; while for minimum linkages $\alpha = \pi/2$ and

$$Ex_e = \frac{2\,P\,E_0\,X_e}{J^2}\sin 2\,t\,(A\,F - B\,C)\;\epsilon^{-\sigma_f t}$$
$$+\,\frac{2\,K\,E_0\,X_e}{J^2}\sin 2\,t\,(A\,F - B\,C)$$

Sustained. $$Ex_e = \frac{2\,K\,E_0\,X_e}{J^2}\sin 2\,t\,(A\,F - B\,C)$$

5. Voltage Across the Open Phase

Transient. $$e_1 = \frac{2\,k\,k_1\,B\,e_0}{H^2}\cos\alpha\,[\,H\cos 2\,(t + \alpha)$$
$$+\,B\sin^2 2\,(t + \alpha)\,]\;\epsilon^{-\sigma_a t}$$
$$+\,\frac{k_1\,e_0}{k\,H^2}\,\{\,[H^2\,k - 2\,k\,B\,H\cos 2\,(t + \alpha)$$
$$-\,2\,k\,B^2\sin^2 2\,(t + \alpha)]\cos\,(t + \alpha)$$
$$+\,k\,B\,H\sin\,(t + \alpha)\sin 2\,(t + \alpha)\,\}\;[K - (1 - K)\;\epsilon^{-\sigma_f t}\,]$$

For maximum flux linkages $\alpha = 0$, while for minimum flux linkages $\alpha = \pi/2$ and

$$e_1 = \frac{k_1\,e_0}{k\,H^2}\,\{[H^2\,k + 2\,k\,B\,H\cos 2\,t + 2\,k\,B^2\sin^2 2\,t)]\sin t$$
$$+\,k\,B\,H\cos t\sin 2\,t\,\}\;[(1 - K)\;\epsilon^{-\sigma_f t}K]$$

Steady State. $$e_1 = -\,\frac{k_1\,e_0\,K}{k\,H^2}\,\{\,k\,B\,H\cos t\sin 2\,t$$
$$+\,[H^2\,k + 2\,k\,B\,H\cos 2\,t + 2\,k\,B^2\sin^2 2\,t)]\sin t\,\}$$

Appendix B

Armature Current

Refer to "Premises" and "Fundamental Equations" in the main body of the paper. Equating (5) and (6) and solving for i,

$$i = \frac{2\,k\,e_0\,[\cos\alpha - \cos\,(t + \alpha)]}{(x_D' + x_Q') + (x_D' - x_Q')\cos 2\,(t + \alpha)} \qquad \textbf{(1b)}$$

Resolved into the harmonics, this current becomes,

$$i = \frac{k\,e_0\cos\alpha}{\sqrt{x_D'\,x_Q'}}\,[1 + 2\,b\cos 2\,(t + \alpha)$$
$$+\,2\,b^2\cos 4\,(t + \alpha) + 2\,b^3\cos 6\,(t + \alpha) + \;.\;\;.\;\;.]$$
$$-\,\frac{2\,k\,e_0}{x_D' + \sqrt{x_D'\,x_Q'}}\,[\cos\,(t + \alpha) + b\cos 3\,(t + \alpha)$$
$$+\,b^2\cos 5\,(t + \alpha) + \;.\;\;.\;\;.\;\;.] \qquad \textbf{(2b)}$$

where $$b = \frac{\sqrt{x_Q'} - \sqrt{x_D'}}{\sqrt{x_Q'} + \sqrt{x_D'}} \qquad \textbf{(3b)}$$

k = a factor depending upon the phase which is short circuited. It is equal to 1.0 for line-to-neutral, and $\sqrt{3}$ for line-to-line.

e_0 = peak voltage, line-to-neutral, before short circuit.

When the short circuit occurs at maximum flux linkages, that is at $\alpha = 0$, (**1b**) becomes,

$$\underset{\alpha = 0}{i} = \frac{2\,k\,e_0\,(1 - \cos t)}{(x_D' + x_Q') + (x_D' - x_Q')\cos 2\,t} \qquad \textbf{(4b)}$$

and (**2b**) becomes,

$$\underset{\alpha = 0}{i} = \frac{k\,e_0}{\sqrt{x_D'\,x_Q'}}\,[1 + 2\,b\cos 2\,t + 2\,b^2\cos 4\,t$$
$$+\,2\,b^3\cos 6\,t + \;.\;\;.\;\;.\;\;.]$$

$$-\frac{2\,k\,e_0}{x_D' + \sqrt{x_D'\,x_Q'}}\left[\cos t + b\cos 3\,t + b^2\cos 5\,t + \ldots\right] \qquad \text{(5b)}$$

When the short circuit occurs at zero flux linkages, that is at $\alpha = \frac{\pi}{2}$, (1b) becomes,

$$\underset{\alpha\,=\,\pi/2}{i} = \frac{2\,k\,e_0 \sin t}{(x_D' + x_Q') - (x_D' - x_Q')\cos 2\,t} \qquad \text{(6b)}$$

and (2b) becomes,

$$\underset{\alpha\,=\,\pi/2}{i} = \frac{2\,k\,e_0}{x_D' + \sqrt{x_D'\,x_Q'}}\left[\sin t - b\sin 3\,t\right.$$

$$\left. + \, b^2 \sin 5\,t - b^3 \sin 7\,t + \ldots\right] \qquad \text{(7b)}$$

It is interesting to note that from equation (**2b**), the reactance which, with the voltage $k\,e_0$, determines the fundamental component of short-circuit current is

$$\frac{x_D' + \sqrt{x_D'\,x_Q'}}{2}$$

and that which, with the same voltage, determines the direct-current component is $\sqrt{x_D'\,x_Q'}$ The peak value of the fundamental is

$$i_1 = -\frac{2\,k\,e_0}{x_D' + \sqrt{x_D'\,x_Q'}}. \qquad \text{(8b)}$$

and the direct-current component is,

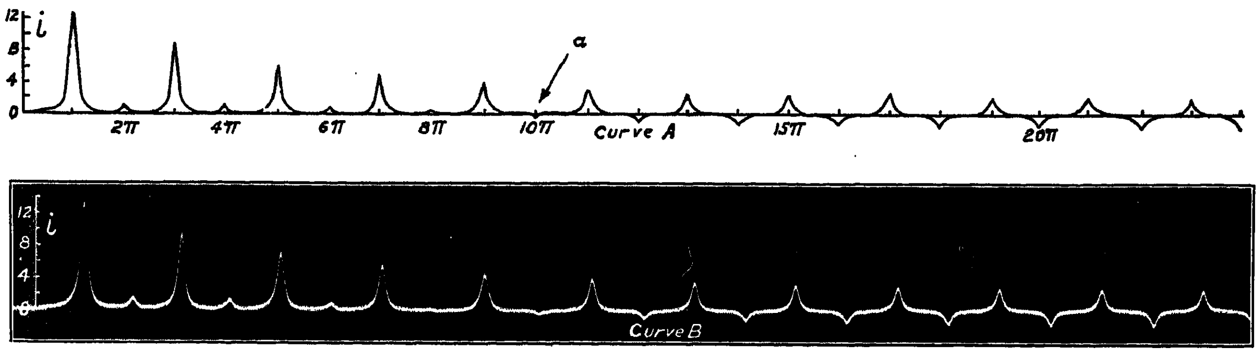


Fig. 11—Transient Armature Current, Maximum Initial Armature Linkages on Single-Phase Line-to-Line Short-Circuit, 20-Hp. Machine

A—Calculated curve
B—Test curve

Equation:—

$$i = \frac{3.46}{3.98 - 3.56\cos 2\,t}\,\epsilon^{-0.0474\,t} - \frac{0.818\cos t}{3.98 - 3.56\cos 2\,t} - \frac{2.64\cos t}{3.98 - 3.56\cos 2\,t}\,\epsilon^{-0.132\,t}$$

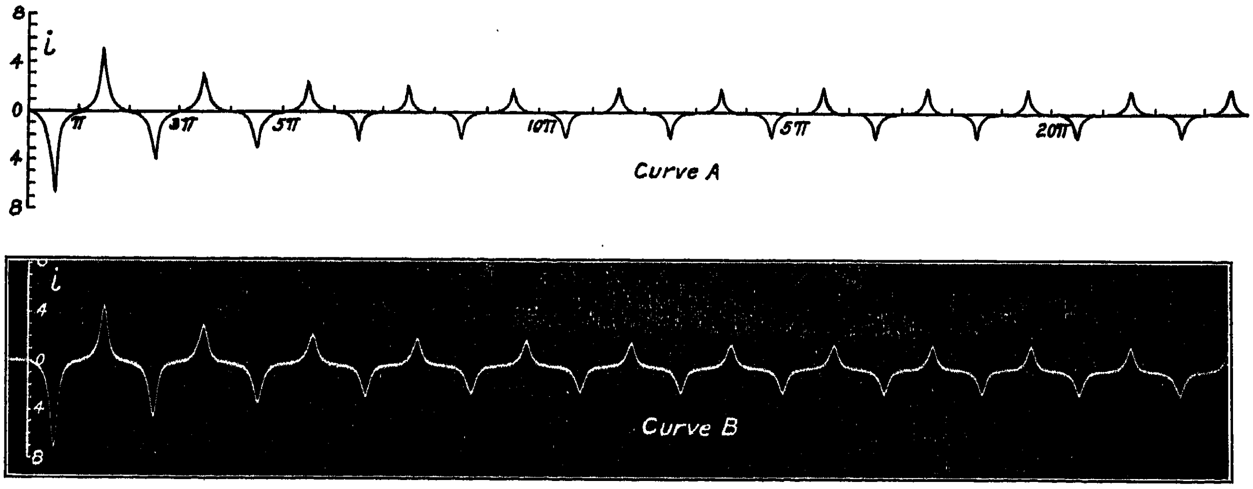


Fig. 12—Transient Armature Current, Zero Armature Linkages on Single-Phase Line-to-Line Short-Circuit 20-Hp. Machine

A—Calculated curve
B—Test curve

Equation:—

$$i = \frac{0.818\sin t}{3.98 + 3.56\cos 2\,t} - \frac{2.64\sin t}{3.98 + 3.56\cos 2\,t}\,\epsilon^{-0.132\,t}$$

$$i_0 = \frac{k\, e_0 \cos \alpha}{\sqrt{x_D' x_Q'}} \tag{9b}$$

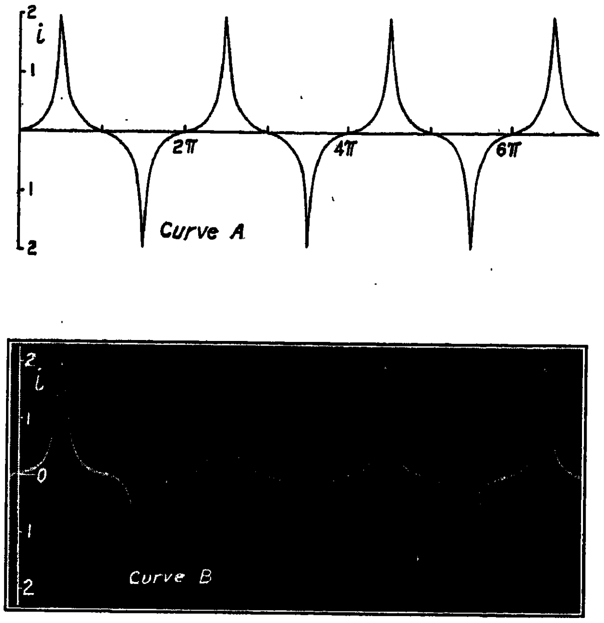


Fig. 13—Sustained Armature Current on Single-Phase Line-to-Line Short-Circuit 20-Hp. Machine

A—Calculated curve
B—Test curve

Equation:— $i = \dfrac{0.818 \sin t}{3.98 + 3.56 \cos 2\,t}$

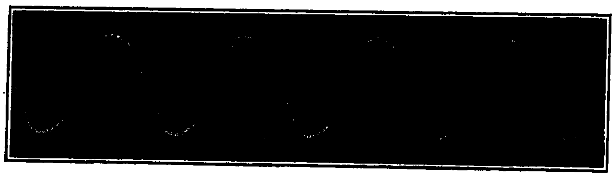


Fig. 14—Sustained Armature Current, Single-Phase Line-to-Line Short Circuit with $x_D' = x_Q'$ 20-Hp. Machine

The effective value of the series in equation (**2b**) will now be derived. This value is the square root of the sum of the squares of the individual effective values. The coefficient of each harmonic term represents the peak value. Thus, dividing each harmonic term by $\sqrt{2}$ and squaring, and taking the square root of the sum, the effective value of the even harmonic series is

$$i_{eff} = \frac{k\, e_0 \cos \alpha}{\sqrt{x_D' x_Q'}} \sqrt{1 + 2\,b^2 + 2\,b^4 + 2\,b^6 + \ldots}$$

or

$$i_{eff} = \frac{k\, e_0 \cos \alpha}{\sqrt{x_D' x_Q'}} \sqrt{\frac{1 + b^2}{1 - b^2}} \tag{10b}$$

And for the odd harmonic series,

$$i_{eff}' = \frac{\sqrt{2}\, k\, e_0}{x_D' + \sqrt{x_D' x_Q'}} \sqrt{1 + b^2 + b^4 + b^6 + \ldots}$$

or

$$i_{eff}' = \frac{\sqrt{2}\, k\, e_0}{x_D' + \sqrt{x_D' x_Q'}} \frac{1}{\sqrt{1 - b^2}} \tag{11b}$$

Equations (**10b**) and (**11b**) give the effective value of the respective series as a fraction of the peak value of normal fundamental current. To place the results in terms of the effective value of normal current, multiply each equation by $\sqrt{2}$.

Turn now to the relations which make it possible to analyze a single-phase short-circuit current wave from only three values taken from the wave. Referring to equation (**2b**), the sum of *each* series is a maximum (but the total is zero) when

$$(t + \alpha) = 2\,\pi$$

$$i = \frac{k\, e_0 \cos \alpha}{\sqrt{x_D' x_Q'}} [1 + 2\,b + 2\,b^2 + 2\,b^3 + 2\,b^4 + \ldots]$$

$$- \frac{2\,k\, e_0}{x_D' + \sqrt{x_-' x_Q'}} [1 + b + b^2 + b^3 + \ldots] \tag{12b}$$

Taking the summation of these series,

$$i = \frac{k\, e_0 \cos \alpha}{\sqrt{x_D' x_Q'}} \frac{1 + b}{1 - b} - \frac{2\,k\, e_0}{x_D' + \sqrt{x_D' x_Q'}} \frac{1}{1 - b} \tag{13b}$$

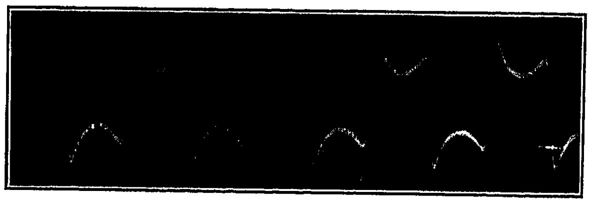


Fig. 15—Sustained Armature Current Single-phase Line-to-Line Short Circuit with $x_D' > x_Q'$ 20-Hp. Machine

Substituting (**8b**) and (**9b**) in (**13b**), the peak value of the odd harmonic series is thus $1/(1 - b)$ times the peak value of the fundamental component i_1, and the peak value of the even harmonic series is $(1 + b)/(1 - b)$ times the direct-current component i_0. Thus, for the sum of the odd harmonics, the peak value is

$$i_{odd} = i_1/(1 - b) \tag{14b}$$

Similarly, the peak value of the even harmonic series is

$$i_{ev} = i_0\,[(1 + b)/(1 - b)\,] \tag{15b}$$

Substituting for b its value from (**3b**), some additional interesting relations are incidentally obtained. Thus

$$\frac{1}{1 - b} = \frac{\sqrt{x_Q'} + \sqrt{x_D'}}{2\sqrt{x_D'}} \tag{16b}$$

and

$$\frac{1 + b}{1 - b} = \sqrt{\frac{x_Q'}{x_D'}} \tag{17b}$$

Substituting (**8b**) and (**16b**) in (**14b**),

$$i_{odd} = \frac{k\, e_0}{x_D'} \tag{18b}$$

and (**9b**) and (**17b**) in (**15b**),

$$i_{ev} = \frac{k\, e_0 \cos \alpha}{x_D'} \tag{19b}$$

The relations are shown graphically in Fig. 9.

Consider now the value of the sum of the two series in (**2b**) at three different values of $t + \alpha$, namely, $\frac{\pi}{4}$, $\frac{\pi}{2}$, and π. First, when $t + \alpha = \frac{\pi}{4}$, or 45 deg.

$$i_{45} = i_0 \frac{1 - b^2}{1 + b^2} - \sqrt{2}\, i_1 \frac{1 - b}{1 + b^2} \tag{20b}$$

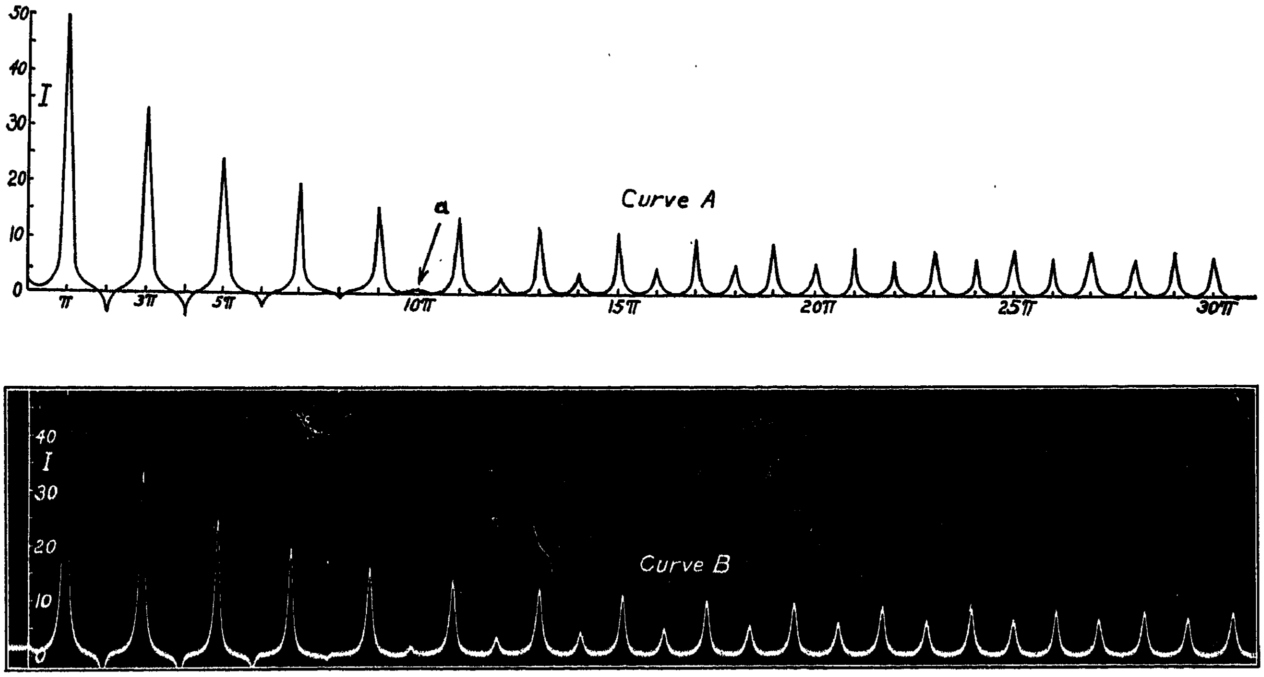


FIG. 16—TRANSIENT FIELD CURRENT, MAXIMUM INITIAL ARAMATURE LINKAGES ON SINGLE-PHASE LINE-TO-LINE SHORT-CIRCUIT 20-HP. MACHINE

A—Calculated curve
B—Test Curve

Equation: —

$$I = \frac{12.3 \cos t}{3.98 - 3.56 \cos t}\, \epsilon^{-0.474t} + \frac{3.08}{3\text{-}98 - 3.56 \cos 2t} + \frac{9.97}{3.98 - 3.56 \cos 2t}\, \epsilon^{-0132t}$$

when $t + \alpha = \frac{\pi}{2}$, or 90 deg.,

$$i_{90} = i_0 \frac{1 - b}{1 + b} \tag{21b}$$

and when $t + \alpha = \pi$, or 180 deg.,

$$i_{180} = i_0 \frac{1 + b}{1 - b} + i_1 \frac{1}{1 - b} \tag{22b}$$

From these three equations, the entire wave can be analyzed. For illustration, let $i_{45} = -0.332$, $i_{90} = 0.5$, and $i_{180} = 6.0$. The three equations thus determine b, i_0, and i_1. They are

$$b = 0.333 \quad i_0 = 1.0 \quad i_1 = 2.667$$

Hence, by (**2b**) the wave is

$$i = 1 + \frac{2}{3} \cos 2\,(t + \alpha) + \frac{2}{9} \cos 4\,(t + \alpha) + \frac{2}{27} \cos 6\,(t + \alpha) + \ldots - 2.667 \left[\cos\,(t + \alpha) + \frac{1}{3} \cos 3\,(t + \alpha) + \frac{1}{9} \cos 5\,(t + \alpha) + \ldots \right]$$

When the wave has no d-c. component i_0, the analysis is much simpler, since only two values from the wave are now needed; namely, the value at 45 deg. and at 90 deg.

Transient Armature Currents. The armature current immediately after short circuit is given by equation (**1b**). The term which disappears when $\alpha = \pi/2$ contains the direct current component and the even harmonics. As time elapses, these currents decay to zero according to the armature decrement factor, σ_a, derived in Appendix H. The other term contains the odd harmonics which are proportional to the field current at every instant. Hence this component of equation (**1b**) decays from the value shown to a sustained value

$$\frac{x_D' + \sqrt{x_D' x_Q'}}{x_D + \sqrt{x_D' x_Q'}}$$

times as great, according to the field decrement factor, σ_f, derived in Appendix H.

The sustained armature current is thus

$$\underset{\text{sustained}}{i} = -2\,k\,e_0 \left[\frac{x_D' + \sqrt{x_D' x_Q'}}{x_D + \sqrt{x_D' x_Q'}} \right] \frac{\cos (t + \alpha)}{(x_D' + x_Q') + (x_D' - x_Q') \cos 2\,(t + \alpha)} \qquad \textbf{(23b)}$$

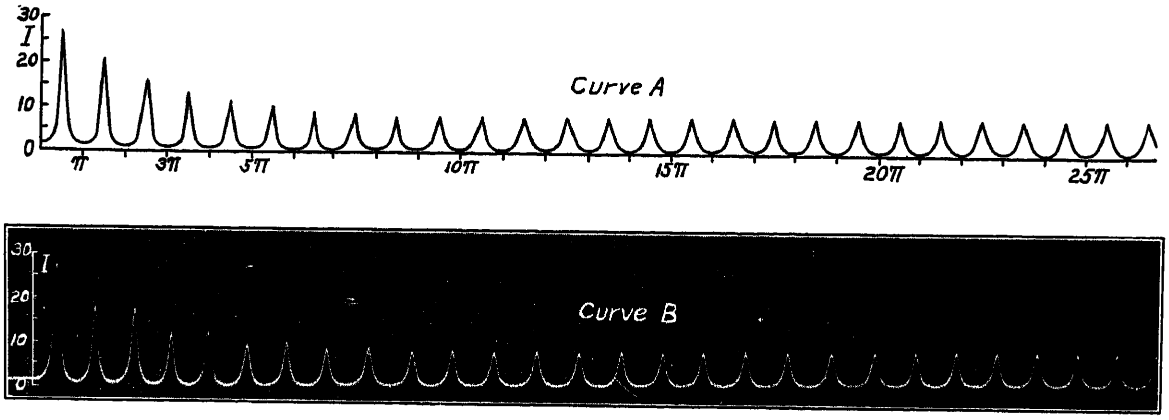


FIG. 17—TRANSIENT FIELD CURRENT, ZERO INITIAL ARMATURE LINKAGES ON SINGLE-PHASE LINE-TO-LINE SHORT-CIRCUIT 20-HP. MACHINE

A—Calculated curve
B—Test curve (not quite zero flux linkages)
Equation:—

$$I = \frac{3.08}{3\ 98 + 3.56 \cos 2\,t} + \frac{9.97}{3.98 + 3.56 \cos 2\,t}\,\epsilon^{-0.132t}$$

The component which decays to zero according to the field decrement factor is then the difference between the second term of equation (**1b**) and equation (**23b**). The total armature current at any instant after short circuit is then

$$i = \frac{2\,k\,e_0 \cos \alpha}{(x_D' + x_Q') + (x_D' - x_Q') \cos 2\,(t + \alpha)}\,\epsilon^{-\sigma_a t}$$

$$- \frac{2\,k\,e_0\,(x_D - x_D') \cos (t + \alpha)}{(x_D + \sqrt{x_D' x_Q'})\,[\,(x_D' + x_Q') + (x_D' - x_Q') \cos 2\,(t + \alpha)\,]}\,\epsilon^{-\sigma_f t}$$

$$- 2\,k\,e_0 \left[\frac{x_D' + \sqrt{x_D' x_Q'}}{x_D + \sqrt{x_D' x_Q'}} \right] \frac{\cos (t + \alpha)}{(x_D' + x_Q') + (x_D' - x_Q') \cos 2\,(t + \alpha)} \qquad \textbf{(24b)}$$

If we substitute from equation (**1b**) and (**2b**) the relations

$$\frac{\cos \alpha}{(x_D' + x_Q') + (x_D' - x_Q') \cos 2\,(t + \alpha)} = \frac{\cos \alpha}{\sqrt{x_D' x_Q'}}\ [\text{even series}] \qquad \textbf{(25b)}$$

and,

$$\frac{\cos (t + \alpha)}{(x_D' + x_Q') + (x_D' - x_Q') \cos (t + \alpha)} = \frac{1}{x_D' + \sqrt{x_D' x_Q'}}\ [\text{odd series}] \qquad \textbf{(26b)}$$

the expression for armature current at any instant after short circuit is

$$i = \frac{2\,k\,e_0 \cos \alpha}{\sqrt{x_D' x_Q'}}\ [\text{even series}]\ \epsilon^{-\sigma_a t}$$

$$\frac{-2\,k\,e_0\,(x_D - x_D')}{(x_D + \sqrt{x_D' x_Q'})\,(x_D' + \sqrt{x_D' x_Q'})}\ [\text{odd series}]\ \epsilon^{-\sigma_f t}$$

$$\frac{-2\,k\,e_0}{x_D + \sqrt{x_D' x_Q'}}\ [\text{odd series}] \qquad \textbf{(27b)}$$

Appendix C

FIELD CURRENT

The equation for the *initial* field current will be obtained in accordance with the plan outlined under "Fundamental Equations."

It is convenient to take the three-phase normal values of current and armature reaction as the bases of the *per-unit* quantities, both three-phase and single-phase. Thus a three-phase *per-unit* current i will produce a rotating space fundamental m. m. f. whose amplitude, expressed as a fraction of the amplitude corresponding to normal current, is numerically equal to the current i. The single-phase m. m. f. and current will be expressed in terms of these.

It will be recalled that the amplitude of the synchronously rotating component of space fundamental m. m. f. due to normal frequency current in one phase of a three-phase machine, is one-half of the amplitude of the *total* pulsating space fundamental produced by that current, the other half rotating oppositely; and that the corresponding synchronously rotating components of the other two phases happen to be in space phase with that of the first. That is, the amplitude of the total three-phase space fundamental m. m. f., *i. e.*, the m. m. f. of armature reaction, is 1.5 times the total amplitude A_1 of the pulsating space fundamental due to one phase only. Or, conversely, the pulsating space fundamental due to a line-to-neutral single-phase current is 2/3 of the m. m. f. of armature reaction A due to a three-phase current of the same value. Since the space fundamental A_1 at any instant is independent of the nature of the variation of the current i which exists at that instant, it follows that, in general, any single-phase current i will produce a space fundamental

$$A_1 = 2/3\,k\,A = 2/3\,k\,i \qquad \text{(1c)}$$

where k = unity for line-to-neutral current, and 1.73 for a line-to-line current. And obviously A_1 will vary in time according as i varies.

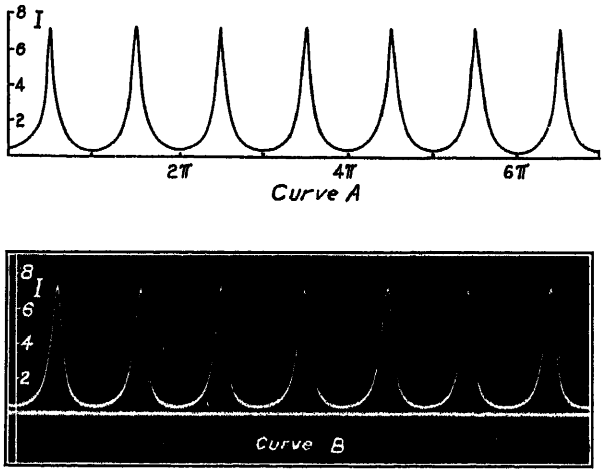


FIG. 18—SUSTAINED FIELD CURRENT ON SINGLE-PHASE LINE-TO-LINE SHORT CIRCUIT 20-HP. MACHINE

A—Calculated curve
B—Test curve

Equation:—

$$i = \frac{3.08}{3.98 + 3.56 \cos 2\,t}$$

When the rotor is moving, the space fundamental m. m. f. A_1 will have relative motion with respect to the rotor poles, but at any instant it may be resolved into the two conventional components; A_D in the direct axis, and A_Q in the quadrature axis. If the single-phase armature current is due to a short circuit at the instant the direct axis is displaced from the axis of the single-phase winding by an angle α, the component of A_1 which remains in the direct axis is

$$A_D = 2/3\,k\,i \cos(t + \alpha) \qquad \text{(2c)}$$

and in the quadrature axis

$$A_Q = 2/3\,k\,i \sin(t + \alpha) \qquad \text{(3c)}$$

It has been shown in *Synchronous Machines—III* that unit m. m. f., in suddenly appearing over the direct axis, will induce a *per-unit* field current

$$\Delta I_d = x_d - x_d' \qquad \text{(4c)}$$

or, since *per-unit* field current is numerically equal to *per-unit* nominal voltage,

$$\Delta I_d = \Delta e_d$$

Thus, a sudden m. m. f. A_d will induce a field current given by

$$\Delta e_d = -A_d\,(x_d - x_d') \qquad \text{(5c)}$$

where x_d and x_d' are respectively the line-to-neutral, three-phase values of synchronous and transient *per-unit* reactances corresponding to the direct axis. (The negative sign is taken in accordance with the convention that the space fundamental m. m. f. due to the armature is considered to be in the same direction as the field m. m. f. when the two axes coincide).

Similarly, for the quadrature axis,

$$\Delta e_q = -A_q\,(x_q - x_q') \qquad \text{(6c)}$$

Since the quantities A_d and A_q in equations (5c) and (6c) each represent the amplitudes of a space fundamental m. m. f., expressed as a fraction of the amplitude of the space fundamental m. m. f., due to normal three-phase current, it follows that the amplitude of any space fundamental m. m. f., so expressed, such as A_D or A_Q, may be directly substituted. Hence, from (2c) and (3c),

$$\Delta e_d = -\frac{2}{3}\,k\,i\,(x_d - x_d') \cos(t + \alpha) \qquad \text{(7c)}$$

and $$\Delta e_q = -\frac{2}{3}\,k\,i\,(x_q - x_q') \cos(t + \alpha) \qquad \text{(8c)}$$

If the main field winding is the only rotor winding in the direct axis, the field current ΔI_d, or Δe_d, will exist entirely in the winding. However, if there are other short-circuited rotor windings in the direct axis, only a part of Δe_d will appear in the main field windings, the remainder appearing in the other closed rotor circuits. For the case of a single rotor winding, the field current ΔI_d which is produced by the armature m. m. f. due to the armature current i, is obtained by substituting in (7c) the value of i given by equation (1b) in Appendix B. Thus,

$$\Delta I_d = \frac{-\frac{4}{3}\,k^2 e_0\,(x_d - x_d') \cos(t + \alpha)\,[\cos\alpha - \cos(t + \alpha)\,]}{(x_D' + x_Q') + (x_D' - x_Q') \cos 2\,(t + \alpha)} \qquad \text{(9c)}$$

It is desirable to express $(x_d - x_d')$ in terms of $x_D - x_D')$.

Referring to the definitions and equations in Appendix I,

$$x_d - x_d' = \frac{3}{2\,k^2}\,(x_D - x_D') \qquad \textbf{(10c)}$$

Substituting this in (**9c**),

$$\Delta I_d = \Delta e_d$$

$$= \frac{-2\,e_0\,(x_D - x_D')\cos(t+\alpha)\,[\cos\alpha - \cos(t+\alpha)\,]}{(x_D' + x_Q') + (x_D' - x_Q')\cos 2\,(t+\alpha)} \qquad \textbf{(11c)}$$

The total *per-unit* field current, or *per-unit* nominal voltage immediately after short circuit, that is the *initial* value, is

$$I_d = (I_0 + \Delta I_d) = e_d = (e_0 + \Delta e_d) \qquad \textbf{(12c)}$$

$$I_d = e_d = \frac{e_0\,(x_D + x_Q') + (x_D - x_Q')\cos 2\,(t+\alpha) - 2\,(x_D - x_D')\cos\alpha\cos(t+\alpha)}{(x_D' + x_Q') + (x_D' - x_Q')\cos 2\,(t+\alpha)} \qquad \textbf{(13c)}$$

When the harmonic series form for i from equation (**2b**), Appendix B, is substituted in (**7c**) and remembering the relation in (**10c**),

$$I_d = e_d = \frac{e_0\,(x_D + \sqrt{x_D'\,x_Q'})}{x_D' + \sqrt{x_D'\,x_Q'}}$$

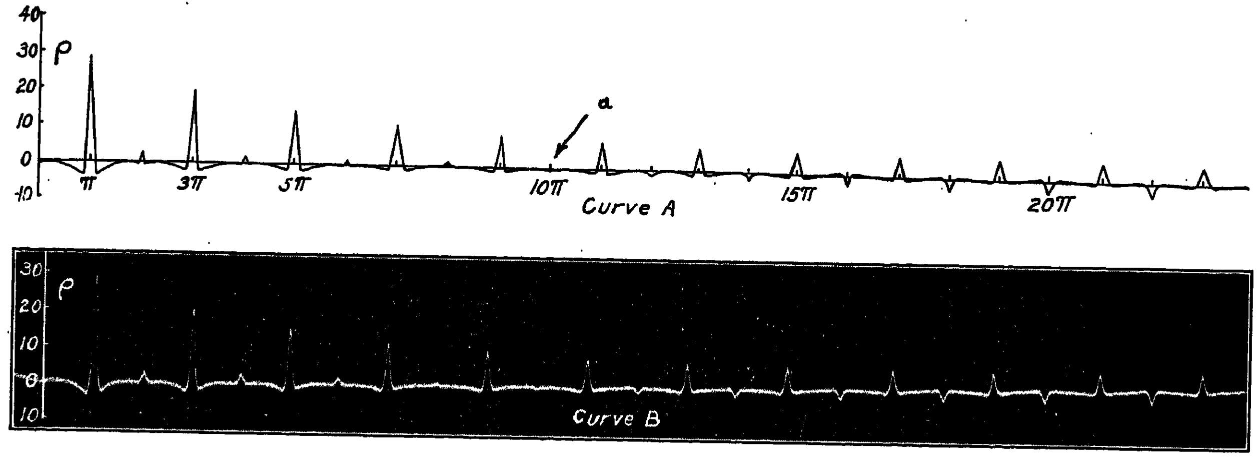


FIG. 19—TRANSIENT OPEN-PHASE VOLTAGE, MAXIMUM INITIAL ARMATURE LINKAGE ON SINGLE-PHASE LINE-TO-LINE SHORT-CIRCUIT 20-HP. MACHINE

A—Calculated curve
B—Test curve
Equation:—

$$e = -7.12\,\frac{(3.56 - 3.98\cos 2\,t)}{(3.98 - 3.56\cos 2\,t)}\,\epsilon^{-0.0474t} - \frac{1.78\cos t - 3.36\cos t\sin^2 t}{3.98 - 3.56\cos 2\,t} + \frac{5.98\cos t\sin^2 2\,t}{(3.98 - 3.56\cos 2\,t)^2}$$

$$-\frac{(5.75\cos t - 10.9\cos t\sin^2 t)}{(3.98 - 3.56\cos 2\,t)^2}\,\epsilon^{-0.132t} + \frac{(19.3\cos t\sin^2 2\,t)}{(3.98 - 3.56\cos 2\,t)^2}\,\epsilon^{-0.132t}$$

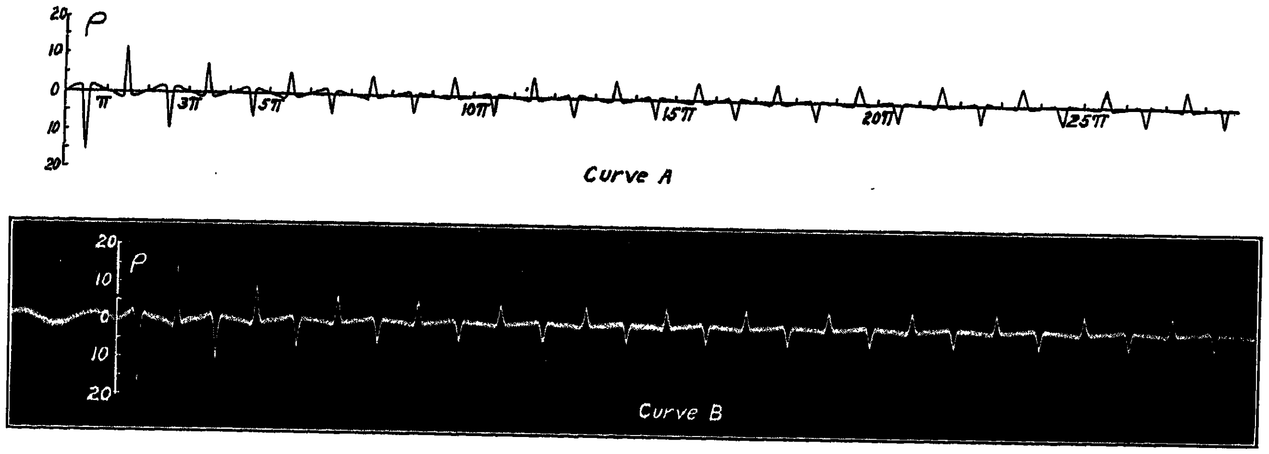


FIG. 20—TRANSIENT OPEN-PHASE VOLTAGE, ZERO INITIAL ARMATURE LINKAGES ON SINGLE-PHASE LINE-TO-LINE SHORT-CIRCUIT 20-HP. MACHINE

A—Calculated curve
B—Test curve

Equation:—

$$e = \frac{-1.78\sin t - 3.36\sin t\cos^2 t}{3.98 + 3.56\cos 2\,t} + \frac{5.98\sin t\sin^2 2\,t}{[3.98 + 3.56\cos 2\,t]^2} - \frac{5.75\sin t - 10.9\sin t\cos^2 t}{3.98 + 3.56\cos 2\,t}\,\epsilon^{-0.132t} + \frac{19.3\sin t\sin^2 2\,t}{3.98 + 3.56\cos 2\,t^2}\,\epsilon^{-0.132t}$$

$$- \frac{e_0 (x_D - x_D') (1 + b) \cos \alpha}{\sqrt{x_D' x_Q'}} [\cos (t + \alpha)$$

$$+ b \cos 3 (t + \alpha) + b^2 \cos 5 (t + \alpha) + \ldots]$$

$$+ \frac{e_0 (x_D - x_D')}{x_D' + \sqrt{x_D' x_Q'}} (1 + b) [\cos 2 (t + \alpha)$$

$$+ b \cos 4 (t + \alpha) + b^2 \cos 6 (t + \alpha) + \ldots] \quad \textbf{(14c)}$$

The quadrature component ΔI_q may be obtained in a similar manner.

The field current comprises a direct current component, a series of odd harmonics, whose amplitudes are in geometric ratio, and a series of even harmonics whose amplitudes are in geometric ratio. The geometric ratio is the same in all cases, for the armature current as well as for the field current, and its value is given by equation (**3b**), Appendix B.

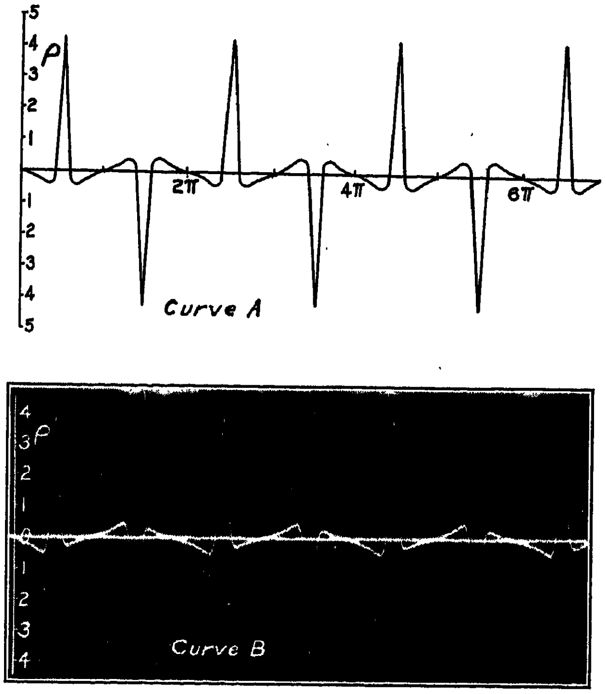


FIG. 21—SUSTAINED OPEN-PHASE VOLTAGE ON SINGLE-PHASE LINE-TO-LINE SHORT-CIRCUIT 20-HP. MACHINE

A—Calculated curve
B—Test curve

Equation:—

$$e = \frac{1.78 \sin t - 3.36 \sin t \cos^2 t}{3.98 + 3.56 \cos 2 t} + \frac{5.98 \sin t \sin^2 2 t}{[3.98 + 3.56 \cos 2 t]^2}$$

Transient Field Current. It will be noticed that the average field current or nominal voltage increases at short circuit in the ratio

$$\frac{x_D + \sqrt{x_D' x_Q'}}{x_D' + \sqrt{x_D' x_Q'}}$$

As time elapses, this average field current will decay to the value existing before short circuit according to the field decrement factor σ_f derived in Appendix H. Since all even harmonics in the field current are proportional to the average field current at every instant after short circuit, they will also decay to their sustained values in the same manner. The odd harmonics in the field current are proportional to the direct-current component in the armature at every instant and thus decay to zero according to the armature decrement factor σ_a derived in Appendix H. In equation (**13c**), the odd harmonics comprise those terms which disappear when $\alpha = \frac{\pi}{2}$, *i. e.*, when there is no direct current component in the armature. The remaining terms comprise the average value and all even harmonics. Thus

$$\underset{\text{odd}}{I_d} = \frac{- 2 e_0 (x_D - x_D') \cos \alpha \cos (t + \alpha)}{(x_D' + x_Q') + (x_D' - x_Q') \cos 2 (t + \alpha)} \quad \textbf{(15c)}$$

and

$$\underset{\text{even}}{I_d} = \frac{e_0 [(x_D + x_Q') + (x_D - x_Q') \cos 2 (t + \alpha)]}{(x_d' + x_Q') + (x_D' - x_Q') \cos 2 (t + \alpha)} \quad \textbf{(16c)}$$

The sustained value of (**15c**) is zero and the sustained value of (**16c**) bears the same relation to (**16c**) as the final average value bears to the initial average value. Hence the sustained field current is

$$\underset{\text{sustained}}{I_d} = \frac{x_D' + \sqrt{x_D' x_Q'}}{x_D + \sqrt{x_D' x_Q'}}$$

$$e_0 \left[\frac{(x_D + x_Q') + (x_D - x_Q') \cos 2 (t + \alpha)}{(x_D' + x_Q') + (x_D' - x_Q') \cos 2 (t + \alpha)} \right] \quad \textbf{(17c)}$$

The differences between (**16c**) and (**17c**) thus decays to zero according to the field decrement factor, and the expression for the total field current at any instant after short circuit is

$$I = \frac{- 2 e_0 (x_D - x_D') \cos \alpha \cos (t + \alpha)}{(x_D' + x_Q') + (x_D' - x_Q') \cos 2 (t + \alpha)} \epsilon^{-\sigma_a t}$$

$$+ \frac{x_D - x_D'}{x_D + \sqrt{x_D' x_Q'}}$$

$$e_0 \left[\frac{(x_D + x_Q') + (x_D - x_Q') \cos 2 (t + \alpha)}{(x_D' + x_Q') + (x_D' - x_Q') \cos 2 (t + \alpha)} \right] \epsilon^{-\sigma_f t})$$

$$+ \frac{x_D' + \sqrt{x_D' x_Q'}}{x_D + \sqrt{x_D' x_Q'}}$$

$$e_0 \left[\frac{(x_D + x_Q') + (x_D - x_Q') \cos 2 (t + \alpha)}{(x_D' + x_Q') + (x_D' - x_Q') \cos 2 (t + \alpha)} \right] \quad \textbf{(18c)}$$

If we substitute the odd harmonic series from equation (**14c**) for (**15c**), and the average value and even harmonics from equation (**14c**) for equation (**16c**), equation (**18c**) may be written

$$
\begin{aligned}
I = {} & \frac{-e_0\,(x_D - x_D')\,(1+b)\cos\alpha}{\sqrt{x_D'\,x_Q'}}\,[\text{odd series}]\,\epsilon^{-\sigma_a t} \\
& + \frac{e_0\,(x_D - x_D')^2\,(1+b)}{(x_D' + \sqrt{x_D'\,x_Q'})\,(x_D + \sqrt{x_D'\,x_Q'})}[\text{even series}]\,\epsilon^{-\sigma_f t} \\
& + \frac{e_0\,(x_D - x_D')}{x_D' + \sqrt{x_D'\,x_Q'}}\,\epsilon^{-\sigma_f t} \qquad \textbf{(19c)} \\
& + \frac{e_0\,(x_D - x_D')\,(1+b)}{x_D + \sqrt{x_D'\,x_Q'}}\,[\text{even series}] \\
& + e_0
\end{aligned}
$$

Appendix D

VOLTAGE ACROSS EXTERNAL REACTANCE IN ARMATURE CIRCUIT

The reactances, x_D' and x_Q', may in general include an external reactance x_e. It is then of interest to find the voltage which exists across this reactance under conditions of sudden short circuit. The per-unit value of voltage will be given by the product of the external inductance and the rate of change of current with respect to time. Since per-unit reactance at normal frequency is numerically equal to per-unit inductance, the voltage is,

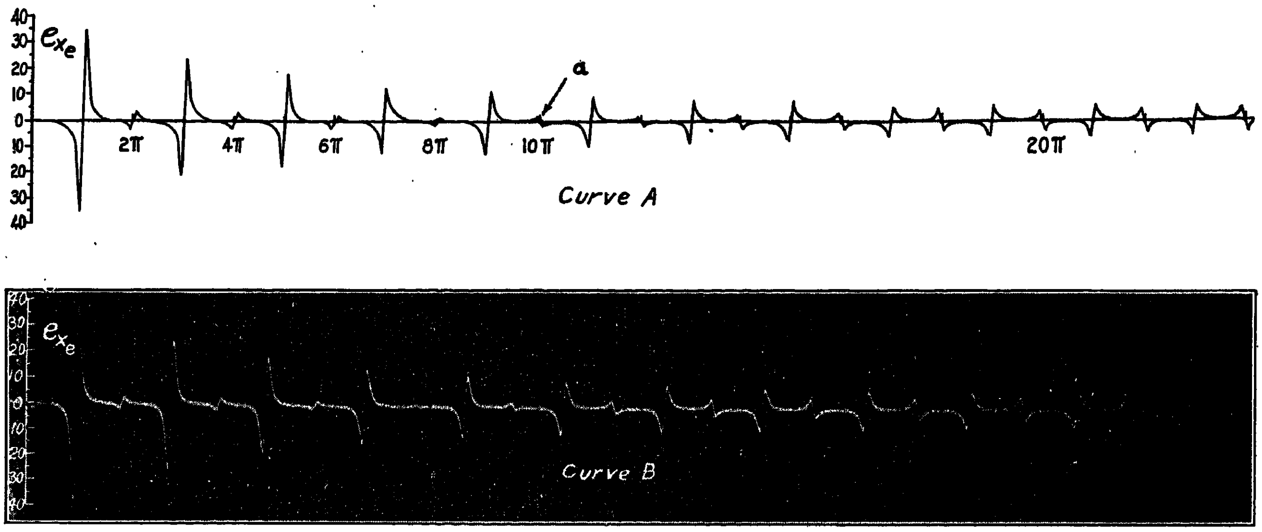


FIG. 22—TRANSIENT VOLTAGE ACROSS AN EXTERNAL REACTOR IN THE ARMATURE CIRCUIT, MAXIMUM INITIAL ARMATURE LINKAGES ON SINGLE-PHASE LINE-TO-LINE SHORT-CIRCUIT 20-HP. MACHINE

A—Calculated curve
B—Test curve

Equation:—

$$
e = -\frac{0\ 164}{3.98 - 3.56\cos 2t}\,\epsilon^{-0.0474t} - \frac{24.6\sin 2t}{(3.98 - 3.56\cos 2t)^2}\,\epsilon^{-0.0474t} + 0.815\,\frac{7.12\cos t\sin 2t + (3.98 - 3.56\cos 2t)\sin t}{(3.98 - 3.56\cos 2t)^2} + 2.64\,\frac{7.12\cos t\sin 2t + (3.98\cos 2t)\,(\sin t + 0.132\cos t)}{(3.98 - 3.56\cos 2t)^2}\,\epsilon^{-0.132t}
$$

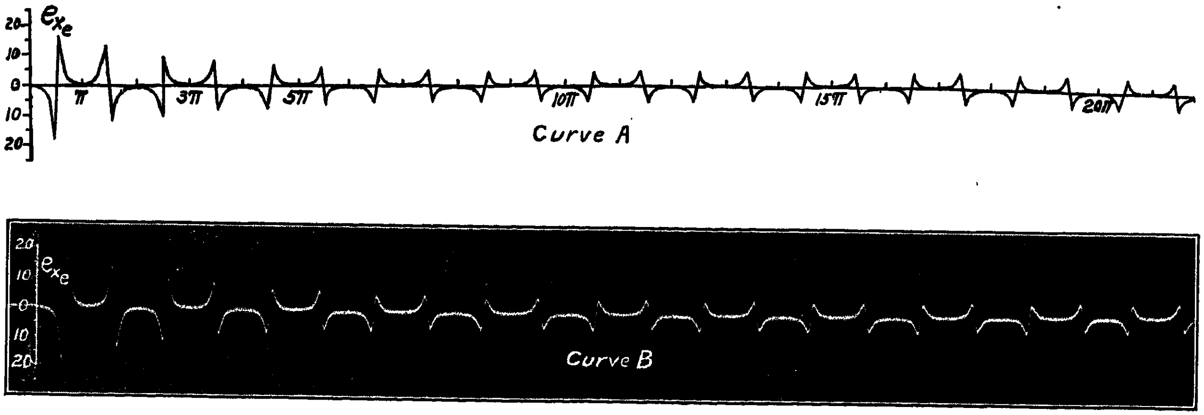


FIG. 23—TRANSIENT VOLTAGE ACROSS AN EXTERNAL REACTOR IN THE ARMATURE CIRCUIT, ZERO INITIAL ARMATURE LINKAGES ON SINGLE-PHASE LINE-TO-LINE SHORT-CIRCUIT 20-HP. MACHINE

A—Calculated curve
B—Test curve

Equation:—

$$
e = -0.815\,\frac{7.12\sin 2t\sin t + (3.98 + 3.56\cos 2t)\cos t}{(3.98 + 3.56\cos 2t)^2} - 2.64\,\frac{7.12\sin 2t\sin t + (3.98 + 3.56\cos 2t)\,(\cos t - .132\sin t)}{(3.98 + 3.56\cos 2t)^2}\,\epsilon^{-0.132t}
$$

$$e_{x_e} = x_e \frac{d\,i}{d\,t} \qquad \textbf{(1d)}$$

The initial value of short-circuit armature current is given by equation (**1b**) of Appendix B. Hence, from this expression, the voltage across the external reactance, assuming no decrements in the current wave, will be

$$e_{x_e} = \frac{2\,k\,e_0\,x_e}{H^2}\,[\,H \sin(t + \alpha)$$

$$+ 2\,B \sin 2\,(t + \alpha)\,[\cos \alpha - \cos(t + \alpha)]\,] \qquad \textbf{(2d)}$$

The symbolic form is that outlined in Appendix A.

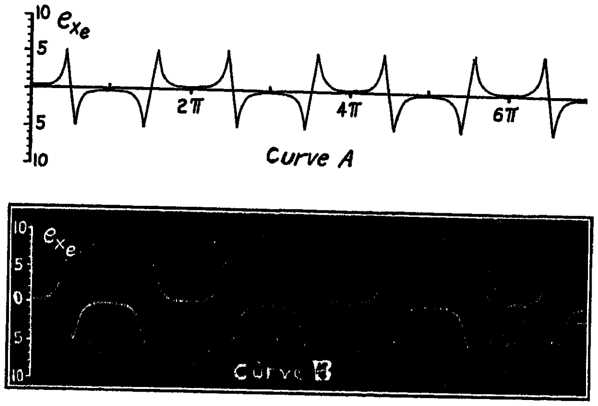


Fig. 24—Sustained Voltage Across an External Reactor in the Armature Circuit on Single-Phase Line-to Line Short-Circuit 20-Hp. Machine

A—Calculated curve
B—Test curve

Equation:—

$$e = 0.815\left[\frac{-\,7.12 \sin 2\,t \sin t\,-\,(3.98\,+3.56 \cos 2\,t) \cos t}{(3.98\,+3.56 \cos 2\,t)^2}\right]$$

Under the development of Transient Armature Current of Appendix B, it was shown that part of the current wave decays to zero according to the armature decrement σ_a, and that a second part decays to zero according to the field decrement σ_f. The third part is the steady-state value. Thus, the above voltage equation will have components which also decay to zero according to these two decrements.

That part of equation (**2d**) which contains the armature decrement corresponds to the first term of equation (**24b**) and is

$$\frac{4\,k\,e_0\,x_e\,B \sin 2\,(t + \alpha) \cos \alpha}{H^2}\,\epsilon^{-\sigma_a t} \qquad \textbf{(3d)}$$

With reference to (**24b**), it will be noticed that a second term, containing the derivatives of the exponential, should be introduced in order to be exact. However, since this quantity is very small in comparison with (**3d**) and will never be noticeable in practical cases, it is not considered here. That is, the voltage due to the rate of change of current which in turn is due to the armature decrement is neglected.

The remainder of (**2d**) is that portion which contains the field decrement σ_f, and which decays from the value shown to a sustained value

$$\frac{x_D' + \sqrt{x_D'\,x_Q'}}{x_D + \sqrt{x_D'\,x_Q'}} = K$$

times as great, according to this decrement. It is divided into the alternating component which finally dies away, and the sustained value, in the same manner as its corresponding currents of equation (**1b**) were formed into the second and third terms of equation (**24b**).

As an alternative to decomposing (**2d**) into the above mentioned components, the transient voltage across an external reactance in the armature circuit may be more easily obtained by differentiating (**24b**). Neglecting those terms concerned with derivatives of the exponents we have,

$$e_{x_e} = \frac{2\,k\,e_0\,x_e}{H^2}\,[\,2\,B \cos \alpha \sin 2\,(t + \alpha)\,\epsilon^{-\sigma_a t}$$

$$+[H \sin(t+\alpha) - 2\,B \cos(t+\alpha) \sin 2\,(t+\alpha)]\,(K + P\,\epsilon^{-\sigma_f t}] \qquad \textbf{(4d)}$$

It is interesting to treat the initial voltage in series form and observe the shapes of the total and component waves. Thus, using the series expression for current as given by equation (**2b**), the voltage across the external reactance becomes,

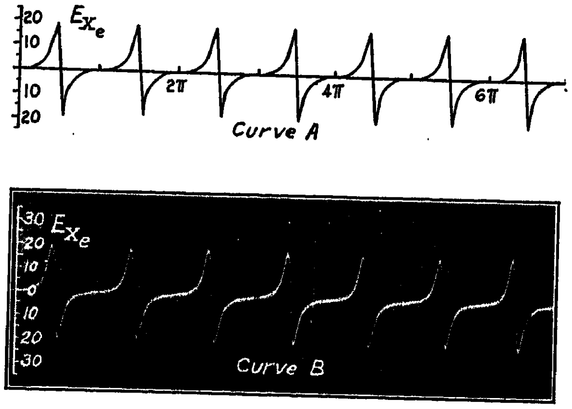


Fig. 25—Sustained Voltage Across an External Reactor in the Field Circuit on Single-Phase Line-to Line Short-Circuit 20 Hp. Machine

A—Calculated curve
B—Test curve (inverted)

$$= \frac{21.9 \sin 2\,t}{(3.98\,+3.56 \cos 2\,t)^2}$$

$$e_{x_e} = -\,\frac{k\,e_0\,x_e \cos \alpha}{\sqrt{x_D'\,x_Q'}}\,[\,4\,b \sin 2\,(t + \alpha)$$

$$+ 8\,b^2 \sin 4\,(t + \alpha) + 12\,b^3 \sin 6\,(t + \alpha) + \ldots]$$

$$+ \frac{2\,k\,e_0\,x_e}{x_D' + \sqrt{x_D'\,x_Q'}}\,[\sin(t + \alpha) + 3\,b \sin 3\,(t + \alpha)$$

$$+ 5\,b^2 \sin 5\,(t + \alpha) + \ldots\,] \qquad \textbf{(5d)}$$

When $\alpha = 0$, the first series will have the general form shown by Fig. 25 in the main body of the paper, while the second series will have the form shown by Fig. 24. The total wave is given by Fig. 22.

Appendix E

VOLTAGE ACROSS EXTERNAL REACTANCE IN FIELD CIRCUIT

Under conditions of sudden short circuit, alternating currents are produced in the field circuit. When this circuit contains some reactance external to the field winding proper, such as the exciter armature, alternating voltages exist across the reactance. It may sometimes be desirable to determine such voltages.

As before, for a pure reactance, the per-unit voltage is given by the product of the per-unit external inductance (which is numerically equal to the per-unit reactance X_e), and the rate of change of current; in this case, field current.

In evaluating this voltage, it is convenient to express the external reactance in field terms. Unit field voltage is defined as the d-c. voltage corresponding, by Ohm's Law, to unit field current. The unit value of either resistance or reactance is thus the ratio of unit field voltage to unit field current. Any other value of ohms could be chosen as the unit base, but this particular value affords special convenience in that the calculated results are in terms of the known field excitation voltage. If, on this basis, the per-unit external reactance in the field circuit is X_e, the per-unit voltage across its terminals under sudden short circuit conditions is the product of X_e and the rate of change of field current.

The initial field current is given by equation (**13c**), hence the voltage across the reactance at the instant of short circuit is

$$E_{X_e} = \frac{2\,E_0\,X_e}{H^2}\,\{\,H\,G\cos\alpha\sin(t+\alpha)$$
$$+\sin 2\,(t+\alpha)\,[\,B\,C - H\,F + F\cos 2\,(t+\alpha)$$
$$-\,2\,G\cos\alpha\cos(t+\alpha)\,]\,\} \qquad \textbf{(1e)}$$

Expressed in series form (**1e**) becomes

$$E_{X_e} = \frac{E_0\,X_e\,G\,(1+b)\cos\alpha}{\sqrt{x_D'\,x_Q'}}\,[\,\sin(t+\alpha)$$
$$+\,3\,b\sin 3\,(t+\alpha) + 5\,b^2\sin 5\,(t+\alpha) + \;.\;.\;.\;]$$
$$-\,\frac{2\,E_0\,X_e\,G\,(1+b)}{x_D' + \sqrt{x_D'\,x_Q'}}\,[\,\sin 2\,(t+\alpha)$$
$$+\,2\,b\sin 4\,(t+\alpha) + 3\,b^2\sin 6\,(t+\alpha) + \;.\;.\;.\;] \qquad \textbf{(2e)}$$

Referring to Appendix B, it will be remembered that, under sustained short circuit, only the odd harmonics are present in the armature current. Hence, only the d-c. component and the even harmonics will exist in the field current for sustained short circuit, as explained in Appendix C. These field currents, given by their respective components of equations (**13c**) and (**14c**) are

$$\underset{\text{sustained}}{I_d} = E_0\,K\left[\frac{C + F\cos 2\,(t+\alpha)}{A + B\cos 2\,(t+\alpha)}\right] \qquad \textbf{(3e)}$$

$$\text{or}\ \underset{\text{sustained}}{I_d} = E_0 + E_0\,P\,(1+b)\,[\cos 2\,(t+\alpha)$$
$$+\,b\cos 4\,(t+\alpha) + \;.\;.\;.\;] \qquad \textbf{(4e)}$$

The sustained voltage across the field reactance is thus

$$E_{X_e} = \frac{2\,E_0\,X_e\,K\sin 2\,(t+\alpha)}{H^2}\,(B\,C - A\,F) \qquad \textbf{(5e)}$$

$$\text{or}\ E_{X_e} = -\,2\,E_0\,P\,(1+b)\,[\,\sin 2\,(t+\alpha)$$
$$+\,2\,b\sin 4\,(t+\alpha) + 3\,b^2\sin 6\,(t+\alpha) + \;.\;.\;.\;] \qquad \textbf{(6e)}$$

Equation (**1e**) can be decomposed into three components; that which dies away according to the armature decrement σ_a, that which decays to zero according to the field decrement σ_f, and the sustained voltage. Since, however, the transient field current is known, equation (**18c**), the voltage across the reactance can more easily be obtained by the product of the derivative of this expression and X_e. Thus,

$$E_{X_e} = \frac{2\,E_0\,X_e\,G}{H^2}\cos\alpha\sin(t+\alpha)$$
$$[\,H - 4\,B\cos^2(t+\alpha)\,]\,\epsilon^{-\sigma_a t}$$
$$+\,\frac{2\,E_0\,X_e\,P}{H^2}\sin 2\,(t+\alpha)\,[B\,C - A\,F\,]\,\epsilon^{-\sigma_f t}$$
$$+\,\frac{2\,E_0\,X_e\,K}{H^2}\sin 2\,(t+\alpha)\,[\,B\,C - A\,F\,] \qquad \textbf{(7e)}$$

As in the case of the voltage across an external reactance in the armature circuit, the terms containing the derivatives of the decrements have been omitted because they are quite inappreciable. It will be observed that the last term of (**7e**), representing the sustained voltage across a field reactance, is exactly equation (**5e**).

Appendix F

VOLTAGE ACROSS THE OPEN PHASE

It has been shown that the current in the short-circuited phase produces a space fundamental m. m. f.

$$A = 2/3\,k\,i \qquad \textbf{(1f)}$$

of which the component in the direct axis is

$$A_D = 2/3\,k\,i\cos(t+\alpha) \qquad \textbf{(2f)}$$

and the component in the quadrature axis is

$$A_Q = 2/3\,k\,i\sin(t+\alpha) \qquad \textbf{(3f)}$$

Under sudden short-circuit conditions, the space fundamental fluxes produced are

$$\varphi_D = 2/3\,k\,i\,x_{ad}'\cos(t+\alpha) \qquad \textbf{(4f)}$$
$$\varphi_Q = 2/3\,k\,i\,x_{aq}'\sin(t+\alpha) \qquad \textbf{(5f)}$$

The components of these fluxes which an open winding whose axis is at an angle β from the axis of the short-circuited winding are respectively,

$$\varphi_1 = 2/3\,k\,i\,x_{ad}'\cos(t+\alpha)\cos(t+\alpha+\beta) \qquad \textbf{(6f)}$$

and $\varphi_2 = 2/3\, k\, i\, x_{aq}' \sin (t + \alpha) \sin (t + \alpha + \beta)$ **(7f)**

The flux linkages due to these fluxes are

$$\psi_1 = 2/3\, k\, k_1\, i\, x_{ad}' \cos (t + \alpha) \cos (t + \alpha + \beta) \quad \textbf{(8f)}$$

and $\psi_2 = 2/3\, k\, k_1\, i\, xa_q' \sin (t + \alpha) \sin (t + \alpha + \beta)$ **(9f)**

where k_1 is a constant by which per-unit fundamental flux must be multiplied to give the flux linkages in the open winding under consideration. For a line to neutral winding, k_1 is unity and for a line to line winding k_1 is $\sqrt{3}$.

The current in the short-circuited winding also produces mutual flux linkages with the open-circuited winding such as those due to space harmonics, mutual slot fluxes, mutual fluxes in the end turn leakage paths, etc. The flux linkages from this source are

$$\psi_3 = i\, x_m \quad \textbf{(10f)}$$

The flux linkages in the open-circuited winding due to the rotor current existing before short circuit are

$$\psi_3 = k_1\, e_0 \cos (t + \alpha + \beta) \quad \textbf{(11f)}$$

The total flux linkages in the open winding at any instant after short circuit are then

$$\psi = \psi_1 + \psi_2 + \psi_3 + \psi_4 \quad \textbf{(12f)}$$

Using the relations, similar to equation **(13i)**, that

$$x_{ad}' + x_{aq}' = \frac{3}{2\, k^2} (x_{AD}' + x_{AQ}') \quad \textbf{(13f)}$$

$$x_{ad}' - x_{aq}' = \frac{3}{2\, k^2} (x_{AD}' - x_{AQ}') \quad \textbf{(14f)}$$

the total flux linkages are

$$\psi = k_1\, e_0 \cos (t + \alpha + \beta) + i\, x_m + \frac{k_1\, i}{2\, k}$$

$$[(x_{AD}' + x_{AQ}') \cos \beta + (x_{AD}' - x_{AQ}') \cos (2\, t + 2\, \alpha + \beta)] \quad \textbf{(15f)}$$

The voltage induced in the open winding when another winding in the machine is short-circuited is given by the negative rate of change of equation **(15f)**, or

$$e_1 = k_1\, e_0 \sin (t + \alpha + \beta)$$

$$+ \frac{k_1}{k} (x_{AD}' - x_{AQ}') \sin (2\, t + 2\, \alpha + \beta)\, i$$

$$- \left\{ x_m + \frac{k_1}{2\, k} [\, (x_{AD}' + x_{AQ}') \cos \beta \right.$$

$$\left. + (x_{AD}' - x_{AQ}') \cos (2\, t + 2\, \alpha + \beta)\,] \right\} \frac{d\, i}{d\, t} \quad \textbf{(16f)}$$

Substituting the value of current from equation **(1b)**, and its derivative, and using the notation of Appendix A, this voltage becomes

$$e_1 = k_1\, e_0 \sin (t + \alpha + \beta) + \frac{2\, k\, e_0}{H}$$

$$(x_{AD}' - x_{AQ}') \sin (2\, t + 2\, \alpha + \beta) [\cos \alpha - \cos (t + \alpha)\,]$$

$$- \frac{2\, k\, e_0}{H} \sin (t + \alpha) \left\{ x_m + \frac{k_1}{2\, k} \right.$$

$$\left. [(x_{AD}' + x_{AQ}') \cos \beta + (x_{AD}' - x_{AQ}') \cos (2\, t + 2\, \alpha + \beta)] \right\}$$

$$- \frac{4\, k\, e_0\, B}{H^2} \sin 2\, (t + \alpha)$$

$$[\cos \alpha - \cos (t + \alpha)\,] \left\{ x_m + \frac{k_1}{2\, k} [(x_{AD}' + x_{AQ}') \right.$$

$$\left. \cos \beta + (x_{AD}' - x_{AQ}') \cos (2\, t + 2\, \alpha + \beta)\,] \right\} \quad \textbf{(17f)}$$

When normal current flows through one winding of the armature, flux linkages will be produced in the other windings. Of these linkages, the larger part is due to space fundamental fluxes. The remainder are due to non-space fundamental fluxes such as mutual slot fluxes, mutual fluxes in the end turn leakage paths, etc. These linkages are represented by x_m. In many cases, this is small compared with the space fundamental flux linkages and by assuming it to be zero many of the equations are greatly simplified.

For ordinary values of β which exist in synchronous machines (60 deg. and 90 deg.) equation **(17f)** essentially has its maximum when $t = 180 - \alpha$. For this value of t, all terms but the first and second disappear and

$$e_{1m} = -\, k_1\, e_0 \sin \beta \left[\frac{x_Q'}{x_D'} (1 + \cos \alpha) - \cos \alpha \right] \quad \textbf{(18f)}$$

When β is very small this value is in error since for $\beta = 0$ deg. there will be a voltage in the open phase corresponding to leakage reactance voltage in the short-circuited phase. For values of β approaching zero it is necessary to plot the voltage curve to find its maximum value.

The open phase voltage can also be expressed in the harmonic form by substituting the current expression from equation **(2b)**, and its derivative. In this harmonic form, if we make the approximation that

$$x_{AD}' - x_{AQ}' = (x_D' - x_Q') \quad \textbf{(19f)}$$

and let $\beta = \frac{\pi}{2}$, then $x_m = 0$, and the *fundamental voltage* in the open phase is found to be

$$e_1 = k_1\, e_0\, (1 + b) \cos (t + \alpha) \quad \textbf{(20f)}$$

The fundamental voltage in an open-circuited phase 90 deg. away from the short-circuited phase will thus be increased by the ratio $(1 + b)$, as mentioned in the main body of the paper.

When the short circuit occurs when maximum flux links the winding, *i. e.*, when $\alpha = 0$, the voltage on the open phases will have the general form shown in Fig. 19

When $\alpha = \frac{\pi}{2}$, no flux links the winding at the instant of short circuit and the voltage on the open phase may be of the form shown in Fig. 20.

If the short circuit occurs line to neutral, the winding comprised of the two unshort-circuited phases in series is in space quadrature with the short-circuited phase. Then

$$\begin{aligned} \beta &= \pi/2 \\ k_1 &= \sqrt{3} \\ x_m &= 0 \\ k &= 1 \end{aligned}$$

For $\alpha = 0$, equation (**17f**) then becomes, (using the simplified notation),

$$\begin{aligned} e_1 = k\,e_0 \cos t &+ \frac{2\,k\,e_0}{H}\,(x_{AD}' - x_{AQ}')\cos 2\,t\,(1 - \cos t) \\ &+ \frac{k\,e_0}{H}\,(x_{AD}' - x_{AQ}')\sin t \sin 2\,t \\ &+ \frac{2\,k\,B\,e_0}{H^2}\,(x_{AD}' - x_{AQ}')\sin^2 2\,t\,(1 - \cos t) \end{aligned} \quad \textbf{(21f)}$$

When $\alpha = \frac{\pi}{2}$, equation (**17f**) becomes

$$\begin{aligned} e_1 = -\,k\,e_0 \sin t &- \frac{2\,k\,e_0}{J}\,(x_{AD}' - x_{AQ}')\sin t \cos 2\,t \\ &- \frac{k\,e_0}{J}\,(x_{AD}' - x_{AQ}')\cos t \sin 2\,t \\ &+ \frac{2\,k\,B\,e_0}{J^2}\,(x_{AD}' - x_{AQ}')\sin t \sin^2 2\,t \end{aligned} \quad \textbf{(22f)}$$

As pointed out before, practically all the difference between x_D' and x_Q' is due to the difference in space fundamental flux, a negligible amount being due to the variation in leakage fluxes in the two axes. Hence, for all practical purposes, $(x_D' - x_Q')$ may be taken equal to $(x_{AD}' - x_{AQ}')$, which simplifies equations (**21f**) and (**22f**), some of the terms combining into more compact terms. Equation (**21f**) has a maximum when $t = \pi$. For this value of t

$$e_1 = -\,k\,e_0\left[\frac{2\,x_Q'}{x_D'} - 1\right] \quad \textbf{(23f)}$$

$$\begin{aligned} \alpha &= 0 \\ \beta &= \frac{\pi}{2} \\ t &= \pi \end{aligned}$$

The voltage on this winding before short circuit is $k\,e_0$. Hence, the voltage at short-circuit line-to-neutral may rise to $\frac{2\,x_Q'}{x_D'} - 1$ times the voltage before short circuit on the open phases.

Equation (**22f**) has a maximum when $t = \pi/2$. Then

$$e_1 = -\,k\,e_0\left[\frac{x_Q'}{x_D'}\right]$$

$$\begin{aligned} \alpha &= \frac{\pi}{2} \\ \beta &= \frac{\pi}{2} \\ t &= \frac{\pi}{2} \end{aligned} \quad \textbf{(24f)}$$

Equation (**17f**) may be decomposed into a form which clearly shows the components affected by the armature and field decrements, and the steady-state term. Thus, substituting for $(x_{AD}' - x_{AQ}')$ and omitting x_m, equation (**17f**) becomes

$$\begin{aligned} e_1 = {} & \frac{2\,k\,k_1\,B\cos\alpha\,e_0}{H^2}\,[\,H\cos 2\,(t + \alpha) \\ & + B\sin^2 2\,(t + \alpha)\,]\,\epsilon^{-\sigma_a t} \\ & + \frac{k_1\,e_0}{k\,H^2}\,\{\,[H^2\,k - 2\,k\,B\,H\cos 2\,(t + \alpha) \\ & - 2\,k\,B^2\sin^2 2\,(t + \alpha)\,]\cos(t + \alpha) \\ & + k\,B\,H\sin(t + \alpha)\sin 2\,(t + \alpha)\}\,[K - (1 - K)\,\epsilon^{-\sigma_f t}\,] \end{aligned} \quad \textbf{(25f)}$$

This is the open phase voltage for any condition of transiency, *i. e.*, for any flux originally trapped in the armature at the time of short circuit. For maximum flux linkages $\alpha = 0$ and all terms appear. For minimum flux linkages $\alpha = \frac{\pi}{2}$ and that term containing the armature decrement does not appear. The steady-state voltage is obtained by neglecting all terms containing either the armature or field decrements.

Appendix G

VARIATION OF ARMATURE INDUCTANCE WITH POSITION

The object of resolving a sine wave of m. m. f. into two components is to obtain fluxes which are in space phase with the m. m. fs. In dealing with synchronous machines, resolution is made about the direct and quadrature axes, and the reactance of a single-phase armature winding in any position with respect to these axes, due to the space fundamental flux produced by the space fundamental m. m. f., may be obtained as follows.

Assume a single-phase armature winding with the axis of the winding at an angle γ from the direct axis of the machine. When a current i of normal frequency flows through this winding a space fundamental m. m. f. of amplitude

$$A = \frac{2}{3}\,k\,i\cos t \quad \textbf{(1g)}$$

is produced. The component of A in the direct axis is

$$A_D = \frac{2}{3}\, k\, i \cos\gamma \cos t \tag{2g}$$

and the component in the quadrature axis is

$$A_Q = \frac{2}{3}\, k\, i \sin\gamma \cos t \tag{3g}$$

The m. m. f. in the direct axis produces a space fundamental flux in this axis proportional to the m. m. f. If there are no short-circuited secondary windings in this axis, the proportionality factor between m. m. f. and line-to-neutral linkages is x_{ad},* or, in general $k\, x_{ad}$.

Then, $$\psi_D = \frac{2}{3}\, k^2\, i\, x_{ad} \cos\gamma \cos t \tag{4g}$$

Likewise, in the quadrature axis

$$\psi_Q = \frac{2}{3}\, k^2\, i\, x_{aq} \sin\gamma \cos t \tag{5g}$$

The component of ψ_D which is in space phase with the armature winding, thus linking the winding, is

$$\begin{aligned}\psi_1 = \psi_D \cos\gamma &= \frac{2}{3}\, k^2\, i\, x_{ad} \cos^2\gamma \cos t \\ &= \frac{2}{3}\, k^2\, i\, x_{ad} \left[\frac{1}{2} + \frac{1}{2}\cos 2\gamma\right] \cos t\end{aligned} \tag{6g}$$

Also, the component of ψ_Q in space phase with the armature winding is

$$\begin{aligned}\psi_2 &= \frac{2}{3}\, k^2\, i\, x_{aq} \sin^2\gamma \cos t \\ &= \frac{2}{3}\, k^2\, i\, x_{aq} \left[\frac{1}{2} - \frac{1}{2}\cos 2\gamma\right] \cos t\end{aligned} \tag{7g}$$

The linkages due to the total flux in space phase with the winding is

$$\psi = \frac{2}{3}\, k^2\, i \left[\frac{x_{ad} + x_{aq}}{2} + \frac{x_{ad} - x_{aq}}{2} \cos 2\gamma\right] \cos t \tag{8g}$$

The three-phase reactances, x_{ad} and x_{aq} may be expressed in terms of the corresponding reactances for the single-phase winding in question. When a current i flows in a single-phase winding, the alternating stationary m. m. f., fundamental in space is $\frac{2}{3}\, k$ times the rotating m. m. f. produced when a three-phase current i flows through a three-phase winding. If these m. m. fs. act on the same reluctance paths, the space fundamental fluxes will have the same relation as the m. m. fs. The flux linkages, produced by the single-phase current i will then be $\frac{2}{3}\, k^2$ times the linkages produced by the same three-phase current i, and

$$x_{AD} = \frac{2}{3}\, k^2\, x_{ad} \tag{9g}$$

and $$x_{AQ} = \frac{2}{3}\, k^2\, x_{aq} \tag{10g}$$

Substituting these relations in (8g)

$$\psi = i \left[\frac{x_{AD} + x_{AQ}}{2} + \frac{x_{AD} - x_{AQ}}{2} \cos 2\gamma\right] \cos t \tag{11g}$$

The induced voltage is $\frac{d\,\psi}{d\,t}$. Thus, remembering that i is taken here as the amplitude of normal frequency current:

$$e = i \left[\frac{x_{AD} + x_{AQ}}{2} + \frac{x_{AD} - x_{AQ}}{2} \cos 2\gamma\right] \sin t \tag{12g}$$

The coefficient of $i \sin t$ is the *per-unit* reactance of the single-phase winding due to the space fundamental flux produced by the space fundamental m. m. f., as a function of position when both the rotor and stator are stationary. Since *per-unit* reactance of a stationary circuit at normal frequency is numerically equal to the per-unit inductance of the circuit, this coefficient gives also the *per-unit* inductance; and when used as an inductance, is independent of the relative motion of the rotor and stator.

As pointed out in a previous paper,[9d] the leakage reactance also varies with position as a second harmonic of the angular displacement of the winding from the direct axis, *i. e.*,

$$x_L = \frac{x_{LD} + x_{LQ}}{2} + \frac{x_{LD} - x_{LQ}}{2} \cos 2\gamma \tag{13g}$$

The total reactance may then be written

$$x = \frac{x_D + x_Q}{2} + \frac{x_D - x_Q}{2} \cos 2\gamma \tag{14g}$$

where, $$x_D = x_{LD} + x_{AD} \tag{15g}$$

and, $$x_Q = x_{LQ} + x_{AQ} \tag{16g}$$

Appendix H

DECREMENT FACTORS

Field Decrement. The theoretical basis for determining this is given in the main body of the paper under "Fundamental Equations." It is shown in Appendix C that the average field current—*i. e.*, the d-c. component, spontaneously increases at short circuit to a value

---

*x_{ad} is the three-phase reactance of armature reaction and applies here since the single-phase space fundamental m. m. fs. are expressed in terms of normal three-phase space fundamental m. m. f.

$$\frac{x_D + \sqrt{x_D' \, x_Q'}}{x_D' + \sqrt{x_D' \, x_Q'}}$$

times the value existing immediately before short circuit. Thus the increase in equivalent magnetic reluctance, as discussed in the above reference, is given by the same ratio, and the corresponding change in inductance is given by the reciprocal. Hence, assuming negligible armature resistance, as may be done without appreciable error in commercial machines, the field decrement factor is

$$\sigma_f = \frac{x_D + \sqrt{x_D' \, x_Q'}}{x_D' + \sqrt{x_D' \, x_Q'}} \, \sigma_0 \qquad \textbf{(1h)}$$

where σ_0 is the open circuit, field decrement factor,

$$\sigma_0 = \frac{\mathcal{R}}{\omega \, \mathcal{L}_0}$$

where $\mathcal{R}$ is the ohmic resistance of the field circuit, and $\mathcal{L}_0$ the *total* inductance, in henrys, of the field circuit, corresponding to the condition of open circuited armature, and ω is $2 \pi f$, where f is normal frequency. These three quantities $\mathcal{R}$, $\mathcal{L}_0$, and ω are not *per-unit* quantities, but σ_0 is.

If appreciable resistance exists in the armature circuit the value of σ_f given by **(1h)** will be decreased to the extent (discussed under "Fundamental Equations") which the equivalent inductance is thereby increased.

It is of interest to note that in the ideal case of a laminated cylindrical rotor where

$$x_Q' = x_Q = x_D$$

equation **(1h)** reduces to $\sigma_f = \sqrt{\dfrac{x_D}{x_D'}} \, \sigma_0 \qquad \textbf{(2h)}$

The decrement factors **(1h)** and **(2h)** give, of course, the average decrement, thus determining the direct component of transient current in the field, the envelope of the even harmonics in the field current, and the envelope of the odd harmonics of current in the armature. It can be shown that these decrement factors are rigorous for the envelopes and average component referred to, for the assumed case of zero armature resistance. Thus the only error involved is the slight difference in wave shape, the peaks being correct. The same thing is true of the armature decrement, derived below, under the assumption of zero field resistance.

Armature Decrement. The armature decrement applies to the treatment flux linkages which happen to be caught and "trapped" in the armature circuit when the latter is short-circuited. It is, of course, altogether a matter of chance of rotor position at the instant of short circuit, whether such linkages shall be zero, maximum value, or some intervening value; that is, there may be or may not be an "armature transient," and when it occurs, it may have any value between the above limits.

The value of trapped linkages is,

$$\psi = k \, e_0 \cos \alpha \qquad \textbf{(3h)}$$

The average current required to sustain these linkages is, by equation **(9b)** in Appendix B,

$$i_0 = \frac{k \, e_0 \cos \alpha}{\sqrt{x_D' \, x_Q'}}$$

Thus, the inductance to be used in the decrement factor is $\psi \div i_0$, or

$$L = \sqrt{x_D' \, x_Q'}$$

and the armature decrement factor is

$$\sigma_a = \frac{r}{\sqrt{x_D' \, x_Q'}} \qquad \textbf{(4h)}$$

where r is the *per-unit* resistance of the short-circuited armature circuit, and x_D' and x_Q' represent the *per-unit static transient* inductance, (and also the corresponding *per-unit* reactance).

The decrement factor σ_a applies to the d-c. component of the total armature current, the envelope of the even harmonic currents in the armature, and the envelope of the odd harmonics in the field current.

Appendix I

REACTANCES—DEFINITIONS AND RELATIONS

x_d = synchronous reactance, three phase, line-to-neutral, direct axis, corresponding to open-circuited rotor windings.

x_{ad} = reactance of armature reaction, thus due to the direct axis space fundamental air-gap flux, produced by three-phase currents. It is one component of x_d.

x_{ld} = armature leakage reactance, three-phase, line-to-neutral, and is the other component of x_d. Thus,

$$x_d = x_{ad} + x_{ld} \qquad \textbf{(1i)}$$

x_d' = transient reactance, three-phase, line-to-neutral, direct axis, corresponding to the total leakage flux between the armature and field windings. It thus corresponds to the total flux linkages of the armature winding occasioned by the sudden application of three-phase currents, when the field winding is closed, such flux linkages being only those which generate voltage of fundamental frequency. It comprises x_{ed}—that is, the leakage flux of the armature—plus additional space fundamental flux which crosses the air-gap, and corresponds in amount to the field leakage flux produced by the induced field current. The latter component is denoted by x_{ad}'. Thus

$$x_d' = x_{ld} + x_{ad}' \qquad \textbf{(2i)}$$

x_D = *total* static reactance, single-phase, corresponding to the condition that, at stand-still, the direct axis coincides with the axis of the single-phase winding, and that the rotor windings are open-circuited. It thus corresponds to the total linkages produced by the armature current under those conditions. It will, of course, have a different value if the single-phase winding is considered to be line-to-line, from that if the winding is taken as line-to-neutral, the per-unit value being about three times as much in the former as in the latter case. It is analogous to the three-phase total reactance x_d.

x_{AD} = static reactance, single-phase, corresponding to the space fundamental flux, direct axis under the above conditions. It is a component of x_D, and is analogous to the three-phase reactance of armature reaction x_{ad}.

x_{LD} = static armature leakage reactance, single phase, direct axis, corresponding to the conditions specified under x_D. It is analogous to x_{ld} and is the other component of x_D, that is, corresponds to all flux linkages not contained in the space fundamental. Thus,

$$x_D = x_{AD} + x_{LD} \qquad (3i)$$

x_D' = static transient reactance, single-phase, direct axis, corresponding to the conditions specified under x_D, except that the rotor windings are short-circuited. It corresponds to the total leakage flux between the armature and rotor windings, and is thus analogous to x_d. It thus comprises x_{LD}, and some additional space fundamental flux crossing the air-gap and corresponding in amount to the field leakage produced by the induced field current. The latter component is denoted by x_{AD}'. Thus,

$$x_D' = x_{LD} + x_{AD}' \qquad (4i)$$

The corresponding symbols for all of the foregoing ten quantities for the quadrature axis are denoted by the subscript q instead of d for three-phase quantities, and by Q instead of D for the static single-phase quantities. Thus small letter subscripts indicate three-phase quantities, and capitals indicate static single-phase quantities.

There are certain additional important relations which exist between the above quantities. These relations will now be derived.

Subtracting equation (4i) from equation (3i),

$$x_D - x_D' = x_{AD} - x_{AD}' \qquad (5i)$$

and, subtracting equation (2i) from (1i),

$$x_d - x_d' = x_{ad} - x_{ad}' \qquad (6i)$$

Now the same space fundamental m. m. f., whether produced by a three-phase or a single-phase current, will produce in the same axis the same space fundamental linkages in a given winding. Thus, since by definition, the product of x_{ad} by the amplitude of a space fundamental m. m. f. produced by a three-phase current direct axis will give line-to-neutral linkages of armature reaction, it follows that the product of x_{ad}' by *any* space fundamental m. m. f. in the same axis will give line-to-neutral linkages. Hence, the line-to-neutral linkages, maximum value, due to a suddenly appearing space fundamental m. m. f. A_D, the latter being produced by a single-phase current, are,

$$\psi_D = A_D\, x_{ad}$$

when the rotor circuits are open-circuited, and are,

$$\psi_D' = A_D\, x_{ad}'$$

when the rotor circuits are closed. The line-to-line linkages would be k times these values, where $k = 1.0$ for line-to-neutral, 1.73 line-to-line. Thus in general

$$\psi_D = k\, A_D\, x_{ad} \qquad (7i)$$

and

$$\psi_D' = k\, A_D\, x_{ad}' \qquad (8i)$$

Subtracting (8i) from (7i),

$$\psi_D - \psi_D' = k\, A_D\,(x_{ad} - x_{ad}') \qquad (9i)$$

But, by definition of x,

$$\psi_D = i_D\, x_D \qquad (10i)$$

and

$$\psi_D' = i_D\, x_D' \qquad (11i)$$

Subtracting these,

$$\psi_D - \psi_D' = i_D\,(x_D - x_D') \qquad (12i)$$

Equating (9i) and (12i), and substituting the general relation from (1c) between the amplitude of the space fundamental m. m. f. produced by single-phase current, and the value of the current,

$$2/3\, k^2\, i_D\,(x_{ad} - x_{ad}') = i_D\,(x_D - x_D')$$

Therefore, substituting (6i),

$$x_d - x_d' = \frac{3}{2\,k^2}\,(x_D - x_D') \qquad (13i)$$

Nomenclature

Per-unit quantities, as outlined under "Discussion of Theory," are listed and defined below. The general scheme of notation involves the use of capital letters for field circuit quantities, small letters for the armature.

For the Armature. All voltages are expressed as a fraction of the peak value of normal armature voltage; currents, as a fraction of the peak value of normal armature current; flux, as a fraction of normal space fundamental flux, the latter corresponding to normal voltage at normal frequency on open circuit; linkages, as a fraction of the linkages corresponding to normal flux; inductance, as a fraction of unit inductance, the latter being defined by the ratio of normal linkages to normal current; and reactances (at fundamental frequency), as a fraction of unit reactance, the latter being defined by the ratio of normal voltage line-to-neutral to normal current.

For the Field. All voltages are expressed as a fraction of unit field voltage, *i. e.*, the direct voltage across the field corresponding to unit field current; currents, as a fraction of unit field current, *i. e.*, the direct current corresponding to normal a-c. terminal voltage on open circuit (no saturation); flux, as a fraction of normal flux, as defined above for the armature circuit; linkages, as a fraction of unit field linkages, the latter being defined as the linkages produced in the field winding by normal flux; inductance, as a fraction of that inductance which is defined as the ratio of unit field linkages to unit field current; reactance, as a fraction of unit field reactance, which is defined as the ratio of unit field voltage to unit field current.

With regard to the per-unit bases for field voltage and field reactance, it should be mentioned that there are other bases which, in certain cases, it might be more convenient to adopt. It is altogether a matter of convenience. For instance, if it should be of greater convenience to have symmetrical equations for the armature and field, than to have, as in the present case, the calculated results of field circuit voltage come out in terms of the known excitation voltage, then unit field voltage might be taken as the voltage which unit field linkages would generate in the field winding at normal frequency. Unit field reactance and resistance would then be changed accordingly.

A = amplitude of space fundamental of armature m. m. f. Specific values of A are:

- A_d = m. m. f. in direct axis, produced by balanced polyphase current
- A_q = corresponding value in quadrature axis
- A_1 = amplitude of m. m. f. produced by current at any instant in a single-phase winding.
- A_D = component of A_1 in direct axis
- A_Q = component of A_1 in quadrature axis

b = geometric ratio defined by the relation following equation (9)

e = instantaneous value of any armature voltage

e_0 = line-to-neutral voltage before short circuit

e_d = nominal voltage due to direct axis excitation

e_q = nominal voltage due to quadrature axis excitation

Δe_d = increment in e_d induced by transient field current

Δe_q = increment in e_q induced by transient field current

E = field voltage in general

E_0 = field voltage preceding short circuit

i = armature current in general

I = field current in general

I_d = field current in direct axis winding

I_q = field current in quadrature axis winding

ΔI_d = increment in I_d induced by transient armature current

ΔI_q = corresponding increment in I_q

I_0 = field current at the instant before short circuit

k = voltage conversion factor for the short-circuited phase. $k = 1$ for line-to-neutral short circuit, and $= \sqrt{3}$ for line-to-line short circuit

k_1 = similar factor for the voltage generated in the open circuited phase

l = inductance, in general, of the armature circuit

l' = transient inductance of the armature, variable with respect to rotor position

L = inductance, in general, of the field circuit

$\mathcal{L}$ = inductance, *in henrys*, of the field circuit, with open circuited armature. That is, the total field self-inductance. Not a *per-unit* quantity

r = per-unit resistance of the armature circuit

$\mathcal{R}$ = resistance, *in ohms*, of field circuit. Not a per-unit quantity

t = time, expressed as a fraction of the time corresponding to one radian at normal frequency. It is thus numerically equal to the elapsed radians

x = armature reactance in general

x_{ad} = three phase, line to neutral reactance of armature reaction, direct axis (See Appendix I)

x_{ld} = three phase line to neutral armature leakage reactance, direct axis

$x_d = x_{ad} + x_{ld}$ = three phase, line to neutral synchronous reactance, direct axis

x_d' = three phase line to neutral transient reactance, direct axis

x_{AD} = static reactance single phase, corresponding to the space fundamental flux, direct axis. (See Appendix I)

x_{LD} = single phase armature leakage reactance, direct axis

$x_D = x_{AD} + x_{LD}$ = total static reactance, single phase direct axis. (See Appendix I)

x_D' = single phase static transient reactance, direct axis

The corresponding reactances in the quadrature axis are denoted by q and Q, instead of d and D.

α = displacement between armature winding axis and axis of field pole at the instant of short circuit. See Fig. 1

β = angle between the axis of the short circuited winding and any open circuited winding considered

γ = angle between armature winding axis and direct axis at any time t. See Fig. 1

ϕ_D = flux in the direct axis due to current in a single phase winding

ϕ_Q = corresponding flux in the quadrature axis

ψ = flux linkages in the armature, in general

ψ_i = flux linkages, produced in the armature by any transient armature current

ψ_m = armature linkages due to current in the field winding

ρ = ratio of open phase armature voltage to peak value of unit armature voltage

σ_0 = decrement factor of the field circuit, corresponding to the *total* self-inductance (*i. e.*, with armature open circuited). It is thus,

$$\sigma_0 = \frac{\mathcal{R}}{\omega\, \mathcal{L}_0}$$

where $\mathcal{R}$ is the total resistance (in ohms), $\mathcal{L}$ is the total inductance (in henrys), of the field circuit; and ω is the angular velocity corresponding to normal frequency—thus $2\,\pi f$.

Although, $\mathcal{R}$, $\mathcal{L}$, and ω are not *per-unit* quantities, σ_0 nevertheless is. The usual simple exponential is of the form

$\epsilon^{-\frac{t'}{t_0'}}$, where t' and t_0' are in seconds. In the present system t represents the elapsed radians—*i. e.*, time is expressed as a fraction of the time of one radian at normal frequency. Thus, referring to the exponent

$$\frac{t'}{t_0'} = \frac{t'}{\mathcal{L}/\mathcal{R}} = \frac{\omega\, t'}{\omega\, \mathcal{L}/\mathcal{R}} = \sigma_0\, t$$

σ_f = decrement factor of the field corresponding to the condition of single-phase short circuit. See equation (2)

σ_a = corresponding decrement factor of the armature circuit. This factor is on a per-unit basis similar to σ_0. It is merely the ratio of per-unit r to per-unit x. If these were each multiplied by unit ohms, and in the resulting ratio of resistance to reactance the factor ω were taken out of the reactance, the decrement factor would be back on the usual basis.

ω = $2\,\pi\, f$, where f = normal frequency

Bibliography

1. Steinmetz, C. P., "Transient Electric Phenomena and Oscillations," 1909, Chapter XIV.

2. Boucherot, P., "Electromagnetic Phenomena Resulting from the Sudden Short-Circuiting of an Alternator," *Trans. International Elec. Congress*, Vol. II, p. 49-93, Turin, 1912.

3. Biermanns, J.,(a) *E. T. Z.*, 1915, p. 579. (b) "Uberstrome in Hochspannungsanlagen," 1926, p. 49 et seq.

4. Diamant, N. S., (a) *Calculation of Sudden Short-Circuit Phenomena* . . . , TRANS. A. I. E. E., Vol. 34, p. 2237, 1915., (b) *Sustained Short Circuit Phenomena*. . ., TRANS. A. I. E. E., Vol. 36, Part 2, p. 1141, 1918.

5. Franklin, R. F., (a) Thesis (Master's Degree), Union College, 1923, *Calculation of Alternator Short-Circuit Currents*. (b) *Short-Circuit Currents of Synchronous Machines*, TRANS. A. I. E. E., Vol. 44, p. 420, 1925.

6. Karapetoff, V., (a) *Initial and Sustained Short Circuits in Synchronous Machines*," TRANS. A. I. E. E., Vol. 44, p. 403, 1925. (b) *Variable Armature Leakage Reactance in Salient-Pole Synchronous Machines*, TRANS. A. I. E. E., Vol. 45, p. 729, 1926.

7. Laffoon, C. M., *Short Circuits of Alternating-Current Generators*, TRANS. A. I. E. E., Vol. 43, p. 356, 1924.

8. Shimidzu and Ito, "Theory of Single-Phase Generators," Memoirs of College of Engineering, Kyoto Imperial University, Vol. 2, No. 7, 1925.

9. (a) Doherty and Shirley, *Reactance of Synchronous Machines and its Application*, TRANS. A. I. E. E., Vol. 37, Part 2, p. 1323, 1918. (b) Doherty, R. E., *Short-Circuit Current of Induction Motors and Generators*, TRANS. A. I. E. E., Vol. 40, p. 547, 1921. (c) Doherty, R. E., *A Simplified Method of Analyzing Short-Circuit Problems*, TRANS. A. I. E. E., Vol. 42, p. 849, 1923. (d) Doherty and Nickle, *Synchronous Machines—I and II*, TRANS. A. I. E. E., Vol. 45, p. 912, 1926. (e) Doherty and Nickle, *Synchronous Machines—III*, Winter Convention 1927.

10. Newbury, F. D., Discussion. TRANS. A. I. E. E., Vol. 37, Part 2, p. 1304, 1918 and Laffoon, C. M., p. 1316.

11. Arnold, E., *Die Wechselstromtechnik*, Vol. 4, p. 37-40, 1913.

12. Bekku, S., "Sudden Short Circuit of Alternator," Researches of the Electrotechnical Laboratory No. 203, June 1927.

13. Lyon W. V., *Transient Conditions in Elec. Machinery*, TRANS. A. I. E. E., 1923, Vol. 42, p. 157.

14. Dreyfus, L., "Freie Magnetische Energie Zwischen Verketteten Mehrphasensystemen," *E. U. M.* 29 S. 891, 1911.

15. Dreyfus, L., "Ausgleichvorgange in der Symmetrischen Mehrphasenmaschine," *E. U. M.* 30 S 25, 121, 139, 1912.

16. Dreyfus, L., "Ausgleichvorgange Beim Plotzlichen Kurzschluss von Synchron Generatoren," *Archiv fur Electrotechnik*, 5 S 103, 1916.

Discussion

C. H. Linder: In *Synchronous Machines—Part IV*, mathematical relations are presented which permit the calculation of the short-circuit currents and voltages at the instant of single-phase short circuit, and at any time thereafter. Experimental data are included in the paper which check the calculated curves

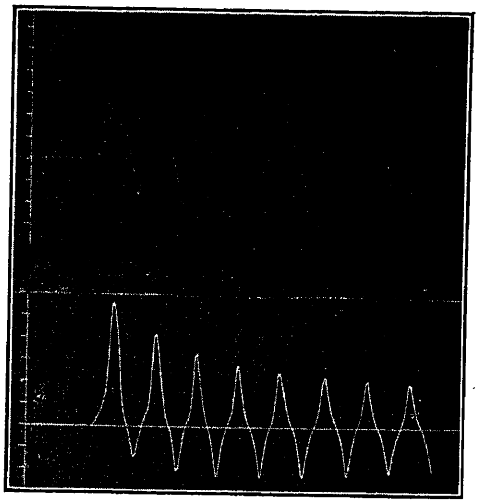


FIG. 1—ARMATURE CURRENT; THE SHORT CIRCUIT OCCURRING AT THE INSTANT OF MAXIMUM LINKAGES WITH THE SHORT-CIRCUITED PHASE

Upper curve—Test
Lower curve—Calculation

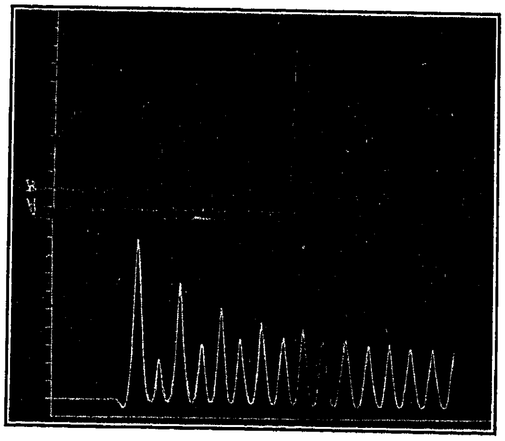


FIG. 2—FIELD CURRENT; THE SHORT CIRCUIT OCCURRING AT THE INSTANT OF MAXIMUM LINKAGES WITH THE SHORT-CIRCUITED PHASE

Upper curve—Test
Lower curve—Calculation

accurately for both round-rotor and salient-pole machines at single-phase short circuit.

Additional evidence is presented in the accompanying Figs. 1 to 5, with particular reference to the correlation of test and calculated curves, for armature current, field current, and open-phase voltage, at single-phase short circuit on a salient-pole synchronous machine. The machine used to obtain the test data was rated: *A T I* 4-pole, 15-kv-a., 1800 rev. per min., 220 volts. All short circuits were made from line to line at a subnormal voltage, thereby eliminating any error in the test curves due to saturation. Open-phase voltages were measured 90 deg. from the short-circuited phase; for example, in case the short circuit was between line 2 and line 1, the open-phase voltage was measured from 3 to zero.

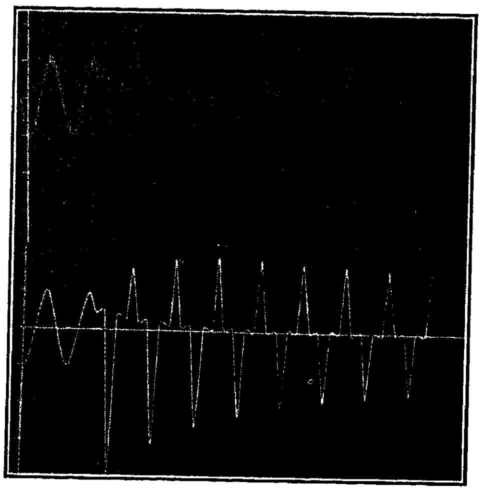


FIG. 3—OPEN-PHASE VOLTAGE; THE SHORT CIRCUIT OCCURRING AT THE INSTANT OF MAXIMUM LINKAGES WITH THE SHORT-CIRCUITED PHASE

Upper curve—Test
Lower curve—Calculation

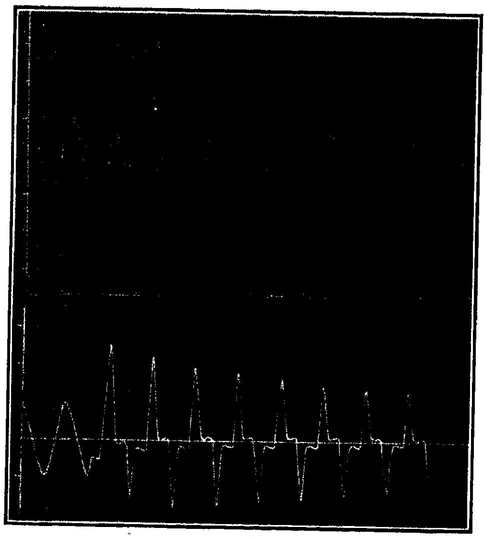


FIG. 4—OPEN-CIRCUIT VOLTAGE; THE SHORT CIRCUIT OCCURRING AT THE INSTANT OF MAXIMUM FLUX LINKAGES WITH THE SHORT-CIRCUITED PHASE

Upper curve—Without the amortisseur winding
Lower curve—With the amortisseur winding

A comparison of the curves shows that the open-circuit voltage is considerably less with than without the amortisseur winding; so that the amortisseur is beneficial in reducing open-phase voltages at single-phase short-circuit.

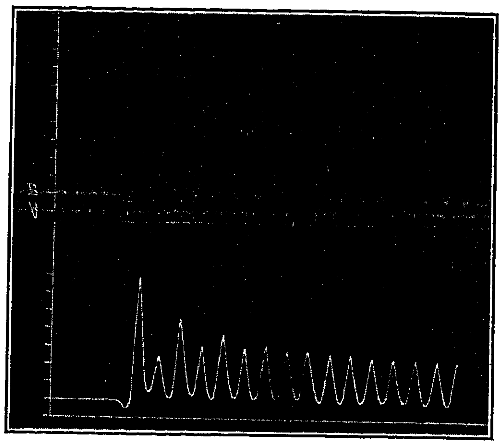


FIG. 5—FIELD CURRENT; THE SHORT CIRCUIT OCCURRING AT THE INSTANT OF MAXIMUM LINKAGES IN THE SHORT-CIRCUITED PHASE

Upper curve—Without the amortisseur winding
Lower curve—With the amortisseur winding

The variation in field current is less with than without the amortisseur winding on the rotor. This is because the amortisseur winding acts as a cushion for the field, having induced in it a voltage which is otherwise induced in the field.

B. A. Behrend: Mr. Doherty has succeeded in elucidating one of the most important and intricate problems in a-c. circuits by a method of approach which makes a great appeal to the engineering mind. Simple and natural as it may seem it was an ingenious suggestion to treat the initial value of the short-circuit current of a synchronous or induction machine by assuming a circuit without resistance and introducing this resistance later to account for the decay of the short-circuit current. The general law which Mr. Doherty has developed and which he calls the "law of linkages" and for which he has given a general proof, is henceforth going to be a great help in solving engineering problems. In this present paper he has shown further how, by introducing a rather large number of factors, he is able to account in every detail, not only for the initial current, but also for the entire form of the decaying current wave. The striking agreement between his calculations and the records of the oscillograph are most interesting.

In actual practical calculations it may be advisable to sacrifice in completeness of calculation in order to gain in simplicity. The total errors involved may be no greater by pursuing such a course than by attempting to take into account every phase of the problem. However, much depends upon the taste of the individual designer and a method suited to one may not suit another.

Robert Treat: These papers on the reactance and behavior of synchronous machinery indicate that we are coming closer to a complete understanding of what goes inside of a synchronous machine. They do not indicate, to my mind, that we shall forever be burdened with the complication which a cursory examination of them would indicate. I believe that further study will show that for certain classes of calculations our solutions can be reduced to practically the same simplicity that they had years ago, but with this important difference; where before omissions were made because of our ignorance, in future they will be made because of our understanding that they have an insignificant effect on the result.

The operating engineer is interested in the internal processes of a synchronous machine from several standpoints. Among others, I might name three particularly important ones: first, from the standpoint of the duty which they are going to impose on oil circuit breakers; second, from the effect they are going to have upon the relaying system; and third, he is recently becoming interested in the effects which the synchronous machine is going to have upon the stability of his system; whether they are going to stay in synchronism with all their fellows on his system at all times or whether they are going to fall out of step sometimes, and if so, when, and under what conditions.

In respect to the first two, particularly, this is a function of the short-circuit current which will flow. The short-circuit current in the system is caused by a voltage and limited by an impedance. The voltage is generated by the synchronous machines. The impedance is furnished partly by these machines and partly by the external circuit.

In regard to the external circuit, we have for some time had a fair comprehension of the various characteristics which are important therein, of static apparatus in general, transmission lines, transformers, and so forth. But what went on inside has been a very considerable mystery. Not so very long ago oil circuit breakers were applied so far as interrupting capacity was concerned in terms of kilowatts installed in the station where the circuit breakers were put. A little later we distinguished between kv-a. supplied from salient-pole generators and from smooth-rotor generators. From that time we have progressed to a point where we now try to calculate as near as we know how the exact amount of current which the breakers may have to interrupt. It is a matter of considerable importance to an operator to know the amount of current which he is going to encounter on a short circuit in or near his power house. I mention only one effect. The electromagnetic stresses which he encounters in the bus supports and so forth vary not directly, but as the square of that current, hence, if his calculations of current are 50 per cent off, the results are over 100 per cent off.

The remarkable agreement reported by Messrs. Doherty and Nickle between their calculated and test results indicates that this mystery of what is going on inside the synchronous machine is disappearing. The results reported in these papers will be simplified, I am sure, so that they can be applied to a determination of what is going to happen in a system during transient disturbances and will, without any question, make the life of the operating engineers a little easier than it is now. If he can know how much short circuit is going to be encountered under all conditions with certainty, not only can his oil circuit breakers be determined with greater precision but, probably of more importance, he will be able to predict what his relay system is going to do under the various conditions to which it will be subjected.

W. V. Lyon: I should like first to congratulate the authors on the analysis they have made. However, I do not regard this analysis as the final word on this problem but only as a stepping stone to a more complete solution. For they, in order to make the solution come within our present mathematical ability, have neglected the losses in the circuit. That is, they have neglected them in determining the magnitudes of the currents and have later introduced them in order to obtain approximate values of the damping coefficients. This treatment seems to be quite satisfactory in the relatively simple problems that they have solved. Now it is a sure prediction that the mathematical difficulty which stands in our way to a more exact solution of the problem today will be removed tomorrow. One of the most promising ways of attacking transient problems is by means of the Heaviside operational method. It has proved of enormous value in solving problems dealing with static circuits and there seems to be no difficulty in extending the method to rotating machines. While speaking of mathematical difficulties it is interesting to point out that they are to a great extent removed if the d-c. exciting circuit is a polyphase one, for example, if the winding is arranged like a 3-phase symmetrical winding and has direct current supplied between one terminal and the other two terminals, which are themselves short-circuited. This system of excitation is assumed by Bekku in his recent analysis to which reference is made, see Fig. 6 herewith.

I do not wish to be understood as detracting in the least degree from the excellence of this paper but I must nevertheless emphasize that it should be regarded as a stepping stone to a more complete solution of the problem.

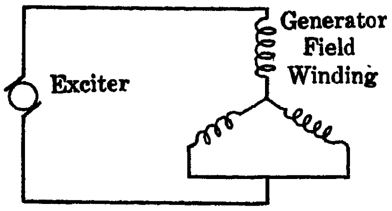
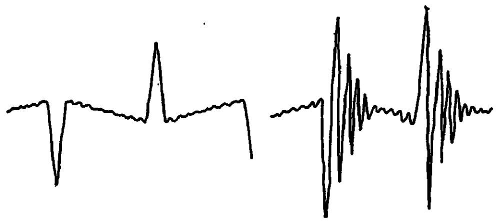


FIG. 6—POLYPHASE EXCITATION

There is one interesting point to which I should like to call attention. You will observe that in the oscillograms shown in Fig. 12 and Fig. 17 the first peak of current is highest, that the next two peaks are lower but are equal, that the next two are still lower but are also equal, whereas in the computed curve of current each peak is lower than the last. It would be interesting to know if the authors have any explanation of this.

I should like to point out that the nearest approach to a uniform air gap is in a properly designed induction motor. A rotor such as used in a turbo alternator does not have a uniform gap. Due to the effect of the slots which contain the field winding the reluctance of the air gap may be somewhat more in the quadrature axis than in the direct axis.

Although it is not particularly important, I believe it might avoid some confusion if the damping coefficients were not represented by the Greek "sigma," inasmuch as this letter is used to represent the leakage coefficient of two coupled circuits,[1] and is thus an important constant in both the steady-state and transient theory of such circuits. In fact, the so-called "transient reactance" which occurs in the solution of a polyphase short circuit of an alternator is the product of this leakage coefficient and the synchronous reactance, *i. e.*, in symbols, $X_t = \sigma X_s$ I suggest that, following a very common practise, the Greek "alpha" be used as the damping coefficient.[2]



7A 7B

FIG. 7—EFFECT OF CONDENSER LOAD

I wish to comment on the high potential that is observed across the open phase under the condition of single-phase short circuit. Two years ago we obtained oscillographic records during a single-phase line-to-line short circuit when the generator was loaded with three condensers which were used to simulate a three-phase cable. We found that with the condensers used both the steady and transient potentials across the open phase were about 50 per cent greater than when the condensers were disconnected. The steady-state oscillogram of this potential without the condensers and with them are shown herewith in Figs. 7A and 7B. Another interesting and more common case is that in which the generator is connected to the line through transformers. Here the high potentials that would otherwise exist would be somewhat reduced due to the increased exciting current that a transformer takes when the applied potential is increased in this manner.

---

[1] B. A. Behrend. Induction Motors.

[2] See Doherty and Shirley, *Reactance of Synchronous Machines and Its Applications*, TRANS. A. I. E. E., 1918, Vol. XXXVII, p. 1248.

I feel very strongly about the use of the term "transient reactance." To speak of the "transient reactance" of a generator as has been done in the past is to my mind very unfortunate. Fundamentally reactance depends upon frequency, $x = 2\,\pi\,f\,L$ or in a more generalized form $x = p\,L$, where p is the so-called generalized angular velocity. If the frequency is omitted the term "reactance" in general is useless. Now in this problem of alternator transients there are in the simplest case armature currents of several frequencies, so that a logical mind is forced to ask what is this "transient reactance"?

In analyzing some new phenomenon we often draw upon our past experience and use modified forms of well known terms with which to describe it, rather than invent new terms which will be distinctive. For example, the impedance of an a-c. circuit was called the apparent resistance and similarly, the product of the potential and current, VI, was called the apparent power. Now to anyone who is familiar with the simple theory of power in the a-c. circuit, it is apparent that the product VI is *not* the power and this appellation of "apparent power" only befogs the understanding. When the first attempts were made to calculate the transient current in an alternator at short circuit the underlying theory was very imperfectly understood and, following precedent, it was perhaps natural that a crude method should be devised which contemplated dividing an electromotive force by a reactance, just as would be done in the problem of the steady-state short circuit.

If at this time we had been fortunate in having an exact analysis based upon reasonable assumptions, it is quite probable that the term "transient reactance" would not have come into general use, but that the transient currents would be computed in terms of resistance, self and mutual inductances, and the so-called leakage coefficients. We are now beginning to realize that the theory of these transient currents cannot be expressed by such a simple formula as was at first proposed. It is interesting to note that Messrs. Durgin and Whitehead who suggested this term, "transient reactance," in 1912 apparently appreciated the complexity of the problem more than many who have since attempted to apply their method. This is what they say:

> "This paper is confined to demonstrating the existence of a characteristic of alternators provisionally named the *transient impedance* and to investigate the influence of the characteristic . . . . . it seems evident that the current is limited by some reactance beside those contemplated in the original theory[3]". "The existence of a transient reactance, or rather the existence of complex reactions which are conveniently grouped and replaced by a single fictitious quality called the transient reactance is shown conclusively by the test results and has, indeed, been more or less clearly appreciated for some time.[4]."

At first the "transient reactance" was taken to be the so-called leakage reactance of the armature, but this conception has gradually changed until it is, as the present authors show, taken to be what might be called the "equivalent reactance" of the armature and field windings. That is, it is the reactance measured on the armature side with the field winding short-circuited in the same way in which the equivalent reactance of a transformer or of an induction motor is measured. To quote from the paper by Messrs. Doherty and Shirley on *Reactance of Synchronous Machines and Its Applications.*[5] the value of the transient current at symmetrical polyphase short circuit is given by:

$$\sqrt{2}\left(\frac{E}{x_t} - \frac{E}{x_s}\right)\epsilon^{-\alpha_f t} + \sqrt{2}\,\frac{E}{x_t}\,\epsilon^{-\alpha_a t}.$$

This is still founded on the idea that the transient current is determined by dividing a generated e. m. f. by a "transient reactance." The symbol E is rather indefinitely defined as "the voltage per leg," but it seems to be apparent that it represents the phase e. m. f. on open circuit. Let us apply this method of analysis to the following problem. An alternator operating in parallel with others loses its field excitation due to a break in the field circuit of the exciter. The field winding of the alternator is still closed through the armature of the exciter. Directly after the armature current of the alternator reaches its steady value, which may not be much in excess of its full-load value, a polyphase short circuit occurs at the terminals of the alternator. The problem is to determine the transient current in the alternator. We have here a case in which there is no e. m. f. due to field excitation and furthermore there is no steady short-circuit current. If the theory of transients is no more perfectly understood than is expressed by the above equation it would be said that the transient current is zero, whereas, as a matter of fact, it may be nearly as great as if the polyphase short circuit had occurred before the alternator lost its field excitation.[6] Obviously if this is true anything that fosters the idea that the transient current can be computed by dividing a generated e. m. f. by a "transient reactance" in a manner similar to that in which the steady current is determined, impedes our progress toward a true understanding of the problem. To the writer's mind the very term "transient reactance" does this by suggestion. Now while it is true that the problem of transient conditions in machinery is a complicated one at best it is also true that approximate solutions can be obtained which are at once relatively simple and accurate. In some of these approximate solutions it so happens that one of the important constants of the machine is what is commonly called the "equivalent reactance." This equivalent reactance is measured by applying potential to one of the windings and short circuiting the other, as is done, for example, in the case of a transformer or of an induction motor. It is now suggested that this same test be used in determining the so-called "transient reactance" of an alternator. It seems but logical that, if the same test procedure is followed in the case of a transformer, an induction motor, and an alternator, and the same use is made of the results of this test in solving steady and transient problems concerning the transformer and the induction motor and in solving transient problems concerning the alternator, this commonly determined constant of the machines should have a common name. If this constant has a certain designation in one case it should have the same designation in all cases. When discussing the transformer and induction motor the term "equivalent reactance" is used. This has come to have a definite and well understood significance. It certainly does not tend to clarify the understanding of the transient problem to call a similarly defined and measured constant of a synchronous machine by another name especially if this name itself suggests an erroneous conception of the problem. If the term, "equivalent reactance" is believed unsuitable for this commonly determined constant we should choose some other appropriate name, but let us at least be consistent in our nomenclature.

Furthermore I predict that as the operational method is extended to the problem of machine transients the terms that will be used will be resistance and self and mutual inductance. At present I can only hope that use of the term "transient reactance" will be as fleeting as its name implies.

---

[3] "Transient Electric Phenomena and Oscillations," Steinmetz.

[4] "Transient Reactions in Alternators," by W. A. Durgin and R. H. Whitehead.

[5] Trans. A. I. E. E., 1918, Vol. XXXVII, p. 1209.

[6] The ratio of the transient currents in these two cases is the ratio of E to V, *i. e.*, the ratio of the generated volts on open circuit in the first case to the terminal volts at the time that the short circuit occurs in the second case with no field excitation. This readily follows from the analysis given in the writer's paper, *Transient Conditions in Electric Machinery*, Trans. A. I. E. E., 1923, Vol. XLII, p. 157.

W. F. Dawson: It is out of the question to go through all of the detailed work indicated by these formulas as a preliminary to each design made, but when a group or line of machines is developed they naturally follow uniform proportions until they look more or less like brothers and behave as such. Of course no line is complete until there is reasonable assurance that detailed calculations of the "reactance" will indicate values approximately as required. After these mathematical investigations, the thought naturally arises, Why do we do it? What application shall we make of the results? It has already been stated it is to give the operators a chance to select their oil switches and to protect their systems. This apparatus is selected from the knowledge of the reactance and on the assumption that the oscillograph waves of short circuit current will be sinusoidal. This assumption is approximately true in the case of turbo alternators having solid fields, but with laminated fields on short circuit, the wave shapes are often contracted considerably at the base. This means that if the oil switches and other apparatus are selected on the basis of thermal capacity they will be larger than necessary.

What I consider as another error in the interpretation of oscillographs is the projection of the crest envelope to the instant of short circuit and assuming the maximum current to be the intersection point instead of the top of the first wave which occurs from ¼ to ½ cycle later and which is usually 10 to 15 per cent less than the intersection point. It is not the intersection point but the altitude of the maximum wave crest which determines the mechanical stresses.

It seems reasonable to expect the Standardizing Committee ultimately to take cognizance of these important details and that the selection of oil switches will depend upon the actual r. m. s. of the wave shape, so far as thermal capacity is concerned, and of the maximum altitude of the first wave so far as mechanical strength is concerned.

H. M. Hobart: To get curves to coincide as nicely as in Fig. 16 between the estimated and the observations by the oscillograms seems to me marvellous. Twenty years ago I never dreamed that any such command of the subject would have been possible. Any price is justifiable to have such a command of predetermination of these characteristics of dynamo-electrical machines.

The comparison given in Fig. 16 between the predetermined curve of the transient field current, and the curve actually obtained with the oscillographs, is extremely interesting and remarkable as proof of the great progress which has been made in obtaining a quantitative knowledge of these complicated occurrences.

H. H. Spencer: Messrs. Doherty and Nickle treat at considerable length decrement factors but their eq. (9) does not take any attenuation into account. I should like to ask if they will explain how these entered into the calculations of curve 11 or subsequent curves which show a very sharp decrement of successive peaks.

R. E. Doherty: If I have interpreted the discussion correctly, Mr. Behrend's impression is that the results, while apparently of an accurate nature, are nevertheless too complicated to be of much use. Now the *results* are really not complicated, even if some of the equations leading up to them are. For instance, the relations shown in Fig. 9 and in eqs. 9 to 15 inclusive do not appear so.

But suppose they are so considered, what is the justification? It certainly is not, as Mr. Behrend suggests, to exact the last per cent of accuracy in those problems which already may be solved within practical engineering accuracy. None of us wishes to increase the engineer's already heavy burden of numerical calculation. On the contrary, my own experience as a designer places me in sympathetic agreement with Mr. Behrend's appeal for simplicity in such matters, and I have spent many hours in affecting simplified calculations. But mere accuracy is not the objective here. Progress makes it necessary o answer new questions and calculate characteristics which have not been extensively dealt with. A sound theory, as indicated by the close agreement of test and calculation, makes it possible to answer such questions and to calculate such characteristics with some degree of confidence. And that is the justification for any relative complication which may appear in this work.

In stating that "damping" is of comparatively little importance, Mr. Behrend is probably right, in so far as he is considering only those results, such as mechanical force, which may depend on the first impulse. But in other connections, such as circuit-breaker ratings and synchronizing power following short circuits, damping is of very great importance.

Mr. Behrend mentioned the oscillograph. The function of this instrument is to record not merely the general shape, but also the magnitude. The short-circuit curves are accurately indicated by the oscillograph, and in the figures which are given in the paper the magnitude checks as well as the form. For much higher frequency—say 10,000 cycles per second—the ordinary oscillograph records would not be reliable, but they are for such phenomena as are shown here.

With reference to Mr. Dawson's remarks: I understand that his statement that "Everything is being based on the assumption that the wave shape is sinusoidal," refers to the usual interpretation of oscillograph records—such, for instance, as dividing the peak value of an off-set wave by $2\sqrt{2}$ to obtain the r. m. s. of the a-c. component. He is right that such a procedure is incorrect unless the wave is sinusoidal, and usually it is not. Relations between the peak current and other values are given in Appendix B.

However, I could hardly agree with his statement that the "theoretical value" at zero time "is no good." It may not be, for selecting oil switches, but there are other uses and reasons for knowing it. This matter is one of the major points of the companion paper by Park and Robertson.

Mr. Dawson's comments and predictions at the close of his discussion would have been in order 15 years ago; but I am not aware that anyone nowadays selects circuit breakers on the basis of the *initial* current, corresponding to what he terms the "theoretical reactance," *i. e.*, transient reactance. Many years ago standard decrement curves were adopted as a first step toward a rational method of selecting circuit breakers. We must keep the transient reactance, but we must use it properly in connection with decrement factors, etc., to obtain the desired result.

In reply to Mr. Spencer's question, the decrement factors are derived in Appendix H, and applied in Appendices B and C, under *Transient Currents*. Also the simplified forms of equations are given in Appendix A.

Unlike Mr. Behrend, Professor Lyon believes there are yet more details which should be included in the theory. In determining the magnitude of the initial currents, we have neglected resistance, and have made an approximation in taking it into account in the decrement factors. While the calculated results appear satisfactory, there is no question about the desirability of including all factors rigorously. As stated in the paper, no one has yet done it in the general case, but we hope and trust that Professor Lyon will be able to do this.

The slight variation of the test curves from calculated curves in Figs. 12 and 17 is due, as explained in the title of Fig. 17, to the fact that the magnetic linkages were not quite zero, as assumed when the short circuit occurred.

Professor Lyon feels strongly that the term "transient reactance" should be abandoned. Perhaps a better name could have been found when this term was first proposed, or even later in 1918 when I endorsed it after consulation with a number of interested engineers, including Dr. Steinmetz; but a more appropriate name did not appear. Nor has any since that time, in my opinion. Certainly "equivalent" reactance is not better. There are already too many "equivalent" things.

The term "transient reactance" is not illogical, inasmuch as it indicates the nature of the phenomena to which it is applicable. In this respect, it seems quite as logical as the term "synchronous reactance." Certainly, no one would interpret the latter to indicate that the reactance is, itself, "synchronous," but merely that it is the reactance which applies to certain conditions of synchronous operation, *i. e.*, steady-state conditions. Until a more significant and appropriate term than "transient reactance" is available, it would seem convenient and desirable to continue its usage.

With regard to the specific problem given by Professor Lyon, the solution in terms of open-circuit voltage, transient reactances, etc., is quite simple. Let

i_0 = armature current while the machine is connected to the bus.

x_d' = transient reactance line to neutral.

x_d = synchronous reactance line to neutral.

If the machine is removed from the bus and immediately short-circuited, the short-circuit current will be the same as if the short circuit occurred while the machine was still connected to the bus. When the machine is disconnected from the bus, the induced field current or nominal voltage is, as shown in *Synchronous Machines III*,

$$e_d = i_0 (x_d - x_d') \qquad (1)$$

Since on open circuit, the nominal voltage is numerically equal to the terminal voltage, assuming no saturation,

$$E = i_0 (x_d - x_d') \qquad (2)$$

Then when short circuit occurs

$$i = \frac{E}{x_d'} = i_0 \frac{(x_d - x_d')}{x_d'} \qquad (3)$$

When the machine is connected to a bus of voltage, E_1, the current, i_0, is

$$i_0 = \frac{E_1}{x_d} \qquad (4)$$

Then

$$i = \frac{E_1}{x_d'} - \frac{E_1}{x_d} \qquad (5)$$

As pointed out in the discussion, this current may be nearly as large as if the machine were short-circuited at no load and at a voltage, E_1. The difference is merely the current which existed before short circuit.

The Calculation of the Armature Reactance of Synchronous Machines

BY P. L. ALGER*

Member, A. I. E. E.

Synopsis.—This paper presents new and simplified formulas for the armature leakage reactance of synchronous machines, and compares the results obtained with tests on 100 machines of varied types. The new formulas are characterized by novel expressions for the end leakage and for the "air-gap" or "differential leakage" reactances. Formulas for zero phase-sequence reactances are also given. The definitions of armature leakage and armature reaction reactances used are those proposed by Doherty and Nickle in 1926, and as a result the leakage reactance has a much smaller value than has heretofore been assigned to it, thus requiring a rather fundamental revision of the ordinary conceptions of the flux densities existing in the various parts of a loaded synchronous machine.

I. INTRODUCTION

THE growing interest of operating engineers in machine reactances, (as system stability and short-circuit phenomena become continually more important), together with the recent advances in the theory of synchronous machines, make renewed consideration of the subject of reactance calculation opportune at the present time. Papers on this subject have appeared in the JOURNAL'S pages at intervals over a period of more than 20 years, each paper marking a further refinement of methods and an advance in accuracy. At first, it was customary merely to estimate the numbers of leakage lines per ampere inch of embedded and free conductor, and multiply them by the respective lengths to obtain the reactance. Later, the reactance was segregated into several distinct elements that were separately calculated. These calculations due to their greater importance and in this case, the possibility of more accurate tests have reached greater precision in connection with induction machines than with synchronous machines. Adams[11,13,14] first developed formulas for induction motor reactances, and later Fechheimer[9] and Doherty and Shirley[7] developed formulas for synchronous machines. In 1926 Doherty and Nickle[3] presented some fundamental extensions of the theory of synchronous machines, and proposed new, more precise, definitions of armature leakage reactance and of armature reaction reactance. The present paper adopts these new definitions and develops simple formulas for the accurate calculation of the two quantities, for salient pole machines.

The reactance of armature reaction is here defined as the reactance due to the fundamental sine wave of air-gap flux produced by the armature current acting alone. It is calculated by means of the curves derived by Wieseman[2] from flux plots. There are two distinct values for it, the direct axis value occurring when the axis of armature m. m. f. coincides with the pole axis; and the quadrature axis value, occurring when the axis of armature m. m. f., is midway of the interpolar space.

The armature leakage reactance is defined as the difference between the total, or synchronous, armature reactance and the above defined reactance of armature reaction. The distinction between the leakage and the armature reaction components of the total reactance is quite an arbitrary one, since no winding can have a definite value of leakage reactance except with respect to another winding. Some distinction of this character is very desirable, however, for convenience in calculating the transient reactance, and for other purposes. The armature leakage reactance as above defined, is as nearly equal as may be, to that part of the total armature reactance which remains the same under transient as under steady conditions of operation.

Following Adams' treatment of the induction motor, the armature leakage reactance of a synchronous machine will be divided into four parts; the slot, end, zigzag, and belt leakages. Formulas for each of these will be derived in turn. As the slot leakage has been quite satisfactorily treated by Adams[11] already, the discussion of it here will be abbreviated as much as possible, consistent with the derivation of a little more exact formula, and its extension to apply to zero phase-sequence currents. None of the other three elements of the leakage has been adequately treated by previous writers, so they will be given more detailed consideration.

The four divisions of the reactance found useful for induction machines are followed for the two reasons that they fit in very well with the new conceptions of leakage reactance, and that their adoption will tend to unify the theory of the two types of machine. In particular, the total "air-gap leakage," due to harmonics of the air-gap flux, is segregated into the two parts called zigzag and belt leakage, respectively, in order to permit the effects of the independent variables, number of slots, and number of phase belts to be separately taken into account.

All previously published formulas for zigzag leakage have involved the consideration of overlapping primary and secondary teeth, and so have inadequately taken into account the effects of flux fringing in the air-gap,

---

*Assistant Engineer, A-C. Engineering Department, General Electric Company, Schenectady, N. Y.

For numbered references see bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

and have been inapplicable to synchronous machines. Also, older formulas for end leakage have been frankly empirical in nearly all cases. All new formula for these elements will be derived by considering the leakage flux to consist of revolving magnetic fields, divided into a fundamental and a series of harmonics. In each case, the coefficient of the resulting formula will be derived from theoretical considerations alone, and no empirical multipliers will be used to bring the results into agreement with practise, although simplifying assumptions will be freely made as the occasion demands.

Before proceeding to the derivation of the formulas, it is desirable to get clearly in mind the distinctions between the four elements of the leakage that have been specified. The end leakage reactance is due to the flux linking the end windings alone. The slot leakage comprises all of the flux crossing the slots due to the armature current, but does not include flux passing from tooth to tooth in the air-gap space. Evidently there is a slight error here, because the flux lines near the mouth of the slot do not pass straight across, but bulge outwards into the gap. The zigzag leakage comprises all of the space harmonics of the air-gap flux, due to the armature current in a one slot per pole per phase winding which induces fundamental frequency voltages in the armature. This includes the flux which crosses from tooth to tooth in the air-gap and interpolar spaces without actually reaching the field surface, and so takes up the leakage flux at the point at which the slot leakage left it. The belt leakage then comprises all the remaining fundamental frequency voltage producing space harmonics of the air-gap flux due to armature current. This last reactance is the additional reactance that an actual winding has above that which it would have if there were as many phases as slots per pole. Thus, the zigzag leakage reactance is due to the deviation of the armature m. m. f. wave from a sinusoid, caused by the limited number of slots, and the belt leakage reactance is due to the further deviation caused by the limited number of phases.

The belt and zigzag reactances together have been appropriately called[5] the "differential leakage," since they are due to the deviations of the armature m. m. f. from the ideal sine wave. They may also be called "air-gap leakage," since they represent the difference between the total and the armature reaction components of the fundamental frequency voltage producing air-gap flux. As the differential leakage reactance varies with the relative positions of the axes of the armature m. m. f. and the poles, the armature leakage reactance varies likewise, and has a slightly higher value in the quadrature than in the direct axis.[3]

Formerly, the air-gap flux due to the armature current was divided into two parts, that due to the armature reaction, and that due to "tooth tip leakage."[7] The armature reaction part was intended to represent the air-gap flux produced by the armature that links the field, but the determination of this flux was erroneously based on the linkages of the armature by the air-gap flux due to the field.[7] The remaining part of the total air-gap flux produced by the armature was then called tooth tip leakage. As the new definition of armature reaction includes most of this latter flux, we have abandoned the name "tooth tip" and called the small remaining part, the synchronous frequency voltage-producing harmonic fluxes, "differential leakage," as just described.

Bearing in mind these conceptions of the leakage reactances which we are going to calculate, we can now proceed to the derivation of the formulas. The four components of the reactance will be taken up in the order of their difficulty, and subsequently the results of their application to actual machines will be compared with tests.

II. SLOT LEAKAGE REACTANCE

Only two-layer barrel-type armature windings in open slots will be considered in this paper, since the inferior electrical and manufacturing characteristics of single layer windings have reduced them to a place of only historical importance in polyphase a-c. machines of American manufacture.

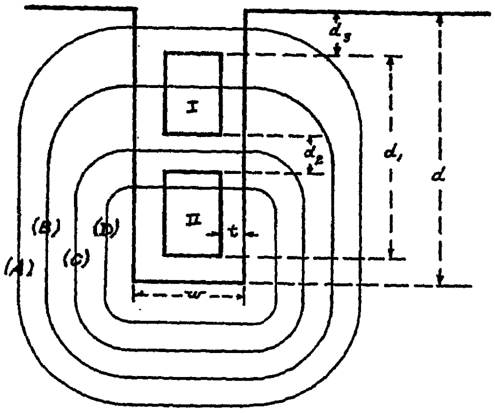


FIG. 1—SLOT LEAKAGE FLUX PATHS

Consider the slot shown in Fig. 1. All the flux crossing the slot returns below its closed end through a path of negligible reluctance, and none of it links any of the other slots. The leakage reactance in ohms per phase, due to this flux is then equal to the product of the following factors:

1. The permeance of the path across unit length of slot.
2. The embedded length of slot, or L
3. The number of slots in series per phase, or S/q
4. The square of the number of series connected conductors in each slot, or $\left(\frac{q\,Z}{S}\right)^2$
5. The rate of change of the flux, or $2\,\pi\,f$
6. A constant to reduce linkages per second to ohms, or 10^{-8}

The permeance of the path for flux A linking all the conductors in the slot of Fig. 1 is:

$$\frac{4\,\pi}{10}\left(\frac{d_3}{w}\right) \text{ lines per ampere per cm. length of slot}$$

And if the winding is full pitch, so that the currents in coil sides I and II are identical in magnitude and phase, the linkages per ampere produced by the flux in paths B and D are equal to the expression:

$$\frac{4\,\pi}{10}\left(\frac{d_1-d_2}{3\,w}\right),$$

since with uniform current distribution over the height d_1, the flux density distribution is linear and the linkage distribution is parabolic, while the average height of a parabola is one-third its maximum height. As all the flux through path C links one-half the total current, the linkages due to it are one-quarter as much as they would be if this flux linked all the current. The linkages due to C are, therefore:

$$\frac{4\,\pi}{10}\left(\frac{d_2}{4\,w}\right),$$

and so the total slot reactance of a balanced, full pitch, polyphase winding is equal to:

$$X_{\text{slot}} = \frac{8\,\pi^2}{10^9}\,(f\,L)\,(S/q)\left(\frac{q\,Z}{S}\right)^2\left[\frac{12\,d_3+4\,d_1-d_2}{12\,w}\right]$$

$$= \frac{0.79\,f\,L\,q\,Z^2}{10^7\,S}\left[\frac{12\,d_3+4\,d_1-d_2}{12\,w}\right] \text{ ohms per phase} \qquad (1)$$

For dimensions measured in inches, the constant 0.79 becomes 2.006. The expression outside the bracket is a quantity containing factors common to many of the elements of reactance, and so it is convenient to give it a special symbol, M, in future equations. The bracketed expression is the "slot constant."

For a fractional pitch winding, the currents in the two coil sides in some slots are not in phase, and so, a more detailed analysis must be made. If the current in coil side II lags that in I by θ^0, then in the slot carrying the return conductor of I, in the bottom of another slot, there will be (under balanced conditions) a top conductor carrying current θ^0 ahead of the current in I. As the mutual inductance of I, with respect to II, is equal to that of II with respect to I, the out-of-phase component of voltage induced in each coil side by the other will be canceled by the corresponding out-of-phase voltage in the return coil side, leaving only the in-phase components of voltage to be considered.

The total self-inductance of I is proportional to:

$$\frac{1}{4}\left[d_3+\frac{d_1-d_2}{6}\right]$$

and that of II is proportional to:

$$\frac{1}{4}\left[d_3+\frac{d_1-d_2}{2}+d_2+\frac{d_1-d_2}{6}\right]$$

$$= \frac{1}{4}\left[d_3+\frac{2\,d_1}{3}+\frac{d_2}{3}\right]$$

while their mutual inductance is proportional to:

$$\frac{1}{4}\left[d_3+\frac{(d_1-d_2)}{4}\right]$$

Hence, the total linkages due to the two currents θ^0 out of phase acting together are proportional to:

$$\frac{1}{4}\left[d_3+\frac{d_1-d_2}{6}\right]+\frac{1}{4}\left[d_3+\frac{2\,d_1}{3}+\frac{d_2}{3}\right] + \frac{(4\,d_3+d_1-d_2)\cos\theta}{8}$$

Or, the slot constant for this case is:

$$\frac{d_3}{2\,w}(1+\cos\theta)+\frac{d_1}{24\,w}(5+3\cos\theta) - \frac{d_2}{24\,w}(3\cos\theta-1).$$

When the winding pitch is 100 per cent, all the slots have $\theta = 0$ deg.; when the pitch is $\frac{q-1}{q}$, all the slots have $\theta = \frac{180 \text{ deg.}}{q}$, and when the pitch is $\frac{q-2}{q}$, they all have $\theta = \frac{360 \text{ deg.}}{q}$. For intermediate pitches, the slot constant varies linearly between these values. Thus, by plotting the function K_S against per cent pitch, where K_S is a broken line function having values equal to

$$K_S = \frac{1}{2}\left(1+\cos\frac{m\,\pi}{q}\right) \text{ at } \frac{q-m}{q} \text{ pitch,} \qquad (2)$$

m being any integer, (as shown in Fig. 2), and substituting it in the expression for the slot constant, we have as the final equation for slot reactance:

$$X_{\text{slot}} = M\left[K_S\left(\frac{d_3}{w}+\frac{d_1}{3\,w}\right)+\frac{d_1}{12\,w}(1-K_S) - \frac{d_2}{4\,w}\left(K_S-\frac{2}{3}\right)\right] \qquad (3)$$

The third term in this expression is negligible, except for zero phase-sequence currents, unless there is a large space between the top and bottom coil sides, as in the case of a multi-speed machine with two primary windings. The second term is very small, unless the pitch is below two-thirds, so that only the first term is ordinarily of importance.

The slot reactance given by (3), with the coefficients K_S of Fig. 2, applies when balanced polyphase currents flow in the windings. It is sometimes desirable to know the slot reactance for other conditions, however, as for single-phase, or for zero phase-sequence currents. These reactances can all be obtained from equation (3), if proper values of K_S are used. The appropriate values can readily be derived by the methods already employed, and therefore to save space the details of the work will be omitted here. The following table of special values of K_S covers most of the cases that are met with in practise.

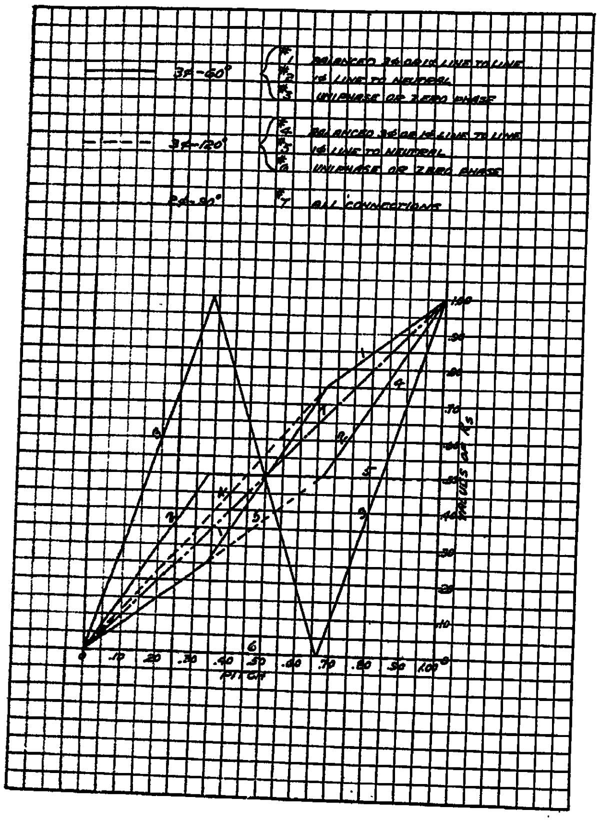


FIG. 2

It will be noted that the single-phase line to neutral slot reactance of a three-phase winding is not half the reactance between lines, unless the winding pitch is exactly 0, 50 per cent, or 100 per cent. This is so because the mutual slot reactance between any two phases, A and B, is only zero when there are as many slots containing an A coil side in the bottom and a B in the top as there are slots containing a B coil side in the bottom and an A in the top; a condition that always exists with two-phase, but that only occurs with pitches of 0, 50, and 100 per cent with three-phase windings. As shown by Park,[1] the line to neutral reactance of a three-phase winding is always equal to two-thirds of the positive phase sequence reactance, plus one-third of the zero phase sequence reactance.

TABLE I
VALUES OF K_S FOR USE IN EQUATION (3)

Connection	Type of winding	Winding pitch 0	1/3	2/3	1
			K_S values		
Balanced, three-phase	three-phase, 60-deg. belts	0	0.250	0.750	1.00
Line to neutral, one-phase	three-phase, 60-deg. belts	0	0.500	0.500	1.00
Line to line, one-phase	three-phase, 60-deg. belts	0	0.250	0.750	1.00
Zero phase	three-phase, 60-deg. belts	0	1.00	0	1.00
Balanced, three-phase	three-phase, 120-deg. belts	0	.375	0.750	0.750
Line to neutral, one-phase	three-phase, 120-deg. belts	0	0.250	0.500	0.500
Line to line, one-phase	three-phase, 120-deg. belts	0	.375	0.750	0.750
Zero phase	three-phase, 120-deg. belts	0	0	0	0
Balanced, two-phase	two-phase, 90-deg. belts	0	0.333	0.667	1.00
Line to line, one-phase	two-phase, 90-deg. belts	0	0.333	0.667	1.00
Zero phase	two-phase, 90-deg. belts	0	0.333	0.667	1.00

For intermediate values of pitch, K_S varies linearly between the tabulated values.

III. END LEAKAGE REACTANCE

The leakage reactance of the end windings will here be considered as due to two kinds of field. The first kind is a revolving magnetic field of the same nature as those that are produced in the core length. This field flows in planes perpendicular to the shaft, and is due to the axially directed and winding currents only. Hereafter it will be designated as the "peripheral leakage field." The second kind is due to the peripherally directed end winding currents, shown in Fig. 4, which produce a leakage field flowing in radial planes, as indicated in Fig. 3. This will hereafter be called the "axial leakage field." The peripheral leakage predominates for synchronous machines with barrel type windings. With single layer chain type windings, the axial leakage flux is greatly increased, and so becomes the greater of the two. For induction machines, the peripheral leakage is greatly reduced by the proximity

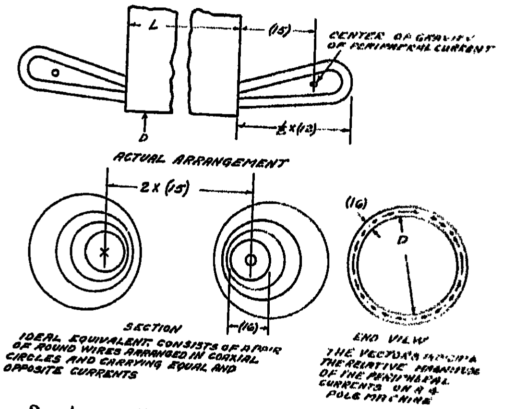

L
(15)
CENTER OF GRAVITY OF PERIPHERAL CURRENT
D
½x(12)
ACTUAL ARRANGEMENT
2 x (15)
(16)
D
SECTION
(16)
IDEAL EQUIVALENT CONSISTS OF A PAIR OF ROUND WIRES ARRANGED IN COAXIAL CIRCLES AND CARRYING EQUAL AND OPPOSITE CURRENTS
END VIEW
THE VECTORS INDICATE THE RELATIVE MAGNITUDE OF THE PERIPHERAL CURRENTS ON A 4 POLE MACHINE

FIG. 3—AXIAL LEAKAGE FLUX OF END WINDINGS

of the primary and secondary windings, and so the axial leakage usually predominates.

The procedure will be, then, first to find the permeance of a cylindrical air core to a sinusoidally distributed encircling m. m. f., second to find the effective m. m. f. of this character produced by the end windings, third to determine thence the reactance due to the peripheral end leakage flux; and, finally, to determine the additional reactance due to the axial leakage. This completes the story for a salient-pole synchronous machine, where there is no mutual inductance between the armature and field end windings. For induction machines, however, the further step of subtracting the mutual reactance of the two windings from the sum of their total reactance is necessary.

As shown by earlier writers[3,6,10], the total flux per pole produced in a circular air core of one cm. axial length by a sinusoidal m. m. f. distributed around the periphery is equal to just twice the maximum m. m. f. in gilberts. Or:

$$\phi_1 = 2\left(\frac{4\,\pi\,A}{10}\right) \tag{4}$$

This equation assumes the flux to lie entirely in air on one side of the periphery of the core, and entirely in iron of high permeability on the other side. It makes no difference whether the air is on the inside or the outside of the core, and neither does it make any difference what the radius of the periphery of the core is. If there is no iron present, so the flux paths lie entirely in air, both inside and outside of the m. m. f., the reluctance is doubled, and so:

$$\phi_2 = \frac{4\,\pi\,A}{10} \tag{5}$$

The flux per pole produced by the same m. m. f. acting on a uniform air-gap of length g is equal to:

$$\phi_3 = \frac{.8\,\pi\,A\,D}{g\,P} \tag{6}$$

And, therefore, the ratio of the flux produced by a given sinusoidally distributed m. m. f. acting on the periphery of an air cylinder of diameter D, with air outside, as well as inside the cylinder, to the flux produced by the same m. m. f. acting on a uniform air-gap of length g, is equal to:

$$\frac{\phi_2}{\phi_3} = \frac{P\,g}{2\,D}, \tag{7}$$

where P is the number of poles of the m. m. f.

The magnetizing reactance due to the air-gap flux of an induction motor with uniform air-gap g is equal to the product of the following factors:

1. The volts per phase due to unit flux per pole;
2. The flux per pole due to unit armature reaction;
3. The maximum of the sine wave of armature reaction per ampere in one phase.

The first of these factors is given by the familiar expression:

$$\frac{V}{\phi} = \frac{\pi}{\sqrt{2}}\,f\,K_p\,K_d\,Z\,10^{-8} \tag{8}$$

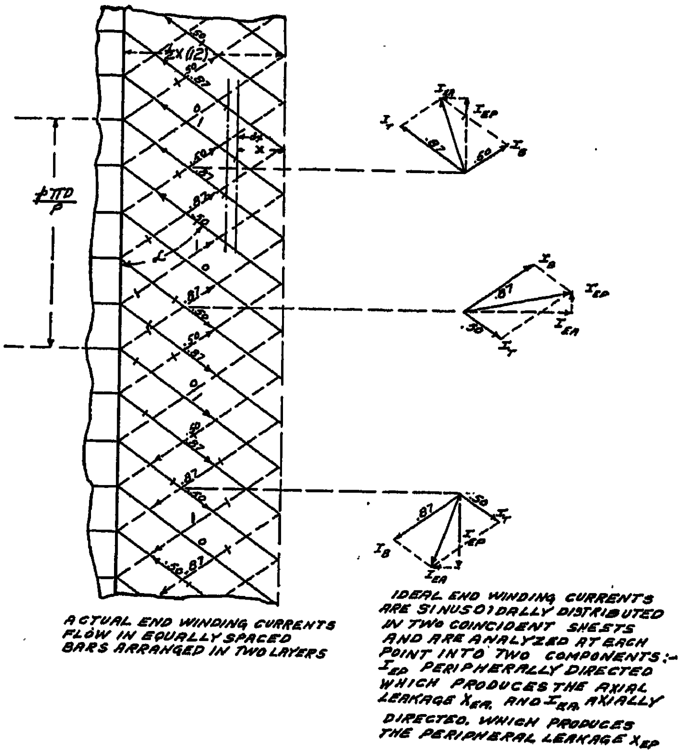
The second factor is:

$$\frac{\phi}{A} = \frac{8\,\pi\,D\,L}{10\,g\,P} \tag{9}$$

and the third* is:

$$\frac{A}{I} = \frac{2\,q\,Z\,K_p\,K_d}{\pi\,P\,\sqrt{2}} \tag{10}$$

The product of these three factors gives the magnetizing reactance of a polyphase armature winding in ohms per phase:



$\frac{\tau p}{P}$
.87
.50
1
0
I_B
I_T
I_{EP}
I_{EA}
ACTUAL END WINDING CURRENTS FLOW IN EQUALLY SPACED BARS ARRANGED IN TWO LAYERS
IDEAL END WINDING CURRENTS ARE SINUSOIDALLY DISTRIBUTED IN TWO COINCIDENT SHEETS AND ARE ANALYZED AT EACH POINT INTO TWO COMPONENTS:- I_{EP} PERIPHERALLY DIRECTED WHICH PRODUCES THE AXIAL LEAKAGE X_{EA} AND I_{EA} AXIALLY DIRECTED, WHICH PRODUCES THE PERIPHERAL LEAKAGE X_{EP}

Fig. 4—End Winding Current Relationship

$$X_a = \frac{8\,\pi\,q\,f\,D\,L\,Z^2\,K_p\,K_d^2}{P^2\,g\,10^9}$$

$$= \frac{0.319\,K_p^2\,K_d^2\,D\,S\,M}{P^2\,g} \tag{11}$$

The ratio of the axial length of the end windings on both ends of the machine to the core length is:

$$\frac{\pi\,p\,D\tan\alpha}{P\,L}, \tag{12}$$

where p is the fractional pitch, and α is the angle indicated in Fig. 4.

---

*R. R. Lawrence, "Principles of Alternating-Current Machines," page 108.

Multiplying together (7), (11), and (12), the leakage reactance of the end windings due to the fundamental of the peripheral field only is:

$$X_{EP} = M\left[\frac{0.319\, K_d^2\, D\, S}{P^2\, g}\right]\left(\frac{P\, g}{2\, D}\right)$$

$$\left[\frac{\pi\, p\, D \tan\alpha}{P\, L}\right] F\,(p) = M\left[\frac{p\, D\, S\, K_d^2 \tan\alpha}{2\, P^2\, L}\right] F\,(p) \tag{13}$$

where $F\,(p)$ is that function of the winding pitch that corresponds for the coil end projection length to the function K_p^2 over the core length. The distribution factor K_d is the same for the end windings as for the core portion, due to the unvarying widths of the phase belts.

To evaluate $F\,(p)$, consider an elementary section of the end windings at a distance x from the outer end, as indicated in Fig. 4. If x is expressed as a fraction of the total axial winding projection on one end, the pitch of the winding at any point is equal to $p\,x$. Therefore, the square of the ratio of the m. m. f. at any point to its value in the core portion is,

$$\sin^2 \frac{p\,\pi\,x}{2},$$

and accordingly the value of $F\,(p)$ is:

$$F\,(p) = \int_0^1 \sin^2 \frac{p\,\pi\,x}{2}\, d\,x = \frac{p\,\pi - \sin p\,\pi}{2\,p\,\pi} \tag{14}$$

To evaluate the additional end reactance due to the irregular peripheral space distribution of the end winding m. m. f., we may proceed as follows: The ratio of the nth harmonic flux per pole in an air core to the fundamental flux per pole is n times greater than the same ratio for a uniform air-gap, (from equations (5) and (6)). For a full pitch winding with a large number of slots per pole, the reactance due to the nth harmonic is therefore approximately $1/n^2$ times that due to the fundamental for an air core, and $1/n^4$ times the fundamental for a uniform air-gap, the latter ratio being derived in the section of this paper on belt leakage. The sum of the values of $1/n^2$ for all the harmonics present in a full-pitch polyphase winding with an infinite number of slots is equal to $1/K_d^2$.† The actual effect of the harmonics is less than this for windings with pitches a little less than 100 per cent, and greater for very short pitch windings, but these errors are not important. The tooth harmonics due to the concentration of the m. m. f. in a small number of conductors are smaller in the end windings than in the core, since the finite slot depth makes the actual length of flux path much greater for the higher harmonics than in the ideal case, for which the formulas are derived, where the m. m. f. is concentrated in a cylinder of zero thickness. Thus, the increase in end leakage reactance due to harmonics may be approximately taken care of by simply omitting the factor K_d^2 in the first member of equation (13).

The component of end leakage reactance due to axial flux may be evaluated by considering the peripheral components of the end winding currents at any fractional distance x from the outer end of the winding, as indicated in Fig. 4. The peripheral currents in the upper and lower layers of the winding are out of phase in time by an angle equal to $p\,\pi\,x$, which differs by π from the phase difference between the axially directed components of the same currents. Thus, the resultant peripheral current at any point is proportional to $\cos \dfrac{p\,\pi\,x}{2}$. The center of gravity of this current area is accordingly located at a distance from the end of the core equal to:

$$\frac{\pi\, p\, D \tan\alpha}{2\, P}\left[1 - \frac{\int_0^1 x \cos \frac{p\ \pi\ x}{6}\, dx}{\int_0^1 \cos \frac{p\ \pi\ x}{2}\, dx}\right]$$

$$= \frac{\pi\, p\, D \tan\alpha}{2\, P}\left(\frac{2 \tan p\,\pi/4}{p\,\pi}\right)$$

$$= \frac{\pi\, p\, D \tan\alpha}{4\, P}\left(1 + \frac{p^2}{5}\right) \quad \text{approx.} \tag{15}$$

We will now make the assumption that the axial end leakage reactance is the same as if the peripheral current were concentrated in a conductor of circular cross-section at each end of the machine at the distance from the core ends given by (15). The proper diameter of cross section to assume for these conductors is evidently greater than the depth of primary slot, and less than the axial length of the end projection on one end. For reasons of simplicity, we will assume this diameter to be one-half the axial projection at each end, or, from (12):

$$\frac{\pi\, p\, D \tan\alpha}{4\, P} \tag{16}$$

As the peripheral current extends over a radial depth equal to the depth of primary slot, the equivalent diameter cannot be less than this. Hence (16) is really based on the assumption that the primary slot depth is about equal to one-third the axial end projection, and if wide variations from this relationship occur, then (16) will be correspondingly in error. The effect of this error is to make the calculated reactance too high for very deep primary slots and too low for very shallow slots. However, since only the logarithm of this effective diameter enters the final equation, a considerable error in the diameter makes only a small error in the result.

---

†Silberstein, "Synopsis of Applicable Mathematics," p. 87.

The current in this conductor varies cyclically around the periphery, its r. m. s. value for a full-pitch winding being $\frac{1}{2\sqrt{2}}$ times the total axially directed current in one pole pitch, or:

$$I_{\text{EP}} = \left(\frac{1}{2\sqrt{2}}\right)\left(\frac{K_p K_d q I Z}{P}\right)\left(\frac{2\sqrt{2}}{\pi}\right)$$

$$= \frac{q I Z K_p K_d}{P \pi} \tag{17}$$

We will neglect the leakage flux that enters the ends of the armature core in an axial direction, as it is largely canceled by opposing eddy currents in the punchings, that are entered broadside on. Then, the peripheral currents at the two ends may be considered as flowing in a pair of parallel wires separated by a distance twice that given by (**15**).

Assuming the air-gap diameter, D, large compared with this distance, the problem reduces to that of calculating the reactive kv-a. of a single-phase transmission line consisting of round wires of a diameter given by (**16**), separated by a distance twice that given by (**15**), of a length πD, and carrying a current per conductor given by (**17**). The inductance of such a line is equal to:

$$\frac{2\pi D}{10^9}\left[\frac{1}{2} + 2\,l\,n\,4\,(1 + p^2/5)\right] \text{henrys}$$

which is practically equal to:

$$\frac{4\pi D\,(8.2 + p^2)}{(5)\,10^9} \tag{18}$$

The total reactive volt-amperes due to the axial end leakage flux are, therefore, equal to:

$$X_{\text{EA}} = \frac{13.1 f q D K_p^2 K_d^2 Z^2 (1 + 0.12 p^2)}{10^9 P^2} \tag{19}$$

Adding (**19**) to (**13**), and dropping the K_d^2 factor of (**13**) as previously explained, we obtain for the total end leakage reactance:

$$X_{\text{End}} = M\left[\frac{p D S \tan\alpha F(p)}{2 P^2 L} + \frac{K_p^2 K_d^2 D S (1 + 0.12 p^2)}{6 P^2 L}\right] \tag{20}$$

We will take $\alpha = 60$ deg., a value higher than the usual angle of the ends, but one that checks the mean length of conductor on most windings, and so one that allows something for the straight portion beyond the core. Equation (20) then reduces to:

$$X_{\text{End}} = \frac{0.3\,M D S\,(3p - 1)}{P^2 L} \tag{21}$$

very closely. The substitution of 0.3 $(3p - 1)$ for the complicated function of pitch given in (**20**) is justified by the comparison of the two values shown in Fig. 5. Over the ordinary range of pitches from 0.5 to 1.2, the agreement is very satisfactory. We will, therefore, adopt (**21**) as our final expression for the end leakage reactance of a synchronous machine with a barrel-type primary winding.

Equation (**21**) gives the end leakage reactance, under the condition of balanced polyphase currents flowing in the armature winding. As the derivation of this equation is based on the assumption that the end leakage fields are due to revolving m. m. fs., with nearly negligible space harmonics, the reactances for other winding connections may be taken as proportional to the corresponding values of armature reaction.

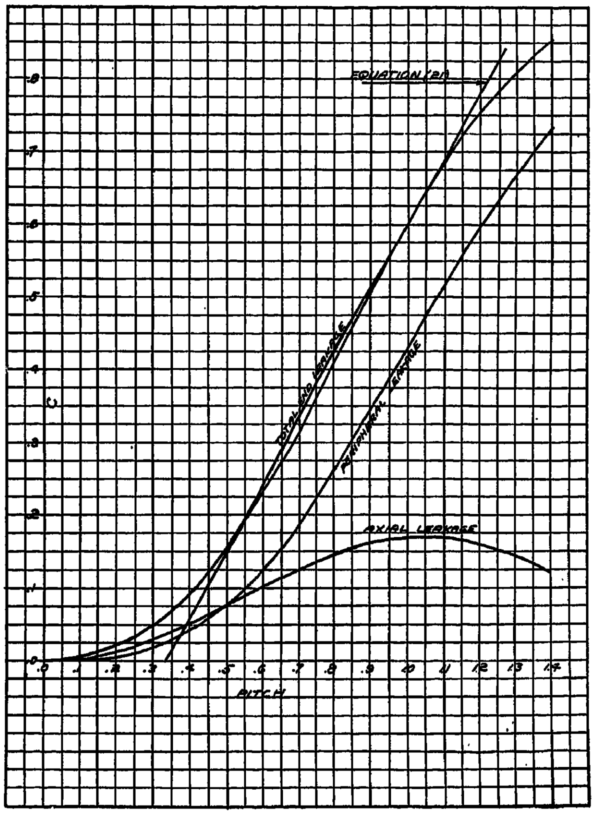


FIG. 5—END LEAKAGE REACTANCE COEFFICIENTS

On this basis, the coefficient 0.3 of equation (**21**) may be changed in accordance with the following table in the special connection cases:

TABLE II

Connection	Winding	Value of coefficient for equation (21)
Balanced polyphase.......	two or three-phase	0.3
Single-phase, line to neutral	two or three-phase	0.2
Single-phase, line-to-line...	two or three-phase	0.3
Zero phase...............	three-phase, 60 or 120 deg...............	0
Zero phase...............	two-phase	0.3

IV. ZIGZAG LEAKAGE REACTANCE

Inasmuch as the stator windings of polyphase machines consist of coils arranged in a finite number of slots and connected in a finite number of phases, usually two or three, the shape of the wave of magnetomotive force along the air-gap surface is not sinusoidal, even though the winding carries balanced sinusoidal polyphase currents. At any particular instant the shape of the m. m. f. wave is stepped, the agreement between the stepped figure and a true sinusoid becoming closer and closer as the numbers of slots and phases are made larger and larger, until perfect agreement results when both the number of slots and the number of phases are infinite.

As shown in reference 3, the air-gap flux produced by this stepped m. m. f. wave in a salient-pole synchronous machine consists of three elements:

1. A fundamental sine wave stationary on the rotor, inducing fundamental frequency voltages;
2. A series of harmonics moving with respect to the rotor and also inducing fundamental frequency voltages; and
3. Other harmonics inducing other than fundamental frequency voltages in the armature winding.

Of these, 1 is the effect of the armature reaction, 2 constitutes an element of the leakage reactance, and 3 may be disregarded. Thus, if we now derive a formula for 2, we will complete the work of determining the armature leakage reactance.

In the present discussion, to begin with, an ideal type of machine will be considered. The air-gap length will be assumed uniform, and the flux will be assumed to cross the gap radially. The slots will be assumed to have no real physical existence which might entail a variation in air-gap permeance, but will be assumed to be merely the points along the periphery of the machine at which the current is concentrated and at which, therefore, the m. m. f. changes. Under these conditions, the air-gap flux will vary along the periphery of the machine at any instant in a stepped wave shape exactly proportional to the m. m. f. at each point.

If the flux wave produced by the stator currents alone be considered, it has been shown that the stepped wave of flux may be analyzed into a fundamental sinusoidal distribution of flux having the same number of poles as that for which the stator winding is connected, plus an infinite number of harmonic flux waves having different numbers of poles.[3] There being no rotor currents by assumption, the magnitudes of the fundamental, and of all of the harmonic fields, are proportional to the current in the windings, the back e. m. f. produced by these fluxes in the stator winding is proportional to the current, and the fluxes are truly self-inductive.

The reactance due to these harmonic fields constitutes the "air-gap leakage," or "differential leakage" reactance of the machine, which is usually divided into two parts, the "belt leakage" and the "zigzag leakage."

The distinction between belt leakage and zigzag leakage is that the former is due to the concentration of the m. m. f. in a definite number of phase belts, while the latter is due to its concentration in a definite number of slots. No accurate dividing line can be drawn between the two kinds of leakage, as they are mutually dependent on each other, and as in the case of one slot per pole per phase, for example, they coalesce into the same thing. The reason for making a distinction is that, for ordinary design proportions, a change in the number of slots affects only the zigzag leakage, while a change in the winding pitch affects only the belt leakage, so that the independent effects of number of slots and of pitch can be more easily dealt with when the two parts are considered separately.

Independent formulas will be developed for the belt and zigzag reactances, therefore, but it must be recognized that, whenever the number of slots per pole is small, the two are not separable, and the formulas become inaccurate. The only accurate way to obtain the total differential reactance is to compute it for each case separately, and tabulate the results. The computation of the total for any regular winding can be simply carried out by the means suggested by Chapman.[8] This has been done in Appendix A, wherein tables for some of the more usual regular windings are given.

The definitions adopted will be as follows:

The per cent differential leakage reactance is the total per cent reactance due to harmonic fluxes crossing the air-gap that are not of the fundamental number of poles. Or, it is the total excess per cent reactance, due to air-gap fluxes, above that of a similar machine, with an infinite number of slots and of phases.

The per cent belt leakage reactance is the total excess per cent differential leakage reactance of a polyphase armature winding above that of a squirrel-cage winding with the same number of slots.

The per cent zigzag leakage reactance is the total per cent differential leakage reactance of a squirrel-cage winding, or of a winding with as many phases as it has slots per pole.

In this section we will consider the zigzag leakage, or the harmonic fluxes produced by a squirrel-cage winding. It is shown in Appendix A by Chapman's method that the total inductance of a full pitch winding with one slot per pole per phase, as a ratio to the inductance of one full-pitch coil is:

$$Q = \frac{1}{s} \csc^2 \frac{\pi}{2 s}, \qquad (22)$$

where s is the number of slots per pole.

And, it is also shown in Appendix A that the total useful inductance of such a winding, due to the fundamental flux it produces, as a ratio to the inductance of one full pitch coil is:

$$Q_u = \frac{4 s}{\pi^2} \qquad (23)$$

Hence, the zigzag leakage, X_t, is equal to:

$$X_t = X_a \left(\frac{\pi^2}{4\, s^2} \csc^2 \pi/2\, s - 1 \right) \qquad (24)$$

where X_a is given by (**11**).

As $\pi/2\, s$ is a small angle, (**24**) may be developed into a series of which only the first two terms need be considered, so that:

$$X_t = \frac{\pi^2\, X_a}{12\, s^2} \left(1 + \frac{\pi^2}{20\, s^2} + \frac{\pi^4}{504 \quad s^4} + \ldots \right) \qquad (25)$$

or, approximately:

$$X_t = \frac{5\, X_a}{6\, s^2} = \frac{5}{6} \left(\frac{P^2}{S^2} \right) X_a \qquad (26)$$

Equation (**26**), then, gives the desired result, the total differential leakage reactance of a regular winding with one slot per pole per phase, or, by definition, the zigzag reactance of any winding with the same number of slots per pole. Clearly the error in the formula is small for large values of s, but it makes the calculated value a little low when s becomes less than 3. The value of X_t thus found applies to a single winding, and to the case of a uniform air-gap length.

For the case of a salient pole synchronous machine, the zigzag leakage can be found from the same formulas if an equivalent uniform value of air-gap length can be ascertained. It has been shown by Doherty and Nickle[8] that the reactance due to the nth harmonic of primary m. m. f. depends on the average permeance, and on the second harmonic of permeance variation only, and that these two permeances have different values for each harmonic. The effect of the variation of permeance due to the salient poles is to make the fundamental armature flux and the belt leakage flux greater in the main axis than in the quadrature axis, but to make the zigzag harmonic leakage fluxes greater in the quadrature axis than in the main axis. This is true because the predominating portions of the high order harmonic fluxes are produced at the points of maximum armature current, and these come over the pole faces (low reluctance) when the poles are in the quadrature axis, while they come over the interpolar spaces (high reluctance) when the poles are in the main axis.

The average permeance increases with the order of the harmonic, since the shorter the harmonic pole pitch, the greater the relative permeance for the peripheral flux that reenters the primary iron without ever reaching the pole face. For the same reason, the second harmonic of the permeance variation is less the greater the value of n.

The same authors have shown that the fundamental magnetizing reactance in the direct axis is proportional to:

$$K_{p_1}{}^2\, K_{d_1}{}^2 (p_0{}^{\mathrm{I}} + 1/2\, p_2{}^{\mathrm{I}}),$$

and that on the same basis the direct axis magnetizing reactance for the nth harmonic is proportional to:

$$\frac{K_{p_n}\, K_{d_n}}{n} \left(\frac{K_{p_n}\, K_{d_n}}{n}\, p_0{}^{\mathrm{N}} + 1/2 \left(\frac{K_{p_{n \pm 2}}\, K_{d_{n \pm 2}}}{n \pm 2} \right) p_2{}^{\mathrm{N}} \right)$$

where the plus signs are taken if the nth harmonic rotates backwards with respect to the fundamental, and vice versa. For the quadrature axis, the signs of the p_2 terms in both the fundamental and the harmonic magnetizing reactance expressions change sign.

It may easily be shown that the product $K_{p_n} K_{p_{n \pm 2}}$ is always either negative or very small, and that $K_{d_n} K_{d_{n \pm 2}}$ is always negative for the low order (belt leakage) harmonics, but is positive for the tooth (zigzag leakage) harmonics. Hence, the numerical values of the two terms in the foregoing expression are additive for the belt leakage, but are subtractive for the zigzag leakage, and the contrary is true for the quadrature axis.

It is quite feasible to carry through the indicated numerical work for any particular case, using these general methods, but to do so for the general case is quite impractical. We will, therefore, make some bold assumptions at this point, with the object of obtaining a simple formula accurate enough for general use. First, we will assume average values for the permeance coefficients, corresponding to a two-thirds ratio of pole arc to pole pitch, a minimum air-gap two per cent of the pole pitch, and a maximum air gap 1.5 times the minimum, as follows:

$P_2{}^{\mathrm{I}} = 2/3\ p_0{}^{\mathrm{I}}$, $p_0{}^{\mathrm{N}} = 1.2\ p_0{}^{\mathrm{I}}$, and $p_2{}^{\mathrm{N}} = 1/3\ p_0{}^{\mathrm{N}}$.

On this basis, the nth harmonic magnetizing reactance varies about 15 per cent, above and below its average, between the direct and quadrature axes. We shall leave this relatively small variation out of account, therefore, and calculate only the average reactance, assuming it to be the same in both axes. Then, the ratio of the magnetizing reactance for the nth harmonic to the direct axis magnetizing reactance of the fundamental is approximately equal to:

$$\frac{X_{a_n}}{X_{a_d}} = \frac{.9\, K_{p_n}{}^2\, K_{d_n}{}^2}{n^2\, K_{p_1}{}^2\, K_{d_1}{}^2}$$

Finally, we must take into account the fact that our ideal assumption of true rectangular m. m. f. waves for each coil is not exact. Actually, the m. m. f. of each slot is distributed across the slot opening, instead of concentrated at a point in the center of the slot, and so the steps in the m. m. f. waves are not vertical, but slightly sloping. This reduces all the harmonics appreciably, especially the higher order ones. We shall assume that the average effect of this error is to reduce the magnetizing reactance to 5/6 of its ideal value. Multiplying the last expression by this factor, therefore, we find the proper average value of X_a to substitute in equation (**26**) is:

$$X_a = \frac{3}{4}\, X_{a_d} \qquad (27)$$

V. Belt Leakage Reactance

The belt leakage reactance is due to the increase in differential leakage caused by reducing the number of phase belts per pole, q, to a small number. When the number of slots per pole is very large, this effect can be readily calculated, but as the slots are made fewer, the belt leakage increases to two or three times its limiting value. When the slots finally become equal to one per pole per phase, the belt leakage becomes zero, by definition. As the effect of the number of slots is further complicated by variations in the winding pitch, it is not considered practicable to present any adequate formula for the belt leakage in the general case. However, as the belt leakage is small by comparison with the zigzag leakage, unless s is large, and as in this case the belt leakage formula is fairly accurate, these errors are not important. For accurate results, it is necessary to compute the total differential leakage reactance, and tabulate the results for all useful windings, as outlined in Appendix A.

This is especially necessary in the case of fractional slots per pole, as here the belt leakage increases greatly, and as it varies considerably, depending on the particular sequence of large and small numbers of slots per phase belt employed.

The harmonic fields corresponding to the belt leakage are of lower orders than those of the zigzag leakage, as the former are primarily due to the phase belts which extend over several of the slots to which the latter are due. Further, since all the harmonic fields must create fundamental frequency voltages in the producing winding, and since the nth harmonic has n times the fundamental number of poles, it follows that each harmonic must revolve with respect to the producing winding at one nth of fundamental speed. Therefore, the harmonic fields due to the primary winding induce high frequency voltages in the secondary, and, if the latter is short-circuited, produce damping currents in it. A regular phase winding, or a salient-pole field winding, has so little admittance for these voltages that this damping effect can be neglected. But, in the case of a squirrel-cage, the induced currents are sufficient to greatly reduce the belt leakage harmonic fluxes. The zigzag leakage fluxes are only slightly reduced in usual cases, as the squirrel-cage admittance for these high order harmonics is small.

Therefore, the primary belt leakage can be practically neglected in all machines with integral slots per pole per phase, having squirrel-cage secondary windings, and so we will assume for this case:

$$X_B = 0 \qquad (28)$$

For regular windings with an infinite number of slots per pole, the belt leakage reactance is the sum of the magnetizing reactances of all the harmonic m. m. fs. For synchronous machines the stator only need be considered, while for induction machines the stator and rotor belt leakage must be added to obtain the total.

As a first step, it will be necessary to calculate the ratio $\frac{X_{a_n}}{X_a}$, where X_{a_n} is the magnetizing reactance of the nth harmonic. A measure of the effectiveness of the stator winding in producing fundamental flux is the number of turns in series per phase times the pitch and distribution factors for the fundamental number of poles, or:

$$N_1 = K_{p_1} K_{d_1} N$$

in the same way the effectiveness of the stator winding in producing nth harmonic flux is measured by

$$N_n = K_{p_n} K_{d_n} N,$$

where K_{p_n} K_{d_n} are the pitch and distribution factors, respectively, of the stator winding for the nth harmonic flux.

If a voltage is applied to the primary winding and is first assumed to be opposed entirely by fundamental flux and then entirely by nth harmonic flux, the ratio of the magnetizing currents in the two cases will be the ratio of X_{a_n} to X_a.

If ϕ is the total flux per pole, then

$$\phi_n = \phi_1 \frac{N_1}{N_n},$$

and if B is the air-gap flux density, then

$$B_n = B_1 \frac{N_1}{N_n},$$

since the area of each pole for the nth harmonic is $1/n$ times that for the fundamental.

Since for the nth harmonic there are n times as many poles to magnetize as there are for the fundamental, the magnetizing current in the second case is:

$$I_{m_n} = I_{m_1} \frac{N_1^2}{N_n^2} n^2$$

And, as the magnetizing reactance is equal to the ratio of the applied voltage to the magnetizing current, the per cent nth harmonic magnetizing reactance is:

$$\% X_{a_n} = \frac{1}{n^2} \left(\frac{K_{p_n}^2 K_{d_n}^2}{K_{p_1}^2 K_{d_1}^2} \right) \qquad (29)$$

The total belt leakage reactance is equal to the sum of the magnetizing reactances of all the harmonics existing with a definite number of phases, a definite winding pitch, and an infinite number of slots. It is well known that a winding with q phase belts per pole produces only those harmonics whose orders are equal to $2kq \pm 1$. And, for all these harmonics, the distribution factor with an infinite number of slots is simply equal to one nth of the fundamental distribution factor. Hence, we may write:

$$X_B = X_a \sum^{n=\infty} \frac{K_{p_n}{}^2}{n^4 K_{p_1}{}^2}, \qquad (30)$$

where n takes only the values $2\,k\,q \pm 1$; $k = 1, 2, 3, \ldots$

For a full pitch two-phase winding with 90 deg. phase belts, this is:

$$X_{B_{90}} = X_a \left(\frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \ldots \right)$$

$$= \left(\frac{\pi^4}{96} - 1 \right) X_a = 0.0147\, X_a \qquad (31)$$

For a full-pitch three-phase winding with 60 deg. phase belts, it is:

$$X_{B_{60}} = X_a \left(\frac{1}{5^4} + \frac{1}{7^4} + \frac{1}{11^4} + \frac{1}{13^4} + \ldots \right)$$

$$= (1 + X_{B_{90}}) \left(1 - \frac{1}{3^4} \right) - X_a = \left(\frac{5\,\pi^4}{486} - 1 \right) X_a$$

$$= 0.00214\, X_a \qquad (32)$$

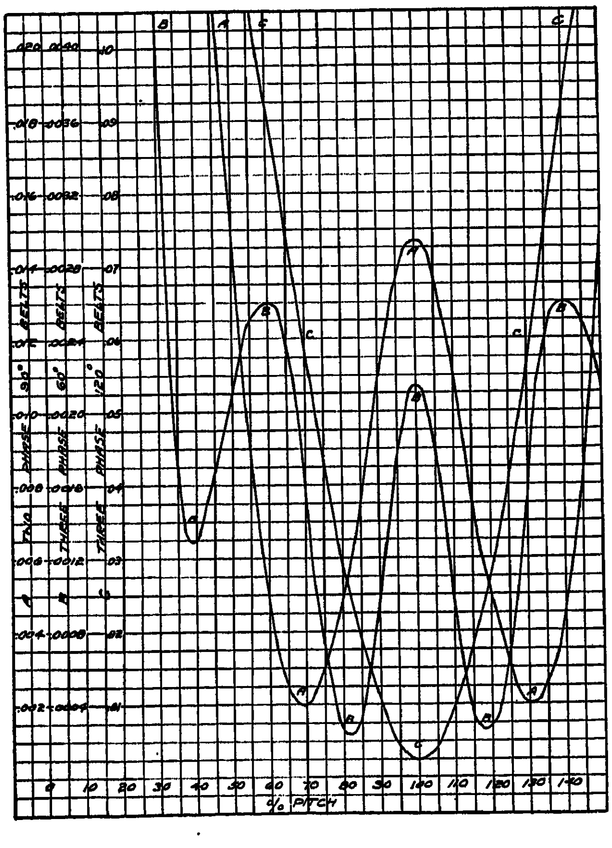


FIG. 6—CHART OF VALUES OF K_B THE BELT LEAKAGE CONSTANT

For a full-pitch three-phase winding with 120 deg. phase belts, it is also 0.00214 X_a, but in this case it increases rapidly as the pitch changes from unity, due to the resulting dissymmetry of the winding, while in the other cases X_B decreases as the pitch changes from unity.

The actual value of X_B for any pitch for each type of winding can not be expressed by any simple formula, but must be found by actually carrying out the summation indicated in (30). This has been done, and the results are shown in Fig. 6.

The value of X_B for a salient pole synchronous machine is also approximately given by the foregoing equation, if 3/4 the direct axis magnetizing reactance is used for X_a, as shown in the previous section. Summing up, therefore, the belt leakage reactance is equal to:

$$X_B = 0 \qquad (28)$$

for any machine with a squirrel-cage winding and integral slots per pole per phase;

$$X_B = \frac{3}{4}\, X_{ad}\, (K_{B_1}) \qquad (33)$$

for salient-pole synchronous machine without squirrel-cage; and with integral slots per pole per phase; and

$$X_B = X_T = \frac{5}{8} \left(\frac{P^2}{S^2} \right) X_{ad} \qquad (34)$$

for a salient-pole synchronous machine with fractional slots per pole per phase. This last equation is arbitrary, as X_B for fractional slot windings varies very widely, but in a general way X_B in such cases is larger the larger X_T is, and it is on the average of the same order of magnitude. From some unpublished work of Professor A. A. Bennett, the differential leakage reactance for six different fractional slot windings averaged 1.82 times X_T. Hence, our assumption of 2 for this factor in the general case is not unreasonable, and should be satisfactory until such time as an adequate table of differential leakage reactance values for irregular windings is available.

Since some of the harmonics of belt leakage flux for fractional slot windings are of fractional, and many are of rather low, orders, a squirrel-cage winding will have demagnetizing currents induced in it, which may reduce the belt leakage reactance considerably. The magnitude of this effect is extremely variable, however, as it depends on many factors. It is the writer's opinion that the demagnetizing action of a squirrel-cage on the belt leakage reactance due to fractional slot windings is small for usual design proportions, and so it will be neglected in the present paper.

In these equations, we have derived expressions for all the elements of the reactance, and we may now turn to their application to actual machines.

VI. APPLICATION TO SALIENT POLE SYNCHRONOUS MACHINES

The leakage reactance in ohms of a salient pole synchronous machine with integral slots per pole, and with a squirrel-cage winding, has been determined by (3), (21), (26), (27), and (28), to be:

$$X_1 = M \left[K_s \left(\frac{d_3}{w} + \frac{d_1}{3\,w} \right) + \frac{0.3\,(3\,p - 1)\,D\,S}{P^2\,L} \right]$$

$$+\frac{5}{8}\left(\frac{P}{S}\right)^2 X_{a_d} \qquad (35)$$

The belt leakage given by equation (**33**) or (**34**) should be added in case the machine has fractional slots or has no squirrel-cage winding. The second order terms of the slot reactance (equation (**3**)) have been omitted as being non-essential for normal synchronous machines.

Expressed as a fractional voltage drop due to full-load current, (**34**) becomes by (**8**), (**10**), and (**11**):

$$X_1 = \frac{7.90\,A}{K_p^2\,K_d^2\,\phi}\left[\frac{P\,L\,K_S}{S}\left(\frac{d_3}{w}+\frac{d_1}{3\,w}\right)\right.$$
$$\left.+\frac{0.3\,(3\,p-1)\,D}{P}\right]+\frac{5}{8}\left(\frac{P}{S}\right)^2\left(\frac{A\,K_a}{F}\right) \qquad (36)$$

for lengths measured in cm., the constant 7.90 becoming 20.0 if inch units are used. The value of F to be used in the last term of (**36**) is the number of normal voltage field ampere-turns, and K_a is the constant required to convert armature reaction ampere-turns into equivalent field ampere-turns. K_a depends on the particular shape of field pole and length of air-gap used, but it has an average value a little less than 0.9, and is given by Fig. 7:

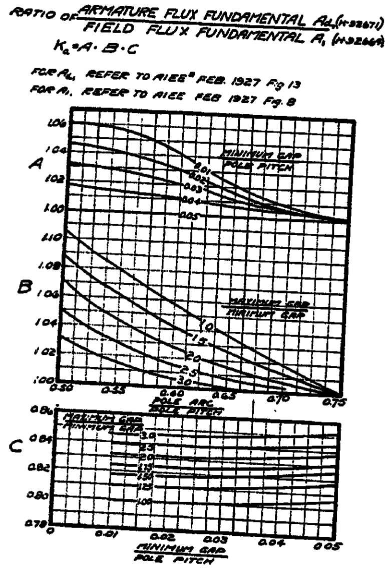


Fig. 7

A further simplification of (**36**) can be made by putting $K_S = \frac{3\,p+1}{4}$, which is exactly correct for three-phase windings with pitches between 2/3 and 1, and which is slightly higher than correct for all other windings, (Fig. 2). As two-phase windings have higher belt leakages, and as the neglected second order terms in the slot reactance become appreciable when the pitch is below 2/3, the small errors so introduced are on the right side.

Finally, therefore, our formula for the per cent leakage reactance, expressed as a decimal, of a salient-pole synchronous machine with a barrel-type armature winding, and fractional slots per pole, in inch units, is:

$$X_1 = \frac{20\,A}{K_p^2\,K_d^2\,\phi}\left[\frac{P\,L}{S}\left(\frac{3p+1}{4}\right)\left(\frac{d_3}{w}+\frac{d_1}{3\,w}\right)\right.$$
$$\left.+\frac{0.3\,(3\,p-1)\,D}{P}\right]+\frac{1.1\,A}{F}\,((P/S)^2+0.6\,K_B) \qquad (37)$$

If integral slots per pole are used, the $(P/S)^2$ term should be multiplied by 0.5 If in addition the machine has a squirrel-cage winding, the K_B term is to be omitted. The reactance is assumed to be the same in both axes, but actually it is greater in the quadrature axis than in the direct axis by about 1/4 of the A/F term.

As the value of X_1 found from test is totally different, depending on how the armature reaction is converted into equivalent field ampere-turns, it is necessary to consider the matter before proceeding to the comparison of test reactances with those given by (37).

Wieseman[2] has derived accurate coefficients by flux plotting for the calculation of the fundamental sine wave of flux produced in a salient pole machine by the concentrated field turns, and by a sinusoidally distributed armature reaction. By taking the ratio of these two coefficients, the necessary factor, K_a, to convert the armature reaction ampere-turns into equivalent field ampere-turns, can be derived. This has been done by Mr. Wieseman, and the result is shown in Fig. 7. By substituting the quadrature axis coefficients, also given in reference No. 2, for those of the direct axis, the armature reaction and the total, or synchronous, reactance in either axis can be accurately calculated.

The usual way to determine the armature leakage reactance by test is first to take open and short-circuit characteristic curves, which give armature voltage on open-circuit and armature current on sustained short circuit, respectively, as functions of field current. Then, the field current corresponding to full load armature current on short circuit, divided by the field current corresponding to normal voltage on the air-gap line, is equal to the synchronous reactance, expressed as a fraction. By subtracting the calculated armature reaction, also expressed as a fraction of the no load air-gap field ampere-turns, from the test value of synchronous reactance, the test value of armature leakage reactance is found. The calculated armature reaction ampere-turns are given by (**10**), multiplied by the value of K_a from Fig. 7.

As the errors of the test value of synchronous reactance probably average one per cent, and as Fig. 7 also involves slight errors, it is evident that this method of obtaining the leakage reactance is inherently very inaccurate. On high-speed machines, the leakage reactance is very small by comparison with the total reactance, so that the results are particularly inaccurate in this case.

It is possible to measure the total leakage reactance of a synchronous machine at standstill, just as for an induction machine, but this requires the inclusion of the reactances of the field winding and squirrel-cage, if present, which do not enter the calculation of steady state performance. It is also possible to measure the armature reactance with the rotor removed and to determine the leakage reactance as the difference between the total and the calculated reactance due to the fundamental of the flux produced in the air core. This last method involves large errors in the amounts of zigzag and end leakage, however, and is not as accurate as the usual short circuit test method of measurement. Finally, it is possible to insert exploring coils on the armature surface and by their means measure the net flux existing during a short-circuit test, which gives a measure of the leakage reactance. The dissymmetry and wave form errors introduced, unless the exploring coils exactly follow the armature coil grouping, together with the fact that the slot leakage part of this flux links only a portion of the winding, make this method of no practical use. For these reasons, the usual method of finding X_1 from open and short-circuit tests has been followed in checking the results obtained with the formulas derived in this paper.

TABLE III
HIGH SPEED MACHINES

Poles	Kv-a.	Freq.	Calculated			Test X_d	Test X_d / Calc. X_d
			X_{ld}	X_{ad}	X_d		
4	80	60	0.105	0.955	1.060	1.043	0.984
4	105	50	0.054	1.220	1.274	1.250	0.981
4	180	60	0.061	1.002	1.063	1.032	0.971
4	600	60	0.060	2.163	2.223	2.195	0.987
4	940	25	0.128	1.770	1.898	1.895	0.998
4	2000	25	0.198	1.562	1.762	1.793	1.018
4	3000	25	0.117	2.300	2.417	2.355	0.974
4	5700	25	0.109	1.857	1.966	1.937	0.980
6	150	25	0.102	1.040	1.142	1.092	0.954
6	187	60	0.063	0.998	1.061	1.068	1.007
6	200	60	0.055	0.804	0.859	0.872	1.015
6	400	60	0.069	1.250	1.319	1.275	0.967
6	435	60	0.077	1.127	1.204	1.172	0.973
6	500	60	0.151	2.065	2.216	2.212	0.998
6	500	60	0.105	1.610	1.715	1.678	0.978
6	1000	60	0.127	1.932	2.059	2.040	0.991
6	1500	60	0.092	2.025	2.117	2.145	1.013
6	3125	25	0.098	1.019	1.117	1.133	1.014
6	5000	25	0.097	1.060	1.157	1.161	1.003
6	5000	25	0.088	1.592	1.680	1.745	1.039
				Average........................			0.999
				Average error from mean.....			±0.017
				Average error from 1.........			±0.012

TABLE IV
MEDIUM HIGH SPEED MACHINES

Poles	Kv-a.	Freq.	Calculated			Test X_d	Test X_d / Calc. X_d
			X_{ld}	X_{ad}	X_d		
8	560	60	0.067	0.815	0.882	0.884	1.002
8	800	60	0.077	1.338	1.415	1.416	1.001
8	1200	60	0.086	1.073	1.159	1.150	0.992
8	2500	60	0.143	2.100	2.243	2.230	0.994
8	5000	60	0.161	1.750	1.911	1.928	1.009
10	1100	60	0.097	0.983	1.080	1.098	1.019
10	1880	60	0.061	1.139	1.200	1.214	1.012
10	7500	60	0.080	0.853	0.933	0.922	0.988
12	1000	25	0.081	0.810	0.891	0.905	1.016
12	1900	25	0.089	0.758	0.847	0.842	0.994
12	2400	60	0.159	1.103	1.262	1.245	0.986
12	2500	60	0.093	1.452	1.545	1.564	1.012
12	5000	60	0.132	1.560	1.692	1.667	0.985
12	5000	60	0.101	0.750	0.851	0.852	1.001
12	5300	60	0.135	1.280	1.415	1.414	0.999
12	7000	50	0.115	0.900	1.015	0.987	0.972
12	12500	50	0.110	1.048	1.158	1.172	1.012
12	15000	60	0.086	0.667	0.753	0.766	1.017
12	15000	60	0.137	1.065	1.202	1.208	1.005
16	3500	50	0.114	0.969	1.083	1.071	0.989
				Average........................			1.000
				Average error from mean......			±0.010
				Average error from 1.........			±0.010

TABLE V
MEDIUM LOW-SPEED MACHINES

Poles	Kv-a.	Freq.	Calculated			Test X_d	Test X_d / Calc. X_d
			X_{ld}	X_{ad}	X_d		
18	1,650	40	0.108	0.844	0.952	0.993	1.043
18	1,650	60	0.123	0.961	1.084	1.100	1.015
20	2,250	40	0.148	0.975	1.123	1.136	1.011
20	14,444	50	0.123	0.896	1.019	1.023	1.004
22	2,000	60	0.123	1.078	1.201	1.188	0.989
22	7,900	60	0.162	1 017	1.179	1.198	1.016
24	1,100	60	0.132	0.992	1.124	1.152	1.025
24	4,000	50	0.136	1.027	1.163	1.178	1.013
24	6,880	50	0.131	0.688	0.819	0.826	1.009
24	10,700	50	0.120	0.648	0.768	0.761	0.991
24	30,000	60	0.128	0.647	0.775	0.803	1.036
28	10,125	60	0.147	0.919	1.066	1.089	1.022
30	3,125	60	0.106	0.973	1.079	1.080	1.001
32	750	60	0.138	0.820	0.958	0.951	0.993
32	2,250	40	0.180	0.918	1.098	1.113	1.014
32	9,375	60	0.142	1.064	1.206	1.202	0.997
36	500	60	0.155	0.853	1.008	1.003	0.995
36	1,500	60	0.117	0.881	0.998	0.988	0.990
40	750	60	0.162	0.756	0.918	0.906	0.987
40	1,375	60	0.088	0.619	0.707	0.716	1.013
				Average........................			1.008
				Average error from mean.......			±0.013
				Average error from 1..........			±0.014

In Tables III to VII inclusive, the calculated and test values of synchronous reactance are compared for five groups of machines of different types. The machines were selected at random, and the tests were all made in the regular commercial routine. The average absolute error for the entire list of 100 machines is 0.4 per cent, and the average numerical error is 1.7 per cent; so that the results are as accurate as could reasonably be expected when the errors of test and those inherent in Fig. 7 are considered. If the value of the armature reaction reactance, X_{ad}, and the test value of total reactance, X_d, are assumed to be exact, the test value of leakage reactance can be found by subtracting the one from the other. This has been done for each group of machines, and the average errors between the test and calculated values of X_l for each case are given in Table VIII, together with the corresponding errors in X_d.

The average absolute error of the leakage reactance for the 100 machines on this basis is 0.6 per cent, and the average numerical error is 14.6 per cent. The worst errors occur for the high speed machines, and inspection of Table VIII indicates that the calculated value of X_d is appreciably too high for high speed machines and a little too low for low speed machines. By empirical adjustments of the coefficients of the end and differential leakage components of the reactance, it is evident that these errors could be reduced. Such adjustments of the constants should be made by each manufacturer for himself, however, as they will depend upon the particular end winding constructions, and so forth, that each one uses. The formulas as given were derived straightforwardly from theoretical considerations, and no attempt has been made to make empirical corrections, although simplifying assumptions have been freely used.

TABLE VI
LOW SPEED MACHINES

Poles	Kv-a.	Freq.	Calculated X_{ld}	Calculated X_{ad}	Calculated X_d	Test X_d	Test X_d / Calc. X_d
56	3,500	60	0.144	0.798	0.942	0.960	1.019
56	4,500	60	0.147	0.860	1.007	1.013	1.006
56	5,720	60	0.159	0.648	0.807	0.775	0.960
60	750	60	0.177	0.699	0.876	0.902	1.030
60	1,500	60	0.155	0.871	1.026	1.041	1.015
60	1,500	60	0.231	0.587	0.818	0.882	1.078
60	2,500	60	0.130	0.724	0.854	0.849	0.994
64	625	60	0.168	0.782	0.950	0.907	0.955
64	6,250	60	0.128	0.870	0.998	1.013	1.015
64	8,000	60	0.254	0.776	1.030	1.043	1.013
68	1,500	60	0.192	0.766	0.958	0.961	1.003
72	750	60	0.239	0.818	1.057	1.065	1.008
72	875	60	0.223	0.818	1.041	1.068	1.026
72	1,000	60	0.228	0.740	0.968	0.962	0.994
72	2,250	60	0.147	0.813	0.960	0.983	1.024
72	12,250	60	0.213	0.782	0.995	1.015	1.020
72	32,500	60	0.166	0.926	1.092	1.130	1.035
80	3,500	60	0.187	0.810	0.997	1.020	1.023
82	2,250	60	0.162	0.745	0.907	0.898	0.990
100	1,250	60	0.127	0.510	0.637	0.652	1.024
			Average				1.011
			Average error from mean				±0.018
			Average error from 1				±0.022

TABLE VII
SMALL LOW SPEED MACHINES

Poles	Kv-a.	Freq.	Calculated X_{ld}	Calculated X_{ad}	Calculated X_d	X_d	Test X_d / Calc. X_d
0e	200	60	0.249	0.970	1.219	1.228	1.007
36	240	60	0.244	0.833	1.077	1.076	1.001
40	80	60	0.281	0.660	0.941	0.962	1.022
44	62	60	0.284	0.576	0.860	0.888	1.033
36	100	60	0.302	0.704	1.006	1.008	1.002
80	120	60	0.245	0.793	1.038	1.017	0.980
40	360	60	0.208	1.013	1.221	1.256	1.029
42	160	60	0.293	0.961	1.254	1.307	1.042
44	50	60	0.430	0.972	1.402	1.460	1.041
44	62	60	0.251	0.619	0.870	0.906	1.041
44	62.5	60	0.143	0.417	0.560	0.552	0.986
46	100	60	0.277	0.698	0.975	0.965	1.010
50	145	60	0.171	0.476	0.647	0.648	1.002
52	140	60	0.207	0.611	0.818	0.838	1.024
60	60	60	0.108	0.301	0.409	0.404	0.988
60	250	50	0.151	0.612	0.763	0.771	1.010
60	250	60	0.290	0.902	1.192	1.183	0.992
62	280	60	0.195	0.657	0.852	0.826	0.969
72	165	60	0.196	0.519	0.715	0.729	1.020
74	375	50	0.224	0.781	1.015	1.005	1.010
			Average				1.010
			Average error from mean				±0.017
			Average error from 1				±0.019

The causes for the varying magnitudes of error shown in Table VIII are brought out more clearly by a study of the relative proportions of the various elements of the total reactance for the different types of machines. Table IX gives the average ratios of slot, end, and differential leakage to the total leakage reactance; and also the average ratio of the total leakage reactance to the armature reaction reactance for each case. Since in the high-speed machines, X_l averages only 6 per cent of X_d, while in the small low-speed machines it averages 25 per cent, it is clear that a given error in the measurement of X_l makes more than four times as great an apparent error in X_l in the former as in the latter case. This partially accounts for the relatively large numerical errors in X_l for high-speed machines.

TABLE VIII

Type of machine	Test / Calc.	Per cent Absolute error	Per cent Numerical error From 1	Per cent Numerical error From mean
	Errors in values of X_l			
High-speed (II)	0.86	−13.6	31	30
Medium high-speed (III)	1.01	1.0	11	11
Medium low-speed (IV)	1.06	6.3	11	10
Low-speed (V)	1.06	5.7	12	10
Small low-speed (VI)	1.04	3.6	8	7
	Errors in values of X_d			
High-speed (II)	0.992	− 0.8	1.9	1.7
Medium high-speed (III)	1.000	0.0	1.0	1.0
Medium low-speed (IV)	1.008	0.8	1.4	1.3
Low-speed (V)	1.011	1.1	2.2	1.8
Small low-speed (VI)	1.019	1.0	1.9	1.7

TABLE IX

Type of machine	Average ratios X_{Slot}/X_l	X_{End}/X_l	X_{Dif}/X_l	X_l/X_d	$X_{l\,new}/X_{l\,old}$
High-speed	0.37	0.60	0.03	0.059	0.56
Medium high-speed	0.57	0.35	0.08	0.087	0.58
Medium low-speed	0.60	0.32	0.08	0.132	0.66
Low-speed	0.67	0.16	0.17	0.180	0.70
Small low-speed	0.58	0.16	0.26	0.252	0.72

The last column of Table IX gives the average ratio of the new value of leakage reactance, to the value given by the older formulas derived by Doherty and Shirley.[7] This ratio varies from a little over one-half for the high speed, to nearly three-quarters for the low-speed machines. The percentage error between test and calculated values of X_l is thus magnified on the new basis, so that a direct comparison of the accuracy given by the two formulas cannot be made. The older formula had an average apparent error of about 16 per cent, so that taking into account the test errors, which cause a considerable part of the dispersion, the new formula is indicated to have about half the true error of the old.

VII. ZERO PHASE-SEQUENCE REACTANCE CALCULATIONS

The zero phase reactance is important in the calculation of single phase short circuits, circulating currents in delta windings, and in other cases. The slot portion of this reactance is given by the complete equation (**3**), using the appropriate values of K_S from Fig. 2. For usual three-phase windings, K_S for the zero phase sequence reactance is equal to $3\,p - 2$. The end leakage portion is very small, and will be neglected. The differential leakage portion is given by the ratio of equation (**12a**) or (**13a**) to (**14a**) in Appendix A. Combining these expressions, we derive the following formula for the zero phase reactance of a three-phase, 60 deg. belt winding, with a pitch between 2/3 and unity, expressed as a decimal:

$$X_{\text{zero phase}} = \frac{20\,A}{K_p^2\,K_d^2\,\phi}\left(\frac{P\,L}{w\,S}\right)$$

$$\left[(3\,p-2)\,(d_3) + (9\,p-5)\,\frac{d_1}{12} - (9\,p-8)\,\frac{d_2}{12}\right]$$

$$+\frac{4\,A\,K_0\,(p-2/3)}{F\,K_p^2\,K_d^2}$$

$$\left[\left(\frac{P}{S}\right)^2 + \frac{1}{27} + 7/18\,(p-2/3) - (p-2/3)^2\right] \quad (\mathbf{38})$$

The corresponding formula for a winding pitch between 1/3 and 2/3 is:

$$X_{\text{zero phase}} = \frac{20\,A}{K_p^2\,K_d^2\,\phi}\left(\frac{P\,L}{w\,S}\right)$$

$$\left[(2-3\,p)\,d_3 + (7-9\,p)\,\frac{d_1}{12} - (4-9\,p)\,\frac{d_2}{12}\right]$$

$$+\frac{4\,A\,K_0\,(2/3-p)}{F\,K_p^2\,K_d^2}$$

$$\left[\left(\frac{P}{S}\right)^2 + \frac{1}{2}\,(2/3-p) - (2/3-p)^2\right] \quad (\mathbf{39})$$

The coefficient K_0 is introduced in (**38**) and (**39**) to allow for the reduction of the harmonic fluxes, especially the third, by induced currents in the rotor circuits. As all the terms except the first in the last brackets of these equations represent third harmonic fluxes, these terms are generally reduced to a fraction of their apparent values, and K_0 should ordinarily be taken as less than 0.5. The coefficient 4 of the last bracket in (**38**) includes the factor of 3/4 for the average permeance of a salient pole machine, as previously derived.

The zero phase reactance has been calculated for two machines, and the values compared with tests, with the following results:

Poles	Kv-a.	Freq.	Winding pitch	Assumed value of K_0	Zero-phase Sequence Reactance		
					Calculated	Test	Per cent error
6	435	60	2/3		0.026	0.021	−24
36	20,000	60	0.80	0.25	0.070	0.0675	4

VIII. CONCLUSIONS

It is believed that equation (**37**) is at once the most accurate and the simplest comprehensive formula for the armature leakage reactance of a synchronous machine that has been published. It requires no curves, no logarithms, and no tables for its use, but only a few slide rule operations, and it gives a value of leakage reactance which, added to the armature reaction reactance derived from flux plots, quite accurately checks the test values of synchronous reactance for the entire range of salient pole synchronous machines in commercial use.

The leakage reactance so determined averages about two-thirds of that given by the widely used formulas derived in reference No. 7, since the latter included as leakage reactance a part of the fundamental sine wave of air-gap flux due to the armature, which links the field winding in the direct axis, and thus constitutes a part of the true armature reaction. The new value of leakage reactance, added to the squirrel-cage reactance, checks observed standstill reactances of synchronous motors; and added to the field reactance checks the values of transient reactance found from oscillographic tests on synchronous generators. With the old reactance formulas, rather arbitrary reductions of the calculated field reactance and tooth tip reactance were necessary before checks with transient and standstill reactances could be secured. Finally, with the old formulas, the calculated increase in "internal voltage" of a synchronous generator under load gave much higher flux densities in low power factor machines than actually do occur, so that arbitrarily reduced values of field leakage under load were used in calculating saturation curves, thus establishing a series of compensating errors.

There is no space to demonstrate the validity of these statements here, and they are simply made to show that the acceptance of the new values of leakage reactance involves a fundamental revision of the generally adopted design constants of synchronous machines. In writing the paper, an effort has been made to make the new formulas and the new definitions such that the further refinements which the future will bring can be added without again altering our conceptions or the orders of magnitude of the characteristic constants of design.

NOMENCLATURE

A = Maximum ampere-turns per pole of armature reaction

d_1, d_2, d_3, w = Slot dimensions shown in Fig. 1

D = Gap diameter

f = Frequency

F = No-load air-gap field ampere-turns per pole

g = Air-gap length (radial)

K_B = Belt Leakage constant, given by Fig. 6

K_p, K_d = Pitch and distribution factors of armature winding (less than unity)

K_S = A function of winding pitch given by Fig. 2

L = Gross core length

M = $\frac{2 f L q Z^2}{10^7 S}$ for inch units

p = Winding pitch expressed as a decimal

P = Number of poles

q = Number of phases

s = Slots per pole = S/P

S = Total number of slots

V = Volts per phase

X_a = Armature reaction reactance

X_B = Belt leakage reactance

X_d = Total, or synchronous, reactance in direct axis

X_l = Total armature leakage reactance

X_t = Zigzag leakage reactance

Z = Series connected conductors per phase

α = Angle of end windings as shown in Fig. 4

θ = Angular phase difference between currents in upper and lower coil sides in one slot

ϕ = Flux per pole in c. g. s. lines

Appendix A

EXACT CALCULATION OF DIFFERENTIAL LEAKAGE REACTANCE

Chapman* has shown that the total inductance of a winding due to the air-gap flux can be very conveniently calculated by adding the inductances due to the rectangles of flux produced by successive pairs of symmetrically located slots. With this method the smallest unit of flux is that produced by a uniform m. m. f. acting over one slot pitch, so that the permeance variations due to the slot openings are quite satisfactorily taken into account by merely using the average permeance, as in calculating the magnetizing reactance. Thus, the point of view leads to the immediate conclusion that all the harmonic fluxes; (hence the zigzag and belt leakage reactances) are exactly proportional to the fundamental magnetizing reactance. And so the elaborate analyses of the permeance for the zigzag leakage flux as distinct from that for the fundamental flux, that have frequently been employed heretofore, are shown to be unnecessary.

We shall now proceed to calculate the total inductance of a regular, distributed, polyphase winding with s slots and q phase belts per pole (s/q a whole number), and a uniform air-gap length, following Chapman's method. In doing this, the self-inductive voltage produced in the set of coils forming the inner halves of two adjacent phase belts in the same phase by their air-gap flux is considered as the sum of the s voltages produced by the fluxes in the s teeth under one pole. The voltage due to any tooth is proportional to the square of the number of turns linking it, as the reluctance of each tooth pitch is the same, while both the flux per ampere and the volts per unit of flux are proportional to the number of turns. It is most convenient to make all the calculations in relative terms, and, therefore, to take as the unit of inductance the self-inductance which the two adjacent half-phase belts of one phase would have if all their turns were concentrated in one pair of slots exactly one pole pitch apart.

Assuming the winding pitch to be b slots short of full pitch, (b being not greater than s/q), and referring to Fig. 8, there will be $\frac{s\,(q-1)}{q} + 1 - b$ central teeth that are linked by all the turns in the two half belts of phase A, and hence the total inductance due to these teeth will be:

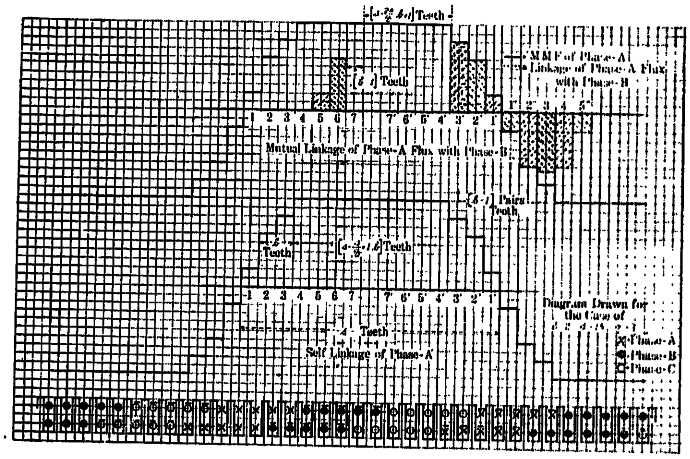


FIG. 8—IDEAL M.M.F. DIAGRAMS FOR POLYPHASE WINDINGS

$$\frac{1}{s}\left(\frac{s\,(q-1)}{q} + 1 - b\right)$$

The next two teeth (3 and 3′) outside these central ones will be linked by $\frac{s-q}{s}$ of the total turns, and will have a permeance $2/s$ of the total permeance, so that they will contribute to the inductance:

$$\frac{2\,(s-q)^2}{s^3}$$

Similarly, consecutive pairs of teeth will each have

---

*Reference No. 13.

q/s less turns than before, until the bth pair is reached. The next pair of teeth after that (1 and 1′) will be linked by $\frac{s - 2q - bq}{s}$ of the total turns, so that they will contribute an inductance equal to:

$$\frac{2}{s^3}\,(s - 2q - bq)^2$$

The following pairs of teeth will have consecutively $2q/s$ less turns than the last, until the outermost pair is reached. Hence, the total self-inductance of Phase A is:

$$L = \frac{1}{s^3}\Bigg[\frac{s^3\,(g-1)}{q} + s^2\,(1-b) + 2\,(s-q)^2 + 2\,(s-2q)^2$$
$$+ \ldots + 2(s-bq)^2 + 2\,(s-2q-bq)^2 + 2\,(s-4q-bq)^2 + \ldots\Bigg]$$

As the sum of the squares of the first n integers is $\frac{n\,(n+1)\,(2n+1)}{6}$, this may be reduced to:

$$\frac{1}{6s^3}\Bigg[6s^2\left(s - \frac{s}{q} + 1 - b\right)$$
$$+ 2q^2\left(\frac{s-1}{q}\right)\left(\frac{s}{q}\right)\left(\frac{2s}{q} - 1\right)$$
$$- 2q^2\left(\frac{s}{q} - b - 1\right)\left(\frac{s}{q} - b\right)\left(\frac{2s}{q} - 2b - 1\right)$$
$$+ 2q^2\left(\frac{s}{q} - 2 - b\right)\left(\frac{s}{q} - 1 - b\right)\left(\frac{s}{q} - b\right)\Bigg]$$

and this reduces to:

$$L_{AA} = 1 - \frac{2}{3q} + \frac{q}{3s^3}\,(2s - 3b^2s - bq + b^3q),$$
$$0 \leqq b \leqq \frac{s}{q}, \qquad \textbf{(1a)}$$

all for a winding pitch not less than $\frac{q-1}{q}$.

The linkages produced by phase A with phase B are similarly found, except that, as the currents in the two phases are out of time phase by π/q degrees, and as conditions of symmetry make the out-of-phase components of the linkages cancel with the corresponding components due to the other phases, the effective current in phase A must be reduced by a factor $\cos \pi/q$.

The number of teeth that are linked by all of phase B as well as by all of phase A is $\left(s - \frac{2s}{q} - b + 1\right)$, and these will create linkages with B equal to:

$$\left(\frac{1}{s}\right)\cos\frac{\pi}{q}\left(s - \frac{2s}{q} - b + 1\right)$$

Of the remaining teeth there are $b - 1$ that carry full flux, but are linked by successively fewer coils of the B phase. These contribute an inductance equal to:

$$\frac{1}{s^2}\cos\frac{\pi}{q}\,[(s-q) + (s-2q)$$
$$+ \ldots + (s - (b-1)\,q)]$$

There are also $b - 1$ pairs of teeth, (3′ and 3″) one of each pair of which carries positive flux and the other an equal negative flux, the former of which is linked by all, and the latter by successively less of the coils of phase B. Each pair is linked by successively less of the coils of phase A. Hence, these teeth contribute an inductance to phase B of:

$$\frac{1}{s^3}\cos\frac{\pi}{q}\,[q\,(s - (b-1)\,q) + 2q\,(s - (b-2)\,q)$$
$$+ \ldots + (b-1)\,q\,(s-q)]$$

The remaining teeth consists of pairs (6, 5″–7, 4″–1′,1″–2′,2″) carrying equal and opposite fluxes and linking the same numbers of coils of phase B, so that they contribute nothing to the inductance of phase B. Hence, the total increase of inductance of phase B due to phase A is:

$$\frac{1}{s^3}\cos \pi/q\Bigg[s^2\left(s - \frac{2s}{q} - b + 1\right)$$
$$+ s\,(s - q + s - 2q + \ldots + s - (b-1)\,q)$$
$$+ q\,(s - (b-1)\,q) + 2q\,(s - (b-2)\,q)$$
$$+ \ldots + (b-1)\,q\,(s-q)\Bigg]$$

which reduces to:

$$L_{AB} = \cos \pi/q\left[\frac{q-2}{q} - \frac{bq^2}{6s^3}\,(b^2 - 1)\right]$$
$$0 \leqq b \leqq s/q \qquad \textbf{(2a)}$$

The bracketed expression in **(2a)** is to be regarded as zero if it is negative, since the inductance is always positive.

The corresponding expressions for values of b between s/q and $2s/q$, corresponding to winding pitches between one and two phase belts short of full pitch, are, if q is greater than 2:

$$L_{AA} = 1 - \frac{1}{3q} - \frac{1}{s^2}(bs - q/3),\ s/q \leqq b\ \pi\frac{2s}{q} \qquad \textbf{(3a)}$$

and

$$L_{AB} = \cos \pi/q\Bigg[\frac{1}{2q} + \frac{q}{2\ s^2} + \frac{3b}{2s}$$
$$- \frac{3b^2q}{2s^2} + \frac{b^3q^2}{3s^3}\ \frac{bq^2}{3s^3}\Bigg] \qquad \textbf{(4a)}$$

$$s/q \leqq b \leqq \frac{2s}{q}$$

And, the value of the self-inductance of one phase for winding pitches greater than zero by not more than one-phase belt width is:

$$L_{A_A} = \frac{q\,(s-b)}{3\,s^3}\left[q + (s-b)\,(3\,s - q\,s + b\,q)\right],$$

$$s - \frac{s}{q} \leqq b \leqq s \qquad \textbf{(5a)}$$

There is no need of considering values of q greater than 3, except for full pitch windings, as three-phase belts per pole are all that are used in practise. A squirrel-cage winding always has one slot per pole per phase, so that its differential leakage ratio is the same for all values of pitch. For this case $s/q = 1$, and (**2a**) reduces to:

$$L_{A_B} = \frac{s-2}{s}\cos \pi/s \qquad \textbf{(6a)}$$

and the corresponding expression for the nth phase is:

$$L_{A_N} = \frac{s - 2\,(n-1)}{s}\cos\frac{(n-1)\,\pi}{s} \qquad \textbf{(7a)}$$

The total inductances per phase are obtained by adding the self and effective mutual inductances, as given by the foregoing expressions. For $q = 2$, (**2a**) is zero, so that the total inductance of a two-phase winding is from (**1a**):

$$L_{90^\circ} = 2/3 + \frac{1}{3\,s^3}\,(4\,s - 6\,b^2 s - 4\,b + 4\,b^3) \qquad \textbf{(8a)}$$

For $q = 2$, (**1a**) and (**5a**) are identical, so that the single equation (**8a**) is valid over the entire range of pitches from 0 to 1, for two-phase windings.

For a three-phase winding, $q = 3$, and the total inductance of one phase is given by the sum of (**1a**) and twice (**2a**) for pitches between 2/3 and 1, and by the sum of (**3a**) and twice (**4a**) for pitches between 1/3 and 2/3:

$$L_{60^\circ} = 10/9 + \frac{1}{2\,s^3}\,(4\,s - 6\,b^2\,s - 3\,b + 3\,b^3),$$

$$0 \leqq b \leqq s/3 \qquad \textbf{(9a)}$$

and

$$L_{60^\circ} = 19/18 + \frac{1}{2\,s^3}\,(5\,s + b\,s^2 - 6\,b - 9\,b^2\,s + 6\,b^3),$$

$$s/3 \leqq b \leqq \frac{2\,s}{3} \qquad \textbf{(10a)}$$

(**9a**) and (**10a**) become identical if $b = s/3$.

For a squirrel-cage winding, the total inductance of one phase is, from (**1a**) and (**7a**), placing $b = 0$ and $q = s$ in the former:

$$L_K = 1 + \frac{s-2}{s}\cos \pi/s + \frac{s-4}{s}\cos\frac{2\,\pi}{s} + \ldots$$

$$= 1 - \frac{2}{s}\cos\left(\pi/s + 2\cos\frac{2\,\pi}{s} + 3\cos\frac{3\,\pi}{s}\right.$$

$$\left. + \ldots\ldots + (s-1)\cos\frac{(s-1)\,\pi}{s}\right)$$

$$= \frac{1}{s}\,c\,s\,c^2\,\pi/2\,s \qquad \textbf{(11a)}$$

It is interesting to compare the total reactances of the several types of windings by means of these formulas. For a regular three-phase winding, equations (**9a**) and (**10a**) show that:

$$L_{60^\circ} = \frac{10}{9} + \frac{.2}{s^2} \text{ for full pitch}$$

$$= \frac{149}{144} + \frac{7}{4\,s^2} \text{ for 5/6 pitch}$$

$$= \frac{5}{6} + \frac{3}{2\,s^2} \text{ for 2/3 pitch}$$

$$= \frac{5}{9} + \frac{1}{s^2} \text{ for 1/2 pitch}$$

$$= \frac{5}{18} + \frac{1}{2\,s^2} \text{ for 1/3 pitch}$$

For a regular two-phase winding (**8a**) shows that:

$$L_{90^\circ} = \frac{2}{3} + \frac{4}{3\,s^2} \text{ for full pitch}$$

$$= \frac{9}{16} + \frac{1}{s^2} \text{ for 3/4 pitch}$$

$$= \frac{1}{3} + \frac{2}{3\,s^2} \text{ for 1/2 pitch}$$

$$= \frac{5}{48} + \frac{1}{3\,s^2} \text{ for 1/4 pitch}$$

For a single-phase winding made by using two legs of a regular three-phase winding, the total inductance is found by substituting 1 for $\cos \pi/2$ in (**2a**) and adding (**1a**) to it, which gives identically the same equation (**9a**) as for the original three-phase winding. Thus, the per cent line-to-line reactance of a three-phase winding is always identical with the per cent three-phase reactance. For a single-phase winding consisting of one leg of a three-phase winding only, the total inductance is given by (**1a**), (**3a**), and (**5a**), whence:

Line to neutral

$$L_{60^\circ} = \frac{7}{9} + \frac{2}{s^2} \text{ for full pitch}$$

$$= \frac{17}{24} + \frac{3}{2\,s^2} \text{ for 5/6 pitch}$$

$$= \frac{5}{9} + \frac{1}{s^2} \text{ for } 2/3 \text{ pirch}$$

$$= \frac{7}{18} + \frac{1}{s^2} \text{ for } 1/2 \text{ pitch}$$

$$= \frac{2}{9} + \frac{1}{s^2} \text{ for } 1/3 \text{ pitch}$$

$$= \frac{5}{72} + \frac{1}{2\,s^2} \text{ for } 1/6 \text{ pitch}$$

Thus, the line-to-neutral air-gap flux reactance of a three-phase winding operated single-phase is 7/10 of its value in three-phase operation for full and half-pitch windings, but is only 2/3 of its three-phase value for 2/3 pitch windings. For a 1/3 pitch winding, it is 4/5 of its three-phase value. These variations are due to the triple harmonic fields that are produced by each phase winding separately, but that are canceled out by a 2/3 pitch, or by a three-phase connection.

Since the zero phase sequence reactance is equal to three times the line-to-neutral, less twice the balanced three-phase reactance, of a three-phase winding, the foregoing expressions lead to the following expressions for the zero phase sequence inductance:

Zero phase

$$L_{60^\circ} = \frac{1}{9} + \frac{1}{s^3}\,(2\,s - 3\,b^2\,s - 6\,b + 6\,b^3), \qquad 0 \leqq b \leqq s/q \qquad \textbf{(12a)}$$

$$= \frac{5}{9} - \frac{1}{s^3}\,(2\,s + 4\,b\,s^2 - 6\,b - 9\,b^2\,s + 6\,b^3) \qquad \frac{s}{q} \leqq b \leqq \frac{2\,s}{q} \qquad \textbf{(13a)}$$

Hence:

$$\text{Zero phase } L_{60^\circ} = \frac{1}{9} + \frac{2}{s^2} \text{ for full pitch}$$

$$= \frac{1}{18} + \frac{1}{s^2} \text{ for } 5/6 \text{ pitch}$$

$$= 0 \qquad \text{ for } 2/3 \text{ pitch}$$

$$= \frac{1}{18} + \frac{1}{s^2} \text{ for } 1/2 \text{ pitch}$$

$$= \frac{1}{9} + \frac{2}{s^2} \text{ for } 1/3 \text{ pitch}$$

All of this zero phase inductance is leakage flux, since the fundamental m. m. fs. of the three-phases cancel with this connection.

The useful, or fundamental, flux produced by the same windings can be readily calculated from the usual pitch and distribution factors, and the difference remaining after the useful is subtracted from the total reactance will give the desired values of total differential leakage reactance.

The ratio of the useful to the total flux generated by a single full-pitch coil is $8/\pi^2$, as this is the ratio of the area of a sine wave to the area of a rectangle whose fundamental it is. This factor, multiplied by the squares of the pitch and distribution constants of the winding, and by the ratio, $q/2$, of the total revolving field produced by all the phases acting conjointly to the field made by a single phase, gives an expression for the reactance due to the useful flux. Or:

$$L_m = \frac{4\,q^3}{\pi^2}\;\frac{\sin^2 \pi/2\,q\,\sin^2 \pi/2\,(1 - b/s)}{s^2 \sin^2 \pi/2\,s} \qquad \textbf{(14a)}$$

Putting $q = 2$ in (**14a**), dividing it into (**8a**), and subtracting unity from the result, we have for the total differential leakage reactance of a two-phase winding, expressed as a ratio to the fundamental magnetizing reactance:

$$\frac{X_{D_{90^\circ}}}{X_m} = \frac{\pi^2\,(s - b)\,(s^2 + b\,s - 2\,b^2 + 2)\,\sin^2 \pi/2\,s}{24\,s\,\sin^2 \pi/2\,(1 - b/s)} - 1 \qquad \textbf{(15a)}$$

If s is large, and b is zero, corresponding to the case of a full-pitch two-phase winding, (**15a**) gives:

$$\frac{X_{D_{90^\circ}}}{X_m}\;\frac{\pi^2\,(s^2 + 2)}{24}\,\sin^2 \pi/2\,s - 1 = \frac{\pi^4}{96} + \frac{\pi^4}{48\,s^2}\left(.412 - \frac{.687}{s^2} + \ldots\right) - 1 \qquad \textbf{(16a)}$$

If s is very large, (**16a**) reduces exactly to (**31**), thus confirming the latter result. For small values of s, equation (**15a**) is the only convenient means of obtaining exact values for $X_{D_{60}}$. Numerical values of it for some usual windings are given in Table X.

TABLE X

DIFFERENTIAL LEAKAGE REACTANCE OF TWO-PHASE WINDING EQUATION (15a)

Pitch deficiency b	$s/2$ = slots per pole per phase					
	1	2	3	4	5	6
0	0 2337	0.0840	0.0468	0.0330	0.0264	0 0229
1	0.2337	0.0583	0.0334	0.0251	0.0213	0.0193
2		0.0840	0.0285	0.0177	0.0147	0.0138
3		0.0840	0.0468	0.0189	0.0116	0.0096
4			0.1019	0.0330	0.0148	0.0080
5			0.2337	0.0649	0.0265	0.0130
6				0.1222	0.0486	0.0229
7				0.2337	0.0839	0.0398
8					0.1382	0.0650

Putting $q = 3$ in (**15a**), dividing it into (**10a**) and (**11a**), and subtracting unity from the result, the total differential leakage reactance for a three-phase, 60 deg. phase belt winding is:

$$\frac{X_{D_{60}}}{X_m}$$

$$= \frac{\pi^2(20\, s^3 + 36\, s - 54\, b^2 s - 27\, b + 27\, b^3 \sin^2 \pi/2\, s}{486\, s \sin^2 \pi/2\,(1 - b/s)} - 1,$$

$$0 \lesseqgtr b \lesseqgtr \frac{s}{3} \qquad \textbf{(17a)}$$

and

$$\frac{X_{D_{60}}}{X_m}$$

$$= \frac{\pi^2(19\, s^3 + 45\, s + 9\, b\, s^2 - 54\, b - 81\, b^2 s + 54\, b^3) \sin^2 \pi 2\, s}{486\, s \sin^2 \pi/2\,(1 - b/s)} - 1,$$

$$s/3 \lesseqgtr b \lesseqgtr \frac{2\, s}{3} \qquad \textbf{(18a)}$$

For a full-pitch three-phase winding with a large number of slots per pole, (**17a**) may be reduced to:

$$\frac{X_{D_{60}}}{X_m} = \frac{2\, \pi^2\, (5\, s + 9)}{243} \sin^2 \pi/2\, s - 1$$

$$= \frac{5\ \pi^4}{486} + \frac{\pi^4}{54\, s^2}\left(0.543 - \frac{0.672}{s^2} + \ldots\right) - 1 \qquad \textbf{(19a)}$$

If s is very large, (**19a**) reduces exactly to (**32**).

Numerical values of (**17a**) and (**18a**) for some usual three-phase windings are given in Table XI.

For a squirrel-cage winding, $q = s$ and $b = 0$, so that by substituting these values in (**15a**) and dividing the result into (**14a**), the differential leakage reactance is found to be:

TABLE XI
DIFFERENTIAL LEAKAGE REACTANCE OF THREE-PHASE WINDINGS
EQUATIONS (17a) AND (18a)

Pitch deficiency b	$s/3$ = slots per pole per phase					
	1	2	3	4	5	6
0	0.0966	0.0284	0.0140	0.0089	0.0064	0.0052
1	0.0966	0.0235	0.0115	0.0074	0.0055	0.0045
2	0.0966	0.0284	0.0111	0.0063	0.0044	0.0035
3		0.0284	0.0140	0.0069	0.0041	0.0030
4		0.0284	0.0143	0.0089	0.0050	0.0031
5			0.0137	0.0092	0.0064	0.0040
6			0.0140	0.0089	0.0068	0.0052
7				0.0083	0.0067	0.0056
8				0.0089	0.0062	0.0055
9					0.0057	0.0052

$$\frac{X_{D_K}}{X_m} = \frac{\pi^2}{4\, s^2}\, c\, s\, c^2\, \pi/2\, s - 1 \qquad \textbf{(20a)}$$

If s is large, this may be developed into a series of which only the first term need be considered:

$$\frac{X_{D_K}}{X_m} = \frac{\pi^2}{12\, s^2}\left(1 + \frac{\pi^2}{20\, s^2} + \frac{\pi^4}{504\ \ s^4} + \ldots\right) \qquad \textbf{(21a)}$$

For large values of s, (**21a**) agrees exactly with (**25**), thus confirming the previously derived formula for zigzag leakage reactance.

Bibliography

1. 1928, Park, R. H., and Robertson, B. L. *The Reactances of Synchronous Machines,* Winter Convention of the A. I. E. E., 1928.

2. 1927, Wieseman, R. W., *Graphical Determination of Magnetic Fields; Practical Applications to Salient-Pole Synchronous Machine Design,* TRANS., A. I. E. E., Vol. XLVI, p. 141.

3. 1926, Doherty, R. E. and Nickle, C. A., *Synchronous Machines—Part I. An Extension of Blondel's Two-Reaction Theory,* TRANS. A. I. E. E., 1926, Vol. 45, p. 912.

4. 1926, Gray, A., "Electrical Machine Design, 2d. ed. Rev." by P. M. Lincoln, p. 523, 1926, McGraw, N. Y.

5. 1922, Siegel, E., "Leakage Calculations for Three-Phase Induction Motors with Phase-Wound Rotors," (in German), *Elekt. u. Masch.,* Vol. 40, pp. 217-22, May 7, 1922. *Sci. Abstr.,* Sec. B, Vol. 25, pp. 502-03, Oct. 1922, Abstract translation.

6. 1920, Alger, P. L. and Eksergian, R., *Induction Motor Core Losses,* A. I. E. E. JOUR., 1920, Vol. 39, October, pp. 906-20.

7. 1918, Doherty, R. E. and Shirley, O. E., *Reactance of Synchronous Machines and Its Applications,* A. I. E. E. TRANS., 1918, Vol. 37, pp. 1209-97; discussion, pp. 1298-1340.

8. 1916, Chapman, F. T., "Air-gap Field of the Polyphase Induction Motor," *Electrician,* Lond., Vol. 77, pp. 663-67, 705-08, 730-34, 904-06, 936-38; Vol. 78, pp. 9-12, 49-50. Aug 18-Sept. 1, Sept. 22-29, Oct. 6-13, 1916.

9. 1912, Fechheimer, C. J., *Self-Starting Synchronous Motors,* A. I. E. E. TRANS., 1912, Vol. 31, pp. 529-85; Discussion, pp. 586-604.

10. 1909, Schenkel, M., "Determination of Leakage Flux in Alternating-Current Machines," (in German), *Elekt. u. Masch.,* Vol. 27, pp. 201-08, Feb. 28, 1909. *Sci. Abstr.,* Sec. B, Vol. 12, pp. 160-61, Apr. 1909, Abstract-translation.

11. 1907, Adams, C. A. and others, *Fractional Pitch Windings for Induction Motors,* A. I. E. E. TRANS., 1907, Vol. 26, pp. 1485-1503; Discussion, pp. 1525-26.

12. 1907, Hellmund, R. E., *Zigzag Leakage of Induction Motors,* A. I. E. E. TRANS., 1907, Vol. 26, pp. 1505-24.

13. 1905, Adams, C. A., *The Design of Induction Motors—With Special Reference to Magnetic Leakage,* A. I. E. E. TRANS., 1905, Vol. 24, pp. 649-84; Discussion, pp. 685-87.

14. 1904, Adams, C. A., "The Leakage Reactance of Induction Motors," Int. Elec. Congress, St. Louis, *Trans.,* Vol. pp. 706-24; Discussion, pp. 725-28, 1904.

15. 1904, Hobart, H. M. and Punga, F., *A Contribution to the Theory of the Regulation of Alternators,* A. I. E. E. TRANS., 1904, Vol. 23, pp. 291-322; Discussion, pp. 323-336, 340-43.

Discussion

B. A. Behrend: In connection with Mr. Alger's very complete investigation of the factors which determine the armature reactance of a synchronous machine the question may be raised whether it is more satisfactory for the designer to derive his characteristic data from comparison with known similar designs or by a fresh calculation for each particular case. As a rule the designs vary comparatively little, thus making it possible to obtain design data by comparison. The analysis of the constituent parts of the total leakage field as carried out by Mr. Alger is in keeping with the desire to leave no part of the theory unexplored and to link together the leakage of synchronous and induction machines. In this very painstaking and intricate procedure I doubt that a method which is evolved by one designer and which suits his method of working is likely to appeal equally to another designer. I am inclined to view such methods as to a great extent individual though notwithstanding this they are of the greatest importance in clarifying obscure problems and showing us the empirical basis of our assumptions. It would be interesting to know just how many factors could be grouped together without entailing errors greater than those ordinarily encountered in the limitations of design and workmanship.

A. A. Bennett: I wish to call attention to Fig. 4. The currents flowing along the bent end windings there are supposed to be distributed in sinusoidal fashion, and they are then analyzed into two components, one flowing peripherally and the other axially.

I object to the idea here expressed of analyzing something into components which have no meaning. We cannot analyze the cosine function into the product of a co function by a sine function. The peripheral current is supposed to be flowing along a conductor, but it isn't a real current in any proper sense. At one point, the current is large and flowing in one direction; at another point, there is no current at all, and, at still another point, it is large and flowing in the opposite direction.

E. B. Shand: Mr. Alger has modified somewhat the methods of separating out the machine fluxes so that the resultant leakage reactance is considerably less than that given by a number of other formulas. I have always considered that these latter formulas, beginning with the classic one of Dr. Arnold, were inconsistent in using fluxes which should properly be classed as cross-magnetizing and thereby gave results which could not be entirely reconciled with all data found on tests. This was pointed out in my discussion on "*Variable Armature Leakage Reactance in Salient-Pole Synchronous Machines*" by V. Karapetoff, A. I. E. E. TRANS., 1926, p. 733. Formulas of the type of this present may be relied upon to give more complete information of machines.

P. L. Alger: Mr. Shand's remark agrees with what many of us have felt for a long time, that the necessity for using a lower value of leakage reactance to check transient performance than for synchronous operation indicated that the old value of synchronous leakage reactance was too high. The new and lower values of leakage reactance will, I am sure, clear up many discrepancies heretofore existing in synchronous-machine theory. It is well to remember, however, that the point of division between the reactances of armature reaction and armature leakage cannot be very exactly defined. For example, my colleague, D. H. Harms, has pointed out that a small part of the fundamental flux wave in the direct axis, due to the armature reaction, passes along the air gap space without entering the field at all, and that this part is truly a leakage reactance, although in the paper it is included with the armature reaction. From Appendix B of reference 3, the magnitude of this part of the reactance is approximately:—

$$\Delta X_{ad} = \frac{P^2 g^2}{3 D^2} X_{ad},$$

which usually amounts to less than 0.5 per cent of X_{ad}, but may be as much as 2 per cent in some high-speed machines. This more exact distinction between the two elements of the reactance makes the calculated values of transient reactance a trifle higher.

I agree with Professor Bennett that my conception of the axial end leakage flux, as due to a discontinuous peripheral current, is a highly artificial one, and that it would be more satisfying to consider the end leakage flux as a whole. I can only say that the device adopted affords a short cut that leads to simpler and more definite results than I could find by any other method.

The Reactances of Synchronous Machines

BY R. H. PARK[1] and B. L. ROBERTSON[1]

Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—Until somewhat recently, synchronous machine theory has been satisfied with a relatively few characteristic constants, or reactances, in terms of which the behavior of machines has been calculated. Present theory, however, requires many more coefficients. There are now generally recognized two values each of leakage, synchronous, and transient reactance which correspond to the two symmetrical axes of magnetization of the armature current and which refer to balanced operation. Negative and zero phase-sequence reactances are also employed to determine operation under unbalanced conditions, and it is possible and desirable to distinguish other reactances. In view of the increasing complexity of the subject it is felt that a critical survey of it is in order and the object of the paper has been to provide that survey.

The paper has been divided into two parts. Part I is descriptive, and treats the subject with regard to those factors which are important to application or operating engineers, and to designers. In particular, the major types of reactances which include the synchronous, transient, and phase-sequence reactances, are discussed. These quantities are defined and their methods of test outlined. It appears necessary to consider a second type of transient reactance, namely sub-transient reactance. Both reactances may be determined from short-circuit oscillograms as illustrated in the paper. A table is included which gives the numerical range of reactances for the various types of synchronous machines.

Part II discusses the theoretical considerations, with a view to broadening and classifying existing conceptions of reactance. It includes the effect of external reactance on negative phase-sequence reactance, and the variation in this latter quantity depending upon whether current or voltage is impressed on the machine. An important aspect of the division of synchronous reactance into armature reaction and leakage reactance is discussed. Transient reactance is shown to be the difference between synchronous reactance and the ratio of the mutual reactance between armature and field and the total field reactance. Calculations are included to show that the short-circuit and open-circuit time-constants are related to each other in a simple manner.

The appendixes cover the following subjects:

a. Application of the Principle of Superposition to Synchronous Machine Analysis.

b. Replacing the Effect of Induced Field Currents by Employing Transient instead of Synchronous Reactances.

c. Significant Rotor Circuits in Addition to the Main Field Winding (which affect transient reactances).

d. The Negative Phase-Sequence Reactance of a Synchronous Machine with Negative Phase-Sequence Voltage Impressed.

e. Construction of Equivalent Circuits: Concept of Field Leakage Reactance.

f. Calculation of Total Field Reactance.

g. Relation of the Mutual Reactance Between Armature and Field to the No-Load Excitation Current.

h. Relation Between Three-Phase and Single-Phase Reactances.

i. Discussion of the System of Notation Used in the Paper.

j. Per-Unit Representation of Quantitities.

* * * * *

THE REACTANCES OF SYNCHRONOUS MACHINES

IN the course of the development of the theory of synchronous machines there has come into use, along with an increased knowledge of the behavior of such machines under varying conditions of operation, a correspondingly increased number of characteristic constants or coefficients referring generally to reactive voltages, in terms of which this behavior is calculated.

Thus, in place of the concept which was formerly held of a single leakage reactance and armature reaction, there are at present generally recognized two values each of leakage, synchronous, and transient reactances, corresponding to the two symmetrical axes of magnetization of armature current. In addition to these reactances, which refer to balanced operation, there are also values of negative and zero phase-sequence reactance, which are employed to determine operation under unbalanced conditions. It is also possible and desirable to distinguish other reactances, particularly those which obtain with the rotor stationary, and those which are encountered by harmonic currents. At the same time, improvements have recently been made in methods of calculating leakage reactance and armature reaction. Therefore, on account of the increasing complexity of the subject it is felt that a critical survey of it is in order. The object of the paper has been to provide a survey of this character which would be of value both from the standpoint of system analysis and from that of fundamental theory.

Since the requirements of these two points of view are to a considerable extent distinct, the paper has been divided into two parts. Part I treats the subject with regard to those factors which are important to designing and application engineers, and to operating engineers. Part II discusses in detail the theoretical considerations involved, with a view to broadening and classifying existing conceptions of reactance.

PART I. REACTANCES PARTICULARLY AFFECTING OPERATION

In the analysis of system stability, and in the calculation of the effect of short-circuits, the factors of interest to operating engineers are those which relate to the behavior of the machine as viewed from the armature terminals. The most significant of these factors are the armature reactances of machines to normal frequency current having any distribution between phases, any power-factor, and whether transient or sustained; also, in connection with transient components of current, their rates of decay, or decrements.

Types of Reactances. The major types of armature reactances then are to be distinguished according to:

---

[1] Both of the Genl. Engg. Dept., General Electric Co., Schenectady, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-17, 1928.

28-21

A. Distribution; that is, the relative distribution of current between phases.

Any distribution of armature current may be expressed as the superposed sum of three symmetrical components:[3]

a. Balanced three-phase currents of normal phase rotation, or *positive phase-sequence,*

b. Balanced three-phase currents of reverse phase rotation, or *negative phase-sequence,* and

c. Balanced three-phase currents of equal time phase, or *zero phase-sequence.*

B. Method of application in time of positive phase-sequence currents.

Here it is desirable to distinguish between:

a. Steadily applied, or sustained currents,

b. Suddenly applied, or transient currents.

In the case of armature reactances these differences are due to the transient currents induced in the rotor circuits when armature current is suddenly applied. If, as is generally so, there are closed circuits on the rotor in addition to the field winding, as for example an amortisseur, it is found that some of the currents in the rotor circuits may die away very rapidly. In order to distinguish between the effect of these currents and the effect of those which die away slowly, it is desirable to establish two (or more) transient reactances. The higher reactance, *i. e.*, the reactance which depends on currents that die away slowly, is then referred to as the *transient reactance* of the machine. The lower reactance may be referred to as the *sub-transient reactance* on account of its lower value.

C. Position of the rotor with respect to axis of magnetization of positive phase-sequence currents.

When the rotor is moving synchronously, the positive phase-sequence current can be resolved into two components, one of which magnetizes in the axis of the poles, and the other in the inter-polar space. Accordingly, these components are referred to as *direct* and *quadrature,*[4] and the corresponding reactances are:

a. Direct, or

b. Quadrature.

Thus, to summarize, the types of armature reactance which have been considered so far are to be distinguished according to whether they are:

A. Positive, negative, or zero phase-sequence,

B. Sustained, transient, or sub-transient,

C. Direct or quadrature.

The determination of a suitable notation for these reactances should depend more upon present and future requirements than merely upon previous practise.

After careful consideration of the subject, it seemed that a consistent notation could be obtained by denoting the various types of reactance according to the method outlined below:

A. Distribution.
    Armature reactances
        Positive phase-sequence x_1, or no subscript
        Negative phase-sequence x_2
        Zero phase-sequence x_0
B. Application in time.
    Sustained—No special indication.
    Transient—One prime, *i. e.*, x'
    Sub-Transient—Two primes, *i. e*, x''

If it is desired to distinguish other degrees of transiency, additional primes may be added.

C. Position of rotor.
    Direct x_d
    Quadrature x_q.

The various armature reactances of the types discussed are then represented by the notation shown in Table I.

TABLE I

ARMATURE REACTANCES

x_d	Direct synchronous, positive phase-sequence.
x_q	Quadrature synchronous, positive phase-sequence.
x_d'	Direct transient, positive phase-sequence.
x_q'	Quadrature transient, positive phase-sequence.
x_d''	Direct sub-transient, positive phase-sequence.
x_q''	Quadrature sub-transient, positive phase-sequence.
x_2	Negative phase-sequence.
x_0	Zero phase-sequence.

Definitions and Methods of Test. Synchronous reactance: The problem of determining the relations between fundamental components of armature voltage and current during steady operation has been thoroughly analyzed by Blondel,[5] Arnold,[6] and Doherty and Nickle.[7] The accepted theory may be briefly summarized as follows:

First, the balanced three-phase system of armature currents is resolved into two component three-phase systems: one in which the current in each individual phase reaches a maximum at the instant that the axis of the field pole coincides with the axis of magnetization of the phase under consideration; and another in which the current in each individual phase reaches maximum at the instant the axis of magnetization is in line with the axis midway between poles, that is, one-quarter cycle later. The former is called the *direct* component because it produces direct component of armature reaction. The latter is the *quadrature* component.

Then coefficients are defined expressing the ratio of reactive component of voltage to armature current for each type of current. These coefficients are the *direct* and *quadrature* synchronous reactances of the machine.

These reactances may be expressed either in ohms, or as a ratio of their reactance in ohms to normal ohms, where normal ohms is the ratio of normal voltage and current. When so expressed, they are referred to as *per-unit reactances*.[8] Thus:

---

[3] Reference (11).

[4] A considerably broader conception of direct and quadrature quantities is developed in Part II.

[5] Reference (13).

[6] Reference (12).

[7] Reference (8).

$$\text{Per-unit reactance} = \frac{\text{reactance in ohms}}{\text{normal ohms}}$$

$$\text{Normal ohms} = \frac{\text{normal line-to-neutral voltage}}{\text{normal line current}}$$

Consequently, *per-unit direct synchronous reactance* is defined as the *per-unit* fundamental component of reactive armature linkages, due to unit sustained direct component of armature current. Quadrature synchronous reactance is defined similarly. The resultant terminal voltage is then found by subtracting the reactance drop in each axis from the no-load terminal voltage corresponding to the existing field current, assuming no saturation. Hence, if the machine is dead short-circuited at normal voltage, the reciprocal of the per-unit sustained armature current is equal to the per-unit direct synchronous reactance.

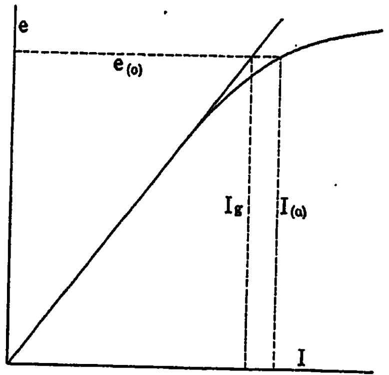


FIG. 1—SATURATION CURVE

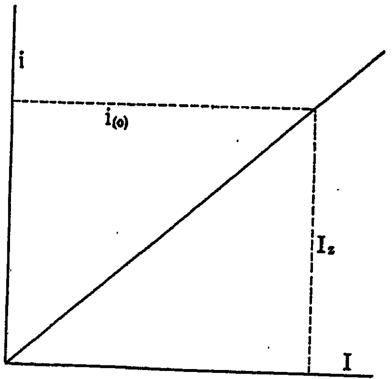


FIG. 2—SYNCHRONOUS IMPEDANCE CURVE

Synchronous reactances can be measured accurately by test. Thus, direct synchronous reactance, x_d, is conveniently determined from the saturation and synchronous impedance curves of the machine, which Figs. 1 and 2, respectively, represent.

In the figures:

$e_{(0)}$ = normal armature voltage.[9]
$i_{(0)}$ = normal armature current.
$I_{(0)}$ = field current required to produce $e_{(0)}$ at no-load.
I_g = field current required to force normal flux across the air-gap.
I_s = field current required to produce normal armature current at short-circuit.

Synchronous reactance, in accordance with established practise, is understood to be calculated on the basis of no saturation, and all quantities are referred to the air-gap line. Hence, the direct component of synchronous reactance is measured as the ratio of I_s and I_g, *i. e.*,

$$x_d = \frac{I_s}{I_g}$$

The quadrature synchronous reactance, x_q, of a three-phase machine may be determined by the following method.[10] The machine, coupled in electrical quadrature to a relatively very large synchronous motor and connected to the motor lines, is operated unexcited. The ratio of the armature voltage, e, to the corresponding armature current, i, is the quadrature synchronous reactance in ohms. Expressed as a per-unit quantity, it is:

$$x_q = \frac{e}{i} \cdot \frac{i_{(0)}}{e_{(0)}} \, .$$

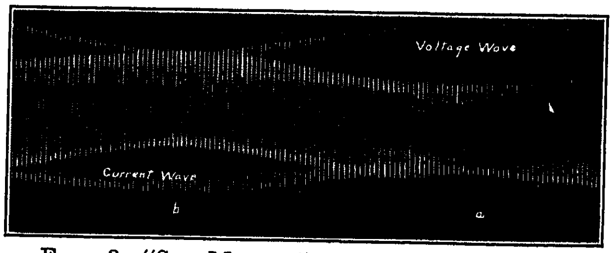


FIG. 3—"SLIP METHOD" OF OBTAINING x_d AND x_q

The disadvantage of this scheme is the limitation it imposes on the sizes of the machines which may be tested, due to the fact that the driving motor must be of sufficient size so that its angular displacement will not be appreciable.

A method, referred to as the "slip method," affords, however, a practical means of obtaining x_q, and may also be used to obtain x_d. The machine to be tested is left unexcited and a three-phase voltage is applied to its armature. The rotor is coupled to a driving motor, of sufficient capacity to overcome the reluctance torque at the reduced voltage, and is run at a low value of slip. In this manner the poles are slipped past the m. m. f. wave. The magnetizing current is then a function of the rotor position with respect to the m. m. f.

The oscillogram of Fig. 3 is the result of such a test.

---

[8] The desirability of using the term *per-unit* to characterize reactance, when expressed in terms of unity as a base, is discussed in Appendix J.

[9] The subscript, (0), enclosed in parentheses, refers to normal values, and should not be confused with the subscript, 0, used to indicate zero phase-sequence quantities.

[10] Two alternative methods of obtaining x_q are given in References (3) and (7) respectively.

It will be noticed that both the current and voltage are modulated harmonically as the pole position changes, the variation in voltage being due to reactance drop in the source of power employed.

When the poles line up with the axes of the phases, position (a) on the above curve, the current is a minimum. At this position the ratio of voltage to current is the direct synchronous reactance. Similarly, when the axes of the phases are midway between the poles, position (b), the ratio of voltage to current is the quadrature synchronous reactance.

This last means of testing for the synchronous reactances is simple and has several points of advantage. Thus, the power supply to the tested machine can be small, just enough so that the meter readings are easily obtained. Corresponding readings, or oscillograms, of voltage and current can be taken, and the calculated reactances plotted as a function of time or pole position. Also, the driving motor need only be of sufficient capacity to supply the reluctance torque. Moreover, if the motor speed is well regulated, so as to produce a low value of slip, there will be no difficulty in taking readings directly from indicating instruments, since the swings of the meter needles can be easily followed. In some cases, it may be desirable to take oscillograms of both voltage and current.

Transient reactance. When a machine is subjected to a three-phase short-circuit from an initial condition of no-load, the flux-linkages in every rotor circuit must initially stay constant. But, since the armature current tends to demagnetize these circuits, it is necessary for the currents in them to increase, in order that the condition of constant flux-linkages in each circuit may be fulfilled.

This increase in rotor m. m. f. is responsible for the familiar fact that the initial short-circuit current of a machine is greater than that obtained under sustained conditions, after the induced d-c. currents in the field and additional rotor circuits have died away. The resultant armature current is calculated in terms of the voltage before the short-circuit, as the ratio of that voltage to a value of reactance referred to as the *transient* reactance of the machine.

More specifically, this reactance is, of course, the direct component of transient reactance, since it involves only reactive or direct axis component of current. Thus, direct transient reactance, x_d', is

$$x_d' = \frac{e}{i}$$

where e is the voltage preceding short-circuit and i is the symmetrical component of armature current just after the short-circuit.

The exact interpretation of this definition would imply that the current i is to be measured as the value of the envelope of the wave of symmetrical component of current, as projected to the instant of short circuit. But, in machines with closed rotor circuits in addition to the field, this initial value may not satisfactorily represent the performance of the machine on account of the fact that the current induced in some or all of these additional circuits may die away very rapidly. It is, therefore, desirable to establish the conception of transient reactance proper, as the value of apparent transient reactance which applies to the current variation after the rapidly decaying components of current have died away. It can be obtained, as shown in Fig. 4, by projecting the envelope of the current wave to the instant of short circuit, neglecting the first one or two peaks. Point (b) of Fig. 4 corresponds to the transient reactance proper. The reactance determined by the projected value of the envelope of the first few peaks of the current wave, point (a), is then referred to as the sub-transient reactance. In practise, the low value of this reactance may be due in part to saturation in the leakage paths.

Practically, the value of short-circuit current can be expressed as the sustained value plus one or more components, each of which die away exponentially. Transient reactance proper is then the reciprocal of the current that would exist at $t = 0$ if the high-speed terms were absent. These relations may be made clear by reference to Fig. 5, which shows the observed values of symmetrical component of armature current from a three-phase short-circuit test on a large water wheel generator. The test was taken at reduced voltage so as to exclude saturation, and the per-unit values of current have been multiplied by a factor so as to refer to a test at normal voltage without saturation.

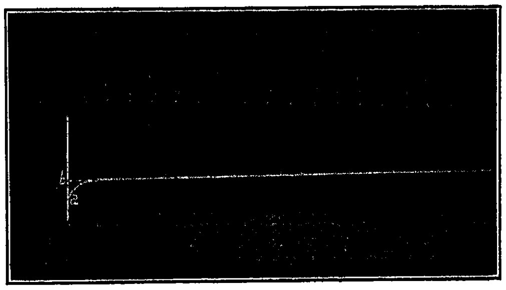


FIG. 4—DETERMINATION OF x_d' AND x_d'' BY METHOD OF PROJECTION

The per-unit current is expressible in this case as,

$$i = 1.54 + 1.83\,\epsilon^{-.653\,t} + 1.06\,\epsilon^{-51\,t},$$

the term with high decrement factor being due to the presence of a copper short-circuiting collar at the top of the pole. The values of direct transient reactance are then,

$$x_d'' = \text{direct sub-transient} = \frac{1.0}{1.54 + 1.83 + 1.06}$$

$$= 0.226$$

and,

$$x_d' = \text{transient} = \frac{1.0}{1.54 + 1.83} = 0.297$$

So far, transient reactance has been interpreted only in terms of the no-load short-circuit current of the machine. But, clearly, it may also be regarded as the reciprocal of the ratio of suddenly applied balanced current, to the reactive voltage due to this current. It is, therefore, similar to synchronous reactance, except referring to suddenly applied instead of sustained currents. From this standpoint it will of course have both a direct and quadrature value, each of which in turn can be transient or sub-transient. If, however, as is often the case, there is no really effective winding in the quadrature axis of the machine, the transient reactance in this axis should be taken equal to synchronous. Thus, the term *sub-transient reactance* will always refer to the "high-speed" phenomena in either axis. Moreover, if, as in the case of the machine to which the data of Fig. 5 refer, there happens to be no rotor winding in the quadrature axis, the sub-transient quadrature reactance will also equal the synchronous in that axis. Hence, in this case,

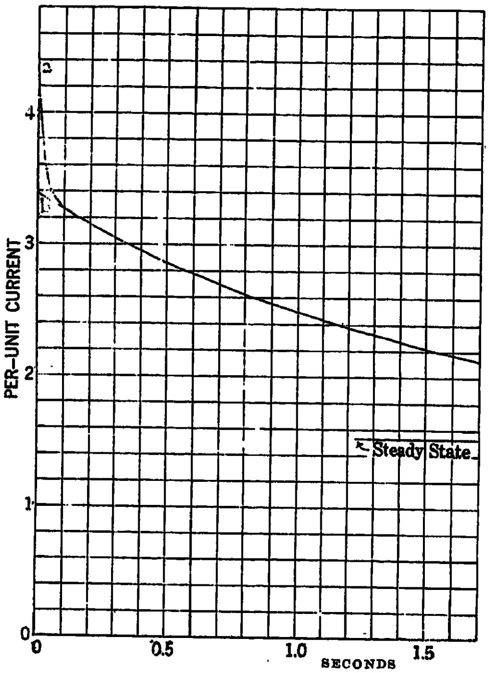


FIG. 5—DECREMENT OF SYMMETRICAL COMPONENT OF CURRENT THREE-PHASE SHORT CIRCUIT ON A 20,000-KV-A. WATERWHEEL GENERATOR

$$x_q'' = x_q' = x_q.$$

On the other hand, for machines equipped with amortisseurs, the value of the sub-transient quadrature reactance will be different from the corresponding synchronous value, although, as before,

$$x_q' = x_q$$

since there is no really effective winding in this axis. In practise, an amortisseur always has a decrement sufficiently high so that its reactance is sub-transient.

Thus, the per-unit sub-transient reactance of a machine, in either the direct or quadrature axis, is defined as the per-unit value of positive phase-sequence voltage, due to the sudden application of a positive phase-sequence current of normal amount; while transient reactance is the corresponding apparent value that obtains after the "high-speed" transients have died out. The four coefficients will, therefore, be

x_d' direct transient reactance
x_q' quadrature transient reactance
x_d'' direct sub-transient reactance
x_q'' quadrature sub-transient reactance

It has been pointed out that the direct sub-transient reactance can be determined from short-circuit tests. It is also possible to measure it by suddenly opening up a sustained short circuit, that is, applying the equivalent of a negative current equal in magnitude to the sustained current. Also, either direct or quadrature transient reactance can be found by impressing a direct or quadrature voltage, respectively, on the terminals, directly or through a reactance. In all of these cases the ratios of voltage to current, as shown on oscillograms, vary progressively from an initial value equal to sub-transient reactance, through transient, to a final value equal to synchronous reactance.

Another method which is sometimes useful in determining x_d' is to determine oscillographically the time-constant of the decay of armature voltage and current, respectively, with the armature open-circuited and short-circuited. If these time-constants are respectively T_0 and T_0', x_d' may be found from the relation[11]

$$\frac{x_d'}{x_d} = \frac{T_0'}{T_0}$$

provided that x_d is already known.

A convenient method, which may be used in testing for both x_d''[12] and x_q''[13] is to apply a single-phase line-to-line voltage to the machine, the rotor being held still. The field is shorted through an ammeter and the rotor moved until the meter reading is a maximum, at which position the rotor is directly under the center of the phase. The direct sub-transient reactance, in ohms, is one-half[14] the ratio of the armature voltage to the armature current under these conditions.

The pole is moved slowly until the field ammeter reads zero, showing that the pole is in quadrature with the axis of magnetization. Then, the quadrature sub-transient reactance, in ohms, is again one-half[14] the ratio of the armature voltage to the armature current.

For delta-connected machines the line-to-line reactances are similarly divided by two and the equivalent Y-circuit used.

---

[11] See Part II.

[12] If there are no auxiliary windings on the rotor this reactance is transient, *i. e.*, x_d'. Without such windings the method may also be used to obtain synchronous reactance by leaving the field open-circuited.

[13] If amortisseur windings are not present this reactance is the quadrature synchronous reactance.

[14] See Appendix H.

Negative phase-sequence reactance. In the case of a machine rotating synchronously the application of fundamental negative phase-sequence current gives rise to negative phase-sequence fundamental and positive phase-sequence third harmonic voltages from line-to-line and from line-to-neutral. The per-unit negative phase-sequence reactance of a machine is thus equal to the per-unit fundamental phase voltage, or the per-unit line-to-line voltage, due to normal negative phase-sequence current supplied at the machine terminals.

On the other hand, if the rotor is not moving exactly synchronously the line-to-line voltages will not be purely fundamental, but, as is shown in Part II, they will contain slip frequency components. On this account, the direct determination of negative phase-sequence reactance by test is often inconvenient.

However, it can be determined very easily by static test since the negative phase-sequence reactance of a machine is very nearly equal to the average[15] of x_d'' and x_q'', *i. e.*,

$$x_2 = \frac{x_d'' + x_q''}{2}$$

Zero phase-sequence reactance. The application of fundamental zero phase-sequence current to a synchronous machine gives rise to pulsating third harmonic m. m. f. in the air-gap and end-windings, and to a slot flux the magnitude of which varies widely with the winding pitch. Thus, with other than full pitch coils, the slot flux is diminished by the presence, in the slots, of coil sides carrying current in opposite directions. Zero phase-sequence slot reactance is thus very sensitive to pitch, and because of the fact just previously mentioned is a minimum at $^2/_3$ pitch. The zero phase-sequence air-gap leakage also varies accordingly and disappears at $^2/_3$ pitch, since in this case there is no air-gap m. m. f. There is no armature reaction m. m. f. due to zero phase-sequence currents. Hence zero phase-sequence reactance is very small, say, from 15 to 60 per cent of the direct sub-transient reactance. The effect of the motion of the rotor is very small, and, as a consequence, there are no appreciable harmonic voltages. *Per-unit zero phase-sequence reactance* is defined as the per-unit phase voltage with normal zero phase-sequence current applied. While its definition implies a condition under which the rotor is moving at normal speed, the difference in test result, if the rotor is stationary, is inappreciable.

DECREMENTS

Symmetrical component of current. As pointed out in the discussion of transient reactance, the decrement curve of the transient armature current of a machine on short-circuit will, in general, be composed of several simple decrement terms, or exponentials. That is, the current will be expressible as a series

$$= a + b\,\epsilon^{-\frac{t}{T_b}} + c\,\epsilon^{-\frac{t}{T_c}} + \dots.$$

where a is the sustained component of current, and b, c, etc., are the transient components of current with time-constants T_b, T_c, etc., respectively. In practise all but one of these time-constants will be small, *i. e.*, less than about one-twentieth sec., while for dead short-circuits the remaining time-constants will vary in large machines from about 0.5 to 2.5 sec.

Thus, a few cycles after the occurrence of a short, from an initial condition of no-load and voltage e, the armature current will be

$$i = e\left[\frac{1}{x_d} + \left(\frac{1}{x_d'} - \frac{1}{x_d}\right)\epsilon^{-\frac{t}{T_0'}}\right]$$

where T_0' is the short-circuit time-constant of the machine.

If the machine is operating at no-load and its field winding is short-circuited, the variation of armature voltage, shortly after the beginning of the transient, will follow a decrement having a time-constant T_0, which is larger than the short-circuit time constant T_0'. These time-constants are related by the simple expression

$$T_0' = \frac{x_d'}{x_d}\,T_0.$$

As it happens that the open-circuit time-constant of most large machines is about five sec., the above relation provides an easy means of determining T_0' when the other two quantities are known. The time constant T for a short circuit through an external reactance x is then given as

$$T = \frac{x_d' + x}{x_d + x}\,T_0.$$

This also applies to the case of single-phase short circuits. Thus, on single-phase short circuits, the positive phase-sequence component of current is to be calculated as the current that would exist when an equivalent[16] three-phase short circuit is applied.

In the case of short circuits under load the time-constants of the rotor circuits in the quadrature axis are also involved. Except for solid rotor turbo alternators, these are all so fast, however, as not to merit attention from an operating standpoint.

The case of turbo alternators is also complicated to some extent by the large amount of saturation which exists in the rotor leakage paths. The effect of saturation in such paths is not confined to turbo alternators only, but is encountered to some extent in short circuits of salient pole machines. Its effect does not, however, greatly modify the general conclusions stated.

---

[15] See Appendix D.

[16] The equivalent reactance of a line-to-line short is the negative phase-sequence reactance viewed from the point of short, while for line-to-neutral shorts it is the sum of the negative and zero phase-sequence reactances. The negative and zero phase-sequence currents can, of course, be found by applying the conditions which hold at any instant,—that for line-to-line faults, the positive and negative components of current are equal in the fault; and that for line-to-neutral faults, all three components are equal in the fault.

D-c. component. On dead short circuits, the time-constant of the d-c. component is

$$\tau = \frac{1}{2\,\pi f}\;\frac{x_2}{r}\text{ seconds}$$

while if the machine is short-circuited through an external reactance, x, the time-constant is

$$\tau = \frac{1}{2\,\pi f}\;\frac{x_2 + x}{r}\text{ seconds}$$

where r is the armature resistance as a per-unit quantity.

The time-constant on single-phase short-circuit is calculated as for the equivalent three-phase short-circuit.

Range of magnitude of reactances and time-constants. It is noticeable that in any certain type of machine the average value of each of the particular reactances appears to be a rather constant quantity, the deviation from which is not very great. Thus it is possible to obtain tables showing the general limits and the average values of the reactances, such as Table II.

The first figures in any group of three quantities indicate the lower limit, the second figures are the average, and the third figures, the upper limit. It is understood that they are not absolute values, but are representative of most machines. Where only two quantities are mentioned, the lower and upper limits are meant. If only one term is given it is the average reactance.

PART II

THEORETICAL CONSIDERATIONS

The general problem of calculating the performance of a synchronous machine does not admit of any but the most difficult forms of analysis, unless the effects of saturation and hysteresis are neglected. Therefore, they are not considered in the analysis which follows. Practically, these effects are small in a large number of cases. In those cases where their consideration becomes necessary, the final result, or the coefficients, may be "shaded."

Since saturation and hysteresis are neglected, linear relations exist between current and magnetic flux. It then follows, as is shown in detail in Appendix A, that *for a given motion or position* of the rotor, the currents and voltages in any part of the machine, due to any currents or voltages impressed at its terminals, may be found by superposing the effects that would result from the action of each separately. Thus, there will exist factors of proportionality between the voltage or current impressed at any terminal, and the voltages and currents at all other points of the machine.

For instance, if the armature of the machine is open-circuited, a steady voltage impressed on the field will give rise to a steady direct current in the field, and to fundamental and harmonic voltages at the armature terminals. On the other hand, if, say, one phase of the

TABLE II
TYPICAL VALUES OF REACTANCE

	x_d	x_q	x_d'	x_q'	x_d''	x_q''	x_2	x_0*	T_0 (sec.)
Synchronous Motors									
High-Speed....................	0.65–0.80–0.90	0.50–0.65–0.70	0.15–0.25–0.35	0.50–0.65–0.70	0.10–0.18–0.25	0.12–0.20–0.30	0.11–0.19–0.25	0.02–0.15	2–4
Low-Speed....................	0.80–1.1 –1.5	0.60–0.80–1.10	0.40–0.50–0.70	0.60–0.80–1.1	0.25–0.35–0.45	0.30–0.40–0.50	0.25–0.35–0.50	0.04–0.27	
Synchronous Condensers............	av. 1.60	av. 1.00	0.40–0.50	av. 1.00	0.25–0.30	0.25–0.35	0.25–0.32	0.04–0.10	5–7
Waterwheel Generators.............	0.60–1.0 –1.25	0.40–0.65–0.80	0.20–0.35–0.45	0.40–0.65–0.80	0.15–0.22–0.35	0.40–0.65–0.80	0.25–0.45–0.60	0.02–0.21	3–6
Turbo Alternators									
Solid Rotor....................	av. 1.15	av. 1.0	0.15–0.25	0.15–0.25	0.08–0.15	0.10–0.20	0.08–0.13	0.01–0.08	4–7
Laminated Rotor..............	av. 1.15	av. 1.0	0.15–0.25	av. 1.0	0.08–0.12	0.20–0.30	0.11–0.21	0.01–0.08	

*x_0 varies from about 15 per cent to 60 per cent of x_d'', depending upon winding pitch.

armature is short-circuited, a steady voltage applied to the field will give rise to steady and harmonic currents in the field, and harmonic currents and voltages, respectively, in the shorted and open armature phases. The ratios of these various currents or various voltages to the impressed field voltage may be referred to as the characteristic coefficients of the machine with reference to the particular condition of operation and the particular terminal quantity impressed, which in the illustrative case under consideration happens to be field voltage.

In general, characteristic coefficients can be established to cover any combination of impressed and resultant quantities, of any frequency, in any of the four distinct terminal circuits[17] of the machine comprising the three armature phases and the field winding. These coefficients may be further distinguished from each other according to the rate and time-phase of application of the impressed quantities, the character of the terminal circuits on which these quantities may be impressed, the motion and position of the rotor, and the time-phase of the resultant quantities relative to the impressed quantities.

The general problem of classifying and establishing a nomenclature for all of the various types of coefficients would be difficult, especially as there is no generally accepted name for the voltage, due to a voltage, or the current due to a current.[18] Moreover, the necessity for a careful study of the broad subject is not pressing at present, although it is anticipated that as progress is made in the analysis of machine performance, the recognition and employment of an increasing number of characteristic coefficients will take place.

However, the specific problem of classifying and studying that particular class of coefficients, which refers to the reactive voltages and currents in synchronous machines, is relatively straightforward: at the same time it is important, in view of the practical significance of this type of coefficient in the study of the phenomena occasioned by short circuits. It is this comparatively restricted aspect of the general subject which is considered in the present paper.

Following conventional practise, these particular characteristic coefficients which express the factors of proportionality between reactive voltage and current, are referred to, generally, as reactances.

Dependence of Reactance on Terminal Circuit. Unlike a simple reactance, however, these reactances are not independent of the character of the terminal circuit. Thus, when negative phase-sequence voltage is applied directly at the terminals of a three-phase machine, the phase current contains a negative phase-sequence fundamental and a positive phase-sequence third harmonic. Neglecting the effect of the resistance of the closed rotor circuits as regards double frequency current, the per-unit value of the fundamental component of current with normal negative phase-sequence voltage applied is approximately

$$\frac{1}{2}\left(\frac{1}{x_d''}+\frac{1}{x_q''}\right) \tag{1}$$

corresponding to a reactance:

$$x_2 = \frac{2\,x_d''\,x_q''}{x_d''+x_q''}\,. \tag{2}$$

If this voltage were applied through an external reactance, x, the total reactance of the circuit would be:

$$\frac{2\,(x_d''+x)\,(x_q''+x)}{x_d''+x+x_q''+x}\,.$$

Thus, the effective negative phase-sequence reactance of the machine would be:

$$x_2 = \frac{2\,(x_d''+x)\,(x_q''+x)}{x_d''+x_q''+2\,x} - x$$

$$= \frac{2\,x_d''\,x_q''+x\,(x_d''+x_q'')}{x_d''+x_q''+2\,x}\,. \tag{3}$$

If x is sufficiently great this reactance becomes:

$$x_2 = \frac{1}{2}\,(x_d''+x_q'')\,. \tag{4}$$

Physically, the effect of the external reactance is to force the current in the machine to be sinusoidal, and to permit the existence of a third harmonic voltage across each phase. This results in a change in the effective negative phase-sequence reactance of the machine. This change may be considerable. For instance, the ratio of the two extreme types of reactance given by equations (2) and (4) might be as high as five to one.[19]

In the case of sustained single-phase line-to-line short circuit, on the other hand, or in the equivalent case of fundamental voltage impressed from line-to-line, the per-unit *fundamental* component of current is shown by Doherty and Nickle to be:[20]

---

[17] Three-phase machines are referred to. The armatures of single-phase machines may be regarded as three-phase armatures with one leg left idle in operation.

[18] The term *short-circuit ratio*, expressing the per-unit armature current on sustained short circuit due to the application of no-load field current, is an exception to this rule.

[19] In the case of the small machine employed in the tests illustrated in Doherty and Nickle's paper *Synchronous Machines IV* the negative phase-sequence coefficient is 0.40 when negative phase-sequence voltage is impressed directly, and is 1.99 when the voltage is impressed through a large reactance, or a ratio of five, based on the smaller quantity. This, however, is a very extreme case.

In machines of normal design, the ratio would be approximately,

Water-wheel generators	1.30
Synchronous Motors and Condensers	1.04
Turbine Generators	1.00

The exact difference in these two types of reactances is discussed in detail in Appendix D.

[20] This form follows from equation (**27b**) of the paper mentioned by substituting $x_D = 2\,x_d$, $x_D' = 2\,x_d''$, and $x_Q' = 2\,x_q''$.

$$\frac{\sqrt{3}\, e}{x_d + \sqrt{x_d'' \, x_q''}}$$

where e is the per-unit voltage before the short. Thus e is also the per-unit value of the equivalent line-to-line voltage impressed. But, in the analysis provided by the method of symmetrical coordinates, this current is:

$$\frac{\sqrt{3}\, e}{x_d + x_2} \, . \qquad (5)$$

Thus, for the particular type of short-circuit under consideration, the effective negative phase-sequence reactance is

$$x_2 = \sqrt{x_d'' \, x_q''} \qquad (6)$$

that is, the geometric average of x_d'' and x_q''. If, however, the short circuit is effected through an external reactance, x, per phase, the reactance of the complete circuit is:

$$\sqrt{(x_d'' + x)\,(x_q'' + x)}$$

and the effective reactance of the machine is:

$$x_2 = \sqrt{(x_d'' + x)\,(x_q'' + x)} - x \qquad (7)$$

Expanding (7) by the binomial theorem gives:

$$x_2 = x \left[1 + \frac{1}{2} \left\{ \frac{x_d'' + x_q''}{x} + \frac{x_d'' \, x_q''}{x^2} \right\} - \frac{1}{8} \left\{ \frac{x_d'' + x_q''}{x} + \frac{x_d'' \, x_q''}{x^2} \right\}^2 \; . \; . \; . \; . \text{ etc.} \right] - x$$

When x is sufficiently large this becomes

$$x_2 = \frac{1}{2} \, (x_d'' + x_q'') \qquad (8)$$

as in (4).

In practise short circuits are usually effected through some external reactance. For this reason, and in order to retain the principle that the reactance of a machine and any external reactance in series[21] is the sum of the machine and external reactances, it is desirable to normally regard the reactance of a machine, when referred to without qualification, as that obtaining with a large external reactance in each phase, or the reactance that obtains with fundamental *current* rather than voltage impressed. Thus, negative phase-sequence reactance is to be interpreted normally as the effective phase-sequence reactance with current rather than voltage impressed and, numerically, may be taken as the average of x_d'' and x_q''.

So far, it has been pointed out that the effective negative phase-sequence reactance of a synchronous machine varies with the nature of the circuit external to the machine. This effect, however, is not limited to the negative phase-sequence reactance, but applies, to a slight extent, to positive and zero phase-sequence reactances. Thus, when fundamental current is "impressed" on a synchronous machine, the rotor of which is moving normally, harmonic as well as fundamental terminal voltages are produced. If, however, the machine is supplied from an infinite bus these harmonic voltages are short-circuited. But an nth harmonic current of proper phase-sequence, taken in combination with a winding which does not exclude nth harmonic m. m. f., results in the rotation of this m. m. f., in synchronism with the poles. Also, with a salient pole machine, an nth harmonic m. m. f. over the pole can give rise to a flux of fundamental pitch, and thus generate fundamental voltage. It follows that if the harmonic voltages which exist with current impressed are short-circuited by impressing fundamental voltage at the terminals of the machine, the effect of the resulting harmonic currents will be to generate some fundamental voltage. This slightly modifies the proportion between positive phase-sequence current and voltage from the value which would obtain with current impressed at the machine terminals.

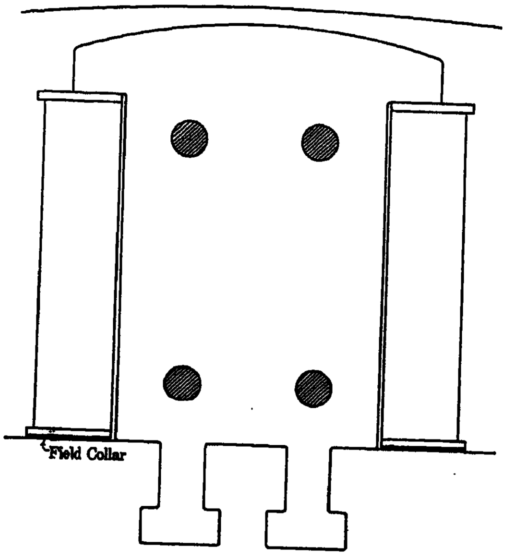


Fig. 6—30,000-Kv-a. Waterwheel Generator. Rotor Circuits: Field Collar

Division of Synchronous Reactance into Armature Reaction and Leakage Reactance

Armature reaction. The synchronous reactance of a machine is ordinarily calculated as the sum of the armature reactance due to armature reaction flux, and the armature reactance due to leakage flux, but the exact line of demarcation between reactive and leakage flux has not always been very carefully drawn.

Originally, armature reaction was apparently considered equivalent merely to the action of a lumped coil moving along the armature surface and subtracting its ampere-turns directly from the ampere-turns on the pole. Later, it was realized that it was desirable to modify the effective amount of armature reaction to take account of the facts;—that the m. m. f. of armature reaction existed at the armature surface and, hence, a given amount of it did not have the same effect as if it were due to the field winding; and that the principal component of m. m. f., and the only one that is stationary with respect to the poles, is distributed sinusoidally along the armature surface.

---

[21] When reactances are to be taken in parallel the principle is reversed. However, it is felt that usually the series principle is the more useful.

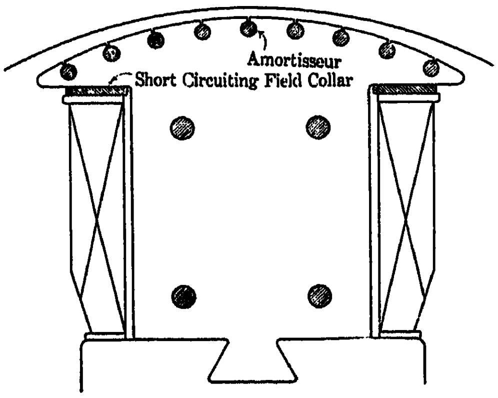


Fig. 7—150-Kv-a. High-Speed Synchronous Motor. Rotor Circuits: Amortisseur, Field Collar

Thus, in recent theory, armature reaction is understood to refer to the space fundamental or stationary component of m. m. f. due to the armature. However, some question arises as to whether the term should be interpreted to apply to the total synchronous m. m. f., due to the armature, thus including the synchronous m. m. f. in the end-windings, or whether it should be limited so as to refer only to the m. m. f. along the stacked length of the machine.

Obviously, the answer to this question should depend on the relative value of each alternative as regards calculating the performance of the machine. But the chief purpose of separating out the voltage due to armature reaction from the total synchronous reactance drop is to consider separately those fluxes which can normally penetrate the rotor windings, and which can, under abnormal conditions, induce direct current or slip frequency in these circuits. Therefore, it appears logical that, in general, armature reaction should be taken to include all of the m. m. f. that is stationary with respect to the rotor, since, and especially in induction motors or turbo alternators, the end-winding m. m. f. can produce a flux which links the rotor circuits. Hence, this point of view, *i. e.*, that armature reaction exists in the end windings as well as along the stacked length of the machine, is favored by the authors.

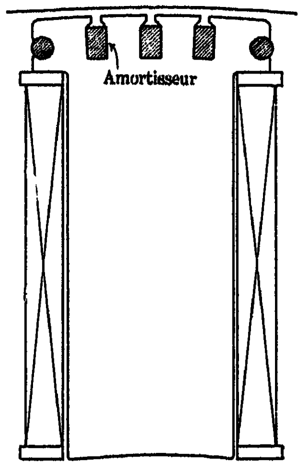


Fig. 8—240-Kv-a. Low-Speed Synchronous Motor. Rotor Circuits: Amortisseur

However, from a practical point of view, the net effect of the armature reaction in the end-windings of a salient pole machine is usually negligible as far as regards the induction of flux in any of the rotor circuits. Consequently in the present state of the art, at least, it is legitimate to regard all the end-winding flux of salient pole machines as leakage flux.

In the case of turbo alternators, this assumption is less correct, although the action of the iron end-rings employed on these machines tends to reduce the mutual flux between armature and field, and, thus, lessen the degree of approximation involved. However, these machines have a relatively small leakage reactance, so that the error involved, practically, in treating end-flux as all leakage is small here also.

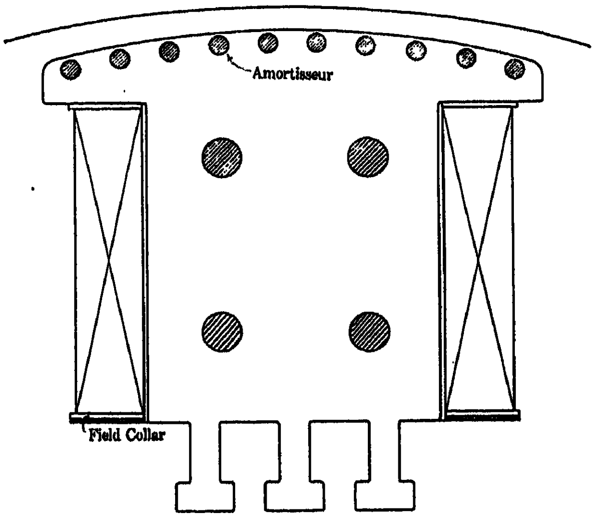


Fig. 9—25,000-Kv-a. Synchronous Condenser. Rotor Circuits: Amortisseur, Field Collar

On the other hand, in the case of induction motors, there can exist a comparatively large amount of end-flux. Since a considerable part of this flux may be mutual with the rotor winding, the approximation that end-flux is all leakage cannot be satisfactorily employed.

Thus, on account of physical appropriateness, and to obtain consistency, it is desirable, normally, to consider all armature m. m. f. which is stationary with respect to the rotor as armature reaction. In practical computations, however, it is desirable, at present, to limit the term to the m. m. f. in the stacked length of the machine, except in the case of induction motors and special synchronous machines.

In addition to establishing the conception of *armature reaction* as denoting a particular type of m. m. f., it has appeared desirable to broaden the scope and application of the term so that it may apply not only to m. m. f. but also to fluxes and their linkages. Thus, it is proposed to characterize by the qualifying term *armature reaction* all those phenomena which depend upon the action of the synchronous component of armature m. m. f. In this way there are:

A. *M. m. f. of armature reaction,* or armature reaction proper; all the synchronous armature m. m. f.

B. *Flux or flux density of armature reaction;* that component of flux, or of flux density at any point of the air-gap or in the rotor, which is due to the armature reaction m. m. f.; under normal conditions, all that flux or flux density which is stationary with respect to the rotor.

C. *Linkages of armature reaction;* the linkages with any circuit on the armature, or rotor, of the fluxes of armature reaction.

D. *Reactances of armature reaction;* the reactive coefficients expressing the voltages of armature reaction (*i. e.,* linkages) in any circuit on the armature, or rotor, due to normal armature current.

The reactance of armature reaction, or in the case of induction motors, the magnetizing reactance, will, however, when referred to without qualification, mean the per-unit armature fundamental linkages due to normal positive phase-sequence armature current and will have direct and quadrature components, thus:

x_{ad} = direct reactance of armature reaction

x_{aq} = quadrature reactance of armature reaction.

Methods of calculating armature reaction have been given by R. W. Wieseman.[22]

Leakage Reactance. Leakage reactance is defined as that part of synchronous reactance which is not included in the reactance of armature reaction. Hence, there are two components

$$x_{ld} = x_d - x_{ad}$$

$$x_{ld} = x_q - x_{aq}.$$

They are the reactances due to the flux in the slots and to the flux produced by the moving harmonic of m. m. f., which latter is that flux in the air-gap or end-winding which is not normally stationary with respect to the poles. Actually, the flux due to the *n*th harmonic m. m. f. pulsates over the rotor at $\left(1 - \frac{1}{n}\right)$ times normal frequency and, therefore, is subject to the demagnetizing influence of the field winding and all other closed rotor circuits. Consequently, in determining the permeance[23] of the air-gap to *n*th harmonic m. m. f. the effect of these circuits should be considered. In practise, however, the rotor circuits are adjusted as far as possible so that this effect is small in order to eliminate losses in these windings.

Methods of calculating leakage reactance are given by Mr. P. L. Alger in a companion paper on this subject.

THE IDEAL SYNCHRONOUS MACHINE

The ideal synchronous machine has a sinusoidal open-circuit voltage wave, and leakage reactance unaffected by rotor position, *i. e.,* only armature reaction flux is affected by pole position. Practically, the divergence of machines from this ideal type is often important; for example, as a cause of circulating currents, or, in the case of motors, of locking at part speed. In a large number of cases, however, it is legitimate to neglect these differences. The calculation of the electrical behavior of an ideal machine of this type can be accomplished in terms of a relatively small number of characteristic constants. Thus, it may be shown[24] that the per-unit linkages in phase *a* of an ideal synchronous machine are expressed as

$$\psi a = I_d \cos\theta - I_q \sin\theta - \frac{x_0}{3}(i_a + i_b + i_c)$$

$$- \frac{x_d + x_q}{3}\left[i_a - \frac{i_b + i_c}{2}\right]$$

$$- \frac{x_d - x_q}{3}\Big[i_a \cos(2\,\theta) + i_b \cos(2\,\theta - 120)$$

$$+ i_c \cos(2\,\theta + 120)\Big] \qquad \textbf{(10)}$$

where,

i_a, i_b, and i_c are the instantaneous currents in phases *a*, *b*, and *c*.

I_d is the nominal voltage, or per-unit rotor current in the direct axis.

I_q is the nominal voltage, or per-unit rotor current in the quadrature axis, ahead of the poles.

θ = electrical degrees in direction of normal rotation between axis of poles and the magnetic axis of phase *a*.

Axis of phase *b* is 120 deg. ahead of the axis of phase *a*.

Axis of phase *c* is 120 deg. behind the axis of phase *a*.

The linkages in phase *b* are similar except that $(\theta - 120)$ is substituted for θ in the first two terms, i_b for i_a in the fourth term, and $(120 - \theta)$ for θ in the fifth term. The linkages in phase *c* are obtained in a corresponding way. From this, the voltage of any phase can be obtained from the relation

$$e_a = \frac{d\,\psi_a}{d\,t} - r\,i_a \qquad \textbf{(11)}$$

In order to determine the reactance between the armature and rotor, it is of great benefit to resolve the armature quantities under consideration with respect to the two axes of symmetry of the poles, a procedure introduced by Blondel and further developed by Arnold, and Doherty and Nickle.

---

[22] Reference (5).

[23] Reference (8).

[24] Reference (14).

To this end, and in analogy with their recognized significance under steady conditions of operation, let the direct and quadrature components of current at any instant, and under any condition of operation, be taken equal to the instantaneous amplitudes of the direct and quadrature components of armature m. m. f. It may be shown[25] that, at any instant, the magnitudes of these m. m. fs. are equal to the quantities

$$i_d = \frac{2}{3}\left[i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)\right] \qquad \textbf{(12)}$$

$$i_q = \frac{2}{3}\left[i_a \sin\theta + i_b \sin(\theta - 120) + i_c \sin(\theta + 120)\right]$$

The currents, i_d and i_q, then determine the reaction of the armature on the rotor at any instant. In general, there will be more than one rotor circuit. Consider, however, any particular circuit, say circuit *h* of the direct axis; that is, circuit *h d*. There will be factors of proportionality between the linkages, Ψ_{hd}, in this circuit, the currents in every other rotor circuit in the direct axis, and the instantaneous direct component of armature current. Thus, there is

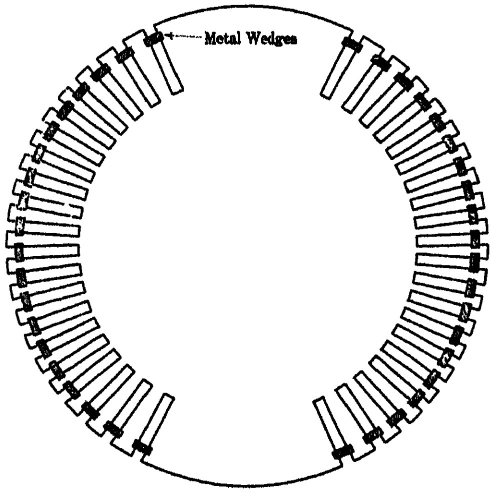


FIG. 10—10,000-KV-A. TURBO ALTERNATOR. ROTOR CIRCUITS: ROTOR IRON, METAL WEDGES

$$\Psi_{hd} = X_{hfd}\, I + X_{h1d}\, I_{1d} + \dots\dots\dots \\ + X_{hhd}\, I_{hd} + \dots\dots X_{hnd}\, I_{nd} - X_{hmd}\, i_d \qquad \textbf{(13)}$$

where

I = per-unit field current

I_{hd} = per-unit current in circuit *h* of the direct axis

X_{hfd} = per-unit mutual reactance of field with circuit *h d*

X_{hnd} = per-unit mutual reactance of circuit *h d* with circuit *n d*.

X_{hmd} = per-unit mutual reactance of armature with circuit *h d*.

Similar relations exist in the quadrature axis except that field current is not involved.

If there is only one rotor circuit, equation **(13)** becomes merely

$$\Psi = X\, I - X_m\, i_d$$

in which Ψ, X, X_m, and I refer to this circuit.

In order to retain the valuable principle that the per-unit nominal voltage is equal to the total rotor current, it is desirable to define unit current in any circuit as that current which acting by itself produces normal space fundamental flux at the armature surface. In this way the nominal voltage at any instant is always to be identified as the instantaneous rotor current. Thus, in the direct axis, as

$$I_d = I + I_{1d} + I_{2d} + \dots + I_{nd} \qquad \textbf{(14)}$$

Transient Reactance. In analogy with the procedure employed in determining the effect of the armature circuit on the rotor, it is desirable to establish the conception of instantaneous direct and quadrature components of armature linkages. If these components are defined as

$$\psi_d = \frac{2}{3}\left[\psi_a \cos\theta + \psi_b \cos(\theta - 120) + \psi_c \cos(\theta + 120)\right] \qquad \textbf{(15)}$$

$$\psi_q = -\frac{2}{3}\left[\psi_a \sin\theta + \psi_b \sin(\theta - 120) + \psi_c \sin(\theta + 120)\right]$$

it may be verified that the simple relations, which obtain between armature linkages and armature and rotor currents under steady conditions, now hold at any instant. That is, under all conditions,

$$\psi_d = I_d - x_d\, i_d \qquad \textbf{(16)}$$
$$\psi_q = I_q - x_q\, i_q$$

Transient reactance will then be the constant of proportionality between ψ and i when i is suddenly impressed. Thus, if $\Delta\,\psi_d$ is the sudden change in ψ_d when a current $\Delta\, i_d$ is suddenly impressed, there is

$$x_d' = -\frac{\Delta\,\psi_d}{\Delta\, i_d} \qquad \textbf{(17)}$$

But, $\Delta\,\psi_d = \Delta\, I_d - x_d\, \Delta\, i_d$

where $\Delta\, I_d$ is the corresponding increase in direct axis rotor current. Therefore,

$$x_d' = x_d - \frac{\Delta\, I_d}{\Delta\, i_d} \qquad \textbf{(18)}$$

and similarly in the quadrature axis. The value of $\Delta\, I_d$ may be found by solving the *n* equations

$$\Psi = \Psi_{1d} = \Psi_{2d} = \dots\dots \Psi_{nd} = 0 \qquad \textbf{(19)}$$

subject to the conditions $i_d = \Delta\, i_d$. $\Delta\, I_q$ may be found similarly. We shall only consider in detail here the case of a single rotor circuit. The equations **(19)** become in this case merely

$$\Psi = X\,\Delta\, I - X_m\,\Delta\, i_d = 0$$

from which it follows that, in this case,

$$x_d' = x_d - \frac{X_m}{X} \qquad \textbf{(20)}$$

where X_m is the mutual reactance of the main field wind-

---

25. Reference (14).

ing with respect to the armature, and X is the total field reactance.

Calculation of Rotor Reactances. Before calculating these quantities in per-unit terms it is necessary to decide on an appropriate value of normal or unit linkages. It has been found convenient to specify normal linkages in each rotor circuit as the product of normal flux by the number of series turns in the circuit. On this basis it is shown in Appendix G that, for any rotor circuit h,

$$X_{hm} = \frac{A_{(0)}}{F_{h\,(0)}} \tag{21}$$

where,

$A_{(0)} = 2.12 \times$ (effective full-load armature ampere-turns per pole, per phase).

$F_{h\,(0)}$ = normal no-load ampere-turns of circuit h with no saturation.

Thus, the determination of the mutual reactances of X_{mh} can be reduced to the determination of no-load excitation ampere-turns in each rotor circuit.

The self and mutual rotor reactances at present require flux plots for their determination, except in the case of the total field reactance.

As shown in Appendix F, the latter is given by the expression

$$X = K_\phi + \frac{F_{(0)}}{\phi_{(0)}}\, l\, L \tag{22}$$

where K_ϕ and L are given by formulas and curves in Doherty and Shirley's 1918 paper on reactances, l is the stacked length of the machine, and $\phi_{(0)}$ is normal no-load armature flux. The transient reactance of a machine with no additional rotor circuits is then given as

$$x_d' = x_d - \frac{A_{(0)}}{F_{(0)}}\,\frac{1}{K_\phi + \dfrac{F_{(0)}}{\phi_{(0)}}\, l\, L} \tag{23}$$

Equivalent Circuits and Field Leakage Reactance. This general subject is treated mathematically in Appendix E. It is shown that any number of equivalent circuits can be constructed to represent the relation between armature and rotor currents and linkages. Of the various circuits which may be so constructed there is one in which the armature branch is x_{ld}, and the magnetizing branch is x_{ad}. The field branch of this particular circuit has been referred to as the field leakage reactance, and is denoted by X_l.[26] It is shown in Appendix E that X_l may be expressed in terms of the quantities already described by the relation,

$$X_l = x_{ad}\left(\frac{x_{ad}\,X}{X_m} - 1\right) \tag{24}$$

Field Voltage. It sometimes is desirable to define normal field voltage as the voltage which will just force normal current through the resistance of the field, but in order to obtain symmetry with the equations for armature voltage a different procedure is here employed. In the following, normal voltage of any rotor circuit is that voltage which would exist in this circuit if normal flux changed through it at normal frequency. Normal resistance is then the ratio of normal $I\,R$ drop to this voltage. In this way the per-unit voltage of any field circuit, say $h\,d$, is

$$E_{hd} = \frac{d\,\Psi_{hd}}{d\,t} + R_{hd}\,I_{hd} \tag{25}$$

while for the field circuit proper there is

$$E = \frac{d\Psi}{d\,t} + R\,I \tag{26}$$

Decrements. Only the case of a machine with the field as the only rotor circuit will be considered. In this case, if the armature is open-circuited, there is

$$\Psi = X\,I$$

and

$$E = X\,\frac{d\,I}{d\,t} + R\,I$$

so that the open-circuit time-constant of the field is

$$T_0 = \frac{X}{R} \text{ (radians)} \tag{27}$$

If the machine is short-circuited through a reactance x, and the effect of the resistance of the armature circuit is negligible, as is normally the case, there is

$$\Psi = \left[X - \frac{X_m}{x_d + x}\right] I = \frac{x_d' + x}{x_d + x}\,X\,I$$

$$E = \frac{x_d' + x}{x_d + x}\,X\,\frac{d\,I}{d\,t} + R\,I$$

so that the time-constant of the field, in this case, is

$$T = \frac{x_d' + x}{x_d + x}\,T_0 \tag{28}$$

The time-constant on dead short-circuit is

$$T_0' = \frac{x_d'}{x_d}\,T_0 \tag{29}$$

The effect of armature resistance, or of machines operating in parallel, is to modify these results in a complicated way which will not be considered here.

If, as is ordinarily the case, the resistance of the rotor circuit is negligible, the time-constant of the direct current in the armature depends upon the inductance of the armature to direct current in the same way as the time-constant of direct current in the field depends on the inductance of the field to direct current. The inductance of the armature to direct current is, however, the average of x_d'' and x_q'', and is thus equal to the negative phase-sequence reactance. Thus the time-constant for the d-c. component in the armature is simply

---

[26] Reference (6).

$$\tau = \frac{x_2}{r} \text{ (radians)}$$

as stated in Part I.

ACKNOWLEDGMENTS

The authors wish to thank Messrs. R. E. Doherty, C. A. Nickle, P. L. Alger, and Professor W. V. Lyon for their valuable suggestions, and Mr. R. G. Lorraine for his assistance in the preparation of the figures.

Appendix A

APPLICATION OF THE PRINCIPLE OF SUPERPOSITION TO THE ANALYSIS OF SYNCHRONOUS MACHINES

In a synchronous machine, as in a static device, the flux density at any point will be in linear relation to the total number of currents in every circuit of the machine, providing saturation and hysteresis are neglected. However, in a synchronous machine the factors of proportionality between flux density and current depend upon the angular position of the rotor. Thus, if the circuits in the machine are numbered 1, 2, etc., the component of flux density in a given direction at any point of the machine will be related to the currents in a manner shown by the following equation:

$$\beta = f_1(\theta)\, i_1 + f_2(\theta)\, i_2 + \ldots \text{ etc.}$$

Consequently, for a given position of the rotor, the flux at any point in the machine may be found by superposing the fluxes due to the current in every circuit acting separately. Also, the flux due to the current in any circuit may be found by superposing the fluxes due to any components into which this current may be resolved, taken separately.

The same rule will of course apply to linkages. Thus, the linkages in circuit 1 will be represented as

$$\psi_1 = \lambda_{11}(\theta)\, i_1 + \lambda_{12}(\theta)\, i_2 + \ldots \text{ etc.}$$

where λ_{11} is the self-inductance of circuit 1, and λ_{12} is the mutual inductance of circuit 1 with respect to circuit 2. The voltage in circuit 1 is then

$$e_1 = r_1 i_1 + \frac{d\psi_1}{dt}$$

$$= \left(r_1 + \frac{d\lambda_{11}(\theta)}{d\theta}\frac{d\theta}{dt}\right) i_1 + \lambda_{11}(\theta)\frac{di_1}{dt} + \frac{d\lambda_{12}(\theta)}{d\theta}\frac{d\theta}{dt} i_2$$

$$+ \lambda_{12}(\theta)\frac{di_2}{dt} + \ldots \text{ etc.}$$

But, since if $i = i_x + i_y$

$$\frac{di}{dt} = \frac{di_x}{dt} + \frac{di_y}{dt}$$

it follows that, for a given *position* and *speed* of the rotor, the voltage in any circuit of the machine may be found by superposing the voltages due to the current in every other circuit acting separately. Also, under the same conditions, voltage due to the current in any circuit may be found by superposing the voltages due to any components into which this current may be resolved.

To illustrate, the armature voltage due to a field current I, and an armature current i, may be found as the sum of the armature voltages due to the currents I and i acting separately. That is, the voltage under load may be found by subtracting from the no-load voltage corresponding to the existing field current, the voltage due to the impedance drop caused by the armature current. Or, the voltage due to a current

$$i = i_a \sin(t + \alpha)$$

can be found by superposing the voltages due to currents

$$i_x = i_a \cos\alpha \sin t$$

and,

$$i_y = i_a \sin\alpha \cos t$$

acting alone.

So far it has merely been pointed out under what conditions the voltages due to a given current distribution can be found by superposing the voltages due to any components of the distribution. Suppose, however, that a given distribution of voltage is impressed at the machine terminals. Let this distribution be characterized by the symbol $e(t)$, and let $e_x(t)$ and $e_y(t)$ be component distributions into which $e(t)$ may be resolved so that $e(t) = e_x(t) + e_y(t)$. Let the currents due to these distributions be $i(t)$, $i_x(t)$, and $i_y(t)$. Then there must be

$$i(t) = i_x(t) + i_y(t)$$

since this result fulfills all the conditions imposed by the differential equations and terminal conditions of each circuit of the machine, provided that the position of the rotor as a function of time, $\theta(t)$, is the same with $e(t)$, $e_x(t)$, and $e_y(t)$ impressed.

Thus, it follows that the current distribution due to a given distribution of armature voltage may be found by superposing the current distributions due to any components into which the voltage distribution may be resolved, provided that the motion of the rotor, as a function of time, is the same for the components as for the resultant.

For example, if a machine is supplying current to a reactor and is then short-circuited, the current at any time after the short circuit may be found by superposing on the initial current distribution the fictitious currents which would be due to the application of a negative voltage at the point of short circuit. This negative voltage is equal and opposite to that which would have existed at the point of short circuit had the short circuit not occurred, and it is assumed to be applied with the machine unexcited.

On the other hand, if the machine is supplying a kilowatt load, it would be only possible to calculate the currents at the instant of short circuit in such manner. This is because of the fact that generally, in this case, the motion of the rotor would be different before and after the short circuit on account of the change in load effected by the short circuit. In the first instant, however, the inertia of the rotor would prevent a change in its motion.

When current is impressed on the open circuits of a machine, in which closed circuits also exist, the same limitations apply as in the case for voltage. In both events the currents and voltages due to the impressed quantities depend on the complete variation in time of those quantities. Superposition is still legitimate, subject to the condition that the motion is the same for the components as for the resultant.

The general statement of the possibility of superposition is, then, that *for a given motion or position* of the rotor, the currents and voltages in any part of the machine, due to any currents or voltages impressed at its terminals, may be found by superposing the effects that would result from the action of each taken separately.

Appendix B

REPLACING THE EFFECT OF INDUCED FIELD CURRENTS BY EMPLOYING TRANSIENT INSTEAD OF SYNCHRONOUS REACTANCES

Equation (**10**), of Part II, gives the complete expression for the phase linkages of a synchronous machine under all conditions of operation. However, in the case of rapidly changing or suddenly impressed currents, there is a definite relation between the currents in the field and armature circuits.[27] As a consequence, the field current terms may be replaced by terms involving armature currents.

Thus, if the field circuit is closed, and the effects of a given armature current are to be calculated

a. only in the first instant after their sudden application,

b. under sustained conditions of such a nature that all m. m. fs. over the poles are pulsating, or moving at sufficient speed that the demagnetizing action of the rotor circuits is practically complete,

it is merely necessary to substitute x_d'' and x_q'' or x_d' and x_q' for x_d and x_q in order to eliminate the terms I_d and I_q.

The linkages in phase a thus become

$$\psi_a = -\frac{x_0}{3}(i_a + i_b + i_c) - \frac{x_d'' + x_q''}{3}\left[i_a - \frac{i_b + i_c}{2}\right] - \frac{x_d'' - x_q''}{3}\Big[i_a \cos(2\,\theta) + i_b \cos(2\,\theta - 120) + i_c (\cos(2\,\theta + 120)\Big] \qquad (1b)$$

x_d' and x_q' replace x_d'' and x_q'' when the m. m. f. over the poles is

a. Suddenly impressed, but steady thereafter, and it is desired to determine the effects of currents after the currents in the "additional rotor circuits" have died away.

b. Alternating so slowly that the demagnetizing action of the "additional rotor circuits" is slight.

But if, with closed circuits on the rotor, the armature currents are sustained and of such a nature that the armature m. m. f. is stationary and steady over the poles, x_d and x_q are used as before.

Finally, if there are closed circuits on the rotor, but neither of the above conditions is fulfilled, it is generally possible to divide the currents into two parts; first, those which meet sub-transient or transient reactance, and second, those which meet synchronous reactance. The appropriate factor is then used with each type of current. For example, a single-phase line-to-line current

$$i_a = -i_b = i_0 \cos t$$

is resolvable into positive and negative phase-sequence components as

$$i_a = \frac{1}{\sqrt{3}} i_o [\cos(t - 30) + \cos(t + 30)] = i_0 \cos t$$

$$i_b = \frac{1}{\sqrt{3}} i_0 [\cos(t - 150) + \cos(t + 150)] = -i_0 \cos t$$

$$i_c = \frac{1}{\sqrt{3}} i_0 [\cos(t + 90) + \cos(t - 90)] = 0 \qquad (2b)$$

For normal motion of the rotor, the positive phase-sequence component of current meets synchronous reactance, and the negative phase-sequence meets sub-transient reactance. Thus,

$$\psi_a = -\frac{i_0}{3}\left[\begin{array}{l} \frac{x_d + x_q}{2} \cos(t - 30) \\ + \frac{x_d - x_q}{2} \cos(t + 2\,\theta_0 + 30) \\ + \frac{x_d'' + x_q''}{2} \cos(t + 30) \\ + \frac{x_d'' - x_q''}{2} \cos(3\,t + 2\,\theta_0 + 30) \end{array}\right] \qquad (3b)$$

Appendix C

SIGNIFICANT ROTOR CIRCUITS IN ADDITION TO THE FIELD

The principal circuits in the rotor of a machine, besides the field winding proper, which are capable of significantly affecting the behavior of the machine under practical operating conditions, are listed in Table III.

---

[27] That is, $I_d = (x_d - x_d'')\, i_d$ and $I_q = (x_q - x_q'')\, i_q$.

TABLE III

Type of Machine	Significant rotor circuits in addition to field
Water-wheel Generators	Amortisseur, if used.
Synchronous Motors and Condensers.	Field collars, if used. Spider, unless laminated.
Turbine Generators	Metal Wedges Rotor Iron, unless laminated.

Figs. 6 to 10 show these circuits pictorially for representative machines of each type.

Appendix D

THE NEGATIVE PHASE-SEQUENCE REACTANCE OF A SYNCHRONOUS MACHINE WITH NEGATIVE PHASE-SEQUENCE VOLTAGE IMPRESSED

With normal negative phase-sequence current impressed on the terminals of a machine the phase currents are,

$$i_a = \cos t$$
$$i_b = \cos (t + 120)$$
$$i_c = \cos (t - 120)$$

It follows from equation (10), of Part II, and from the fact that all the resultant fluxes are varying rapidly over the rotor,[28] that the flux linkages in each phase are, under these circumstances,

$$\psi_a = \frac{x_d'' + x_q''}{2} \cos t + \frac{x_d'' - x_q''}{2} \cos (2\,\theta + t) \qquad \textbf{(1d)}$$

$$\psi_b = \frac{x_d'' + x_q''}{2} \cos (t + 120) + \frac{x_d'' - x_q''}{2} \cos (2\,\theta + t - 120) \qquad \textbf{(2d)}$$

$$\psi_c = \frac{x_d'' + x_q''}{2} \cos (t - 120) + \frac{x_d'' - x_q''}{2} \cos (2\,\theta + t + 120) \qquad \textbf{(3d)}$$

The variation in linkages will give rise to a voltage of fundamental frequency and a harmonic of $(2\,d\,\theta/d\,t + 1)$ times fundamental frequency. For normal motion of the rotor these voltages will be fundamental and third harmonic of which the fundamental may be found by analyzing the voltage wave over one cycle. However, if the speed of the rotor is not exactly normal, the third harmonic in the terminal voltage will pulsate at the slip frequency.

If normal third harmonic positive phase-sequence current is impressed, *i. e.*, if

$$i_a = \cos (3\,t + \alpha)$$
$$i_b = \cos (3\,t - 120 + \alpha)$$
$$i_c = \cos (3\,t + 120 + \alpha)$$

the phase linkages become,

$$\psi_a = \frac{x_d'' + x_q''}{2} \cos (3\,t + \alpha) + \frac{x_d'' - x_q''}{2} \cos (2\,\theta - 3\,t - \alpha) \qquad \textbf{(4d)}$$

$$\psi_b = \frac{x_d'' + x_q''}{2} \cos (3\,t - 120 + \alpha) + \frac{x_d'' - x_q''}{2} \cos (2\,\theta - 3\,t - 120 - \alpha) \qquad \textbf{(5d)}$$

etc.

If fundamental negative phase-sequence voltage is maintained at the armature terminals, and if the resistance of the armature is negligible, the armature linkages will also be negative phase-sequence. This will require that both negative phase-sequence fundamental and positive phase-sequence third-harmonic currents exist, having a definite ratio to each other. Thus, let $i_{2(1)}$ be the magnitude of the negative phase-sequence *fundmental* current, and $i_{1(3)}$ be the magnitude of the positive phase-sequence *third-harmonic* current. Also, suppose that the two are acting together, and let $\theta = \theta_0 + t$. Then, combining equations (1d) and (4d),

$$\psi_a = i_{2(1)} \frac{x_d'' + x_q''}{2} \cos t + i_{1(3)} \frac{x_d'' + x_q''}{2} \cos (3\,t + \alpha) + i_{2(1)} \frac{x_d'' - x_q''}{2} \cos (3\,t + 2\,\theta_0) + i_{1(3)} \frac{x_d'' - x_q''}{2} \cos (2\,\theta_0 - t - \alpha) \qquad \textbf{(6d)}$$

If,
$$\alpha = 2\,\theta_0$$
and,
$$i_{2(1)} \frac{x_d'' - x_q''}{2} = - i_{1(3)} \frac{x_d'' + x_q''}{2} \qquad \textbf{(7d)}$$

ψ_a will contain only a fundamental component, *i. e.*,

$$\psi_a = i_{2(1)} \left[\frac{x_d'' + x_q''}{2} - \frac{(x_d'' - x_q'')^2}{2\,(x_d'' + x_q'')} \right] \cos t = i_{2(1)} \frac{2\,x_d''\,x_q''}{x_d'' + x_q''} \cos t \qquad \textbf{(8d)}$$

If
$$\psi_a = \cos t$$
then
$$i_{2(1)} = \frac{1}{2} \left(\frac{1}{x_d''} + \frac{1}{x_q''} \right)$$
and
$$i_{1(3)} = - \frac{1}{2} \left(\frac{1}{x_d''} + \frac{1}{x_q''} \right) \left(\frac{x_d'' - x_q''}{x_d'' + x_q''} \right)$$

But under the conditions specified,

---

[28] See Appendix B.

$$\psi_b = \cos(t + 120), \quad \psi_c = \cos(t - 120),$$

so that the current distribution,

$$i_a = \frac{1}{2}\left(\frac{1}{x_d''} + \frac{1}{x_q''}\right)\left[\cos t - \frac{x_d'' - x_q''}{x_d'' + x_q''}\cos(3\,t + 2\,\theta_0)\right] \qquad \textbf{(9d)}$$

$$i_b = \frac{1}{2}\left(\frac{1}{x_d''} + \frac{1}{x_q''}\right)\left[\cos(t - 120) - \frac{x_d'' - x_q''}{x_d'' + x_q''}\cos(3\,t + 2\,\theta_0 - 120)\right] \qquad \textbf{(10d)}$$

$$i_c = \frac{1}{2}\left(\frac{1}{x_d''} + \frac{1}{x_q''}\right)\left[\cos(t + 120) - \frac{x_d'' - x_q''}{x_d'' + x_q''}\cos(3\,t + 2\,\theta_0 + 120)\right] \qquad \textbf{(11d)}$$

is just adequate to give rise to balanced normal negative phase-sequence terminal voltage. It follows that the application of balanced normal negative phase-sequence voltage will give rise to a fundamental negative phase-sequence current of amount

$$\frac{1}{2}\left(\frac{1}{x_d''} + \frac{1}{x_q''}\right)$$

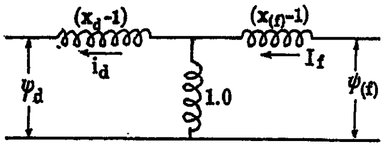


Fig. 11—Equivalent Circuit of Synchronous Machine Direct Axis, Main Field Only

The voltage may be applied either from line-to-line or from line-to-neutral.

Appendix E

Construction of Equivalent Circuits: Concept of Field Leakage Reactance

Consider a machine without any "additional rotor circuits." The equations relating per-unit armature and field linkages are, in this case:

$$\psi_d = I - x_d\,i_d \qquad \textbf{(1e)}$$

$$\Psi = X\,I - X_m\,i_d \qquad \textbf{(2e)}$$

The second equation may be put in the form,

$$\Psi_{(f)} = X_{(f)}\,I - i_d \qquad \textbf{(3e)}$$

where,

$$\Psi_{(f)} = \frac{\Psi}{X_m} \qquad \textbf{(4e)}$$

$$X_{(f)} = \frac{X}{X_m} \qquad \textbf{(5e)}$$

But, equations (**1e**) and (**3e**) are satisfied by the equivalent circuit of Fig. 11.

This circuit also holds good when the linkages $\Psi_{(f)}$ are determined by an external circuit such as an impedance, for example, a reactance and resistance in series. Thus, it is possible to calculate the power factor of a current of any frequency induced in the field, or to determine the effect of external field reactance, etc.

The field voltage equation is

$$E_{(f)} = \frac{d}{d\,t}\Psi_{(f)} + R_{(f)}\,I$$

where $E_{(f)} = \dfrac{E}{X_m}$ and $R_{(f)} = \dfrac{R}{X_m}$

Reactance and resistance of the field circuit are, of course, to be calculated in per-unit and divided by X_m. Normal or per-unit resistance is the product of normal field current and resistance in ohms, divided by the voltage due to unit rate of change of normal flux linkages.

While the circuit just discussed is perfectly practical, it is not at all the only circuit that can be constructed, nor is it in every way the best, since it may involve negative reactances. These latter, while not inconvenient mathematically, are not entirely satisfactory from a physical standpoint.

A particular circuit previously employed[29] is that in which the armature branch of the equivalent circuit is made equal to the leakage reactance of the armature. In this case write:

$$\psi_d = A\left[\frac{I}{A}\right] - x_d\,i_d \qquad \textbf{(6e)}$$

$$B\,\Psi = A\,B\,X\left[\frac{I}{A}\right] - B\,X_m\,i_d \qquad \textbf{(7e)}$$

where A and B are arbitrary factors to be determined in such a way that i_d and $\dfrac{I}{A}$ can be employed in the equivalent circuits. Then, it is necessary that

$$x_d - A = x_l \qquad \textbf{(8e)}$$

or,

$$A = x_d - x_l = x_{ad}$$

But, also there must be, on account of the nature of the equivalent circuit:

$$A = B\,X_m \qquad \textbf{(9e)}$$

$$B = \frac{A}{X_m} = \frac{x_{ad}}{X_m}. \qquad \textbf{(10e)}$$

The equations may then be written:

$$\psi_d = x_{ad}\,I_{[f]} - x_d\,i_d \qquad \textbf{(11e)}$$

$$\Psi_{[f]} = X_{[f]}\,I_{[f[} - x_{ad}\,i_d \qquad \textbf{(12e)}$$

where,

$$I_{[f]} = \frac{I}{x_{ad}} \qquad \textbf{(13e)}$$

$$\Psi_{[f]} = \frac{x_{ad}}{X_m}\,\Psi \qquad \textbf{(14e)}$$

---

[29] Reference (6).

$$X_{[f]} = \frac{(x_{ad})^2}{X_m} X \qquad \textbf{(15e)}$$

The equivalent circuit in this case is shown in Fig. 12. In this case the field voltage equation is

$$E_{(f)} = \frac{d}{d\,t} \Psi_{[f]} + R_{[f]} I_{[f]}$$

where $E_{[f]} = \dfrac{x_{ad}}{X_m} E$ and $R_{[f]} = \dfrac{x_{ad}}{X_m} R$

The term $X_{[f]} - x_{ad}$ has been referred to as field leakage reactance—thus,

$$X_l = X_{[f]} - x_{ad} = x_{ad} \left[\frac{x_{ad}}{X_m} - 1 \right] = \frac{x_{ad}}{X_m} [x_{ad} - X_m] \qquad \textbf{(16e)}$$

The formula for transient reactance may be obtained by shorting the field side of the equivalent circuit. Referring to Fig. 12 this gives:

$$x_d' = x_d - 1 + \frac{(X_{(f)} - 1)}{X_{(f)}} = x_d - \frac{1}{X_{(f)}} = x_d - \frac{X_m}{X} \qquad \textbf{(17e)}$$

or in terms of field leakage reactance

$$x_d' = x_l + \frac{X_l\, x_{ad}}{X_l + x_{ad}} \qquad \textbf{(18e)}$$

The form $\left(x_d - \dfrac{X_m}{X} \right)$ is probably the easiest for calculation since;

x_d is calculated by standard methods.

X_m is calculated from a knowledge of no-load normal field current.

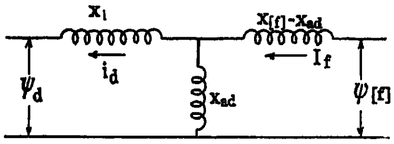


Fig. 12—Equivalent Circuit of Synchronous Machine, Direct Axis, Main Field Only

X may be calculated without great difficulty. It is merely the ratio of the total field linkages to the linkages the field would enclose if normal space fundamental armature flux linked all of the field turns.

The construction of equivalent circuits when additional rotor circuits are to be considered is more difficult than in the simple case discussed and will not be treated here.

Appendix F

Calculation of Total Field Reactance

The calculation of total field reactance, *i. e.*, the quantity or quantities X, is conveniently accomplished by considering separately the fluxes at the armature surface, in the slots or inter-polar space, and at the ends of the poles. This corresponds to the division of armature leakage into air-gap, slot, and end leakage. The reactance X is then expressible as the sum of three factors relating to the armature, inter-polar space, and pole ends respectively. Thus:

X = Armature factor + inter-polar, or slot-factor + end factor.[30]

Armature Factor. That component of flux, due to current in any field circuit, which enters the armature surface will be assumed to link all of that circuit, a condition which, it happens, is very generally fulfilled. Then the armature factor in field reactance, expressed as a per-unit quantity, is the ratio of the total flux to the space fundamental component of flux entering the armature along one pole pitch. This factor was first employed by Doherty and Shirley in their paper on Reactance,[30] and has received the designation K_ϕ. As pointed out by them, it is capable of being expressed as the sum of the per-unit harmonics of flux, due to normal current in the circuit under consideration, divided by the order of the harmonic. Thus:

$$K_\phi = 1 + \frac{k_3}{3} + \frac{k_5}{5} + \ldots\ldots + \frac{k_n}{n}$$

where the corresponding per-unit flux density wave is:

$$\beta = 1 + k_3 \sin 3\,\gamma + k_5 \sin 5\,\gamma + \ldots \text{etc.,}$$

γ being the electrical angle measured from the quadrature axis of the machine.

It is interesting to realize, then, that harmonics in the no-load flux wave of a machine are of the nature of leakage flux. This leakage flux can, however, be positive or negative according to whether the wave of no-load flux is flat-topped or peaked. This would be indicated by negative constants in the above equation. Charts for determining K_ϕ for ordinary field windings are available in the above mentioned paper on Reactance.

Interpolar, or Slot, and End Factors. These factors comprise the field linkages due respectively to all the flux in the stacked length of the machine that does not penetrate the armature surface and to all the end-flux.[31] A formula for the field-flux linkages of salient-pole machines, due to flux which does not penetrate the armature surface, has been given by Doherty and Shirley.[32] Using the quantity L given in formula **(33a)** of that paper, the inter-polar and end factor becomes equal to:

$$\frac{N\, I_{(0)}\, L\, l}{\phi_{(0)}}$$

Where $I_{(0)}$ = no-load field current in amperes
$\phi_{(0)}$ = no-load space fundamental flux per pole
l = gross stacked length of core.

---

[30] Reference (10) p. 1242.

[31] More strictly, all the end-flux which is not space fundamental at the armature winding.

[32] Reference (10), p. 1279.

Thus, there is:

$$X = K_\phi + \frac{N\, I_{(0)}\, L\, l}{\phi_{(0)}}$$

Appendix G

RELATION BETWEEN X_m AND NO-LOAD EXCITATION CURRENT

Ordinarily, the leakage reactance of a machine does not have a transient value perceptibly different from its sustained one because the currents induced in the various rotor circuits by the leakage fluxes are sensibly reactive in character, *i. e.*, are nearly out of phase with the voltage producing them. Consequently, it is possible to neglect the changes in leakage reactance with method of application of current and to regard all the change that is necessary to account for the difference in synchronous and transient reactance as being due to the difference in the reactances of armature reaction in the two cases. Therefore, in calculating the values of the constants X_m, X_{m1d}, etc., it is only necessary to consider the flux linkages of armature reaction, that is, the flux linkages due to the space fundamental component of armature m. m. f.

But the effect of normal space fundamental m. m. f. acting in one axis of the rotor is just two-thirds[33] of the effect of the maximum value of normal phase current. That is, if $i_{(0)}$ is the current flowing in a single-phase winding when the rotor is stationary and its axis in question is directly in line with the axis of magnetization of the excited armature phase, the effect of normal space fundamental m. m. f. is $\frac{2}{3}\, i_{(0)}$.

Now consider any rotor circuit, for example, circuit *h*. The linkages in this circuit, due to a current of normal peak value $i_{(0)}$ in one armature phase in line with circuit *h*, are

$$\frac{2}{3} N_h\, \phi_{(0)}\, X_{hm} \qquad \textbf{(1g)}$$

Thus the mutual inductance of circuit *h* and the armature phase is

$$\frac{2/3\, N_h\, \phi_{(0)}\, X_{hm}}{i_{(0)}} \qquad \textbf{(2g)}$$

The linkages in the armature phase due to a normal value of current $I_{h(0)}$ in circuit *h* are, from the definition of normal current in any rotor circuit, simply[34]

$$n\, \phi_{(0)} \qquad \textbf{(3g)}$$

Thus the mutual inductance of the armature phase and circuit *h* is

$$\frac{n\, \phi_{(0)}}{I_{h(0)}} \qquad \textbf{(4g)}$$

But since (**2g**) must be equal to (**4g**) it follows that

$$X_{hm} = \frac{3}{2}\, \frac{n\, i_{(0)}}{N_{h(0)}\, I_{h(0)}} \qquad \textbf{(5g)}$$

or,

$$X_{hm} = \frac{A_{(0)}}{F_{h(0)}} \qquad \textbf{(6g)}$$

where,

$A_{(0)}$ = 2.12 (effective armature times per pole per phase) × (normal r. m. s. armature current).

$F_{h(0)}$ = no-load air-gap ampere-turns of any circuit .*h*

In particular,

$$X_m = \frac{A_{(0)}}{F_{(0)}} = \text{per-unit mutual field reactance} \qquad \textbf{(7g)}$$

where,

$F_{(0)}$ = no-load air-gap ampere-turns of field.

Using this relation, it is possible to reduce the calculation of transient reactance to the following items:

A. Determination of synchronous reactance,

B. Determination of the "no-load field currents" of each rotor circuit, *i. e.*, determination of current necessary to give normal fundamental component of armature flux, and

C. Determination of self and mutual coefficients of the rotor circuits between themselves.

Appendix H

THE RELATIONS BETWEEN THREE-PHASE AND SINGLE-PHASE REACTANCES

In accordance with the procedure discussed in Part II, the impedances of a machine will be regarded as the ratios of the sinusoidal components of voltage and current due to sinusoidal current in the windings. Since only sinusoidal currents of definite frequency, and only sinusoidal components of terminal voltage of corresponding frequency, are to be considered, it follows that it is legitimate to employ vector notation.

Thus, any current $i = i_0 \sin(t + \alpha)$ will be representable as a vector $\bar{i} = i_0\, \underline{/\alpha}$, and the phase currents will be representable as

$$\begin{aligned} \bar{i}_a &= \bar{i}_1 + \bar{i}_2 + \bar{i}_0 \\ \bar{i}_b &= \bar{i}_1\, \overline{\backslash 120} + \bar{i}_2\, \underline{/120} + \bar{i}_0 \\ \bar{i}_c &= \bar{i}_1\, \underline{/120} + \bar{i}_2\, \overline{\backslash 120} + \bar{i}_0 \end{aligned} \qquad \textbf{(1h)}$$

The phase voltages are similar.

It may be readily verified that with a line-to-line connection between phases *b* and *c*,

$$\bar{i}_1 = -\bar{i}_2 = \frac{\bar{i}_{sbc}\, \underline{/90}}{\sqrt{3}} \qquad \textbf{(2h)}$$

where $\bar{i}_{sbc}$ = line-to-line current of phases *b* and *c*, while with a line-to-neutral connection from phase *a* to neutral,

$$\bar{i}_1 = \bar{i}_2 = \bar{i}_0 = \frac{\bar{i}_{oa}}{3} \qquad \textbf{(3h)}$$

where $\bar{i}_{oa}$ = current from phase *a* to neutral.

---

[33] The effect of fundamental m. m. f. only is to be considered.

[34] Since only the sinusoidally distributed component of the armature winding is considered, harmonic linkages are neglected.

Under normal conditions of operation, however, it follows from the symmetry of every phase, that the fundamental components of the voltages due to any phase-sequence current distribution must be of the same phase-sequence as the current. Thus, subject to the condition of normal motion of the rotor, there is,

$$\begin{aligned} \bar{e}_1 &= \overline{z_1 \dot{i}_1} \\ \bar{e}_2 &= \overline{z_2 \dot{i}_2} \\ \bar{e}_o &= \overline{z_0 \dot{i}_0} \end{aligned} \qquad \textbf{(4h)}$$

but the line-to-line voltage of phases b and c is

$$\begin{aligned} \bar{e}_{bsc} &= \bar{e}_b - \bar{e}_c = 3\,(\bar{e}_1 - \bar{e}_2)\ \diagdown\overline{90} \\ &= (\bar{z}_1 + \bar{z}_2)\,\bar{i}_{s\,bc} = \bar{z}_s\ \bar{i}_{s\,bc} \end{aligned} \qquad \textbf{(5h)}$$

or line-to-line impedance $= \bar{z}_s = \bar{z}_1 + \bar{z}_2$

Similarly it follows that, subject to the conditions stated, line-to-neutral impedance is

$$z_\sigma = \frac{\bar{z}_1 + \bar{z}_2 + \bar{z}_0}{3} = \frac{\bar{z}_s + \bar{z}_0}{3} \qquad \textbf{(6h)}$$

However, these relations do not obtain without special interpretation unless the rotor is moving normally. In case of an "ideal machine" with the rotor stationary, the values of single-phase reactance are given, by the value of $\psi_a - \psi_b$ with $i_a = -i_b = 1$, $i_c = 0$, and ψ_a with $i_a = 1$, $i_b = i_c = 0$, respectively, for a line-to-line or line-to-neutral connection.

Thus, with all rotor circuits closed, the line-to-line static reactance is

$$\begin{aligned} x_s &= x_d'' + x_q'' + \frac{x_d'' - x_q''}{3}\Big[\cos(2\,\theta) - 2\cos(2\,\theta - 120) \\ &\quad + \cos(2\,\theta + 120)\Big] \\ &= (x_d'' + x_q'') + (x_d'' - x_q'')\cos(2\,\theta + 60) \end{aligned} \qquad \textbf{(7h)}$$

The value of x_s when the axis of magnetization of the armature coincides with the direct axis of the rotor is

$$x_s = x_{sd} = 2\,x_d'' \qquad \textbf{(8h)}$$

When the axis of the armature winding coincides with the quadrature axis the value of x_s is

$$x_s = x_{sq} = 2\,x_q'' \qquad \textbf{(9h)}$$

The corresponding line-to-neutral reactances are, with all rotor circuits closed,

$$x_\sigma = \frac{x_0}{3} + \frac{x_d'' + x_q''}{3} + \frac{x_d'' - x_q''}{3}\cos 2\,\theta \qquad \textbf{(10h)}$$

When the axis of the armature winding is in line with the poles

$$x_\sigma = x_{\sigma d} = \frac{x_0 + 2\,x_d''}{3} = \frac{x_0 + x_{sd}}{3} \qquad \textbf{(11h)}$$

and when the armature axis is in quadrature with the poles

$$x_\sigma = x_{\sigma q} = \frac{x_0 + 2\,x_q''}{3} = \frac{x_0 + x_{sq}}{3} \qquad \textbf{(12h)}$$

These relations are useful in determining the values of x_0, x_d'', and x_q''. Thus

$$x_0 = 3\,x_{\sigma d} - x_{sd} = 3\,x_{\sigma q} - x_{sq} \qquad \textbf{(13h)}$$

$$x_d'' = \frac{x_{sd}}{2} \qquad \textbf{(14h)}$$

$$x_q'' = \frac{x_{sq}}{2} \qquad \textbf{(15h)}$$

as mentioned in Part I.

The values of per-unit single-phase reactances which obtain with no closed rotor circuits may also be identified as the per-unit single-phase armature inductances. Thus, in treating the subject of short-circuit currents, Messrs. & Doherty and Nickle[35] employ the relations,

$$\text{line-to-line inductance} = \frac{x_{SD} + x_{SQ}}{2} + \frac{x_{SD} - x_{SQ}}{2}\cos(2\,\theta')$$

line-to-neutral inductance

$$= \frac{x_{\Sigma D} + x_{\Sigma Q}}{2} + \frac{x_{\Sigma D} - x_{\Sigma Q}}{2}\cos(2\,\theta)$$

where, $\theta' = \theta + 30^0$

The idea of transient inductance has also been introduced. For example, the transient line-to-line inductance is,

$$\frac{x_{SD}' + x_{SQ}'}{2} + \frac{x_{SD}' - x_{SQ}'}{2}\cos(2\,\theta')$$

In case machines with auxiliary rotor circuits were to be considered, the conception of the subtransient inductance and the use of the double primed notation would also be in order. Thus, for the transient reactances,

$$x_{SD}' = 2\,x_d', \qquad x_{SQ}' = 2\,x_q'$$

$$x_{\Sigma D}' = \frac{x_0 + 2x_d'}{3}, \qquad x_{\Sigma Q}' = \frac{x_0 + 2\,x_q'}{3}$$

and for the sub-transient reactances,

$$x_{SD}'' = 2\,x_d'', \quad \text{etc.}$$

Appendix I

NOTATION EMPLOYED

The problem of devising a simple scheme of notation which will be in accord as much as possible with existing nomenclature has not been easy. The difficulty is due to the many terms which pertain to synchronous machines.

The notation of the paper follows the somewhat generally established principle that all armature quantities are denoted by small letters or symbols, and all field terms by large letters or symbols. The subscripts d and q refer to the *direct* and *quadrature* axes of magnetization. The corresponding single-phase line-to-line and line-to-neutral quantities are recognized by the subscripts s and σ, respectively. Positive phase-sequence terms may be indicated by the subscript 1, but are usually given no special notation. Negative and zero phase-sequence terms are specified by the subscripts 2 and 0, respectively, following the notation introduced by Mr. C. L. Fortescue.[36] All transient quantities are signified by primes.

---

[35] Reference (2).

These compose, in the main, the group of indexes required, but various modifications and special designations were necessary. The notation is not peculiar to this paper but is largely the same as the system used by Doherty and Nickle in their recent treatments of synchronous machines. Although it is not urged that this notation be standardized, since the primary purpose has been to fill the requirements of the present article, the system may be helpful to others.

A. MAJOR QUANTITIES TO BE IDENTIFIED

	Circuits	
	Armature	Rotor
Voltage	e	E
Linkages	ψ	Ψ
Current	i	I
M. m. f.	A	F
Reactance	x	X
Resistance	r	R
Time-constant	τ	T
Flux density	β	
Time (Electrical radians)	t	t

Electrical space angle along armature surface with respect to phase "a" γ

Electrical degrees in direction of normal rotation between axis of poles and magnetic axis of phase "a" θ

B. USE OF SMALL SUBSCRIPTS IN NOTATION

a. Direct and quadrature quantities.
direct d
quadrature q

These apply to:
positive phase-sequence armature quantities
rotor quantities
single-phase static quantities

b. Distribution.

1. Armature quantities.

positive phase-sequence	1, or no subscript
negative phase-sequence	2
zero phase-sequence	0
single-phase line-to-line	s
single-phase line-to-neutral	σ
separate phases	a, b, c

2. Rotor quantities.
direct d
quadrature q

Where there are several circuits in any axis they are to be distinguished from each other by numeral subscipts, as X_{2d}, except that the symbol, f, may be retained for the field winding proper. If no subscript is employed reference to the main field winding is implied.

c. Use of zero as subscript.

Except in connection with time-constants, 0 will indicate zero phase-sequence quantities, and (0) will imply normal, or per-unit quantities.

T_0 = field time-constant with all armature phases open.
τ_0 = armature time-constant with all rotor circuits open.

C. PRIME QUANTITIES

a. With reference to armature reactances:
sustained—no special indication.
transient—one prime, *i. e.*, x'.
sub-transient—two primes, *i. e.*, x''.

b. With reference to time-constants:
field time-constant with all armature phases shorted—T_0'
armature time-constants with all rotor circuits closed—τ_0'

D. ALL SUBSCRIPTS CAPITALIZED

In the present scheme of notation, this form is reserved for the particular single-phase static reactances used by Doherty and Nickle in their paper *Synchronous Machines IV*, and discussed here in Appendix B. These are:

total static reactance	x_D
static reactance due to space fundamental	x_{AD}
static leakage reactance	x_{LD}
static transient reactance	x_D'
static transient fundamental reactance	x_{AD}'

As these reactances are single-phase they are also characterized by S and Σ to indicate whether they are line-to-line or line-to-neutral, thus

$$x_{SD}, \text{ or } x_{\Sigma D}.$$

Corresponding quadrature values are designated similarly by the subscript Q instead of D.

E. MISCELLANEOUS TERMS

It is necessary to include a number of special terms which have not been discussed above. The notation used for these quantities is more or less arbitrary, but an attempt has been made to employ accepted symbols as much as possible.

h = a subscript denoting any rotor circuit. In conjunction with d or q it signifies that that circuit is in the direct or quadrature axis.

K_ϕ[37] = a per-unit quantity expressing the ratio of the total flux to the space fundamental component of flux entering the armature along one pole pitch.

l = used as a subscript to denote leakage reactance, or used alone to denote length of armature stacking.

L[37] = effective leakage lines per unit length of stacking, linking the entire field winding when the field m. m. f. is one ampere-turn.

---

[36] Reference (11).

[37] First employed by Doherty and Shirley in their paper Reactance, reference 10.

m = a subscript denoting mutual reactance between armature and any field winding.

n = a subscript, similar to h, denoting any field circuit, or used alone to denote effective armature turns per pole per phase.

N = turns per pole of the main field winding.[38]

Δ = a coefficient employed to denote increment change in any quantity.

In Appendix E it has been found necessary to devise special notations for certain quantities in order that they fit the construction of the equivalent circuits. These notations employ parentheses and brackets. Their use may be made clear by reference to that appendix.

PRINCIPAL DERIVED UNIT QUANTITIES

F. The principal derived unit quantities are

$\psi_{(0)}$ = normal armature flux-linkages

$$= \frac{e_{(0)}}{2\,\pi\, f}\, 10^8$$

$\phi_{(0)}$ = normal armature flux

$$= \frac{\psi_{(0)}}{n}$$

$\Psi_{(0)}$ = normal flux-linkages of the main field winding $= N\,\phi_{(0)}$

$E_{(0)}$ = per-unit field voltage produced by the variation at normal frequency of normal flux-linkages in the main field $= 2\,\pi\, f\, \Psi_{(0)}\, 10^{-8}$

Appendix J

PER-UNIT REPRESENTATION OF QUANTITIES

Messrs. Doherty and Nickle, in their recent series of papers on Synchronous Machines, have employed the practise of representing quantities as a decimal fraction of some normal or base value. This method shares with the percentage method of representation, the marked advantage that it eliminates the necessity of employing unwieldy conversion factors in purely analytical work. At the same time, it is definitely superior to the percentage system in that it is unnecessary to continually consider the factor "one hundred." In the percentage system this factor must be specially inserted or divided out whenever the operations of division or multiplication are performed. Therefore, on account of the convenience of the decimal fraction system it will be used hereafter in the paper.

Thus, in the paper, each quantity will have a normal value, and the value of the quantity at any time will be expressed as a decimal fraction of its normal value. For example, normal armature linkages are defined as the linkages which generate normal voltage when varied at normal frequency. Thus, $\psi = 0.45$ means that the armature linkages at the instant of consideration are 45 per cent of this normal value.

Originally, the method of representation used was referred to as "percentage representation of quantities, as a decimal," but this name was to some extent a source of confusion in view of the accepted meaning attached to the word "percentage." Recently the term "per-unit representation of quantities" has been suggested as a designation for the decimal percentage system. As this name appears to solve the difficulties previously encountered, it has been used throughout the paper.

References

1. 1928, Alger, P. L., *The Calculation of the Reactance of Synchronous Machines.* See p. 493.

2. 1928, Doherty, R. E. and Nickle, C. A., *Synchronous Machines IV.* See p. 457.

3. 1927, Schaefer, R. A., *A Study of Transverse Armature Reaction in Synchronous Machines*, JOURNAL A. I. E. E., August, 1927, pp. 811-814.

4. 1927, Wagner, C. F. and Dovjikov, A., "Impedance of a Rotating Synchronous Machine to a Negative Sequence Voltage" *Electric Jl.*, March 1927, pp. 117-21.

5. 1927, Wieseman, R. W., *Graphical Determination of Magnetic Fields; Practical Application to Salient-Pole Synchronous Machine Design*, JL. A. I. E. E., Vol. 46, 1927, pp. 430-31; Discussion, pp. 614-20.

6. 1927, Doherty, R. E. and Nickle, C. A., *Synchronous Machines III, Torque-angle Characteristics Under Transient Conditions.* TRANS. A. I. E. E., Vol. XLVI, 1927, p. 1.

7. 1927, Douglas, J. F. H., *Transverse Reaction in Synchronous Machines*, JL. A. I. E. E., February 1927, pp. 109-15.

8. 1926, Doherty, R. E. and Nickle, C. A., *Synchronous Machines I and II, An Extension of Blondel's Two Reaction Theory*, A. I. E. E. TRANS., Vol. 45, pp. 912-47.

9. 1926, Biermanns, J., Uberstrome in Hochspannungsanlagen, 1926, p. 154.

10. 1918, Doherty R. E. and Shirley, O. E., *Reactance of Synchronous Machines and Its Application*, A. I. E. E. TRANS., 1918, Vol. 37, part 2, pp. 1209-97; discussion pp. 1298-1340.

11. 1918, Fortescue, C. L., *Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks*, A. I. E. E. TRANS., 1918, Vol. 37, p. 1037.

12. 1913, Arnold, E., *Die Wechselstromtechnik*, 1913, Vol. IV, p. 214.

13. 1913, Blondel, A., *Synchronous Motors and Converters*, a translation by C. O. Mailloux, 1913, Part III.

14. General Equations of an "Ideal Synchronous Machine." An article by one of the authors to appear shortly in the *G. E. Review.*

Discussion

A. E. Kennelly: At the present time, according to the information which has been presented to us, there are possibly thirty-one varieties of reactance in electrical machinery.

It seems important that these useful concepts referred to in these papers as "reactance," "leakage reactance," "differential leakage reactance," and so on, should be clearly set forth and tabulated, because evidently, in the future, when a speaker uses the word "reactance," his audience will want to say, "Which?" It is increasingly important that this matter be thoroughly classified.

The concept which has been presented to us of the per-unit value seems to me of great importance. Percentage, of course, is an expression that we are constantly employing and is very useful, but in computations, percentage is not so convenient, not so simple as per-unitage, and there is no reason why the unit should not have at least as much recognition as the centum, so I hope that method of expression will continue to find favor.

---

[38] Effective turns in the case of turbo alternators and induction motors.

H. H. Spencer: The papers treat generally of single-phase short circuits and their resulting phenomena, but the trouble which we run into in operating practise is not solely confined to single-phase disturbances. I should like to ask the writers if it is possible simply to superpose the conditions of subsequent short circuits upon the initial equations to determine what may happen later and if so, how the constants here presented must be shaded, and how constants for the three-phase fault can be found.

S. L. Henderson: Under the subject of Zero Phase-Sequence Reactance, the statement is made that there is no armature-reaction m. m. f. due to zero phase-sequence currents. This is true for the fundamental current. However, for a third-harmonic current, the currents in the three phases are in time phase, and there is an m. m. f. of three times the number of pairs of poles on the rotor. The effect of this third harmonic m. m. f. can be seen on a delta-connected machine, without $^2/_3$ pitch. The temperature of the field and the losses on short circuit are higher with the machine connected in delta than when connected in star.

I understand, therefore, by the statement in the paper, that there may be an m. m. f., but that there will be no reaction of this m. m. f. on the exciting circuit. Is this because this m. m. f. cancels out in a pair of poles?

R. H. Park: Mr. Spencer has inquired how the condition of a three-phase fault occurring shortly after a single-phase fault is to be calculated. A fault of this type may be calculated by determining from the solution for the single-phase short circuit the values of field and armature flux linkages at the instant when the three-phase fault occurs, and then adjusting the a-c. and d-c. components of the three-phase fault to correspond with these calculated initial conditions. Thus, neglecting saturation and extra rotor circuits, the initial value of the a-c. component of the three-phase short-circuit current will be the same as though the three-phase short circuit had occurred at a voltage equal to the product of the voltage before the single-phase short circuit by the decrement of the a-c. component of the single-phase short-circuit current, at the instant at which the three-phase short circuit occurs.

Thus, if the three-phase short circuit begins at time (t_o) after the occurrence of the single-phase short circuit the symmetrical component of three-phase short-circuit current will be,

$$i = e\left[\frac{1}{x_d} + \left(\frac{e^{-a t_0}}{x_d'} - \frac{1}{x_d}\right)\epsilon^{-\frac{t - t_0}{\tau_0'}}\right]$$

If there are several rotor circuits or if it is desired to determine the d-c. component the answer is not as easy. The general method which may be employed is to calculate the effect of impressing, on the open phases of the machine, a voltage equal and opposite to that which would have existed under the condition of single-phase short circuit if the three-phase short circuit had not occurred. If the currents which are found in this way are added to those found from the single-phase short-circuit calculation the sum will be that for the three-phase short circuit.

Mr. Henderson's difficulty in regard to our statement that "there is no armature reaction due to zero-phase-sequence currents" arises from the special interpretation of this term which we have introduced. According to the concept advanced in the paper we would regard as armature reaction only those magnetomotive forces which are stationary with respect to the rotor. The advantage claimed in this view is, that it clearly delineates between those magnetomotive forces which can freely penetrate the rotor, and which do not induce eddy currents in it, and those magnetomotive forces which move relative to the rotor and are thus subject to the demagnetizing action of the currents which they induce. These latter magnetomotive forces are regarded as producing leakage flux.

As stated in the paper the exact line of demarcation between armature reaction and armature reactance must necessarily be drawn arbitrarily. It is hoped, however, that the distinction made will prove to be a useful one.

As to the possibility of the reaction of the third-harmonic m. m. f. due to zero-phase-sequence current on the exciting circuit, it is of course understood that m. m. f's need not be considered as directly reacting on each other, but that we are entitled to find the effect of two m. m. f.'s by adding the effects that would be due to each acting separately. Now in general the d-c. current in the field will produce an armature voltage of fundamental and all harmonic frequencies. A third-harmonic zero-phase-sequence current will develop armature m. m. f. of third and multiples of third harmonic frequency in space, varying in time at three times normal frequency. To the extent that the reluctance of the air gap is not uniform these m. m. f.'s can give rise to fluxes of frequency in space different from their own, and it is to just this extent that they can give rise to other than third-harmonic armature voltage. Since the third-harmonic m. m. f. in question is pulsating at three times normal frequency, half of it may be regarded as moving forward at synchronous speed, thus stationary with respect to the poles. In the case of a salient-pole machine this third-harmonic m. m. f., stationary over the poles, can give rise to a fundamental component of flux, also stationary over the poles, and thus generating voltage of normal frequency in the armature. Ordinarily, however, this effect is very small. Were it present in a delta-connected machine, not of $^2/_3$ pitch, a slight change (increase or decrease) in the fundamental component of voltage would be expected when the delta is opened or closed.

The fact that the third-harmonic circulating current of a delta-connected machine, not of $^2/_3$ pitch can cause very appreciable losses makes it desirable that a means of calculating these currents be available. If the third-harmonic voltage of the open delta is first calculated, the current that will result when the delta is closed can be found by dividing this voltage by three times the third-harmonic zero-phase-sequence reactance, which in turn will, in general, be slightly more than three times the corresponding fundamental zero-phase-sequence reactance. The reason that the third-harmonic zero-phase-sequence reactance is slightly greater than three times the corresponding fundamental value is that, as mentioned above, in the case of third-harmonic current, half of the space third-harmonic m. m. f. is stationary with respect to the poles, and thus is not subject to the demagnetizing action of the rotor. Usually this effect is not very great. It can, however, be corrected for by modifying the value of (K_o) in Mr. Alger's formula for zero-phase-sequence reactance.

The Thermal Volume Meter

BY G. W. PENNEY[1] and C. J. FECHHEIMER[1]
Associate, A. I. E. E. Fellow, A. I. E. E.

Synopsis.—The thermal volume meter was originally brought out by Professor Thomas, and is frequently called a Thomas meter. The principle is that the temperature of the gas is raised by means of an electrical heater, the change in temperature being accurately measured either by means of resistance thermometers connected in a wheatstone bridge network, or by means of a thermopile. It is believed that the thermocouple method is preferable to resistance thermometers, and most of the paper deals with that construction.

The usual construction has walls of wood or micarta, which serve as an insulating support for the heater coils and thermocouples. The general proportions of the volume meter as to section and length are dealt with, the former being determined mainly by the mean velocity of the air, and the latter by the distance needed to give the gas an opportunity to mix after passing the heater. A sufficient number of thermocouples is required to give a good reading of the potentiometer with a comparatively low temperature rise of the gas and also to obtain a good average of the gas temperature. The details of construction are described. The equations that are suitable for calculating the volumes of the gas are then given.

Then follows considerable discussion on the possible sources of error in the meter, the principal ones being

The non-uniform heating of the gas.
Non-uniform distribution of velocities.
Heat escapement.

Methods are suggested for determining the magnitude of these various conditions experimentally or by calculation. There are several minor sources of error, such as:

Time lag of meter.
Unsteady source of e. m. f.
Instrument errors.
Heat flow along thermocouple wires.
Loss of pressure in passing through the meter.
Variation of specific heat and humidity.

Most of these are shown to be of negligible effect.

Some notes on design which include means of calculating the proportions of the heater are given.

In Appendix I the advantages and disadvantages of the various methods of measuring gas volumes are included. In Appendix II will be found some notes on comparison of resistance thermometers and thermocouples for temperature measurement. In Appendix III consideration is given to the electrical machine as a thermal volume meter.

INTRODUCTION

THERE are many ways of measuring the volume of flow of gases, each having advantages and disadvantages. The most important of these methods are given in Appendix I, together with reasons why they are not generally well adapted for the measurement of air volumes passing through electrical machinery. With most of the more accurate methods the principal objection is that the pressure drop is so great as to reduce the volume considerably (*e. g.*, the orifice method). In the thermal volume meter, this objection is met by using a short duct in which the velocities are only moderate and the obstructions very small, so that the resistance to flow is very slight.

The thermal volume meter was first brought out by Prof. Carl Thomas, and is frequently called a Thomas meter.

The principle of operation of the instrument is that the temperature of the gas is raised, the temperature increase and amount of heat required to produce that change in temperature being accurately measured. As the specific heat at constant pressure has been accurately determined, the mass of the gas flowing per unit of time can readily be calculated. As most engineers are more interested in the volume than in the mass, the change to volume can easily be made if one has a knowledge of the absolute pressure and temperature. In practise, the gas is heated electrically. The temperature increase is measured by one of two methods: (1) by means of resistance thermometers placed before and after the heater, they being used as two arms of a wheatstone bridge; and (2) by means of thermocouples connected in series (frequently called a thermopile). A comparison of these two methods of temperature rise measurements is given subsequently. It is believed that the latter method using thermocouples is preferable, and the following description pertains to that type.

In this paper are given the experiences of the writers and their co-workers at the Westinghouse Elec. & Mfg. Company; there is very little in the technical literature on the subject with which to make comparison.

Some of the data in this paper have appeared elsewhere, but as they were scattered, it was thought best to combine them with those which have not been previously published.

CONSTRUCTION OF THE THERMAL VOLUME METER

As electrical conductors are used for the heater and for the thermocouples, it simplifies the construction to use for the walls material that is an electrical insulator. That has an additional advantage, as an electrical insulator is usually a poor heat conductor, and to secure accuracy it is essential that escape of heat from the outside be avoided. In all but the smallest sizes, pine or maple wood has been used; and for the smaller sizes, such as is shown in Fig. 1, micarta plate was the material adopted. The walls are then thermally "insulated" with cork board. In the larger sizes with wooden walls about 1 in. thick, and with internal temperatures of the air not over 5 deg. cent. above atmospheric temperature, it was found that the heat loss from the outside was negligible, even though

---

[1] Both of the Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

there was no heat insulating material added to the exterior. This heat loss can be calculated close enough to determine whether or not insulating material should be added, and how much.

In Fig. 1 is shown a small meter with one side removed for inspection. The inside dimensions are 3½ by 3½ by 18 in. It is suited for handling from about 60 to 300 cu. ft. per min. This meter had micarta walls, fastened together with machine screws. In Fig. 2 is shown a medium size volume meter in which the sides are made of planed pine lumber about one in. thick.[2]

It has been found desirable to wind the heater element on a mandrel with the wires close together, and then stretch them like a spiral spring. These are arranged in a number of coils of equal numbers of turns, and are supported by means of screw-eyes in the walls. This scheme of construction for the heater coils is shown in Fig. 1. It is desirable to keep the axes of the spirals vertical; otherwise, with the axes horizontal, when the coils heat the wires may sag and there is then the possibility of part of a coil short circuiting on another coil. In the larger volume meters it is desirable to support the heater coils say half way, or one-third and two-thirds the way up, for which glass rods have been found to be suitable. In Fig. 1 four rows of heater coils were used; in all but the very small meters, two rows of heaters are sufficient, and these are then staggered with respect to each other.

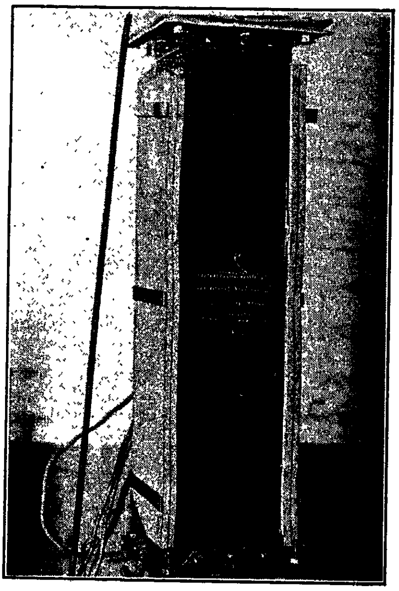


Fig. 1—A Small Volume Meter With One Side Removed for Inspection

The distance between the heater coils and the thermocouple junctions is largely a matter of how far the gas must flow to mix fairly well; it has been the practise of the writers to make that distance at least as great as the mean of the two cross-sectional dimensions for the large meters, and about two to three times as great for the smaller meters. It would be of interest to learn from others what their practise has been.

It is desirable to use small screw-eyes in order that the heater wires may be close to the walls, thereby securing as uniform distribution of heat as possible throughout the cross-sectional area.

The number of thermocouples used has never been less than 16, and usually the number is more. With a large number of thermocouples a good average of the temperature increase of the air is obtained and a fairly high reading of the potentiometer is secured. Thus, with 40 microvolts per degree per couple, and with 25 couples, a reading of 2 millivolts is obtained with only 2 deg. cent., increase in air temperature. In arranging the locations of the thermocouple junctions, the rectangular area is broken into as many equal squares as there are thermocouples, and a junction is placed in each square. The scheme of arrangement for 25 thermocouples all in one plane, in a square duct, is indicated in Fig. 2. In the small volume meter shown in Fig. 1, for simplicity of construction, each set of thermocouple junctions is arranged in four planes. It is well to select the wires of such size that, with the arrangement adopted, the flow of heat along the wire to wall will not be sufficient to alter appreciably the temperature of the junction. The thermocouple wires within the meter are all bare. The scheme of connections is shown in Fig. 3, where for simplicity two thermo-

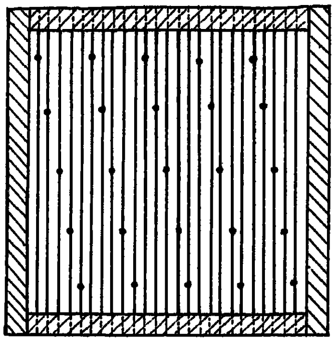


Fig. 2—Cross-Section Showing Location of Thermocouple Junctions

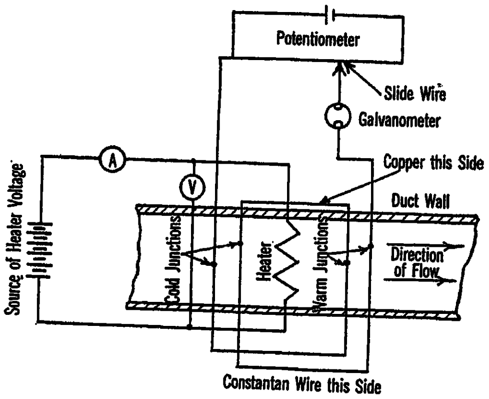


Fig. 3—Wiring Diagram of a Volume Meter

---

[2] See paper by Laffoon and Calvert: *Additional Losses of Synchronous Machines*, Trans. A. I. E. E., Vol. XLVI, 1927, p. 84.

couples are shown. The junctions are usually made by slightly overlapping the wires, and silver soldering; or, in the larger meters, they may be made by twisting the wires and soft soldering. It is interesting to note that all wires on one side of the meter are of one kind of metal, say constantan, and opposite another kind such as copper. Thus, only copper wires are brought to the potentiometer. It has been found simplest, in making the required number of junctions, to bring out the leads through suitably drilled holes in the walls of the meter and make the connections on the outside. These may be simply twisted, preferably soldered.

It is important that the thermocouples placed in the cool stream be at the same distance from the heater as those in the warmed gas. Otherwise, heat radiation will play its part unequally, thereby introducing a large error, even when the heater coils are at a comparatively low temperature.

In dry weather, static electricity has been known to affect the reading of the potentiometer, and it is well, therefore, to ground one of the thermopile leads.

EQUATIONS FOR COMPUTING VOLUMES

It may readily be shown that

$$Q = \frac{0.03155}{\gamma C_p} \frac{W}{\theta} \tag{1}$$

Here Q = cu. ft. per min. of the gas.
γ = density in lb. per cu. ft.
C_p = specific heat at constant pressure.
W = watts absorbed by the gas.
θ = temperature rise of the gas.

For dry air, the density at 25 deg. cent. and 29.92 in. of mercury may be taken as 0.074, and the specific heat as 0.2418. Then for such conditions,

$$Q = 1.765 \frac{W}{\theta} \tag{2}$$

For any other than standard conditions, the law of a perfect gas may be used, so that then

$$Q = 1.765 \left(\frac{273 + t}{273 + 25} \right) \frac{29.92}{P} \frac{W}{\theta}$$

$$= 0.177 \left(\frac{273 + t}{P} \right) \frac{W}{\theta} \tag{3}$$

Here t is the temperature of the air, deg. cent., and P is the total pressure in in. of mercury.[3] The influence of humidity is so small that it may well be neglected. The order of error is discussed under "Sources of Error."

In the volume meter, there are n thermocouples for each of which there are E microvolts per deg., the reading on the potentiometer being e millivolts. Then

$$\theta = 1000 \frac{e}{n E} \tag{4}$$

which substituted in equation (3) gives:

$$Q = \frac{0.177\, n\, E\, W}{1000\, e} \left(\frac{273 + t}{P} \right) \text{cu. ft. per min. for air} \tag{5}$$

$$\text{or } Q = \frac{0.03155\, n\, E\, W}{\gamma\, C_p\, e} \text{ cu. ft. per min. for any gas} \tag{6}$$

Q = cu. ft. per min.
E . = microvolts per deg. from calibration curve.
W = watts input to heater (absorbed by the gas).
γ = density of gas in lb. per cu. ft.
C_p = specific heat of gas at constant pressure (about 0.2418 for air).
e = millivolts as read on potentiometer.
t = gas temperature in volume meter, deg. cent.
P = absolute pressure in in. of mercury.

The calibration curve of the thermocouples is nearly, but not quite, a straight line. In general, its equation may be written as:

$$V = a\, t + b\, t^2 + c\, t^3 \tag{7}$$

where V is the millivoltage, t the temperature, and a, b, and c constants.

For most cases the third term may be dropped. The microvolts per deg., E, will then be:

$$E = 10^3 \frac{d\,V}{dt} = 1000\,(a + 2\,b\,t) \tag{8}$$

Equation (8) is that of a straight line which may be plotted; or a chart may be used combining the constant and the terms $(273 + t)$ and P from equation (5) with E.

It is important to secure an accurate calibration curve, and if there are not facilities available, a sample of the wire may be sent to any of the well-recognized laboratories, such as the Bureau of Standards, where a very accurate calibration can be obtained. One of the manufacturers of constantan wire has a standard calibration, and it keeps its products so close to that curve that it is usually sufficiently accurate to accept its data without further checking.

POSSIBLE SOURCES OF ERROR

The principal sources of error are non-uniform heating of the gas; non-uniform distribution of the gas; and heat escapement.

Non-uniform Heating of the Gas has been previously mentioned. To discuss this, it is assumed that the velocities are uniform throughout the section. It is manifestly impossible to secure uniform distribution of loss in the heater coils, for to secure that would mean that the heater covered the entire section. By making the volume meter fairly long the gas has more of a chance to mix after it has passed the heater. It is believed that the heater coils tend to produce some turbulence in the gas stream, which assists in mixing the gas. Then by employing a large number of thermocouples the error becomes practically negligible. In some of the tests the e. m. f. per thermocouple was measured, which was easily done by removing the insulation from the joints of the series connections on one side of the meter, using a sensitive potentiometer and galvanometer for balancing. It was usually found that there was considerable variation in e. m. f. per couple, but with a large number of couples, the average of the readings was so close to the true average that the error was very small. For example, the volume meter with 20 thermocouples, shown in Fig. 1, was checked against a converging nozzle, designed in accordance with data in the paper by Dr. S. A. Moss.[4] In three sets of readings, in which the air was heated 3 deg. to 6.8 deg., the difference between the nozzle and volume meter readings were + 0.4, − 0.8, and − 1.5 per cent, the plus sign meaning that the volume meter read high. Yet readings of individual thermocouples varied from 0.40 to 1.05 millivolts, with an average of 0.827 millivolts, thus showing 51.5 per cent below, and 27 per cent above, the mean. In this case the velocity distribution was very good, the error due thereto being probably less than 0.3 per cent.

---

[3] It will readily be seen that the volume per unit time changes with the density as the gas passes through the apparatus. The particular value of t and P to be used in equation (3) are those corresponding to the location where the value of volume is desired: this might be ahead of, behind, or within the meter. In most cases, the differences in density at various locations are so small as to be negligible.

Non-Uniform Distribution of the Gas. In the above, where non-uniform heating was considered, it was assumed that the velocities were uniform throughout the meter. In general, that condition cannot be obtained but can be approached close enough to reduce the error, due thereto, to negligible magnitude. The effect of non-uniform velocities is different from non-uniform heating. Thus assuming uniform heat distribution, the temperature increase of the gas as it passes through the heater coil is nearly inversely as the velocity. On the other hand, if the velocities are uniform, the temperature increase is directly proportional to the heat per unit of area. It is because of the reciprocal relation with velocities that great departures from uniformity should be corrected. In a number of the earlier tests it was found that the air in some small areas actually moved in the reverse directions to the main air streams. Then the cool and warm thermocouple junctions were reversed, and a very large error was introduced. It is not essential that the velocities be uniform, but the departures should not cause the errors in measurement to be more than about one per cent. Then there may be considerably greater irregularities than for, say, pitot tube measurements.

If the sectional area of the duct be broken into a number of small but equal areas, through any one of which the velocity be considered as uniform and be assumed to obtain throughout the length of the volume meter, then the ratio of the volume that would be measured were the velocities uniform to the volume that is measured given by:[5]

$$R = \frac{(\Sigma V)\left(\Sigma \frac{1}{V}\right)}{n^2} \tag{9}$$

Here V is the velocity in the small section, and n is their number, that is, multiply the volume as measured by R to obtain the correct volume.

To determine the velocity distribution an impact tube has been used. In some cases the air discharges from the meter directly to the atmosphere so that the impact tube can be used conveniently; although in some cases the duct just beyond the meter must be removed. A suitable impact tube may then be used without considering the differential pressure, as the surrounding pressure is atmospheric, the same as on the open side of the manometer. The section may conveniently be divided into a suitable number of equal squares by means of small wires drawn taut, and the impact tube placed in turn at the approximate center of each square. A suitable, rigid support should be provided far enough back from the issuing stream so as not to interfere with streams before they issue.

In some cases the duct system beyond the volume meter cannot readily be removed, and then slots can be cut in the duct preferably before and after the meter. As used by the authors, the explorations were made in horizontal and vertical directions, (with the flow horizontal), but only along the center lines. The static pressure was usually fairly high, and the differential pressure was then read using an ordinary pilot tube which was passed through the slot. It was necessary to hold the tube rigidly, suitable clamping being provided and tightened for every setting of the tube.

While uniformity of velocities is not essential it has been our practise to make the velocities sufficiently alike to reduce the error to about one per cent, or less, as given by equation (9). When the air is to be taken directly into the volume meter from the atmosphere, the *vena contacta* arising from sharp entrance conditions should be avoided and a suitably curved, converging intake should be provided such as is shown in Fig . 4. It is often convenient to provide a curved, converging intake when the volume meter is made part of a duct system. This scheme was shown in Figs. 3 and 6 in a previous paper.[6] There are times when there is liklihood of having poor distribution of velocities, and when a converging duct cannot conveniently be used. Then, suitable "air straighteners" in a duct ahead of the volume meter have been employed. These consist of about three vertical and three horizontal planes of sheet metal (assuming the air flow to be horizontal). These planes are hinged at one end and the other end can be moved externally by means of suitable rods, or the like, which can be clamped in any position.[7] Adjustments are made until the distribution is satisfactory.

---

[4] "The Impact Tube," American Society of Mech. Eng. 1916.

[5] *The derivation of this equation is given in the Appendix of paper, "Performance of Centrifugal Fans for Electrical Machinery," A. S. M. E., 1924.

[6] *Ventilation of Turbo Alternators; Concluding Study*, A. I. E. E. TRANS., Vol. 45, pp. 255, 256.

It should be especially noted that an obstruction within or just ahead of the meter, or a sudden bend, etc., which may tend to produce poor velocity distribution, should be avoided. On the other hand, if there is a fairly long, straight duct ahead of the volume meter the velocity distribution will probably be sufficiently satisfactory. Generally, the distribution just after the air leaves the volute around a centrifugal fan is not of enough uniformity.

Heat Escapement. To secure accurate measurements all of the heat generated in the heating coils must be absorbed by the gas passing through the meter, and the heat loss from the walls must be of no consequence.

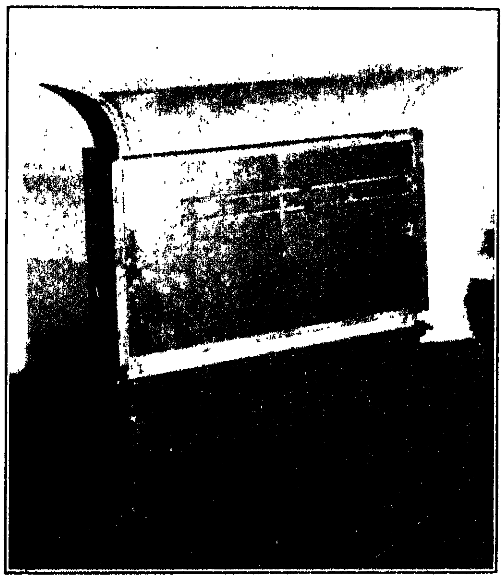


FIG. 4—MEDIUM SIZED VOLUME METER

In the earlier calculations tests in which moderate size volume meters with wooden walls were used, and the difference between the internal and external temperatures were generally less than 4 deg. cent., the losses of heat from the outside surface, were negligible. But in later tests, when small volumes were to be measured with perimeters that were large in relation to the cross-sectional areas and with their walls of higher thermal-conducting material, the heat loss was sufficient to introduce an appreciable error. The air entered the meter not far from the discharge of a centrifugal fan, which was operated at low volumes, high pressures, and inefficiently. The temperature of the air entering the volume meter was consequently serveral deg. cent above the temperature of the surrounding atmosphere. By adding several layers of corkboard to the outside, the heat loss was made practically negligible. Another way of remedying the difficulty is to surround the meter with a jacket of air at about the same temperature as the air inside, which in this particular case could have been done by letting a little of the air, as discharged from the fan, go through the jacket and then escape to the atmosphere. If the meter is measuring the air discharged from an electrical machine, the air entering the meter may be considerably warmer than the surrounding air, which in some cases may give a correspondingly large heat dissipation in passing through the meter. A test for this error is to pass the initially heated air through the volume meter with no current in the heater coils and measure the decrease in temperature using the thermocouples and potentiometer. If a temperature change can be detected there is an error due to heat dissipation.

It is believed to be desirable to maintain fairly low temperatures of the heater, the design being such that it is of the order of 50 deg. cent. above the air. With high heater temperatures, there will be appreciable transfer of heat by radiation to the walls, and thence by conduction through the walls to the outside air. Such loss of heat, which cannot be recorded by the temperature-measuring device may introduce a considerable error.

Although the heater temperature is only about 50 deg. cent. above the air, the watts per sq. in. of wire surface are generally considerably higher than for, say, insulated conductors in an air stream, or bare conductors in still air. The reasons are that the wires are of small diameter; are bare; and are right in the air stream. In one instance the mistake of bringing one of the heater wires as a lead through the wooden wall, was made and the wood was burned by the high temperature that the wire attained. There was a small error introduced because some of the heat generated in the wire buried in the wood was not taken up by the air passing through the volume meter. It has been found satisfactory to place a short loop of the wire under a small brass bolt head or washer, the bolt being used as a terminal. There may then still be a small error due to very small heat escapement through the bolt, but that is generally negligible.

MINOR SOURCES OF ERROR

Time Lag of Meter. As in any thermal device, time is required to reach a steady state of temperature. That time constant[8] is practically $\frac{\text{rate of storage heat}}{\text{rate of dissipation of heat}}$.

Unlike many other thermal devices, the time constant seems to be greater the smaller the meter. In the larger thermal volume meters, the potentiometer connected to the thermocouples may be read in considerably less than a minute after the current value is fixed. In some cases, it seemed unnecessary to wait at all after the value of heater current was fixed. But in some tests made with a small volume meter (1½ in. square in section) it was sometimes necessary to wait more than ten min. until the potentiometer reached a steady value. Fig 5 shows time-temperature curves for a small volume meter. The temperature as indicated by the thermocouples is plotted as a function of the time in minutes after closing the heater switch. This is done for a normal rate of air flow through the meter, and also for a low rate of flow, and shows that the sluggishness increases as the volume decreass. The percentage of error as derived from the temperature curves is also plotted against the time. The sensitivity of the meter was indicated by a sharp break in a cooling curve when a door leading out of the building was opened in cool weather. These latter data are not given in this paper.

---

[7] This scheme is shown in Fig. 1 in paper, "Performance of Centrifugal Fans for Electrical Machinery, A. S. M. E., 1924. See also text following "The Test Rig."

[8] This refers to the time constant in the usual exponential equation.

Unsteady Source of E. m. f. From the above it will be seen that it is important to have a steady source of e. m. f. for the heater, particularly when there is liable to be a considerable time lag. Often, to simplify tests, the heater is connected through a rheostat to a fluctuating shop circuit, and then it is found nearly impossible to duplicate one's readings.

Instruments. If suitably checked, and direct current is used, ordinary indicating ammeters and voltmeters are suitable for measuring the power input to the heater. If the heater material be of negligible temperature coefficient, dividing the volts by the amperes should give a constant, an additional check on the readings. If alternating current is used, an accurate watt-meter may be employed, or usual ammeter and voltmeter if it is found that the circuit is non-inductive. Very sensitive potentiometers are obtainable for measuring the thermocouple e. m. f. However, most of the galvanometers used for balancing are not sufficiently sensitive. Recently a small portable reflecting galvanometer has been used, and the sensitivity has been much improved.

The Thermocouples. As previously pointed out, the walls, plus heat insulation, if used, should limit the flow of heat through them to a negligible quantity. This should imply that the inner wall surface temperature equals that of the air stream adjacent to it. A short distance from the inner-wall surface the air temperature may be slightly different from that right at the wall, and a thermocouple junction may be placed there. If the thermocouple wire is large enough heat may flow from the junction to the wall to affect the temperature at the junction. Therefore, it is desirable to select wire that is just large enough to possess the necessary mechanical strength, and yet prevent enough heat-flow longitudinally to affect the junction temperatures. To secure an accurate average of temperature some junctions must be near the walls, following the arrangement indicated in Fig. 2. It is also evident that to read the correct temperature a little farther away from the wall, where the air temperature is higher, by thermocouple, the cooler air stream near the wall should not affect the junction temperature.

As previously stated, there may be a very considerable error if the cold and warm thermocouples are not symmetrically placed with respect to the heater, (due to unequal radiation), even if the heater is worked at a comparatively low temperature.

In taking observations it is usually well to take at least two sets of readings with different watts input to the heater, for the same volume. The watts divided by the millivolts should not differ more than about one per cent. This is a means of checking unequal radiation to the cold and warm junctions.

Effects of a Drop in Pressure in Passing Through the Meter. When using a volume meter for measuring a much larger volume of air than normal, there may be a considerable pressure drop in the meter. Since this represents a dissipation of potential energy it may be thought that this may result in an increase in air temperature with a corresponding error in the indicated volume. Taking an extreme case, if the energy dissipated went to increase the temperature of the air, a pressure drop of one in. of water would give an increase in temperature of 0.21 deg. cent., which would be a serious error when as ordinarily used the rise in the volume meter is only two to four deg. cent. However, the expansion of the gas tends to cool it and the energy dissipated tends to heat it, so that these two effects compensate for each other. Or, stated differently,

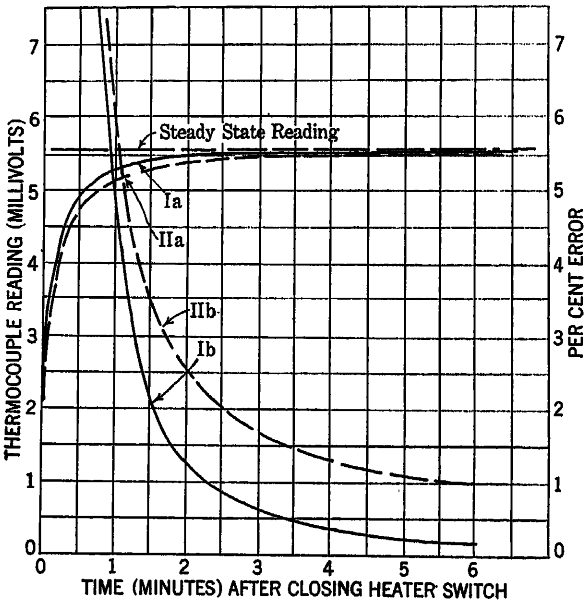


Fig. 5—Time-Temperature Curves for a Small Volume Meter

Ia. Thermocouple reading in millivolts plotted against time in min. after closing the heater switch, with a normal rate of air-flow through the meter.

IIa. Same as Ia except a low rate of air-flow through the meter.

Ib. Per cent error plotted against time at which the temperature is read for normal rate of air-flow through the meter.

IIb. Same as Ib except a low rate of air-flow through the meter.

when gas is expanded by a throttling process the total energy content of the gas is unchanged so that for a perfect gas there would be no change in the temperature. In actual gases there may be some very slight change in temperature which may either heat or cool the gas depending on the initial temperature and pressure; but under conditions prevailing in the thermal volume meter, they are negligible.

Influence of Variation in Specific Heat and Humidity. Errors introduced by neglecting the influence of change of specific heat with temperature, and of various percentages of humidity are very small, and it is believed that for practically all engineering work, their effects upon volume meter readings may be ignored. According to Mark's Mechanical Engineers Hand Book, the specific heat of dry air at 10 deg. cent is 0.2412, and at 50 deg. cent. it is 0.2422,—a difference of only 0.41 per cent, over a probable greater range of temperatures than will usually be encountered.

As to humidity, the density decreases and the specific heat increases the greater the humidity, the product of the two changing very little. At 25 deg. cent and 29.92 in. of mercury, for dry air the product is 0.07403 by 0.2418 = 0.0179. For 100 per cent moisture it is 0.07317 by 0.2462 = 0.01801. The difference between these two products is only 0.6 per cent, and this is for greater extremes than will be found in test.

SOME NOTES ON DESIGN

Let W = watts input to heater.
d = diameter of wire in heater, in.
l = length of wire in heater, in.
ρ = resistivity of metal used in heater in ohms per square in. per in.
S = surface of wire in heater, sq. in.
V = mean velocity of gas, thousands of ft. per min.
E = E. m. f. across heater.
θ = temperature rise of wire above air.

Then
$$W = \frac{\frac{\pi}{4} d^2 E^2}{\rho\, l} \tag{10}$$

$$S = \pi\, d\, l \tag{11}$$

The formula that has been used for the rate that heat is transferred from the wire to the air is:[9]

$$\frac{W}{S\,\theta} = \frac{\left(\frac{1.21}{d} + 595\right)\sqrt{1 + 20.3\,V}}{10^4} \tag{12}$$

The first term in the numerator may be dropped without much error, and the three equations may then be combined and solved:

$$l = 2.82 \sqrt[3]{\frac{W}{\rho\,(1 + 20.3\,V)}\left(\frac{E}{\theta}\right)^2} \tag{13}$$

$$d = 1.13 \sqrt{\rho\, l\, W} \tag{14}$$

from which the length and diameter of wire may readily be calculated. When these are determined, the arrangement of the various parts may be determined by means suggested in other parts of the paper. The watts loss is determined from a knowledge of the approximate volume of air and the temperature rise of the air.

Velocities of air from about 400 to 4000 ft. per min. have been used in volume meters. When using the above equations, one should estimate on the highest velocity anticipated, as, for a given temperature rise of the gas, the higher the velocity the higher the temperature of the heater coils.

Appendix I

ADVANTAGES AND DISADVANTAGES OF VARIOUS METHODS OF MEASURING GAS VOLUMES

The measurement of air volumes is very difficult because air cannot be weighed by the methods ordinarily used for weighing or measuring solids or liquids, nor does its flow follow simple laws such as for the flow of an electric current. For these reasons, rather indirect methods must be used for measuring air volumes. There are many different methods, and each has advantages which make it applicable to certain cases. The following are some of the most commonly used methods:

1. Direct measurement of total volume by means of a gas-holder or gasometer.
2. Measurement by displacement of volume per unit of time (*e. g.*, an ordinary gas meter).
3. Measurement by means of an anemometer of the velocities of the air passing a given area.
4. Measurement of the pressure drop across a given orifice.
5. Measurement of the velocity of discharge from a nozzle, using an impact tube to measure the velocity head.
6. Measurement of velocities in a duct by means of a pitot tube.
7. Measurement of pressure change in a converging (subsequently diverging) channel, known as a Venturi tube.
8. Measurement of volume per unit of time by means of a thermal volume meter.

A few of the advantages and disadvantages of these eight methods are given below, in brief:

1. The gasometer is the absolute method for accuracy. Its very large size prohibits its use except for calibrating other devices, and then only for the smaller rates of flow. For example, the air from a 10-hp. motor, if caught for one minute, would probably occupy about 100 cu. ft.

2. The ordinary gas meter is also too large, except for small volumes of flow. Also, in most displacement meters the pressure drop is prohibitively high for use

---

[9] Taken from paper by Kennelly, Wright and Van Bylevelt, TRANS. A. I. E. E., 1909, p. 388.

with electrical machines. It is an accurate means of measurement.

3. With an ordinary rotating-vane anemometer, it is usually found very difficult to get accurate results. The instrument itself is liable to disturb the flow; the resistance that it offers to the flow may reduce the volume of air through that part of the section in which the instrument is placed. If held by hand, the arm and hand disturb the flow and seriously affect the reading. The anemometer is a delicate instrument, and its calibration is liable to change. It may be used with a fair degree of accuracy when the total volume of air passes through it,[10] but that is necessarily a limited application. So far, the hot-wire anemometer has not been successful, as it is usually too sensitive for direction of flow.

4. The measurement of pressure drop across an orifice is very satisfactory where the pressure drop necessary to obtain a reading of reasonable magnitude is not objectionable. The orifice requires calibration, but fairly complete published data are available giving the calibrations.[11] A considerable length of straight duct is required, both ahead of, and past the orifice, as otherwise the measured pressure drop across the orifice may be changed. For these reasons an orifice is seldom, if ever, used for electrical machines. The method has been used successfully in work on models, but even then the applications were rather special.

5. When a pressure drop of the order of an inch of water is permissible, the air may be discharged through a nozzle or stack, and the velocity head of the stream may be measured by means of an impact tube and an ordinary manometer. Some rather extensive data are contained in a paper by Dr. S. A. Moss,[4] but as the conditions may be different in the electrical machine, it may be advisable to do some experimental work to find a form of nozzle which will give approximately uniform discharge velocities. An example of a stack is shown in Fig. 2 in a recent paper by Laffoon and Calvert.[12] The stack is applied to a turbo alternator, and in many such high speed machines the volume is affected but little by the additional pressure needed for the stack; and if the fan characteristic is approximately known the volume can readily be corrected so as to allow for the reduction.

6. With a pitot tube, it is essential to have a long straight duct so as to eliminate most of the eddies, and the velocities must be high enough to secure high enough reading to insure reasonable accuracy. The long straight duct is usually very inconvenient, and there may be an appreciable pressure drop due to the length of the duct, fairly high velocity and losses at the bend usually essential at the ends of the duct. Even so, unless the pitot tube is used with considerable care, errors will be introduced. For these reasons this method is seldom applicable to electrical machines.

7. The Venturi tube is used frequently with liquids, but seldom with gases. The authors have not had practical experience with this device, but it appears that unless the converging channel is made carefully, and the taps for pressure drop placed with precision, the meter is not accurate. Venturi meters were used in some tests a few years ago with apparently considerable success.[13] There will be some loss of pressure, as, with the divergent channel following the convergent duct, some head must be lost. It is believed not to be in general suitable for use with electrical machinery.

8. One of the principal advantages of the thermal volume meter is that the pressure drop through it is very small, as the obstructions offered by the heater coils and the temperature-measuring devices are not of great magnitude. Furthermore, the velocity can be low as compared with that needed with most of the other methods enumerated above, which also means low pressure drop.

Appendix II

SOME NOTES ON COMPARISON OF RESISTANCE THERMOMETERS AND THERMOCOUPLES FOR TEMPERATURE MEASUREMENT

In some of the earlier work of one of the writers, (about ten years ago), resistance coils were used, consisting of fine copper wires stretched zigzag across the air passage, the wires in the cold and heated air forming two arms of a wheatstone bridge, the other two arms being of constant resistance external to the volume meter. No connections were changed once the circuit was formed, so that there could be no error introduced due to a change in contact resistance. The network was so arranged that with a predetermined temperature rise of the air the galvanometer read zero. The loss in the heater was altered until this condition was obtained. However, this way of operating is very slow and tedious because there is a time lag between a change of the heat input and the resulting change in resistance. This makes it very difficult to secure a balance if there are fluctuations in voltage or air flow. A much more convenient method is to adjust the heater current to a given value and read the resulting temperature rise of the air, but this requires changing the contacts when measuring resistance.

An inherent difficulty in using resistance coils is that the rise in temperature is small, so that the change in resistance is very slight and, therefore, extreme accuracy is required in measuring the resistance. Thus, for two deg. cent. change in air temperature the change in resistance is only 0.8 per cent; consequently the maximum allowable error in measuring this resistance is about 0.008 per cent for one per cent error in volume.

---

[10] An instance of this was described in paper *An Experimental Study of Ventilation of Turbo Alternators*, A. I. E. E. TRANS., Vol. 43, 1924, p. 480.

[11] J. M. Spitzglass, "Orifice Coefficients, Data and Results of Tests, *Trans. of A. S. M. E.*, 1922. Also report of A. S. M. E. Special Committee on Fluid Meters, Part I, 1924.

[12] *Additional Losses of Synchronous Machines*, TRANS. A. I. E. E., Vol. XLVI, 1927, p. 84.

[13] "The Operation of Blowers in Parallel for Forced Draft in Naval Service," M. C. Stuart, *Jl. of Amer. Soc. for Naval Engrs.*, Nov. 1920.

Even though resistance can be measured with great accuracy with a wheatstone bridge network, it is believed that such order of precision is too much to expect. On the other hand, each thermocouple measures the temperature rise of the air between the two points in the cool and warm air where the two junctions are located. A given percentage error in measuring the e. m. f. of the thermocouple produces the same percentage error in the air volume. But when using resistance thermometers a given percentage error in measurement of resistance is multiplied many times in the air volume. Higher air-temperature rises than two degrees are frequently used, but that means proportionately greater power consumption in the heater which is quite an item with large volumes. The smallness of this error with thermocouples is one of their important advantages.

In the tests referred to above, the wires in the resistance coils got covered with dirt and the added weight, combined with the effect of the air streams impinging on the enlarged surfaces, caused the wires to sag, thereby changing the resistance of the thermometer coils. As the resistance did not change equally, a very serious error was introduced. While it is true that in some meters this objection is met by more substantial construction, it also may mean greater obstructions in the paths of the gas, and usually considerably more expensive construction.

There is an advantage in the use of resistance thermometers, in that a better average of the temperatures is obtained. However, as previously explained, it is believed that with a large number of thermocouples, the error is so small as to be negligible.

Each source of error applicable to volume meters equipped with thermocouples, holds also for resistance thermometers, except that due to the escapement of heat along the thermocouple wires to the walls of the meter. However, there may be even a larger error with resistance coils, because the supports afford paths for heat flow to or from the walls, which may affect the temperatures of the coils.

It will be of value to obtain an expression of opinion from others regarding the merits of the two methods of measuring the gas temperature rise.

Appendix III

THE ELECTRICAL MACHINE AS A THERMAL VOLUME METER

In most electrical machines in which forced convection is used for cooling, an accurate knowledge of the temperature rise of the air and of the losses which produce it, should enable one to calculate the volume. Conversely, if one knows the volume and the temperature rise of the air, one can calculate the losses. Both ideas have been used. In most tests it is too expensive to provide separate volume meters. Frequently the machine losses and air-temperature rise in the machine can be used to advantage to determine the approximate volume. Even though the method is not accurate, it is extremely helpful at times when one is searching for the cause of high temperatures.

There are errors that may creep in with this method which do not appear as such in the thermal volume meter.

1. It is difficult to determine the amount of the losses that are absorbed by the ventilating air. Usually the bearing loss has little effect on the air-temperature rise, and, as it is uncertain in magnitude, it is very difficult to correct when making a single test. A method has been suggested by E. Roth[14] in which the losses and corresponding air-temperature rise are measured for two different conditions at the same speed (*e. g.*, tests at two voltages). If the friction losses are the same for both tests, the bearing loss can be eliminated by using the difference in air-temperature rises and the difference in losses. Since the result depends on the difference of quantities of the same order of magnitude, any error in the readings is magnified in the result. This may be a serious source of error because a machine has a high thermal capacity, and it takes considerable time to determine the steady state air rise. An accurate determination of the air rise is nearly impossible if the room temperature is fluctuating. Therefore, this method may lead to erroneous results.

If only one temperature test is made, and the losses determined separately, we are never certain just how much the electrical losses are. This, of course, is quite beyond the scope of the paper, and is only mentioned to indicate some of the uncertainties.

2. We usually assume that all losses go to heating the ventilating air. A small part of the generated heat escapes from the frame, or from other parts of the machine, and all that can be done is to estimate roughly the extent of such heat escapement. This heat escapement is generally by natural convection and radiation, neither of which admit of accurate calculations. Fortunately, most of such heat loss is relatively small, so that a great error is not introduced.

3. If the air temperature rise is measured where the air leaves the machine the air velocity and temperature usually vary widely at different parts of the cross-section. Under such conditions a true average temperature rise cannot be obtained. To correct this, either the velocity or the temperature must be substantially uniform over the cross-section where measurement is made. This requires special precautions in measuring the temperature rise of the air. One method of securing uniform air velocities at the point

---

[14] A. I. E. E. TRANS., Vol. 44, 1925, p. 905

where the temperature is to be measured is described in the paper by Laffoon and Calvert.[2] A stack was used, thereby securing nearly uniform velocities.

It has been the writer's experience that it is desirable to measure the temperature rise of the machine's ventilating air by means of a thermopile, just as for measuring the temperature rise through a volume meter. This means that a large number of wires must be used and for every thermocouple, the wires must be run from the intake to the outlet. Not less than 8 thermocouple junctions should be used at each intake assuming two intakes, meaning that there are 16 junctions at the discharge. It is desirable to increase this number.

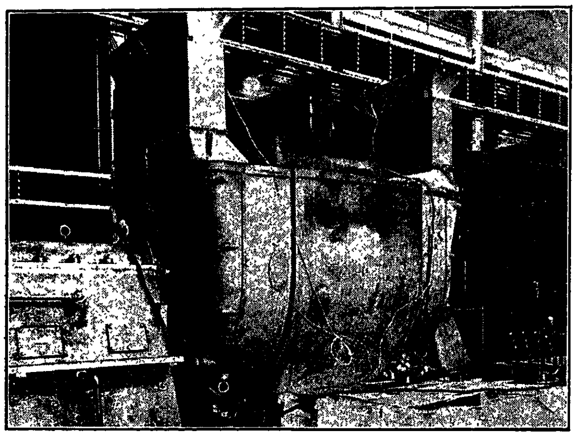


Fig. 6—A Small Turbo Alternator with Volume Meters Attached

Thermocouples were also provided for measuring temperature rise of the air through the machine.

A turbo alternator (Fig. 6) was rigged up for testing, with volume meters and with thermocouples to measure the rise in temperature of the air passing through the machine. For simplicity of test, the machine was inverted. The large number of small wires are parts of the thermocouples.

Experience indicates that it is not well to use cotton covered wire for these thermocouples as they get covered with dirt and there is liable to be a small leakage current from one wire to another which cannot be indicated readily, and all of the readings become worthless. It is well to use higher grade insulation, such as rubber, on the wires. The thermocouple wires in Fig. 6 were cotton covered, but were subsequently changed.

Bibliography

"Measurement of Losses and Efficiency by the Temperature Rise of the Ventilating Air," W. F. Dawson, *General Electric Review*, 1920.

Three-Phase, 60,000-kv-a Turbo Alternators for Gennevilliers, E. Roth, A. I. E. E. Trans., Vol. 44, 1925, p. 905.

A Contribution to Research on the Experimental Determination of the Losses in Alternators, E. Roth, A. I. E. E. Trans., Vol. 45, 1926, p. 221.

"Some Elements of Air Flow in Electrical Machinery," C. J. Fechheimer, *Electric Jl.*, Aug. 1922, p. 348; also, May 1923, p. 174.

"The Influence of Density upon the Weight and Volume of Air which passes through an Electric Machine," C. J. Fechheimer, *Electric Jl.*, Feb. 1926.

Additional Losses of Synchronous Machines, Laffoon and Calvert, A. I. E. E. Trans., Vol. XLVI, 1927, p. 84.

"The Heat Paths in Electrical Machinery," Symons and Walker, I. E. E., (British) 1912.

"The Impact Tube," S. A. Moss, S. A. M. E., 1916.

"Orifice Coefficients, Data and Results of Tests," J. M. Spitzglass, A. S. M. E., 1922.

Report of A. S. M. E. Special Committee on Fluid Meters. Part I, 1924.

"The Orifice Tests of Spitzglass Compared with German Measurements," L. Dorgerloh, *Ver. Deut. Ing. Zeitsch.*, May 21, and June 11, 1927, pp. 703-8, and 851-6.

"Performance of Centrifugal Fans for Electrical Machinery," C. J. Fechheimer, A. S. M. E., 1924.

"The Operation of Blowers in Parallel for Forced Draft in Naval Service," M. C. Stuart, *Jl. of Amer. Soc. for Naval Engrs.*, Nov. 1920.

An Experimental Study of Ventilation of Turbo Alternators, C. J. Fechheimer, A. I. E. E. Trans., Vol. 43, 1924, p. 476.

The Cooling of Electric Machines, G. E. Luke, A. I. E. E. Trans., Vol. 42, 1923, p. 636.

Ventilation of Turbo Alternators—Concluding Study, C. J. Fechheimer and G. W. Penney, A. I. E. E. Trans., Vol. 45, 1926, p. 253.

Discussion

W. F. Dawson: My experiences have been very much along the lines of these authors, except that I have used resistance thermometers rather than thermocouples.

When we started we didn't use the calorimetric idea at all. We started in by attempting to measure the air velocity, and thence the air volume with manometers, and we found, as you will see by referring to an article I wrote for the *General Electric Review*, February 1920, enormous irregularities of air velocity across the inlet section. The air velocity across the outlet circuit was not quite so bad, but you will find how that might be by referring to the paper by Barclay and Smith,[1] which was prepared almost at the same time that I prepared my article.

At the suggestion of Dr. S. A. Moss, I adopted the calorimetric method. We have been going ahead with that; in fact, I make practically all of my efficiency and loss tests on turbo alternators by this method, and I cannot over-emphasize how valuable and how convenient it has been. We never could have accumulated the information for the analysis of our losses on turbo alternators if it hadn't been for this exceedingly satisfactory scheme.

I feel that a comparatively large increase in the temperature of the air as it is passing the heater coils contributes so much to the accuracy of our information that we can afford a little effect from radiation.

We did find soon after we started the experiment with the heaters that immediately they were switched on, the temperature of both inlet and outlet air was increased; I mean by that the air entering the heater and that leaving it.

Quite contrary to the arrangement shown in the illustrations in the paper, I placed this meter in the discharge pipes or conduits and when I found that we had to take account of radiant heat, we were very careful to place that heater midway between the two resistance thermometers so that the heat would affect both equally.

---

[1] "Determination of the Efficiency of the Turbo Alternator," by S. F. Barclay and S. P. Smith, *Journal I. E. E.*, April 1919.

We make a log showing the effect of the radiant heat from the heater from which final corrections are made.

There is one other thing, the question of averaging the air temperatures, whether it is H_1, H_2, or H_3 (H_1 being the temperature of the air entering the machine or dynamo under observation, H_2 the temperature of the air as it leaves the dynamo, and H_3 the temperature of the air after it has passed the heater). I finally made resistance thermometers in which the copper resistance wire was wound upon wooden struts placed arbitrarily about every 4 in. and having stream line sections, spaced equally across the air passage. We pass the resistance wire between loosely cogged gears in order to corrugate it and prevent it from being strained as it is heated and cooled and then we wind it on those struts as a helix of small insulated copper wire, shellac it in place, connect all in series and calibrate the whole frame, in the Standardizing Laboratory. Then by special indicating galvanometers we read our air temperatures directly and accurately within 0.1 deg. cent.

J. Ormondroyd: The temperature read on the potentiometer of the air-flow meter is an average value got by a "summation of ordinates" integration. The milli-volts which would be generated in any couple depends on the position of the couple in the flow meter cross-section $V = f(x, y)$

where the center of the cross-section can be considered the origin of the x, y coordinate system. The average milli-voltage $\overline{V}$ in the case where the couples are arranged in rows and columns equally spaced over the meter cross-section is

$$\overline{V} = \frac{\Sigma\,[\,\Sigma\, V_x\,\Delta\, x\,]_y\,\Delta\, y}{\Sigma\,\Sigma\,\Delta\, x\,\Delta\, y} = \frac{\Sigma\,[\,\Sigma\, V_x\,\Delta\, x\,]_y\,\Delta\, y}{\text{Cross-section area}}$$

Where the distribution is actually known $\overline{V}$ can be determined accurately by means of an experimental integration by building a solid whose upper surface represented the known $f(x, y,)$ and weighing it, for instance. In the flow meter the actual distribution depends on the disposition of the heaters, and the character of the air flow; but the couples are distributed in a fixed manner over the cross-section. The accuracy of the integration performed by the potentiometer depends on the distribution.

Tests on the small flow meter show that the distribution of the temperature over the cross-section can be represented approximately by the empirical equation $V = 65 - 7x^2 - 7y^2$

With twenty-five couples arranged in five rows in the staggered fashion used, average values in each row of couples and over the whole area are calculated by adding ordinates, and by analytical integrating. Assuming the analytical integral to be correct, the errors are as follows:

Couple plane	Per cent error
Center..........	+ 0.5
Half way out.....	+ 0.6
Outside..........	− 0.9

The average over the whole area is = 50.6 analytically, and = 51 by summing ordinates. The per cent error is 0.8.

Doing the same with 20 couples gives 0.6 per cent error; a somewhat smaller error than the larger number of couples gives, probably due to a more fortunate distribution of the couples.

When continuous resistance wires are used instead of distributed thermocouples, the errors in the planes of the wires may be considered to be zero. But in summing up five parallel wires (to compare with the 25-couple arrangement), the error is still about 0.6 per cent. The difference is not enough to be of practical importance.

The probable error in the small flow meter is in the order of ± 1 per cent. In the larger meter the distribution of temperature is not so uniform and the error may be two or three times larger.

B. L. Barnes: The authors have very wisely called attention to a number of sources of errors in the set-up and operation of a thermal volume meter but I believe that it is also advisable to mention a very important condition that should be taken into consideration when using a volume meter of this type to make efficiency or loss tests on large apparatus. That condition is that the temperature of the air entering and leaving the machine as well as the temperature of the machine must reach a constant value before readings can be taken for determining the losses or efficiency for the load that the machine is carrying. Large machines will require from five to ten hours to reach constant temperature at a given load and if the temperature of the cooling air entering the machine happens to be changing toward the end of that period the change in temperature of the cooling air passing through the machine is not a true measure of the losses. If the temperature of the air is rising the machine losses go to raise the temperature of the machine as well as to raise that of the air passing through. The rise in the air would be less than under a constant condition. If the temperature of the air is dropping the temperature of the machine will drop and the heat energy stored in the machine will be given out to the cooling air so that its temperature will be raised more than under a constant condition.

Several attempts were made by the engineers of the Hydro-Electric Power Commission of Ontario and by myself to carry out tests of this kind on some large generators on the Canadian side at Niagara Falls. We were never quite satisfied with the results because the temperature both in the power house and out of doors was constantly changing, and there was no way of controlling the temperature of the air as it entered the generator.

C. J. Fechheimer: It is interesting to learn that Mr. Hobart was one of the first to propose the idea of the thermal volume meter. It is also well to recall at this time a paper presented before the Institute in 1909 on *The Convention of Heat from Small Copper Wires.*[2] As a result of the investigations given in that early paper the hot-wire anemometer was developed.

Referring to Mr. Dawson's contribution, our views on the question of resistance thermometers versus thermocouples are covered in the paper. There is just one point on which we would comment. He claims that he measures the air-temperature rise within 0.10 deg. If the total rise is of the order of 2 deg., the possible error is then 5 per cent, which is considerable. If the air rises are much higher, the power consumption in the heater becomes considerable with large volumes.

Replying to the question raised by Mr. Barns, we also have had difficulties from variations in cooling-air temperature. This frequently is due to opening and closing of doors, windows, or the like. In some tests in a turbo alternator, the cold thermocouple junctions used in connection with the temperature rise of the air through the machine, were placed on relatively heavy copper which had fairly large thermal capacity, and somewhat reduced the effects of alterations in air temperature. But generally, where accuracy is important, the time for making tests should be so chosen as to eliminate the effects of unsteady draughts, or equivalent, upon the ingoing air temperature. The gradual changes in air temperature are usually not the causes of disturbances; the sudden changes are the troublesome ones.

It is of value to estimate the probable loss of heat from the heater coils when they are worked at high temperatures. We shall assume a heater wire 0.04 in. diameter, and calculate by means of Eq. 12 in the paper the watts per sq. in. at two temperatures and two velocities; 500 and 4000 ft. per min. The heat escapement by radiation is given by the well known equation for dull black surfaces (the wires tarnish and get dirty after being in operation for some time):

---

[2] Paper by A. E. Kennelly, C. A. Wright, and J. S. Van Bylevelt, TRANS. A. I. E. E., Vol. XXVIII, Part I, p. 363.

$$\text{Watts per sq. in.} = 36.9 \left[\left(\frac{T_1}{1000} \right)^4 - \left(\frac{T_2}{1000} \right)^4 \right]$$

Here T_1 and T_2 are the absolute temperatures in deg. cent. of the hot and cold surfaces respectively. The surface of the wire here is only the projected area, whereas that computed by means of Eq. 12 is the entire outer area. That is, for comparison, the radiated watts should be divided by π. One extreme would be if all the radiated heat were finally transmitted to the duct walls whence it escaped by conduction through the walls to the outside air. That is hardly justifiable. We shall assume a duct width equal to the distance between cold and warm thermocouple junctions, in which case there might be a possibility of about ⅛ of the radiated heat escaping. The following table is based upon all of the above assumptions, the ingoing air temperature having been taken as 25 deg. cent.

Heater rise deg. cent.	Air vel. ft./min.	Per cent. error
50	500	0.255
600	500	1.980
50	4000	0.094
600	4000	0.740

Thus it will be seen that at low velocities the error introduced by running the heater, at high temperature may be more than should be allowed. The error is practically negligible if the heater rise is around 50 deg. cent.

Recent Improvements in Turbine Generators

BY STUART L. HENDERSON[1] and C. RICHARD SODERBERG[1]
Associate, A. I. E. E. — Non-member

Synopsis.—During the last few years there has been a very rapid increase in the rating of turbine generators. That these ratings might be possible, a large amount of investigation has been necessary; and this paper covers, in a general way, some of the problems that have been studied and solved by the engineers of the Westinghouse E. & M. Co. The present status of the design of turbine generators is given, and what the future developments must be if still larger generators are to be built.

INTRODUCTION

THE object of the present paper is to describe some of the problems that have been encountered in the development of the turbine generator during the last five years, and to illustrate the means by which the recent increases in the ratings of the individual units have been made possible. These five years have witnessed a very rapid expansion of the size of the individual units. In 1922, the largest four-pole, 1800-rev. per min., single generator in operation, or under construction, had an output of 37,500 kv-a.; at the present time, individual units of the same speed having a capacity of 94,000 kv-a., are being built.

The general application of the cross-compound principle has brought about units of power generation having enormous outputs. The amount of power generated in the present combinations of two or three generators has reached a value corresponding to the output of a complete power plant of a few years ago. The advance made in the ratings of the individual generators is not always apparent in reviewing the outputs of the cross-compound units. The figures given in Table I indicate, however, that the increases in the rating of individual generators have been fully as striking as the increases in the ratings of complete units. In 1925, in the two-pole, 1500-rev. per min. class the maximum output of a single generator was 66,700 kv-a.; at the present time there is a generator of this type under construction having an output of 160,000 kv-a. In the four-pole, 1800-rev. per min. class, the maximum rating in 1924 was 44,400 kv-a.; at the present time, a unit of 94,000 kv-a. is being built. Thus, in both classes, the output has been more than doubled in less than four years. The figures in Table I have been given to indicate the trend of the development and do not represent a complete record of all large units that have been built or are under construction.

The construction of a turbine generator of these large sizes is a problem that encompasses practically all branches of mechanical and electrical engineering, and the achievements of today are the results of years of intensive development work of several firms and many individuals. This chapter of the history on the development of turbine generators, which is presented in this paper, will be told mainly in terms of the development work carried out by the Westinghouse E. and M. Co. It is hardly necessary to emphasize that this does not imply that this company has been the sole agency in this development, because the majority of the problems presented have been encountered and brought to a satisfactory solution by all firms active in this branch of engineering.[2]

THE VENTILATION PROBLEM

For 1800-rev. per min. machines, the diameter of the rotor is limited (with materials commonly used), to about 55 inches, so that the increases in capacity which have taken place in the last five years have been obtained chiefly by increases in the core length. Even in the largest units built five years ago, it became increasingly difficult to supply all the cooling air from the two ends of the machine, not only because of the excessive pressure required, but also because of the increase in temperature of the cooling air as it passed through the machine. This latter factor was particularly important in long axially ventilated machines.

Multiple radial ventilation solved this difficulty, and it is now possible to obtain fully as uniform a temperature distribution for core lengths of 250 in. as for 100 in. The multiple radial system of ventilation is illustrated in Fig. 1. The stator laminations and vent plates are built up in a similar manner to the simple radial type. The multiple radial system differs from the simple radial in that not only can the air enter from the ends of the machine by way of the air-gap, but air can also enter the machine from the back of the core in one or more zones. In this manner a number of paths in multiple is provided. Air passes through the air-gap from the ends and is discharged through the radial vents at the end of the stator. The air entering from the back of the core flows radially inward toward the rotor, then axially along the air-gap, and then, radially outward in the discharge zones. As the length of the stator is increased, the number of intakes from the rear is increased in order to obtain uniform air velocities along the core. The introduction of cool air at a number of points along the core from the back of the core serves to give the result of a number of short machines in series, as far as air distribution is concerned;

---

[1] Both of Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

[2] See Bibliography.

TABLE I

Size of unit—kw.	No. of generators	Rating of each gen.—kv-a.	P. F. %	Speed rev. per min.	Year	Manufacturer
60,000	1	66,700	90	1500	1925	G. E.
80,000	2	44,000	90	1800	1924	Westinghouse
60,000	1	70,600	85	1800	1925	G. E.
50,000	1	59,000	85	1800	1927	Allis Chalmers
88,000	2	38,000 64,700	85	1800 1200	1927	G. E.
90,000	1	100,000	90	1500	On order	G. E.
200,000	3	89,400 72,940 72,940	85	1800 1800 1800	On order	G. E.
110,000	2	68,750 68,750	80	1800 1800	On order	Westinghouse
160,000	2	94,100	85	1800 1800	On order	Westinghouse
160,000	1	160,000	100	1500	On order	G. E.
160,000	2	88,250 100,000	85	1800 1200	On order	Brown-Boveri

and it is no longer necessary to restrict the core length on account of ventilation. The multiple radial ventilation system was extensively investigated several years ago by Fechheimer, Bratt, and others[3] and while the system was not new at that time, an exact predetermination of the dimensions of the air circuit was not possible until the work of these men had been completed.

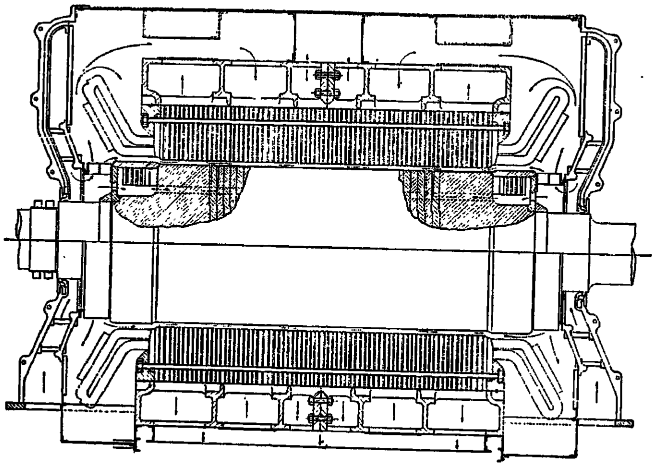


FIG. 1—THE MULTIPLE RADIAL VENTILATION—SKELETON FRAME

The detail arrangement of the stator ventilation has required considerable attention. The minimum number of ventilating ducts required to give satisfactory air distribution may result in iron packages of too great width and, in consequence, too high temperature drop through each package. Fig. 2 shows a section of the stator including two iron packages, one radial vent, and two coils with their insulation. For the distance L of the coil, there are two paths in parallel for the heat to flow to the surrounding air. For the portion of the coil in the radial duct, there is a temperature drop through the insulation and a surface temperature drop; and for that part of the coil embedded in the iron, there is an additional temperature drop through the punchings. Therefore, the part of the coil in the duct will dissipate more than its proportion of the heat. In order to calculate these various temperature drops, it is necessary to determine the thermal conductivity of the

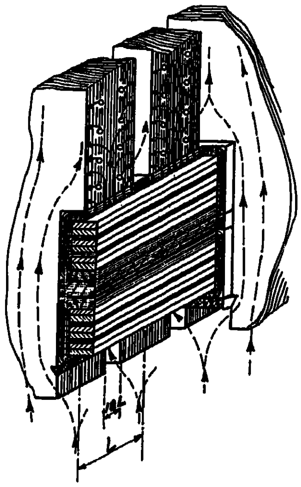


FIG. 2—HEAT FLOW IN STATOR COILS AND PUNCHINGS

built-up stack of punchings, the thermal conductivity of the insulation, and the surface drop in relation to air velocity. For the particular case shown, 30 per cent of the heat generated in the coil escapes directly to the air in the duct, although only 18 per cent of the coil is in the air duct. For a total drop from the copper to the air of 53 deg. cent., 33 deg. cent. represents the drop through the insulation, 12 deg. cent. drop through the iron, and 9 deg. cent surface drop. For that portion of the coil in the air duct, the drop through the insulation is 51 deg. cent. and the surface drop 2 deg. cent. These figures are representative for one particular type of generator and will vary for different proportions in machines, but they do show in a general way the magnitude of the different temperature drops.

---

[3] C. J. Fechheimer, TRANS. A. I. E. E., Vol. 43, 1924, p. 476, *An Experimental Study of Ventilation of Turbo Alternators.*

D. Bratt, TRANS. A. I. E. E., Vol. 43, 1924, p. 467. *The Multiple Radial System of Cooling Large Turbine Generators.*

C. J. Fechheimer and G. W. Penney, TRANS. A. I. E. E., Vol. 45, 1926, p. 253, *Concluding Study of Ventilation of Turbo Alternators.*

C. J. Fechheimer, A. S. M. E., 1924, "Performance of Centrifugal Fans for Electrical Machinery."

For more than five years the project of ventilating with hydrogen generators and other large machines, has been under active consideration by designing engineers. The most important advantage connected with this use of hydrogen is reduction in windage loss; in an 1800-rev. per min. generator of the maximum sizes under consideration, this is the largest single loss in the generator, and may amount to as much as one percent of the kw. rating. But there are obvious practical difficulties in building a sufficiently gas-tight structure and these practical problems have, up to a very recent date, prevented the accomplishment of this interesting development.

While there have been no machines actually installed using hydrogen as a cooling medium, several turbine generators have been sold with the provision that hydrogen may be used later, and a large synchronous condenser has recently been sold for immediate installation with hydrogen cooling. Experimental work is being actively conducted, and the Westinghouse Co. has an experimental 7500-kv-a. generator operating with hydrogen cooling. The decrease in friction and windage loss, and the possible increase in rating due to better heat transfer in a hydrogen atmosphere, make the use of hydrogen most attractive. It is the opinion of the authors that it is only necessary to demonstrate to operating engineers the safety of machines ventilated by hydrogen, when there will be another general step upward in efficiency and rating. In order to gain the most from hydrogen cooling, the design proportions must be changed to suit the new conditions. The arrangement of the ventilating ducts should be different, since higher velocities can be used advantageously. Then too, the proportions of rotor and stator strength must be altered if the same short circuit ratio and stability is to be maintained. For these reasons it would not be possible to change existing air-cooled machines to hydrogen-cooling, and materially increase the rating without making other fundamental design changes.

DESIGN OF STATOR COILS

In order to build machines of extreme core lengths for these large ratings, it became necessary to modify the design of the stator coils. On shorter machines, it has been customary to build the coils in one piece as shown in Fig. 3. This type of coil is continuous from the front, without any connections in the rear. Such a coil has the advantage of being turned over in the rear-end connections; that is, the top conductor of the top coil in one slot becomes the bottom conductor in the bottom coil in the return slot. This provides partial transposition against eddy currents and when additional transposition is provided at the front of the coil, the additional losses due to eddy currents can be reduced to less than 30 per cent of the copper loss. Long coils of this construction may weigh as much as 350 lb. and are therefore difficult to handle in the insulating process and in winding the coils in the core. This makes it desirable to construct them in halves and connect them on the front and rear. If it were possible to connect each strand individually at the rear, the structure of the half coils would be the same as full coils so far as eddy current loss is concerned; but for deep coils with many strands, such construction is impossible because of insufficient room to make all the connections. It is necessary, therefore, to transpose the strands somewhere between the front and the rear of the machine. This is most easily carried out in the straight or buried part of the coil.

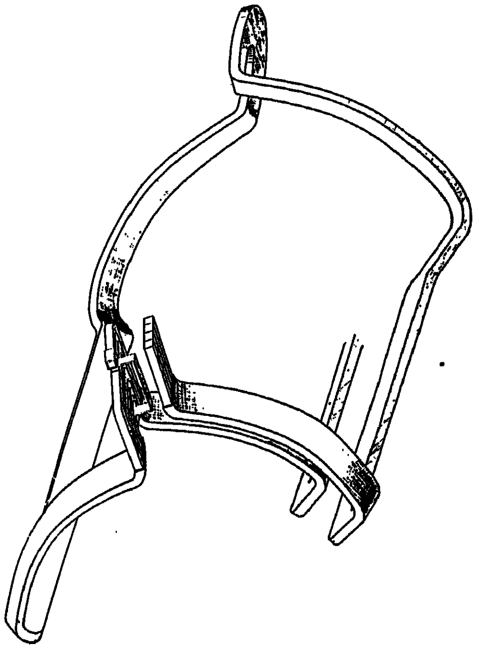


FIG. 3—ONE PIECE STATOR COIL

There are several different methods by which this can be accomplished. The method used on the large machines designed by Westinghouse engineers is a combination of transposition of strands within the slot, and transposition outside the slot at the ends of the coils. Any one conductor in the coils is split up into groups of several strands in depth and the strands in each group are transposed within the slot to eliminate the eddy currents between the strands in the group. The groups are then transposed with respect to each other at the front end of the coil to eliminate the eddy currents between the groups. The strands in a group may then be soldered together at the front and rear using a single joint. This arrangement reduces the transposition of the strands within the coil and simplifies the construction. One group and the detail of one strand is shown in Fig. 4. The transposition points are stamped out with a compound die. Each strand is insulated with mica tape in the same manner as when the coil had no internal transpositions. At the points of transposition, half the section of the strand is removed, but because the amount is a small percentage of the total and also because of the high heat conductivity of the copper, the temperature at these points is not materially different from that at other parts of the conductor. Tests have been made with thermocouples located at different points on the coil, but the variation was no greater than could be expected in a simple conductor of uniform section.

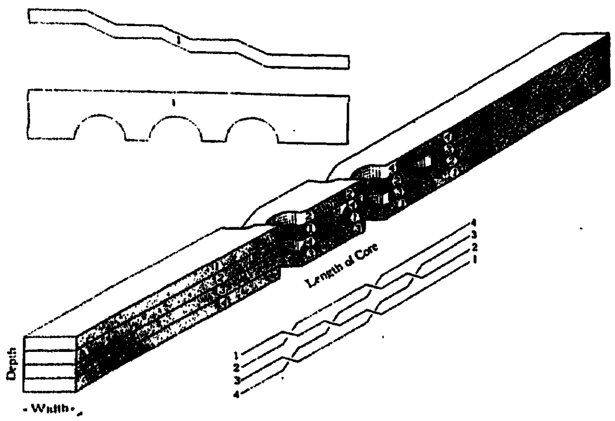


Fig. 4—Internal Strand Transposition

The question of armature coil insulation has been given considerable study in preparation for these larger units. The stator coils on Westinghouse machines have been insulated for a number of years with a mica-folium wrapper on the straight parts and treated tape on the ends of the coils. The individual strands in the coils and the conductors have been insulated with mica tape throughout their entire length. The insulation on the end turns has given good service and the same type of insulation has been used on waterwheel machines up to 18,000 volts. The advantage of the treated tape on the end is that it is flexible and can withstand the slight movements of the coil ends that will occur on short circuit. The seal between the mica-folium wrapper and the treated tape has proved satisfactory. While it has been criticized as a point of possible weakness, there has never been a failure at this point.

An improvement in the mica folium has been made to correct swelling and wrinkling of the insulation. Research work revealed the cause of swelling to be due to the vapor pressure caused by the alcohol in the shellac bond of the mica and, to some extent, to moisture in the paper used for backing the mica. A large number of sample bars was wrapped under different manufacturing conditions and with different classes of material. The bars were then placed in an electrically heated oven, some with voltage applied and others with no voltage. Readings were taken to determine the amount of swelling and temperature. The difference in temperature of the coils with voltage impressed and the temperature of the oven gave a relative measure of the dielectric loss.

In Fig. 5, there is shown one of these sample bars in the oven as arranged for test. The bars were made up of square, hollow tubes and thermocouples were applied to the inside surface of these tubes. Plates were clamped on the coils, to represent the iron laminations, and gaps left between the end of the plates to represent the radial ducts.

The result of these tests led to the development of a new bond that practically eliminated the cause of insulation swelling. A comparison of the dielectric loss, as expressed by temperature rise above the oven temperature for coils with shellac bond and with the new bond, at various oven temperatures, is shown in Fig. 6. Three curves are shown covering two grades of shellac and the so-called black bond. It is evident that the dielectric loss for the black bond increased but slightly with temperature, while the two bars with mica using a shellac bond showed an increasing dielectric loss with rising oven temperatures.

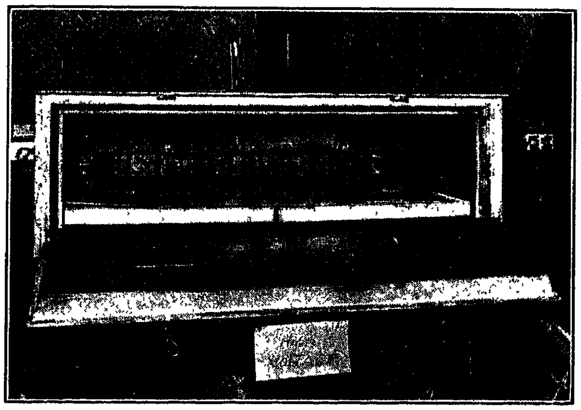


Fig. 5—Experimental Specimen of Coil on Test

The use of this new bond resulted in a wrapper which was more flexible. This characteristic made it possible to build in additional sheets of mica which increased the dielectric strength of the wrapper. Coils wrapped with insulation for 22,000-volt operation have withstood a momentary test of 160,000 volts.

In preparation for higher operating voltages than 13,800, the use of a special coating for the outside of the coils has been developed, to decrease the corona between the sides of the coils and the sides of the slots. If a perfect contact could be made between the coil and the sides of the slot throughout the length of the slot, corona would not exist. This ideal condition, however, cannot be reached because of manufacturing variations in the width of the slot, and in the width of the coil. If the coil touches the slot only at points, then there will be corona from those points of the coil which do not touch. A semi-conducting film is therefore required on the surface of the coil so that all points on the surface will be at ground potential. Fig. 7 shows an experimental set-up to demonstrate the effectiveness of such a conducting film. On each coil there was placed a stack of punchings to represent the stator iron, and 30,000 volts were impressed between the copper and the iron. The bottom coil (I) shows the corona on a coil without grounding film. The middle coil (II) shows a comparison of a coil with and without the protecting film, the middle of the coil being coated. The top coil (III) shows a method of damping out the corona at the ends by using a coating of somewhat higher resistance beyond the core. This test was made at higher voltage to ground than normally obtained in the machine in order to demonstrate more effectively the benefit obtained from the use of a grounding film.

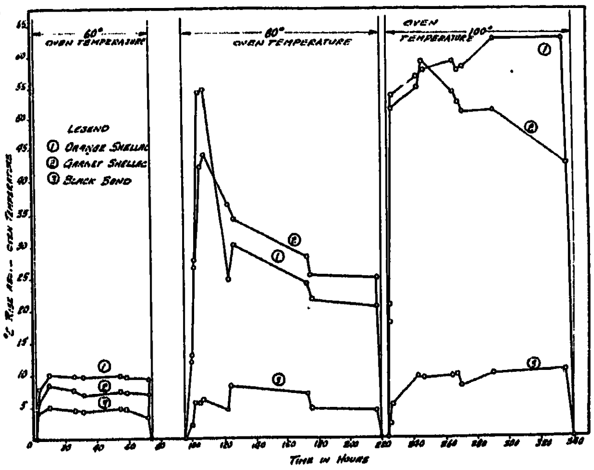


Fig. 6—Temperature Tests on Turbo Alternator Coils

The question of higher voltages for generators is receiving attention on these larger units. A unit of 100,000 kv-a. can be built for 13,800 volts, but the cost of circuit-breakers, switches, and bus-bars may make it desirable to build such machines, especially in new stations, for terminal voltages as high as 22,000. The machines for one station are being built for this voltage. Such machines must meet a final insulation test of at least 45,000 volts before going into service. To prepare for this final test, it is necessary to test the coils at considerably higher voltages after they are placed in the slots, but before they are connected. For lower-voltage machines these preliminary tests are made with the ends, where the connectors are applied, uninsulated. With these higher voltages, however, the discharge will creep along the surface of the coil from the exposed ends to ground if the voltage is applied for a minute. With such high voltage tests in air, a certain amount of fireworks must be expected.

General Design. With the increase in the size of the individual units has also come improvement in efficiencies. In the past the stator laminations were made from non-silicon steel. There was a prejudice against silicon steel in rotating apparatus because of brittleness, the fear being that the stator teeth would break off due to vibrations. Improved methods of building stator iron and better design of finger supports have removed the danger of loose iron and consequently, the danger of vibration. Furthermore, vibration life is a function of the endurance limit of the material and because this is higher for silicon steel, it also has a greater vibration life. The early troubles in the use of silicon steel were associated with the use of fillets with too small radius at the bottom of the slots, which resulted in incipient cracks during punching. These fillets have been increased in radius and this trouble has been eliminated.

Many machines have been built with two per cent silicon steel; and more recently four per cent silicon steel has been generally used. In other words, turbine generator stators are now built with a grade of steel which is equal to the best steel used in transformers. The high-grade, four per cent, silicon steel sheets have more scale than the two per cent and non-silicon steels and, therefore, must receive special treatment to remove the scale. It is possible, by special treatment, to obtain the same space factor with the four per cent steel as with the lower grades.

To illustrate the reduction in core loss that results from the use of these higher grade steels, if the loss in non-silicon steel be represented by 100, the loss with two per cent silicon steel will be 70, and 45 with four per cent silicon steel. The use of four per cent silicon steel improves the efficiency slightly more than one per cent at full load, and two and a half per cent at half load, as compared with the efficiency of machines using non silicon steel. When this steel is used the

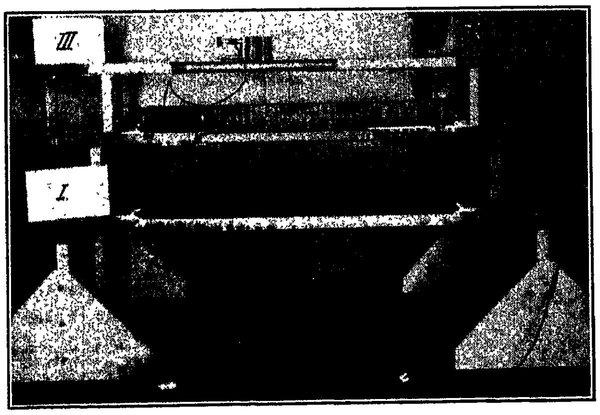


Fig. 7—Corona Elimination

core loss is reduced from 50 per cent of the total loss to 30 per cent of the total loss.

A new trouble that has appeared in these larger machines, is heating of the finger plates which support the stator teeth. This heating is most noticeable on machines with high ampere-turns per slot, and on machines with full pitch or two-thirds pitch windings. For these two classes of windings the hottest fingers will be those between phase groups; and this might be expected since these are the teeth in which the leakage flux flows in a radial direction along the teeth. The flux distribution is also affected by the nearness of the rotor retaining ring and magnetic saturation of the retaining ring. A study of the flux distribution at the end of the machine has been made, using the methods developed in the papers presented at the 1927 Winter Convention of the Institute. The flux causing this heating can be controlled by proper proportions of the finger plates and retaining rings or, if this is not possible, by the use of non-magnetic manganese steel in the finger plates. The use of non-magnetic steel in the retaining rings is also beneficial under certain conditions.

The questions of short-circuit ratio and regulation of turbine generators have not been of serious moment in the past, largely because turbine generators are usually installed near the center of load, and problems connected with long distance transmission have not arisen. The problem has received attention recently on those systems which do not use automatic voltage regulation, and where the capacity of the individual units is large in relation to connected load. In such a station with only a few units in service, the failure of one unit may throw a heavy load on the other units, and power stability may become important. The investigation has been made on the basis of constant field current and, consequently, decreasing voltage. Whether machines will be unstable on the basis of constant field current depends on the design of the generator and also on the system characteristics. Short-circuit ratio and degree of saturation in the generators are of importance, but it is just as necessary to know the variation of the load with change in voltage. With a lighting load, the current and kilowatts will decrease with voltage; while with an induction motor load the kilowatts will remain practically constant, the current will increase, and the power factors will improve with decrease in voltage to a point above the pull-out point of the motors. Usually, an estimate must be made of the proportions of lighting and power load and this estimate has an important influence on the conclusions. Machines are in operation in this country with short-circuit ratios as low as 0.8 and large machines in general have short-circuit ratios between 0.9 and unity and, so far as we are aware, there have been no cases of instability which could be traced to generator design proportions. The cost of generators increases, with increase in short-circuit ratio. In Europe, short-circuit ratios as low as 0.5 are in use and this fact partially explains the high ratings that are obtained on 50-cycle 3000-rev. per min. machines. It is desirable, therefore, that specifications should not call for unnecessarily high short-circuit ratios, because this will limit the maximum possible size of generator, and will increase the cost per kilowatt. More definite information is required concerning the performance of systems with change in voltage, in order to establish more definitely the safe lower limit of short-circuit ratio. The performance of machines can be readily calculated if the load conditions are known, but studies, so far, have been made with assumed load conditions. The results are no more accurate than the degree of accuracy of these assumptions.

Stator Construction. In the earlier constructions, turbine generator frames were made of cast iron. The section of the frame was determined chiefly by the demands of the stator-ventilation system, because the casting included the ducts for conducting the cooling air to and from the core. A frame of this type is shown in Fig. 8.

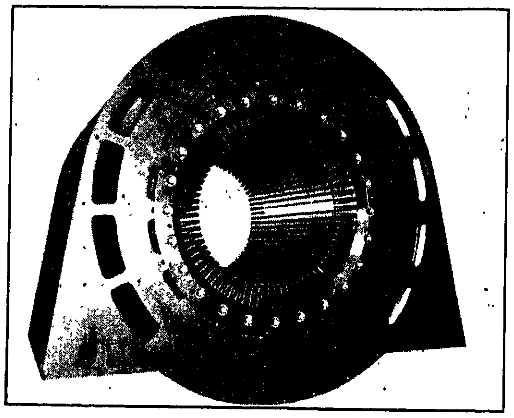


FIG. 8—CAST IRON FRAME WITH PUNCHINGS

One of the most serious objections to this construction is that it leads to frame widths too great for shipment with usual railroad clearances. For the longer core lengths, it would be necessary to ship the stator in parts and carry out the assembly and the winding at the power station. Another objection to the cast frame is the excessive weight. The rigidity and the strength of the frame are far greater than required by the stress conditions to which it is subjected in handling and in operation.

These objections led to the separation of the frame in two parts; a super-structure of light, structural steel for the air passage and a skeleton frame for carrying the core. An analysis of the requirements of strength and rigidity was made in 1925[4] and it was found that the requirements from this point of view permitted a considerable reduction in the depth of the cast frame section and in the total weight.

---

[4] M. Stone, "Stresses and Deflections in Large Dynamo Frames," A. S. M. E., Dec. 1926.

Fig. 9 shows a frame of this type, with a part of the structural steel super-structure mounted on the outside. This super-structure is fitted at the factory, removed for shipment, and re-installed at destination.

This improvement in stator construction reduced the weights of the heaviest pieces to be shipped approximately 30 per cent, and the installed weight of the stator approximately 15 per cent.

With this construction it is possible to build the core and complete the winding at the factory, and still maintain the width within shipping dimensions. The Pennsylvania Railroad assisted in solving this problem of shipment by building two depressed cars, each capable of carrying a load of 275,000 lb.

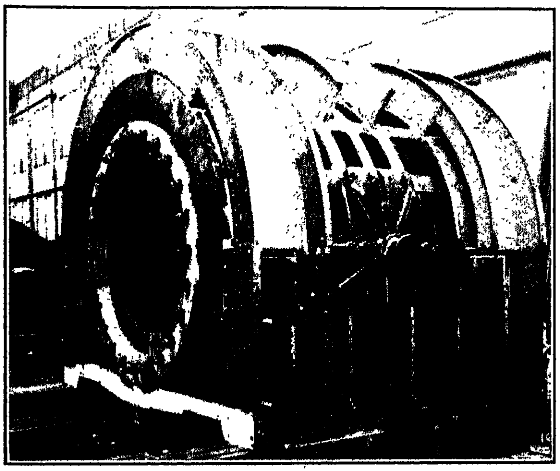


FIG. 9—SKELETON FRAME CONSTRUCTION

It was but another step to build the skeleton frame of structural steel as well, and this has been done in the latest constructions. This structural design has decreased the weight of the stator another ten per cent, and raised the limit of capacity of machines that can be built and wound at the factory from 75,000 kv-a. to approximately 100,000 kv-a.

MECHANICAL CONSTRUCTION OF THE ROTOR BODY

General Conditions of Stress. Working Stresses. The rotor body is subjected to stresses from three major causes: The centrifugal forces, producing radial and tangential stresses; the bending forces (gravity and dynamical forces), producing longitudinal stresses; and the torque, producing shearing stresses. The relative importance of these three causes varies with the type of machine, but up to the present state of the development only the centrifugal stresses, and the bending stresses, have played an important role in limiting rotor dimensions.

The centrifugal forces alone cause critical stresses, either in the radial direction at the bottom of the teeth, or in the tangential direction at the inner bore. The relative magnitude of the former is usually much greater than the latter, but when the rotor is made of a one-piece forging the latter may be more important than the former, on account of the uncertain properties of the material at the inner bore. In the built-up plate rotors, where this uncertainty of the material does not exist, the radial stress at the bottom of the rotor teeth is the true critical stress.

This cross section must carry the centrifugal forces from the teeth themselves, from the coils, and from the wedges, the latter groups being magnified by the wedging effect of the rotor wedges. This magnification can be eliminated almost completely by the use of rectangular wedges (Figs. 10A-B), but the gain is offset, to a certain extent, by the increased amount of material necessary above the wedge.

The stresses in the coil retaining rings, due to centrifugal forces, are almost exclusively tangential so that, although the nature of the design demands a very high stress in these rings, there is less difficulty in establishing the actual stresses in the coil retaining rings than in the rotor body; and the selection of a safe working stress is easier.

The bending stress in the various parts of the rotor body is alternating and for this reason very important, even if the value of the stresses, in comparison with the centrifugal stresses, is small. The building of long rotors, necessitating higher values of these bending stresses, has further accentuated this problem. Before the rotor length increases of the last five years had taken place, the maximum bending stress occurred at some part of the shaft near the bearings, and the bending stresses in the middle body were very small. The gradual lengthening of the rotors has changed this situation somewhat; the bending stresses near the bearings have not been reduced, but the bending stresses in the middle body are now of the same order.

This is particularly true of the one-piece rotors where circumferential grooves are turned down to the bottom of the slots to provide radial exhaust ducts from the axial air passages. This increases the bending stresses and causes stress concentration. Probably the most important aspect of these bending stresses in the middle body of solid rotor forgings is the fact that they exist in conjunction with a certain tangential centrifugal stress. This introduces a two-dimensional condition of stress, of which one component is variable and the other stationary (Fig. 11). The failure of the material

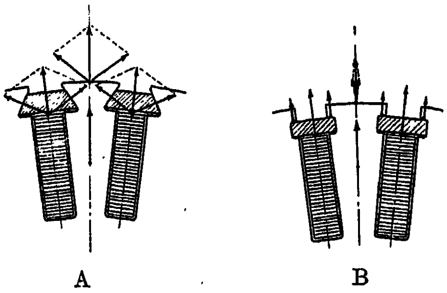


FIG. 10—CONDITIONS OF LOADING ON ROTOR TEETH
A—DOVE TAIL WEDGE
B—RECTANGULAR WEDGE

is dictated by the extreme value of shear stress in the plane of maximum shear. In this plane the stress conditions may be considered as consisting of an average stress, upon which is superimposed a variable stress. (Fig. 12.) By this reasoning the state of stress is reduced to a form, for which failure can be predicted in terms of the properties of the materials in pure tension.

In the built-up plate rotors, air outlets can be provided without material reduction in the section modulus, so that the longitudinal bending stresses and the tangential stresses do not occur at the same points. In these rotors the critical bending stress will probably occur at the sections next to the middle body (under the end windings) for rotors of great length. Here the tangential stresses due to centrifugal forces are small, so that the application of stress is practically one-dimensional, with complete reversals.

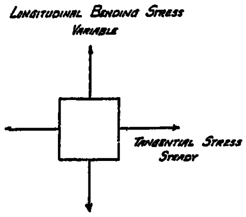


Fig. 11—Two-Dimensional Stress Applications

The question of safe working stresses is a most important one, both from the point of view of safety and economy. A great deal of progress has been made but a number of features still remain under discussion. The scope of this problem comprises not only the considerations of the practical designer, and the results of the theory of elasticity, but also the point of view of the metallurgist, and it is only through complete coordination of these three branches of engineering that a satisfactory point of view can be attained. The following represents the attitude taken by Westinghouse engineers at the present time:

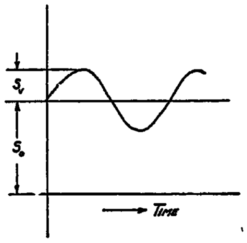


Fig. 12—Stress Cycle

In cases of steady stress in ductile materials, the *yield point* is regarded as the fundamental basis for the selection of the working stress. The factor of safety is thus expressed as the ratio of the yield point of the material, to the stress. The failure of the material at the yield point is caused by shear (maximum shear theory), so that in cases of combined stresses the maximum shear stress is the determining factor in expressing the factor of safety. Conditions of stress concentration are generally disregarded for all cases of steady stress in ductile materials.

In cases of variable stresses in ductile materials, the *endurance limit* for complete reversals is regarded as the fundamental basis for the selection of the working stress, and the factor of safety is expressed as the ratio of the endurance limit to the maximum value of the alternating stress. Failure is attributed to maximum shear in this case also, but here it is necessary to introduce the conditions of *stress concentration*, so that the working stress must be expressed as the maximum stress existing at any point of the structure. This stress is generally two to three times that calculated by elementary formulas.

In cases of combination of a static stress, and a variable stress, (Fig. 12), the available experimental results[5] are used with certain modifications. It has been found that these experimental results are applied in a logical manner if the actual factor of safety is regarded as the combination of an imaginary factor of safety n_0, existing when the static stress is acting alone, and another imaginary factor of safety n_v, existing when the variable stress is acting alone. The actual factor of safety n, is then expressed by the relation

$$\frac{1}{n} = \frac{1}{n_0} + \frac{1}{n_v}$$

In this manner it is possible to define the maximum static stress, which can be applied when its exists in conjunction with a certain variable stress. Fig. 13 is an example of such a curve of working stresses for a 3½ per cent nickel steel.

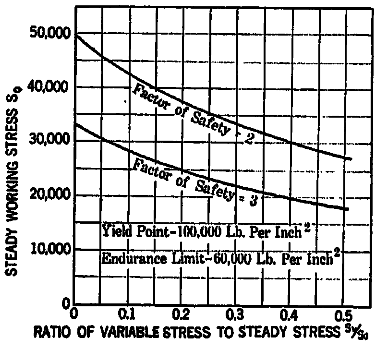


Fig. 13—Example of Working Stresses for Simultaneous Application of steady and Variable Stress

It is evident from the above that the only material properties, upon which the designs are based, are the yield point and the endurance limit.

This by no means implies that the remaining properties of the materials (ultimate strength, elastic limit, elongation, reduction of area, and impact tests are left out of the picture); the consideration of some of these quantities is equally vital to the soundness of the design, but unfortunately, it is only in a qualitative manner and they find expression in the design proportions and in the material specifications.

---

[5] See, for example, Timoshenko and Lessells, "Applied Elasticity," p. 477.

Probably the most important of these "qualitative" considerations are those which refer to the ductility and toughness of the materials. The elongation, measured on two-in. gage length, is still the chief element in this connection, but, since the distribution of the flow in the test piece does not find its expression in this term, it is equally important to consider the reduction of area, which expresses the maximum value of the unit elongation when failure occurs. But even these two terms in conjunction give insufficient information about the true cohesive properties of the materials; these are expressed by the impact test on notched bars. The controversy among metallurgical experts on the true value of the impact test still makes the designer hesitate in placing implicit faith in the impact test values, but there is very little doubt that they will play an important role in the future. At the present time the chief value of the impact tests is to furnish a check on the uniformity of the heat treatment of the material, not readily obtained by any other method.

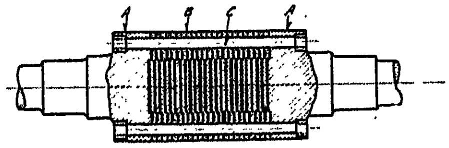


Fig. 14—Longitudinal Section of Plate Rotor

The present attitude of the designers is to place the lower limit of the elongation of the materials in the rotor body at about 20 per cent, and the reduction of area at about 40 per cent. These limits have been established by experience, and when the arbitrary nature of the constants (particularly the elongation) is considered, there is a certain justice in raising the question whether these limits really do represent the lowest safe values that can be used. The question is not without its importance because the above limits represent the chief obstacle in obtaining one-piece forgings for large rotors. On the other hand, the risk is too great to propose radical changes in these limits until more knowledge is available.

A similar situation exists in connection with the location of the test pieces on solid rotors. The attitude of Westinghouse engineers with regard to solid forgings is to judge the quality of the material by radial, tangential, and longitudinal test specimens. This severe specification again adds to the difficulty of obtaining large one piece forgings, but in view of the statements above on the principal stresses in the rotor, it does not seem wise to modify this demand.

The designer who desires to employ very large forgings is faced by difficulties. He can insure reasonable safety by specifying complete radial, tangential, and axial tests and by maintaining fairly high limits for elongation and reduction of area; the result will be a high price for the material; or he can buy on routine, material specifications and assume the liability for the cost of unsuitable forgings.

The Westinghouse Plate Rotor. The limitations of one piece forgings for large rotors were anticipated at an early date by the Westinghouse Co., and their present construction was originated by Behrend and Fields[6] over eighteen years ago. It will be referred to as the *plate rotor*.

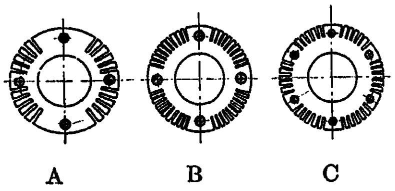


Fig. 15—Arrangements of Through Bolts

The rotor consists of two forgings *A*, (Fig. 14), which furnish the shaft ends, and a certain small portion of the middle body. The remainder of the middle body is made up of plates, *B*, which in the earlier constructions had a thickness of about two in. In some of the latest designs, plate thicknesses up to 8 in. have been used. These plates are spigoted into the end forgings, and into each other, so that adequate centering is obtained. The portions of the plate inside this spigot-fit are relieved, so that only the outer portions of the plates carry pressure. The entire structure is held together by through bolts, *C*, tightened to give a certain initial pressure. The number of bolts is four for two- and four-pole machines, the extra pair of bolts in the former (usually 25-cycle machines) is placed in the center of the slotted portion (Figs. 15A, 15B). Six pole machines have six bolts, placed in the centers of the poles. Fig. 15C.

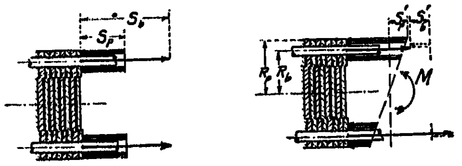


Fig. 16—Load Conditions on Cross Section of Plate Rotor

In order to appreciate the possibilities and advantages of this construction, it is necessary to discuss the fundamental principles of its performance.

Fig. 16A shows schematically the loading conditions on a section of the rotor, when no external bending moment is acting. The compressive stress of the plates, S_p, counterbalances the tensile stress of the bolts, S_b, so that if A_p and A_b are the respective areas (Figs. 17, 18) it is necessary that

---

[6] *Some Difficulties of Design of Turbine Generators*, A. B. Field, Jl. A. I. E. E., Vol. 54, 1915, p. 65.

B. A. Behrend, Trans. A. I. E. E., Vol. 36, 1917, (Discussion) p. 883.

$$S_p = \frac{A_b}{A_p} S_b$$

Fig. 16B represents the loading of the same cross-section when it is subjected to an external bending moment M. The compressive stress on the plates is now increased by S_p' at the top and decreased by the same amount at the bottom. The stress of the top bolt has been reduced by S_b', and that of the bottom bolt increased by the same amount. The stresses of the other two bolts have not been changed materially. It is evident that one of the most important limitations of the structure is that the diminution in stress of the plates and the bolts must not reduce the stress to zero at any point, otherwise a condition of discontinuity will occur. It was found in the earliest applications of the structure, however, that there are no difficulties in securing this condition. As a matter of fact, no machine has been constructed where the variation in plate stress exceeded 20 per cent of the initial stress, or the variation in bolt stress exceeded 4 per cent of the initial stress.

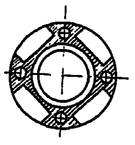


FIG. 17

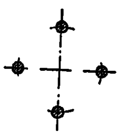


FIG. 18

FIGS. 17 AND 18—PLATE AREA AND BOLT AREA

It is evident from Fig. 16B that the external bending moment M is counteracted by two internal moments, a moment from the plates, which will be denoted by M_p, and another from the bolts, which will be denoted by M_b. Both of these act in the same direction, so that the rigidity of the rotor depends upon both the plate area and the bolt area.

In order to determine the rigidity accurately, it is necessary to take into account the variation of the external bending moment along the length of the rotor. It has been found, however, that it is sufficiently accurate to assume that the bending moment is constant for the entire middle body, that is, for the entire length of the bolt. With this assumption it is easy to compute separately the extension of a bolt, and the extension of a corresponding fiber in the plate column. The values of the bending moments M_p and M_b are obtained from the fact that these extensions are equal. The plate stress, at the radius R_b is $M_p/I_p . R_b$, where I_p is the moment of inertia of the plate area (Fig. 17). The extension of a fiber along the middle body of length a is consequently $M_p/E\, I_p \times R_b \times a$. The modulus of elasticity E is that of the plate column; so long as the compressive stress exceeds about 5000 lb. per sq. in., this has been found to be the same as that of the plate material. The extension of a bolt is similarly $M_b/E\, I_b \times R_b \times a$ where I_b is the moment of inertia of the bolt area (Fig. 18). Consequently

$$M_p = \frac{I_p}{I_b} . M_b$$

Now, since M_p and M_b counteract the external bending moment

$$M = M_p + M_b$$

so that

$$M_p = M \frac{\frac{I_p}{I_b}}{1 + I_p/I_b} \text{ and } M_b = M \frac{1}{1 + I_p/I_b}$$

For the ordinary four-pole machines, $I_p \cong 8\, I_b$, so that the bending moment carried by the bolts is actually very small; about 11 per cent. The bolts furnish mainly the static pressure between the plates. As a matter of fact, a bolt configuration having practically no bending rigidity of its own, such as the case of a single bolt in the middle, would be entirely practical from this point of view. The only difference would be that the plates would then carry the entire bending moment, that is $M_p = M$.

The actual deflection that is obtained under the external bending moment M is now easily calculated. For simplicity we may calculate the actual extension of a bolt, or a fiber of the plates at the radius R_b. This is

$$\frac{M_p}{E\, I_p} \times a \times R_b = \frac{M}{E\, I_p} \times \frac{\frac{I_p}{I_b}}{1 + \frac{I_p}{I_b}} \times a \times R_b$$

$$= \frac{M}{E\,(I_p + I_b)} \times a \times R_b$$

showing that the rigidity of the rotor is determined by the moment of inertia of the plate area, plus that of the bolt area. This is an important feature of the plate rotor, in that the rigidity is not reduced by cutting out the holes for the bolts. In cases of very large variations of the bending moment over the rotor length (very long rotors), this statement must be modified slightly; but the error in using the entire plate area, including the bolt holes, in computing shaft deflections, is never more than about 2 per cent.

It is evident from this that the critical speed of a plate rotor is practically the same as that of a solid rotor having the same effective area. In view of the practise of turning circumferential grooves in the solid rotors for ventilation purposes it may even be said that the plate rotor, for the same ventilation arrangement, furnishes greater rigidity than the solid rotor.

From this fact it follows that the variation in stress, to which the bolts are subjected, is also very small. It is evident that the variation in bolt stress is exactly the same as that of a corresponding fiber of a solid rotor of the same cross-section. This stress is always very small,—it is only in the very longest rotors that it reaches 1000 lb. per sq. in. If it is remembered that the assembly stress is usually about 40,000 lb. per sq. in., it will be understood why this stress variation can be neglected.

The variation in plate stress is somewhat larger, but even this is negligible. The maximum bending stress of the middle body, recorded so far, is about 1500 lb. per sq. in., while the assembly stress is usually above 8000 lb. per sq. in.

The theory has been advanced several times that the plate rotor would furnish appreciably damping to vibrations, due to friction between its parts. It is evident from the above that the only friction occurs between the bolts and the plates, and only when the bending moment varies an appreciable amount over the middle body. Special tests have been made to determine this difference in damping between a solid rotor and a plate rotor, but no such difference has been observed. In fact, cases are on record where the internal damping of a plate rotor was less than that of a solid rotor of the same size. Nor has any change in plate rotors been observed in this respect after several years of service.

As a conclusion to the above discussion of the mechanics of the plate rotor, it can be stated that the plate rotor does not possess any limitations to increases in rotor length other than those furnished by the limitations of shaft deflections and critical speeds. In this respect, it is on par with the solid rotor. Its inherent advantages are due to the possibility of using high grades of steel, independently of the size of the rotor body, and the better possibilities of rotor ventilation.

Development of Rotor Materials: Large One-Piece Forgings. The attitude of the manufacturers with regard to the relative merits of one piece forgings and the sectional constructions has undergone important changes in the last two or three years. It is only a short time ago that most turbo designers were convinced that a satisfactory one-piece forging could be obtained for any of the rotor dimensions within sight, if the price were forthcoming. The latest advances in the size of individual units has changed this condition; the present maximum sizes of rotors can no longer be built of one-piece forgings, regardless of price and specifications.

The development of the plate rotor has been brought to a point where it is competitive in price even for rotors of about 75,000 lb. in weight; so that the conclusion that a plate rotor is necessarily more expensive than a one-piece forging is no longer true.

The art of making large one-piece forgings will continue in its development, however, and a few advances may easily upset these relative positions. Then, too, the plate rotor demands forgings of a respectable size for the shaft ends, so that no manufacturer of large turbine generators can afford to be out of touch with the forging situation. It may be of interest, therefore, to review very briefly some of the problems encountered in preparing a one-piece rotor forging for a 37,500-kv-a. generator, weighing about 130,000 lb.

The first forging attempted for this machine, was brought through in 1923. The ingot was cast from three heats of varying carbon content; the bottom heat having 0.45 per cent, and the top heat 0.29 per cent. The ingot weighed 217,000 lb., and a top discard of 79,000 lb. was made before forging.

After the forging operation it was given a heat treatment consisting of 1600 deg. fahr. normalizing heat, followed by lower temperature treatments.

At this stage, tangential tests, taken from the ends, showed a yield point of 31,000 to 33,000 lb. per sq. in., an elongation of 10 to 19 per cent, and a reduction of area of 12 to 30 per cent. This was unsatisfactory, and two additional heat treatments were undertaken with somewhat higher normalizing temperatures, after which a complete series of tests was made in the radial, tangential, and longitudinal directions. All test pieces, with the exception of one, showed satisfactory properties, with elongation above 20 per cent and reduction of area of about 40 per cent. The exception was a tangential test from the top end of the forging; it gave a yield point of 44,000 lb. per sq. in., an elongation of 9 per cent, and a reduction of area of 11 per cent.

When the forging was turned, subsequently, concentrations of slag pockets were observed near the first shaft fillet on the top end of the forging. The forging was rejected on the strength of this discovery, and subjected to a series of tests. The slag pockets were found to extend into the forging, and a chemical analysis established that carbon segregation had taken place. The carbon content at the top end of the forging was found to be 0.62 per cent, and at the bottom end, 0.26 per cent, showing a reverse condition from that in the ladle analysis.

A second one-piece forging for the same size of machine was brought through in 1925. In the preparation of this forging special care was applied throughout, and the pouring into the ingot was made with basic steel at the bottom, and acid steel at the top. The ingot was not allowed to cool below a cherry red heat, before which it was transferred into the forging furnace. Special precautions were taken in the forging operation, such as v-tooling before necking, and a very careful adjustment of the reductions obtained in each operation. The heat treatment consisted of a high normalizing heat of 1750 deg. fahr. followed by additional lower temperature treatments. One important improvement was hollow boring of the forging before the final heat treatment, not only to facilitate bore search, but to procure greater uniformity of temperature and faster penetration of the heat.

The test results showed uniformly satisfactory properties in the three directions with a yield point of about 30,000 lb. per sq. in., an elongation of about 32 per cent, and a reduction of area of about 55 per cent. The endurance limit was found to be about 27,000 lb. per sq. in.

It is evident that perfectly reliable forgings can be obtained only through the application of the greatest care and the utilization of the latest advances. The performance related for the second forging has been repeated subsequently on smaller forgings, but this special treatment raises the price to a point where one-piece forgings cease to be competitive with the plate construction.

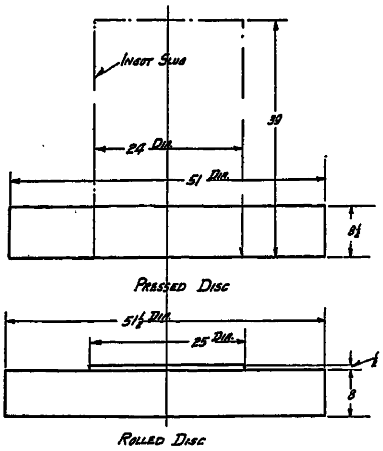


Fig. 19—Forged and Rolled Turbo Plates

The question of test procedure is of vital importance in this connection. If the demand on ductility can be waived, particularly for the tangential direction, the price of one-piece forgings can be reduced considerably, and it may be possible to raise the yield point. Whether this can be done without too much risk, is a question which must remain open until more knowledge of the factors which contribute to failures are available.

Materials for Plate Rotors. The earliest constructions were made up of 2¾ in. *rolled plates*, machined to a thickness of 2 in. The material was rolled out in slabs, large enough to give six rotor plates with ample discard on the sides and on the ends. The material was 30 per cent carbon steel with a yield point of about 40,000 lb. per sq. in., elongation of about 26 per cent, and a reduction area of about 45 per cent. The material itself was fairly inexpensive, but the machining cost was high on account of the great number of surfaces in a rotor. The directional alignment of the structure, due to rolling, was pronounced, so that certain variations in ductility were obtained parallel to, and at right angles to, the direction of rolling. In addition, it sometimes possessed considerable internal stresses.

A nickel steel (about 3½ per cent Ni) was used in a number of machines, where it was desirable to have a yield point somewhat higher than that of the carbon steel.

The development of a thicker plate material, having a more favorable structural alignment, was started a few years ago by W. J. Merton.

The first attempts consisted in obtaining a plate which was pressed out from a cylindrical ingot and subsequently rolled out in a tire mill, (Fig. 19). Probably the most important advancement in the new practise was a heating of the disks above forging temperature, followed by a normalizing treatment, and then by an annealing, relieving the internal stresses.[7] The importance of this heat treatment is seen from the fact, that while the plate, as rolled, had an elongation of four per cent and a reduction area of 8 per cent, after the final treatment these values were brought up to 20 per cent and 36 per cent, respectively. The yield point remains practically unaltered, at about 46,000 lb. per sq. in.

By this method it was possible to produce carbon steel plates of a thickness up to about 8 in., having in some respects properties superior to the flat rolled plates, at a price only slightly higher than that for the flat rolled plates. The increase in price was more than offset by the reduction in machining.

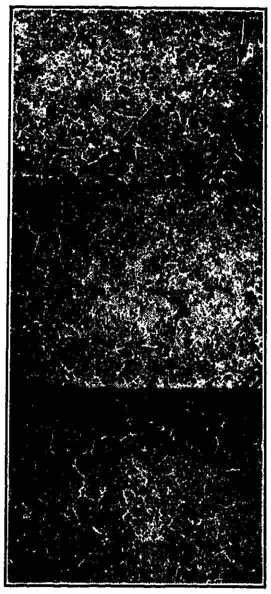


Fig. 20—Micro Photographs of New Plate Material

The next step in the improvement of the plate materials was obtained by piercing the pressed disk and carrying out the remaining operation by forging on a mandrel, instead of rolling. The heat treatment consists of a thorough annealing above finish-forging temperature, a reheating and quenching in water, and then additional reheating for drawing.

---

[7] W. J. Merton, American Society for Steel Treating Ninth Annual Convention, Sept. 1927.

By this process it has been possible to obtain *a yield point of about 65,000 lb. per sq. in.*, an elongation of 25 per cent, and a 50 per cent reduction of area in disks of 0.45 per cent carbon steel. The properties in the tangential and radial directions are practically the same, and the material has an extraordinarily fine grain structure, as shown in Fig. 20. The micro-photographs (from which the figure was made) were taken from three different plates. Fig. 21 gives an external view of one of these plates after rough machining.

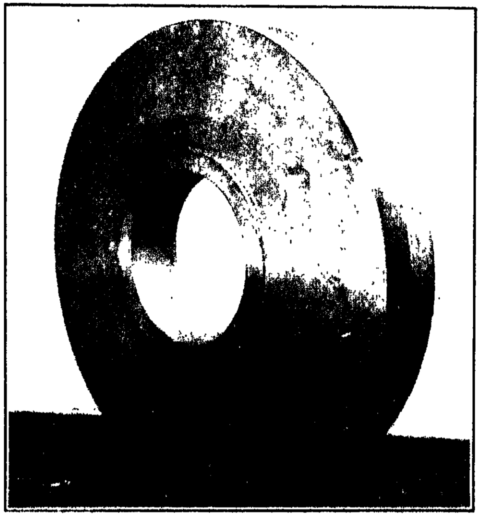


FIG. 21—HEAT TREATED ROTOR STEEL DISKS

The development of this material represents a most important contribution of the metallurgists to the solution of the problem of designing these extremely large size units. Its use has enabled the designer to maintain ample factors of safety, to increase the output from a rotor of given over-all dimensions (because of superior strength of the material), and to lower the over-all cost.

It is not possible, so far as the authors know, to obtain material of this quality, in carbon steel forgings for large size rotors (over about 75,000 lb.), at any price. The possibility of obtaining it at a low price makes the plate rotor of even greater attraction to the designer than heretofore.

The development of the materials for the *end forgings*, may be regarded as a part of the development of solid forgings; although the problems in the shaft ends are not as accentuated as in one piece forgings, due to the relatively small size.

The *bolts* form one of the most vital elements of the plate rotor, and the selection of bolt material is of paramount importance.

The usual material up to a few years ago, was chrome-nickel steel, having a yield point of about 100,000 lb. per sq. in., an elongation of 20 per cent, and a 50 per cent reduction of area. This was superseded by a molybdenum-nickel steel, having somewhat higher yield point and reduction of area. The real virtues of the molybdenum-nickel alloy, however, have been attributed to the difference in Izod-impact test, which is almost twice as high for the molybdenum-nickel steel as for the chrome-nickel steel.

A similar material is used for the *coil retaining rings*, where the demands on stress are of the same order as those of the through bolts. A considerable amount of development has been carried out on the problem of obtaining satisfactory coil retaining rings of non-magnetic steel. Such steels are in use, but their chief objections are high cost and difficulty of machining.

Problems Encountered in Building Plate Rotors: The Bolt Threads. In most of the present four-pole rotors, the bolts have a diameter of 5 in. When it is remembered that these bolts carry a stress at the root of the threads of about 40,000 lb. per sq. in., and that this stress must be maintained indefinitely, it is evident that the design and the cutting of the threads call for more than ordinary care.

The question of stress concentration in the bolts is clearly of great importance, because the average stress must be maintained fairly high. As usual, in cases of ductile materials under steady stress, the existence of a certain stress concentration does not indicate failure when the maximum stress reaches the yield point, because the yielding itself causes a readjustment of the stress. In judging the merits of different types of threads, however, the factor of stress concentration is of sufficient importance to furnish a reliable aid in selecting the most favorable type.

Certain interesting results were obtained in 1925, by R. V. Baud, by photo-elastic measurements on various types of threads. The threads investigated were Whitworth (somewhat modified), Acme, and Buttress threads, (Fig. 22). Table II gives the results of these tests in terms of factors of stress concentration.

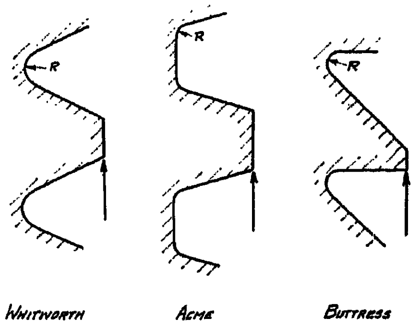


FIG. 22—TYPES OF THREAD

The models tested were made to a certain scale from the actual threads on a five-in. bolt with four threads per in. The factor of stress concentration expresses the ratio of the maximum stress obtained by the photo-elastic experiments to the average stress, calculated by ordinary beam formulas.

TABLE II

Type of thread	Radius of fillet inches	Factor of stress concentration
Whitworth	0.027	1.75
Acme 1	0.003	2.65
Acme 2	0.013	2.00
Acme 3	0.023	1.75
Acme 4	0.045	1.54
Buttress	0.023	2.46

This comparison gives very favorable results for the Acme type of thread with large fillets at the bottom, and this type has been adopted. The threads in use at present are 29 deg. Acme threads with a 0.023-in. fillet at the bottom.

There is another, and more important, source of stress concentration in the bolts, which has been investigated very thoroughly during the last two years. This concerns the distribution of the load on the different threads. It is evident that, due to the extension of the bolt and the compression of the nut, there will occur a certain difference in pitch, which will tend to throw a heavier load on the first threads in the nut. In bolts of ordinary size and moderate loads this question is relatively unimportant, but on large bolts under heavy load it may cause serious stress concentration in the bolt, near the first threads.

An analysis of this problem was made for the first time in 1925 by S. Timoshenko and J. P. Den Hartog; and it was found that if no yielding occurs the load distribution on the threads is very far from uniform. Fig. 23 shows the results that were obtained for a five-in. diameter bolt, cut with Acme threads, four threads per in. The length of the nut is seven in., and the threads are assumed to be cut with no clearances and equal pitch in bolt and nut. It is evident from this result that the major share of the load is carried by about six threads from the bottom of the nut.

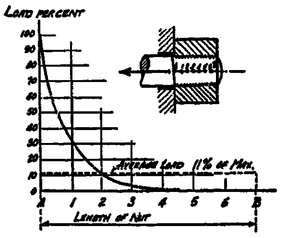


FIG. 23—THEORETICAL DISTRIBUTION OF LOAD ON BOLT THREADS

Several solutions have been advanced to eliminate this condition, among which may be mentioned cutting the threads to unequal pitch in bolt and nut, and arranging the nut in such a manner that the threaded portion is in tension. These solutions had to be discarded, however, since they demanded an accuracy in cutting the threads which could not be attained even with special precision tools.

Further study of the problem has shown, however, that the load concentration on the first threads is so high that yielding will occur. After this yielding has taken place, the load is transferred to additional threads, until the maximum load can be sustained. The load distribution is then of the type shown in Fig. 24. It was found that this yielding begins at about 60 per cent of the maximum load. The yielding is very minute however, and the actual displacements are of the order of machining inaccuracies.

The most important concrete result of this investigation was that it was shown unnecessary to have the nut longer than one-bolt diameter; in fact, where practical conditions so require, the nut may be made considerably shorter than one diameter of the bolt.

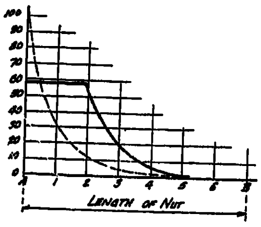


FIG. 24—ACTUAL DISTRIBUTION OF LOAD ON BOLT THREADS

The Problem of Assembly. Before the assembly the plates and the shaft ends are machined, and the bolt holes drilled. The rotor plates are then stacked, and when the stacking is completed, the structure is held together by temporary bolts. The bolt holes are then reamed and the regular bolts inserted and tightened.

In the earlier rotors it was customary to tighten the nuts by a crane, and record the torque required by a scale beam on the crane. This method was found to give erratic results, due to variations in the finish and lubrication of the threads and seats of the nuts. This method was then superseded by determining the bolt stress, by the total elongation of each bolt, which proved to be quite satisfactory as far as final results were concerned. It was comparatively slow, however, and in certain cases considerable variations of the wrench pull, which could not be accounted for, were obtained.

For these reasons, a special pulling machine was developed by G. M. Eaton, whereby it is possible to stretch the bolts to their correct stress (and if necessary to a certain over stress) before the nuts are tightened up. The introduction of this equipment has proved to be a very valuable improvement, and the uniformity of the results is no less an advantage than the increased production.

The arrangement of this bolt puller is shown in Fig. 25. Two bolts, diametrically opposite, are pulled up at one time. The puller is attached to the end of the bolts, outside of the nuts, and when the load is raised to its specified value, the nuts can be run on by a small hand wrench. The actual stress in the bolts is checked by the elongation, after the puller is slackened up. The bolts are tightened in steps until the final load is reached. After machining, the rotor is again returned to the puller, and the bolts are subjected to a temporary over stretch, which is left on for a short time, after which the load is backed off to normal.

DETAILS OF ROTOR CONSTRUCTION

In the present type of machines the maximum rating that can be obtained from certain core dimensions is limited by the heating of the rotor. There are two methods by which this limit can be raised. One is to provide more room for copper by deepening, or widening the slots, or by changing the insulation arrangement. Changes in slot dimensions, within the limits set by saturation of the rotor teeth, can be obtained only by the introduction of better rotor materials. The other is to improve the rotor ventilation. The developments of the last years have been concerned with both of these methods, but the most important change has been accomplished by improving the rotor ventilation.

The problem of rotor cooling is a most difficult one, because of the difficulty of studying the actual conditions with the rotor in motion. This problem is being investigated by the use of stationary and rotating models, and it is expected that the results of these investigations will throw further light on the rotor ventilation phenomena.

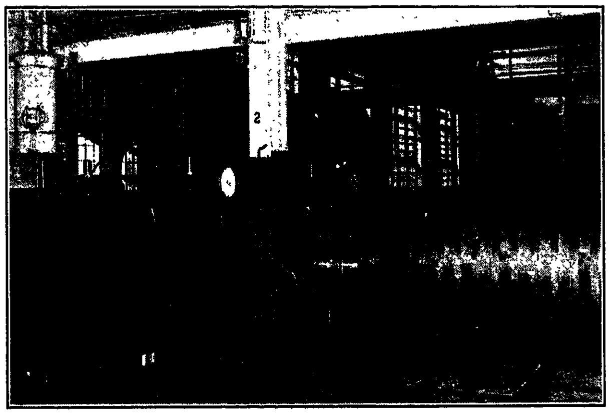


FIG. 25—BOLTS PULLER FOR TURBO-ROTOR BOLTS

One of the most important advantages of the plate construction lies in the possibility of obtaining radial ducts between the plates, communicating with the axial ducts. For this reason the rotors of Westinghouse generators have always been cooled by air circulated through the body. Fig. 26 shows the original arrangement of this system of rotor ventilation. The axial vent ducts are cut at the bottom of the slots, and communicate with the air gap through radial ducts, which are milled out of the rotor plates before assembly.

The introduction of rotor plates with a large central bore made possible a modification, which will increase the volume of air going through the rotor. Fig. 27 shows a cross-section through one of the rotors with the modified system of ventilation. Oblique holes are drilled in the shaft ends before they are assembled with the plates. These holes carry a part of the cooling air to the center of the rotor. The remainder is carried through axial holes through the teeth in the end portion of the rotor. Radial slots are cut down along each rotor tooth before the plates are assembled, and these slots serve to discharge the air from the center of the rotor. In this construction there is no change in direction of the air at high velocities and, therefore, the pressure drop is decreased. In the older design it was necessary to provide a steel cell in the slots to protect the mica lining where the coils were adjacent to the air ducts. In the improved construction, the steel cell can be omitted and the corresponding space made available for copper. With the plate construction it is thus possible to obtain a cooling system without resorting to drilling a number of radial holes in the teeth. The arrangement of the end windings has been subjected to considerable development.

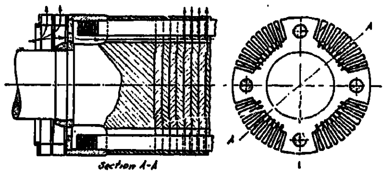


FIG. 26—ORIGINAL SYSTEM OF ROTOR VENTILATION

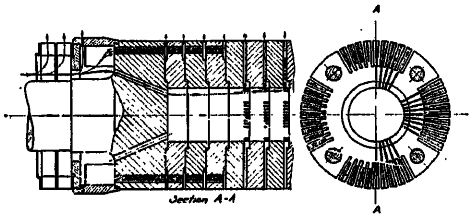


FIG. 27—MODIFIED SYSTEM OF ROTOR VENTILATION

In the earlier constructions the coil retaining ring, Fig. 28, was assembled on the end plate, at *A*, and on the rotor body, at *B*, with an ordinary iron-to-iron fit without shrink allowance. The end plate was usually put on the shaft, at *C*, with a slight press fit. The insulation under the retaining ring consisted of two layers of mica each 1/16 in. thick.

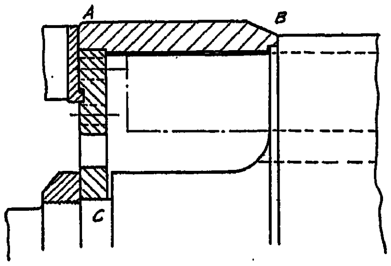


FIG. 28—ARRANGEMENT OF COIL RETAINING RING AND END PLATE

It is evident that this insulation has an unusually heavy duty to perform, being subjected to a continuous pressure of over 5000 lb. per sq. in., in certain cases; to a high temperature; and to frequent shearing action, due to expansion and contraction of the coils. In 1923, looking ahead to these larger units, a new insulation was developed, consisting of channels made of molded asbestos, which were put over the coils, and the retaining ring was lined with the same material. A series of tests was made to investigate the properties of the new insulation, by subjecting it to approximately the same cycles of pressure, temperature, and shearing action as in the actual rotor. It was found that while the old insulation broke down under 365 temperature cycles, the new insulation was still intact after being subjected to 1800 heat cycles.[8]

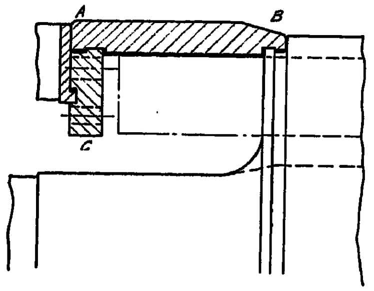


FIG. 29—ARRANGEMENT OF COIL RETAINING RING AND END PLATE

As the size of the individual units increased, and it was necessary to raise the stress in the coil retaining ring, it was found that the expansion of the retaining ring under this stress amounted to as much as 0.050 in. on the larger units. Naturally, with this expansion the centering of the retaining ring was no longer adequate, and the eccentric displacements that were possible were great enough to affect the balance of the rotor to an appreciable extent.

The logical solution for this difficulty would be to provide shrink fits at *A B* and *C*, (Fig. 28); and determine the shrink allowance in such a manner that the fits would have a certain necessary minimum pressure at the over-speed. It is evident, however, that, since the retaining ring then forms a rigid extension to the rotor body, most of the bending rigidity of the rotor between the planes *A* and *B* would be provided by the retaining ring and thus cause very heavy reactions on the end plate, and on the fit at *C*. It was found that these reactions were great enough to disrupt the shrink fit at *B*, and cause the ring to work loose. In order to provide for this possibility, the fit at *A*, between the retaining ring and the end plate, has been made loose, with a clearance great enough to absorb the differential deflection of the rotor between the planes *A* and *B*. The end plate is shrunk on the shaft, and furnishes a rigid base for application of balance weights, and for attachment of the fan.

A further modification of this design is shown on Fig. 29, where the end plate has been left entirely clear of the shaft. Heavy shrink fits are provided at both *A* and *B*, and the fits are made with retaining flanges to prevent the ring from coming loose under the influence of centrifugal forces from heavy balance weights in the end plates. This construction is ideal, in that it completely eliminates the disturbances due to weaving of the retaining ring, and it furnishes an unobstructed passage for the air into the rotor.

It was thought at first that the practise of shrinking the retaining ring on the rotor and on the end plate would cause very large reactions on the fits at stand still. It is true that if the entire radial force of one of the large retaining rings had to be supported by the fits, the ring would probably fail due to shear. It is a fortunate incident, however, that the retaining ring assumes an hour-glass shape at stand still, so that the major part of the radial force is eliminated by contraction in the middle.

The curves in Fig. 30, illustrate this condition for the size of retaining rings used on 1800-rev. per min. machines. It is seen that if both ends of the ring are shrunk on a solid piece, the middle contracts about 80 per cent of the shrink allowance. On the smaller retaining rings for 3600-rev. per min. machines the middle actually contracts to a smaller diameter than that given by the unstressed state. When one end is shrunk on, only about half of the ring is affected. As a result of this, the reactions on the fits at stand still are actually about 25 per cent of the full radial force.

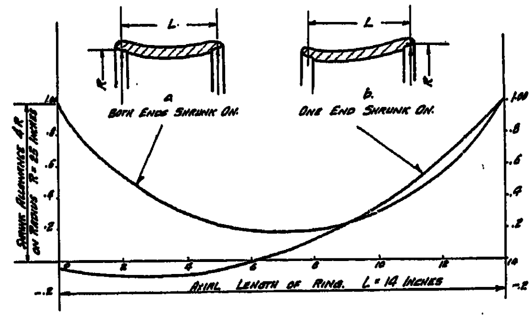


FIG. 30—DEFORMATION OF SHRUNK-ON COIL RETAINING RING AT STAND STILL

This condition produces another important advantage, in that it is possible to press on the rings cold, with reasonable press capacity; about 750 tons being sufficient for the largest rings made so far.

The problem of constructing internal fans has become increasingly difficult for the great amounts of air required for these rotors. A new type of fan with axially curved blades was developed by C. J. Fechheimer in which the bending stresses on the blades, caused by centrifugal forces, were modified into tensile and compressive stresses, (Fig. 31). With the use of this fan it is possible to equip 1800-rev. per min. machines of 75,000-kv-a. rating with internal fans.

---

[8] "Trends in Turbine Generator Development," by F. D. Newbury, *Electric Journal*, July, 1927, p. 334.

CRITICAL SPEEDS

Earlier Conceptions and Design Limitations. The problem of critical speeds in the rotor structure has played a peculiar role in the development of turbo generators; and its influence in placing limitations on the rotor dimensions represents a striking example of the fact that complete absence of theoretical analysis is sometimes to be preferred to unreserved faith in the results obtained by a faulty or partial analysis.

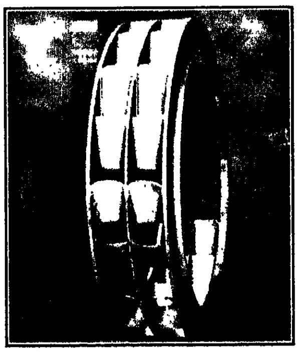


FIG. 31—NEW CONSTRUCTION OF INTERNAL BLOWER

In the early stages of the development it was generally considered impossible, or at least undesirable, to run the machine through any of its critical speeds, and the rotor proportions were definitely limited by the demand that the calculated critical speed must be well above the operating speed. The gradual enlargement of 3600-rev. per min. units soon made it necessary to readjust this conception, and De Laval's demonstrations of the self-centering properties of a flexible shaft furnished proof of the practicability of running through the critical speeds of the rotor without danger. This demonstration was made about 30 years ago, and the theoretical explanation of the phenomenon followed soon after; but it took considerable time before this idea was accepted by practical designers. Thus it is not more than about 15 years ago that the practise of designing 3600 and 3000 rev. per min. machines to run through the lowest critical speeds was firmly established.

At that time there was considerable hesitancy on the part of the designers to extend this reasoning to four- and six-pole designs, primarily because the demand for large units in these types was not yet sufficiently great. It is certain, however, that the old conceptions of the danger of critical speeds played an important role in retarding the natural expansion of the low speed units. When the demand for large units became sufficiently strong, this barrier was finally broken down, and about five years ago the Westinghouse E. and M. Co. no longer felt hampered in the extension of large units to sizes where it was necessary to run through the first critical speed.

Influence of the Flexibility of the Bearing Supports. As the numbers of units in active service increased, the troubles due to vibrations gradually began to form an appreciable portion of the expense of installing and bringing these large machines into satisfactory operation. A survey of these troubles brought about the surprising result that the large units, operating well above the first critical speed, generally caused less vibration troubles than the smaller units of a similar design, believed to operate below the first critical speed of the rotor. Vibration tests on a number of units of the latter type disclosed the fact that the first critical speed of the rotor, when mounted in its actual bearings, may be far below its calculated value. In certain cases the critical speed was found to be less than one-half the value predicted by calculations of the rotor, while in others it was found very near the operating speed.

The quantitative influence of flexibility of the bearing supports is most easily understood by considering the simplest case of a single concentrated mass on a flexible shaft in two supports. It is a well known fact that the critical speed of this arrangement corresponds to the natural frequency of lateral vibrations in one plane. Thus, if the shaft deflection under the influence of gravity is y_1, the critical speed, N_c, corresponds to the natural frequency in cycles per min. of a mathematical pendulum of length y, that is

$$N_c = \frac{60}{2\pi}\sqrt{\frac{g}{y}} = 187.{}_7\sqrt{\frac{1}{y}} \text{ rev. per min.}$$

The stator deflection of the rotor has no other significance than that of being a measure of the flexibility of the shaft. The critical speed remains the same whether the shaft is rotated in the vertical or horizontal position, in fact it would remain unchanged if the intensity of the gravitational attraction were changed, or if it disappeared altogether.

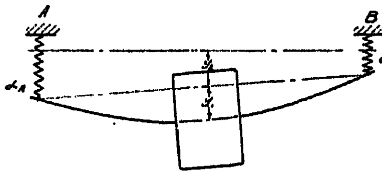


FIG. 32—SINGLE ROTOR SPAN IN TWO FLEXIBLE BEARINGS

As far as the natural frequency of vibration is concerned it is immaterial whether the elastic static deflection occurs in the shaft itself or in elastic members in its supports. We may thus consider the rotor deflection as made up of two components; y_1 due to the flexure of the shaft itself, and y_2 due to the flexure of the bearing supports (Fig. 32). The critical speed is then

$$N_c = 187.{}_7\sqrt{\frac{1}{y_1 + y_2}} \text{ rev. per min.}$$

It is evident at once that this critical speed is lower than that obtained for the rotor in rigid supports; the reduction being dependent upon the relative magnitude of y_1 and y_2.

It is thus a fairly simple matter to obtain an approximate value of the critical speed when the bearing supports are flexible. The actual conditions of motion of the rotor body are quite complicated, however, and several aspects of the problem are not fully known. This applies particularly to the case when the bearing supports have different flexibilities in the horizontal and the vertical directions. The information available at the present time indicates that the natural frequencies in the direction of maximum and minimum flexibility give the critical speeds with sufficient accuracy. This result must be considered in the nature of an empirical result. Ordinarily, the maximum flexibility occurs in the horizontal direction. Thus, most two-bearing machines will be found to have two critical speeds, both somewhat below that calculated for the rotor in rigid supports. Fig. 33 shows the general speed-amplitude curve that is obtained from one of the bearing supports of an arrangement of this kind. Experimental curves of this nature form one of the most important means for scientific analysis of vibration problems. Such curves will be referred to as *resonance curves.*

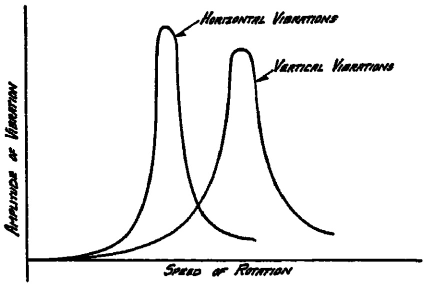


Fig. 33—General Type of Resonance Curves for Machines Operating in Flexible Bearings

The reduction in critical speed due to flexibility of the bearing supports is most marked for machines with a high critical speed of the rotor on rigid supports. It is evident from the above formula that, even if the shaft deflection y_1, could be reduced to zero, corresponding to an infinite value for the critical speed of the rotor in rigid bearings, the actual critical speed, now determined by y_2 alone, may still fall below the operating speed.

This is actually the case for most machines in the 1800-rev. per min. class, having a rotor weight over approximately 50,000 lb. With the most sturdy design of the bearing supports, which have been used so far, it has been found that the actual critical speed cannot be brought above about 1900 rev. per min., no matter how rigid the rotor structure. For rotors of this type, having a calculated critical speed of about 2400 rev. per min., the actual critical speed will usually fall around 1200 rev. per min. A similar situation exists with machines of the 3600-rev. per min. class, when the rotor weight exceeds about 5000 lb. Here the possible upper limit of the critical speed is about 3200 rev. per min.

Thus, while the designer may be able to control the rigidity of the rotor structure to a certain extent by changes in the rotor proportions, he is usually not able to influence the rigidity of the bearing supports to any appreciable extent. The natural limitations of the construction of the stationary parts, and the materials involved, prevent increases beyond certain limits. In addition, the oil film in the bearings provides a certain unavoidable flexibility. The available information on the subject of elasticity of oil films is very scant, but those investigations that have been made[9] indicate that the oil film can be considered as a mechanical spring interposed between the shaft and the bearing.

So far, the discussion has been restricted to the fundamental mode of motion of the rotor, and no reference has been made to the complications arising from the fact that most machines are operated in more than two bearings. The influence of the bearing flexibility upon the second critical speed of the rotor is similar to that of the first, but the lower limits of critical speeds are considerably higher. In the largest machines under construction at the present time the second critical speed of the rotor is of major importance, and it has been found that a reduction of about 30 per cent, from the value calculated for rigid bearings, must be considered. The conditions in multiple-bearing machines, with rigid couplings, are somewhat more complicated but the problem is susceptible of theoretical analysis, the only difficulty being in fixing values for the flexibility of the bearing supports.

It is evident from the above that critical speeds above the operating speed of 1800 rev. per min., as well as most 3600-rev. per min. machines, are practically impossible of realization; notwithstanding the fact that purchasers of electrical machines frequently include a clause of this nature in their specifications.

The General Foundation Problem. The flexibility of the bearing supports is only one of the factors that cause the great discrepancy between calculated and observed critical speeds. A fully satisfactory perspective of the situation is obtained only when the entire unit, with its foundation, is treated as one problem. It is here that the real limitations of mathematical analysis come to light, and the machine designer is made to feel the true importance of intuitive judgment and good luck.

In order to arrive at an understanding of the mechanics of the systems that are obtained when the entire machine, with its foundation, is treated as a unit, it is necessary to represent the system by a dynamical model, where the dynamically essential features are emphasized, and the non-essentials hidden. Such a dynamical model must be constructed separately for both the vertical and the horizontal directions, but the difference is usually only in the magnitude of the various system constants, and not in the arrangement.

---

[9] Stodola, *Schweizerische Bauzeitung*, May 23, 1925. Hummel, *V. D. I.*, *Forchungs Arbeiten*, Heft 287.

Fig. 34 illustrates a typical layout of a foundation for an 1800-rev. per min. machine; and Fig. 35 represents its dynamical model.

It is unnecessary to point out that models of this type represent very rough approximations of the actual systems, and certain parts must be modified further before they yield practical results. A reasonably complete analysis can be made, however; the chief difficulty lies in the lack of knowledge of the system constants involved. It is worthy of note that even if complete mathematical analysis is not made, certain results of importance can be obtained by direct inspection of the dynamical models. As an example of this may be mentioned a condition which is obvious from inspection of the dynamical model shown in Fig. 35. It is evident that the amplitude of vibration of the bearing A, produced by a disturbance in the rotor mass m_1, depends to what extent the nearest stationary mass, m_3, is made to take a part of the motion. This reveals a relation which has not always been observed in the present foundation designs, namely, the necessity for effecting an intimate mechanical connection between the rotating and the stationary masses.

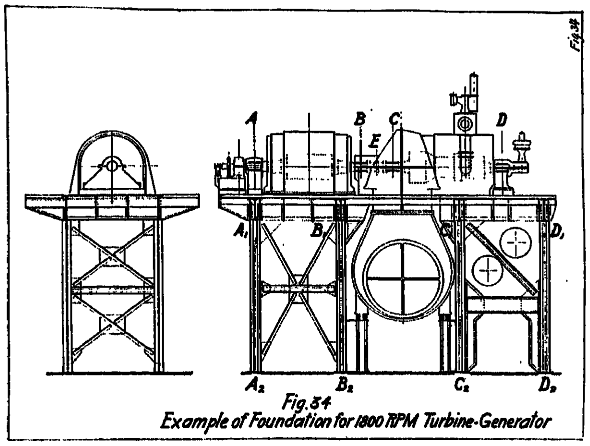
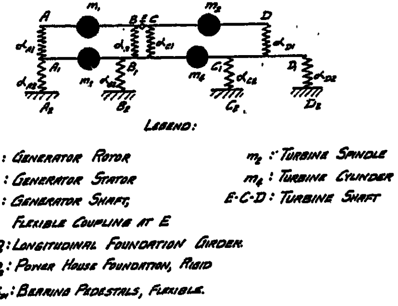


Fig. 34—Example of Foundation for 1800-Rev. per Min. Turbine Generator



A
m_1
B E C
m_2
D
α_{A1}
α_{B1}
α_{C1}
α_{D1}
A_1
B_1
m_3
α_{B2}
m_4
C_1
α_{C2}
D_1
α_{D2}
α_{A2}
A_2
B_2
C_2
D_2
Legend:
m_1 : Generator Rotor
m_3 : Generator Stator
A-B-E : Generator Shaft, Flexible Coupling at E
m_2 : Turbine Spindle
m_4 : Turbine Cylinder
E-C-D : Turbine Shaft
A_1-B_1-C_1-D_1 : Longitudinal Foundation Girder
A_2-B_2-C_2-D_2 : Power House Foundation, Rigid
α_{A1}-α_{B1}-α_{C1}-α_{D1} : Bearing Pedestals, Flexible.
α_{A2}-α_{B2}-α_{C2}-α_{D2} : Foundation Columns, Flexible

Fig. 35—Dynamical Model for Four Bearing Turbine Generator and Foundation

Apart from the general considerations of this nature, however, successful operation always demands that no condition of resonance exist near the operating speed. The system shown in Fig. 35 has four distinct resonance frequencies for the vertical and horizontal directions, making a total of eight resonance peaks; of which some, may overlap and some may fall wholly outside the operating range. In machines of great lengths it may even be necessary to consider the rotational vibrations of the rotor and stator masses as well, thus increasing the number of resonance peaks further.

The term "critical speed" now assumes an entirely different significance from the ordinary conceptions of critical speeds in the rotor alone. The chief object of the analysis is to enable the machine to run in a hollow of the resonance curve; and there are, evidently, no simple rules that will accomplish this result.

It may be of interest to show some of the experimental results obtained from the actual installations which have furnished the justification for this treatment of the problem.

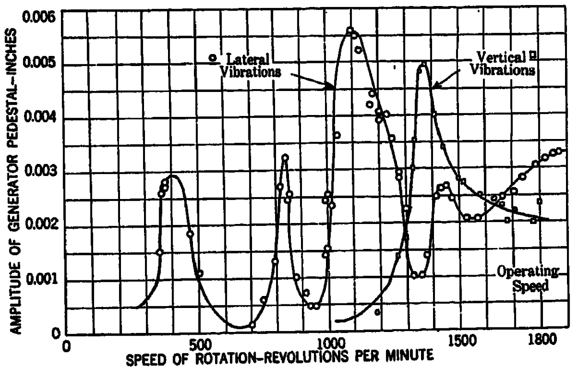


Fig. 36—Resonance Curves for Generator Pedestal of Four Bearing Turbine Generator Experimental Result Obtained from Arrangement shown in Fig. 34.

Fig. 36 shows the resonance curves that were obtained from a 25,000-kv-a. unit of the type shown in Fig. 34. These curves were obtained from the outboard pedestal of the generator end A. Each one of the peaks obtained on these curves could be accounted for, at least approximately, by analysis. The maximum peak for horizontal motion occurred at 1200 rev. per min. This was found to represent the fundamental critical speed of the rotor, which was calculated to 2380 rev. per min. for rigid bearings.

Present Design Limitations. Further progress is necessary before the designer of large turbine generators can take a definite stand with respect to the design limitations made necessary by critical speeds. At the present time two methods of approach are possible. One is to continue along the old channels of thought and discuss the problem in terms of critical speeds of the rotor, making such allowances for the foundation influence as experience will indicate. The other is to attack the foundation problem aggressively, and regard the rotor proportions with no greater emphasis than is demanded by its place in the foundation problem.

Whatever method adopted, it is evident that the old conceptions of placing limitations on rotor dimensions based on the critical speeds of the rotor alone, in rigid bearings, must be modified. The development of large units was needlessly hampered by the old conventions of critical speeds, and by adhering to these conventions, certain designs were actually made with erroneous rotor proportions. This applies to almost all of the larger machines designed to run below the first critical speed.

Apart from these considerations, however, there are certain problems which demand immediate attention. Probably the most important point at the present time is the question of what is the absolute lower limit to which the critical speed of a rotor can be brought without danger. At the present time, very few machines have been constructed with critical speeds below 900 rev. per min., corresponding to a maximum shaft deflection of about 0.045 in. There are, as yet, no indications that the operation of a rotor of this type is accompanied by any dangers or unusual troubles. As the demand for large units increases, however, rotor deflections of 0.100 in., and critical speeds of about 700 rev. per min. will be necessary.

An investigation of the probable trend of the development of Westinghouse machines of the 1800-rev. per min. class has shown that the capacity can be increased to about 120,000 kv-a. before the second critical speed of the generator rotor must be considered. At that capacity, the fundamental cirtical speed will be of the order of 700 rev. per min. It may be necessary to screen off certain core lengths on the present, maximum diameter between about 120,000 and 150,000 kv-a. but above this limit it is still theoretically possible to extend the rotor length and the capacity still further. How far this can be carried is entirely problematical.

It is believed, however, that the bending stress in the rotor might play an important role before the limit in critical speed and rotor deflection is reached. This bending stress is alternating, and, taking into consideration the probable conditions of stress concentration, and the fact that it exists in conjunction with certain other stresses, there will probably be considerable hesitancy on the part of the designers to raise this stress far above 5000 lb. per sq. in. with present rotor materials. This will limit the output to a value around 150,000 kv-a., but a considerable amount of further research is necessary before this figure can be established definitely.

In addition to this limit of the bending stress there are certain other features to be considered. Since no journals are absolutely uniform, and no rotor has an absolutely uniform deflection in all directions, it is evident that, as the rotor deflections are increased, the effects of these errors may also be increased. The danger from this source lies primarily in the danger of producing resonance with the second critical speed. It is not possible to place a limit on the rotor length from this consideration, but it is conceivable that much longer rotors than those built at the present time may give trouble on account of these causes.

The question of instability of the rotor in the air-gap will probably not be of consequence for any of the rotor lengths in sight at the present time. Unless the air-gaps are materially increased it is conceivable that this feature of the problem must be considered when shaft deflections exceed 0.100 in.

In conclusion it may be stated, therefore, that the development towards larger, individual units has not yet reached the point where a definite limit can be established. The results that are obtained when the units, under construction at the present time, have been put into service may conceivably add further elements to the discussion; but it is almost certain that if the demand for large units is maintained, the next decade will witness no less striking undertakings in this respect than the present.

BALANCING AND VIBRATION PROBLEMS IN THE SHOP AND IN THE FIELD

Apart from the problems incidental to the building of the rotor body, its machining, the winding, and the assembly, the vibration problem figures very prominently in the process required to bring a machine into satisfactory operating conditions.

One of the primary requirements for smooth operation (apart from the question of design), is an accurate balance of the rotating parts. The Westinghouse Co. realized the importance of this process for large, high-speed machines at an early stage in the development, and its first balancing machine for this purpose was constructed more than 10 years ago. This machine had a capacity of 250,000 lb. and has been in continuous operation in the Turbine Works at South Philadelphia. In 1923 a similar unit, with a maximum capacity of 300,000 lb., was installed at East Pittsburgh. Similar equipment has been added, subsequently, for the 3600-rev. per min. machines. The construction of these machines, and the method of operation, is practically the same.[10]

With this balancing equipment the process of obtaining a practically perfect balance is a simple matter, but it has been found that the balancing process alone is not sufficient to insure smooth operation, except for the small units. Due to the mechanical imperfections of the coils, and their insulation, certain distortions of the rotor structure take place during the first hours of heating and rotation. It has been found necessary, therefore, to subject each rotor to a definite seasoning process at full speed and maximum field temperature, and sometimes a stable condition is not obtained until after several days of seasoning. The rotor is heated by current through the field windings, and the seasoning is carried out in a special test rig arranged for this purpose. In order to maintain the field temperature sufficiently high it is usually necessary to enclose the rotor in a heater box, which is made of wood in order to eliminate heating by eddy currents. After a stationary state of balance is reached at full speed and maximum temperature, it is not unusual to find that the balance at room temperature and low speed is different. Ordinarily, however, a well seasoned rotor will maintain a permanent balance under all conditions of operation. No particular difference has been observed in this respect between plate rotors and solid rotors, some of the latter type having been very slow to become stabilized.

---

[10] "Balancing Large Rotating Apparatus," L. C. Fletcher, *Electric Journal*, January 1924, p. 5.

Even with the utmost care in these preparations of the rotor, the manufacturer is never completely guarded against troubles in the field, so that field balancing is a very important element of turbine-generator installation.

The cause of these field troubles is occasionally an actual slip in the shop process, and sometimes it may be traced to incorrect alinement; but in the majority of cases the troubles can be traced to unduly sensitive foundations. It is very seldom that a fundamental reconstruction of the foundation can be made, however, and the universal cure (which generally succeeds in eliminating the trouble) is to effect greater precision in the rotor balance by hand-balancing. These field troubles are always a source of inconvenience to the customer, and of unexpected expense to the manufacturer, but they have had one concrete result of importance, in that it is through these field troubles that the available knowledge of the properties of foundations has been gained.

Hand-balancing has always been regarded as more or less of a mystery, the secrets of which have been available only to a select few. The development of the art during recent years has dispelled a considerable portion of this attitude, however, and the systematic methods in use at the present time invariably yield quick results, provided that a cure is obtainable through balancing alone.

It is in this connection that the application of vibration instruments has been particularly valuable. In ordinary cases of balancing it is merely a question of measuring the amplitude of the vibration with a certain degree of accuracy. The vibrometer developed by the Westinghouse Co., has proved to be eminently useful in this respect. For more elaborate measurements of the vibration the Geiger Vibrograph made by Lehmann and Michells in Germany, has been used extensively. This instrument has the advantage of giving a graphical record of the motion, whereby the form of the displacement is also obtained. It is particularly useful in obtaining resonance curves, continuous records of amplitude and frequency, and the results shown in Fig. 36 were obtained by this instrument.

Many vibration problems encountered in the field are due to local conditions of resonance in bearing pedestals, frames, piping, flooring, etc. While the ultimate cause of these vibrations resides in disturbances from the rotor, it is very rarely that a cure can be effected by correcting the cause itself. The most effective cure is usually to change the natural frequency of the element in question, by stiffening or weakening. These local vibrations are not infrequently classed as serious vibrations of the machine as a whole, and a great deal of field trouble can be eliminated by particular care in the design of these parts.

Axial vibrations of bearing pedestals form a very important part of this item. The causes for these axial vibrations are not fully known, and there are cases on record where resonance has definitely been demonstrated not to be the cause. In many cases a cure has been effected by balancing of the rotor, although there is no apparent reason why unbalance should cause axial disturbances. It is probably safe to state that more than 30 per cent of field troubles are due to axial vibrations of bearing pedestals. This represents one example of the many practical vibration phenomena which are not yet fully understood.

Another group of unexplained phenomena has been observed in the vibrations of the steam turbines. Several cases are on record where the amplitude of a certain point would vary periodically with periods of several minutes or even hours. These phenomena are undoubtedly connected in some manner with temperature distortions, and they have been eliminated in several instances by modifying the supports of the turbine housing, but the actual causes are as yet hidden.

The question of alinement is a very important one for large turbine generators. This problem is complicated by the movements in the foundation that take place because of variation in temperature, particularly in cases of exposed steel foundations. Where solid couplings are used, the present practise is to aline the machine in such a manner that no flexural moments are introduced in the coupling, except those produced by gravity. Corrections are then made for thermal expansion. This method of alinement is sometimes modified when flexible couplings are used, but it is probably a safe procedure to aline flexible couplings fully as accurately as solid couplings.

CONCLUSION

From the above it will be evident that the design of large turbine generators is one of extreme complexity, where practically all branches of engineering and science find their application. Now, as in the past, the designer of turbine generators is forced to work very close to the limits determined by available knowledge; and in many innovations, his only guide is the experience gained on artificial models, arranged to test out his theoretical investigations.

A program of development, such as described, is the result of the combined activities of the entire organization of a large company. For many of the results recorded in the paper we are indebted, among others, to C. M. Laffoon, G. M. Eaton, E. Mattman, W. J. Merten, S. Timoshenko, G. M. Lessells, and C. F. Hill, members of the research, metallurgical, and design departments of the Westinghouse Co.

Bibliography

The following bibliography covers a few of the outstanding papers on the subject of turbine generators up to the present time:

B. G. Lamme, TRANS. A. I. E. E., Vol. 32, Part I, 1913, p. 1, *High Speed Turbo Alternators, Designs and Limitations.*

A. B. Field, *Journal I. E. E.*, 1915, p. 65, "Some Difficulties of Design of High Speed Generators."

S. F. Barcley, *Journal I. E. E.*, 1918, p. 472, "The Mechanical Design and Specifications of Turbo Alternator Rotors."

W. J. Foster, E. H. Freiburghouse, M. A. Savage, TRANS. A. I. E. E., Vol. 43, 1924, p. 1249, *Large Steam Turbine Generators.*

E. Wilczek, *World Power Conference*, London, 1924, Vol. III, p. 908, "Hungarian Practice in High-Speed Turbo Alternator Practice."

E. Roth, TRANS. A. I. E. E., Vol. 44, 1925, p. 905. *Three-Phase, 60,000-kv-a. Turbo Alternator for Gennevilliers.*

F. D. Newbury, *Electric Journal*, July 1927, p. 334. "Trends in Turbine Generator Development."

Robert Pohl, 2 *T. Z.*, Feb. 2, 1927, "Neuzeitliche Turbogeneratoren und Luftkuhler."

Discussion

W. J. Foster: I want first to say a word about the voltage. Mr. Henderson takes the position with which I think we will all agree that we do not want any higher voltage in generators than we can well get along with, on account of the space taken up by the insulation, the temperature drop through the insulation, and so forth, and yet at the same time, as pointed out by Mr. Henderson in his paper when he speaks about the amperes per slot and their deleterious effect on losses and so forth, we need the higher voltage to keep the amperes down. We must balance up one effect against another.

There has been pressure brought to bear when we go above 13,800 volts to go to 22,000 or 23,000, since that is considered a standard voltage on transformers and in distribution. A number of intermediate steps has been made such as 18,000, 16,500. We had the 18,000 several years ago on engine-driven generators. The step to 22,000 volts on certain turbine generators was taken by reason of the pressure brought to bear on the manufacturers to go to that voltage on account of the great savings in bus construction, switches, and station wiring. It was found practicable to design the generators for that voltage.

I wish here to make just a little correction in the paper. I think Mr. Henderson and Mr. Soderberg spoke of only one plant where this high voltage is going to be introduced in a generator. There are two such plants. The generator for the first one has already been shipped. It will be installed before the more conspicuous one, which is the 208,000-kw. unit, is installed. The latter consists of three 22,000-volt 85 per cent power-factor generators, 1800 rev. per min., the larger or high-pressure unit 89,411 kv-a. and the smaller ones identical in size, 72,941 kv-a. each. It has been found in developing the insulation and in testing the same that results have been obtained that are quite reassuring. We now have confidence that the 22,000-volt generator is all right.

With reference to the matter of insulation, I am interested in what has been said and what is shown in the paper with regard to the use of shellac, inasmuch as that conforms with my own experience. There are other bonding materials that are far preferable, especially in what happens when they are subjected to higher temperatures.

In the matter of corona discharge, that is another reason why we hesitate to wind for very high voltages. It might be brought forward as a reason why we should hesitate at such a voltage as 22,000. I was especially interested in what the authors have said regarding the development of a covering that arrests or prevents any serious deterioration due to the discharge. I trust that what has been discovered is going to prove permanent. Many years ago in connection with some of the first high-tension generators that were installed at high altitudes where trouble developed due to corona, it appeared desirable to devise a corona-protecting sheathing. The problem was attacked at the factory, and a certain pigment was discovered with which the finished coils were painted, and apparently a cure had been obtained. It was a cure, but only temporary. That protection soon wore away and disappeared. Later the practise became established on the part of the manufacturers with whom I have been associated of putting on a metallic sheathing, which consisted of a metallic ribbon insulated on one side and wound around the fully insulated coil. That was very effective, but in time it was liable to wear out.

I am still of the opinion that the problem is a very important one and should be solved to everyone's satisfaction. I hope what the authors describe will prove the real solution. Whatever corona-protecting sheathing is used must be of such nature as to adhere perfectly to the surface of the completely insulated coil and be a substance that will not deteriorate due to temperature or chemical actions that may take place. As many know, asbestos sheathing offers a solution in many cases.

With regard to hydrogen, the authors point out some of the advantages. I simply wish to add that undoubtedly the benefits obtained by hydrogen are greater on the rotor than on the stator, and as a result it strengthens the weaker part of the generator. It has long been recognized that the rotor is the limiting feature.

What is shown in the paper in regard to the skeleton-frame stator construction is of special interest to me, as the group of men with whom I am associated devised similar stator frames as much as two years ago. In fact, several machines have been built, including the first of the 100,000 kv-a., which I presume at the present time is on its way to the Pacific Coast. That has been a great step forward, for the reasons pointed out in the paper.

B. A. Behrend: The paper is divided into parts dealing with the electrical problems, the ventilation problems, the mechanical design of stator and rotor, and the dynamical balance of the latter. I shall use for this discussion the same headings as those used by the authors in their paper.

II. VENTILATION

The multiple-radial system of ventilation, the principle of which is the invention of Parsons and Holcombe, has been carefully studied and developed and its adoption by the Westinghouse Company marks a step of great progress. We thus see the axial system of ventilation abandoned and supplanted by one whose efficacy has been thoroughly demonstrated. The inherent limitations of the axial system resulted in higher temperatures for the stator coils which entailed both serious electrical and mechanical difficulties. Moreover, we are now no longer confronted with the making of a decision as to the possible superiority of one system of ventilation over another as the principal makers of large turbine units are now using substantially the same system. This marks progress when it is the result of steady evolution and improvement.

The mode of ventilation is naturally connected with the medium of ventilation and the authors have referred to the use of hydrogen gas for cooling. It is my opinion that, given the necessary time to convert the power-plant owners, hydrogen will be freely used on large units raising efficiencies, reducing size, and making possible higher voltages on the individual unit. Water and oil cooling of stators, extensively used abroad, appears to have found little or no favor here. A. B. Field's discussion bears on this subject.

III

In the design of stator coils, sound principles as laid down by A. B. Field, R. E. Gilman, W. H. Taylor, W. V. Lyon, and noted foreign designers, have been adopted. This necessitates cross-overs in the slots which, as Mr. Henderson states, have enabled him to hold down eddy-current losses to about 30 per cent of the corresponding d-c. loss. The method used by Henderson, shown in Fig. 4, appears to be a good mechanical design. With this type of construction, however, care has to be exercised to prevent establishing paths for eddy currents which could be induced by the external field.

The important researches on temperature conditions in stator coils carried on for years by F. D. Newbury and his associates, should have come in for a general review. This was the kind of work which led up to sound principles of design of stator coils and ventilation and was fundamental in character.

Some little time ago (Transactions, A. I. E. E., March 22, 1907) I described a 100-kw. 22,000-volt generator which had then been in operation continuously for some time. I said that "where individual units of very large capacity are installed in power plants, a potential of 20,000 volts is feasible and it is to be recommended in units of 10,000 kw. and above." Today, twenty years later, it might be advisable to weigh these words and to raise the generator voltage with all the advantages which the handling of smaller currents brings. It is right here where hydrogen cooling will be useful in connection with the increased generator voltages. He is a bold man who will predict the limit of capacity of turbo generators, but I am willing to go on record here that this limit lies between 150,000 kw. and 200,000 kw. Twenty years ago (*Cassier's Magazine*, May, 1907) I urged 25,000-kw. units at 22,000 volts as unit sizes for power-generating stations and, having seen this prediction realized and to some extent exceeded, I now go on record stating that we are reaching the technical and economic limit at eight or ten times this size, *viz.*, at 200,000 kw. or 250,000 kw. at the most.

IV. General Design

Here it is interesting to note, first, the adoption of a 4 per cent silicon steel for the core punchings. The enormous advantage in reducing the core loss to one-half of what it would be with ordinary iron is obvious.

The heating of the finger plates is not a new phenomenon. It has had to be met before by the adoption of non-magnetic finger plates, made of brass or bronze. Losses in the finger plates are two-fold, first, those due to induction from the armature, second, those due to induction from the revolving field. In the earlier machines with weak armatures and powerful fields the heating was due principally to the latter; in the present types with powerful armatures and weak fields, the heating is primarily due to the ampere-conductors per slot. It is obvious that a considerable flux must pass through the finger plates with slots twice as deep as they used to be. This flux is roughly proportional to the product of the depth of the slot into the current density. The better the cooling, the greater the latter. Hence the loss in the finger plates, which is proportional to the square of the induction, is easily ten times what it used to be in the older machines. It is, therefore, necessary to study the magnetic circuit of the finger plates carefully so as not to give an opportunity to set up eddy currents. Excessive heating of the finger plates is a source of danger as their mechanical strength is reduced by high temperatures, leading to the possibility of fracture. Thus broken pieces may drop into the air gap and damage the insulation of the coils. The strength of aluminum alloys is generally affected by heat and their use is, therefore, to be avoided for parts liable to be the seat of eddy currents.

The question of generator stability is brought up by the authors. I have elsewhere (Chicago Regional Convention, Nov. 1927) discussed this matter and I wish to emphasize here that I hope that the designers of large generators will not return to their former practise of using strong fields and weak armatures in order to raise the frequency of natural oscillation of the generators. We have gone through all this in the past and it would be painful to see it all gone through again.

V. Stator Construction

The adoption of welded structural-steel frames for turbo generator stators is highly to be commended. Cast iron is unsuited for this purpose, and cast steel is too costly.

VI. Mechanical Construction of the Rotor Body

As referred to by the authors, I developed the radial slot construction and the plate rotor for the Westinghouse Company about nineteen years ago. The reasons for abandoning steel castings or forgings were those given by the authors. The bolted shaft ends, first used by DeLaval on his single wheels in his steam turbines, were used by me in 1907 on the first of thirteen generators of 300 rev. per min. for Niagara Falls (Transactions, A. I. E. E., 1908, p. 811). The first machine of this series was destroyed on its overspeed test on April 28, 1908. A complete description of the accident with copious illustrations was given by me before a joint meeting of the American Societies of Civil and Mechanical Engineers and of the American Institute of Electrical Engineers, in Boston, fifteen years ago. The type of rotor construction is shown in Fig. 5, in the writer's paper already cited. An 8-in. unpierced central disk of 3½ per cent nickel-steel supported two nickel-steel end rings forged like tires on a mandrel. Careful specifications accompanied the order for these forgings.

The fractures revealed a high axial combined with a low radial ductility. The elastic limit of the radial test bars was high and so was their ultimate strength. The central disk had evidently not suffered at all; it remained intact and test bars taken from it showed no signs of excessive stress. The use of bolted-on shaft ends proved entirely successful, thus indicating the way to the bolted ends with unpierced shaftless rotors described by the authors and introduced by Behrend and Field.

Different committees investigating the accident at the time arrived at the conclusion, which was buttressed by tests of models made in a Tinius Olsen testing machine, that, had the forgings possessed proper ductility in a radial direction, there would have been no reason to have expected the accident. In replacing these machines I used on the first one 2-in. rolled carbon-steel plates, and on the second one, 2-in. rolled nickel-steel plates, carbon steel having proved too delicate in its heat treatment. These plates were all quenched in water. Particular attention was paid to uniformity of ductility in two directions at right angles to each other, *i. e.*, in the direction of rolling and at right angles thereto. Samples taken from the plates in these directions has to bend cold flat 180 deg. without showing cracks. (See Transactions, A. I. E. E., 1918, p. 886.) It is this type of construction of which thirteen units have been in continuous operation for twenty years at Niagara Falls and these are the first 2-in. plate rotor types ever built.

The enormous advantage of such a plate construction was irresistibly impressed upon my collaborator A. B. Field and myself and when we joined the Westinghouse Company in 1909, we introduced this type for turbo rotor construction, returning to the unpierced plates but retaining the 2-in. plates of rolled material.

In my paper before this Institute of 1908, I had given a complete stress chart showing both the radial and tangential stresses in revolving disks, both pierced and unpierced. This chart is given in Fig. 2, herewith, and it has been reproduced by Professor Stodola in his book on steam turbines and by C. M. Laffoon in an able paper before the A. S. M. E., 1922, p. 485, Fig. 12. The important point here is the sudden doubling of the tangential stress when the disk is pierced at the center. In disks made of materials of low ductility, with unequal radial and tangential stresses, the tangential stress may easily treble as a direct result of a central hole. We, therefore, avoided such piercing and developed the design shown by the authors in their Fig. 14.

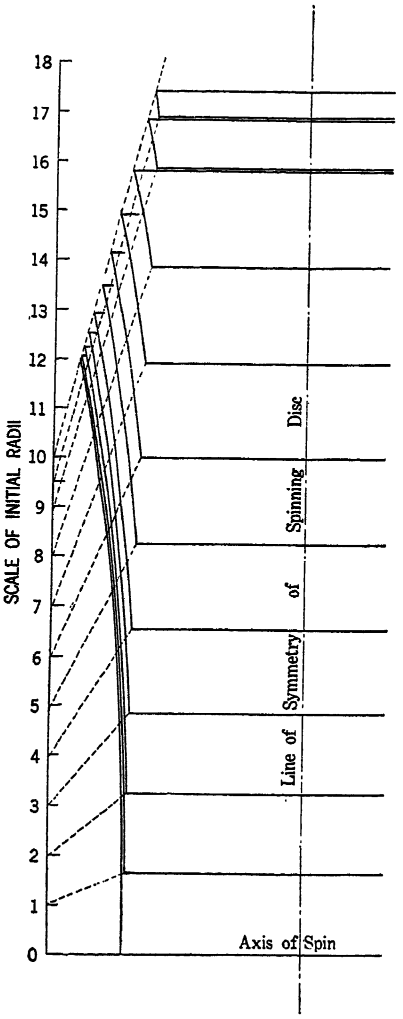


Fig. 1—Deformation and Strain Chart of Rapidly Spinning Disk and Toroid

From the above numbers 1, 2, 3, to 10 indicate initial inside and outside radii of disk before deformation. These points by virtue of the strain distribution travel along the dotted lines according as the inside radius of the toroid is small or large. The disk or toroids develop concave surfaces as boundaries perpendicular to the axis of spin.

The theory makes the cylindrical boundary generated by a line parallel to the axis of rotation, while in reality the generating line is a curve with its center on the line of symmetry making a convex surface for the disk.

Had the accident to the Niagara Falls generator, described in my paper of 1908, been the only one of its kind, or had such an accident occurred only in a machine of my design manufactured in a certain plant, the evolution of a new type might have seemed unnecessary. But in 1922, two generators with cast-steel rotors of twice the rating of our machine of 1908, designed by other engineers and built by another company, were totally destroyed at Niagara Falls. Thus I have felt warranted in urging the Westinghouse Company unceasingly to adopt a type of construction which uses neither castings nor forgings and which permits easy discovery of flaws and defects of material although entailing increased cost of machining.

Now four conditions appeared to me paramount in the design of the plate rotor:

1—The use of a commercial material which could be looked over and inspected in "small bits", so to speak, at the shops of the generator builder.

2—The possibility of using unpierced dis's in which the radial and tangential stresses equal each other at the center.

3—Great ease of effective rotor ventilation.

4—Avoiding excessive tangential stresses on the inside of the bored hole due to shrinking fits on the shaft.

Against these advantages must be cited the greater cost of machining.

Now the authors appear to favor forged rings instead of plates and thus they lose the advantages cited under (1) and (2). In forgings of 8-in. thickness with a central hole the maximum stress is liable to be about twice the stress in the 2-in. unpierced disks or plates. The probability of an invisible flaw remaining undiscovered is 1/4; 1/4 = 1/16 or 16 times as great in an 8-in. forging than in a 2-in. plate, assuming the same probability of occurrence of flaws in either material which is hardly the case. In general, let us assume a forged or solid steel rotor of 100-in. length and let us compare with the same a plate rotor of equal length with plates of 2-in. thickness, then the probability of a flaw remaining undetected is 2500 times greater in the first than in the second case. This is, perhaps, the most graphic way of bringing home the issue between solid rotors, cast or forged, and plate rotors of the type proposed by us. Of the latter, the Westinghouse Company has built considerably over one hundred large units which have been entirely successful.

Forging rings on a mandrel has a tendency to lower the radial ductility on account of the direction of the forging work and, simultaneously, to raise the axial ductility, which is useless; the process of mandrel forging thus operates in the inverse direction to that which is desirable. This statement is made in spite of the micro-photographs shown by the authors in Fig. 20. The tendency toward such variation can be offset only by extensive heat treatment, counteracting the effect of the forging work, and such heat treatment is not in the hands of the builder of the generators.

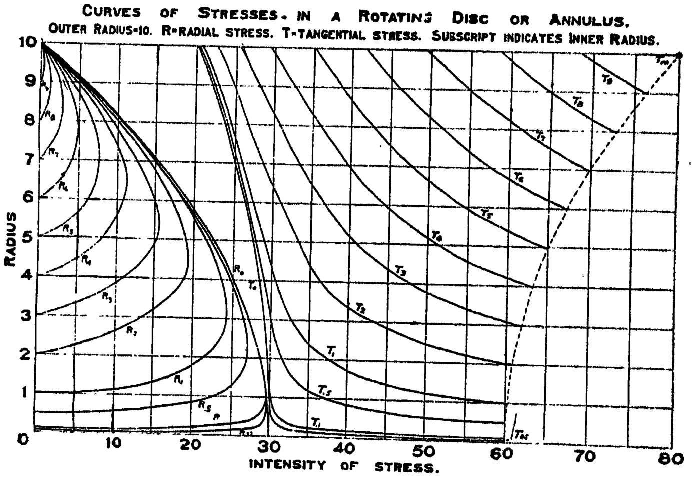


Fig. 2

With rolled plates it is possible to order material for stock in large quantities and at times when the steel mills are not busy on standard specifications; with the rolled or forged rings this cannot be done and every job is a special job. We secured plates from the armor plate department of the Carnegie Steel Company at Munhall, Pennsylvania, of nickel steel with properties which showed that an elastic limit of 55,000 lb., an ultimate of 75,000 lb., and 28 per cent elongation with 45 per cent reduction was obtainable, thus indicating that it can be done. It is, of course, undesirable business for the mills as it is fastidious work.

VIII. Critical Speeds

In his splendid analysis of critical speeds, Mr. Soderberg points out the importance of operating at a speed between the first and second critical speeds or between the fundamental lateral vibration and its first harmonic. He shows the effect of the oil film in the bearings as resulting in a lower critical speed than that which would obtain were the bearings perfectly rigid and he concludes that "it is here that the real limitations of mathematical analysis come to light and the machine designer is made to feel the true importance of intuitive judgment and good luck."

Mathematical analysis cannot solve the intricate problem of dynamic balance but such analysis as Mr. Soderberg has given elsewhere of systems of several degrees of freedom supplies a great and useful guide for which the engineer will be duly grateful to the author.

Edouard Roth: I wish to add to the very suggestive list of great units which have been built or which are under construction in the States, a unit of 40,000 kv-a. at 3000 rev. per min., 13,500 volts, built by the Société Alsacienne de Constructions Mécaniques at Belfort (France) which is, as far as I know, the largest unit at 3000 rev. per min. and of which I append some photographs (Figs. 3 to 5). Fig. 4 shows how the very important question of shipment has been solved; this is a problem on which the authors rightly insist very much. The solution of this problem is easier for the American designers, first because the railway clearances are larger in America than in Europe, and then because the American railway companies allow for a heavier load per axle, than the European companies; for example, in Europe there exists no car that can carry 275,000 lb. The kind of construction represented in Fig. 5 allowed us to build up the sheet-iron core in the factory, but the winding had to be put in place at the power station. The solution by means of a skeleton frame is very ingenious but I do not believe that it can be generally employed in Europe.

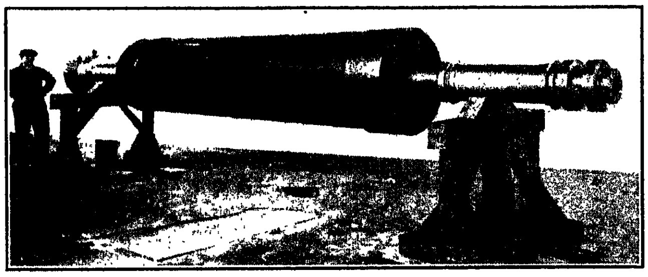


Fig. 3

In the case of the machines built by the Société Alsacienne, it is not easy to use this kind of frame on account of the ventilation system. In my paper, which is referred to by the authors, I gave a detailed description of this system, the advantages of which were so clearly emphasized in the discussion by Mr. Fechheimer. This system has proved to be very efficient and is used by my firm for all turbo alternators.

In this same paper I explained in detail that this system of ventilation is possible owing to the leakage slots, which were so thoroughly discussed on the same occasion. I repeat that these slots make it possible to build alternators with a flux which assures great stability (another problem on which the authors insist) and which at the same time gives a very small instantaneous short-circuit current. These leakage slots are used by my firm for all turbo-alternators of outputs above 5000 kv-a.

Another problem which the authors mention is that of the heating of the finger plates, supporting the stator teeth, which is due to stray load losses; may I add that there are also losses in the end plates and in the extreme portions of the lamination of the stator core. In my opinion, which is also that of the authors, the only and best remedy is the use of non-magnetic steel for the wedge and end plates, as well as for the retaining rings of the rotor.

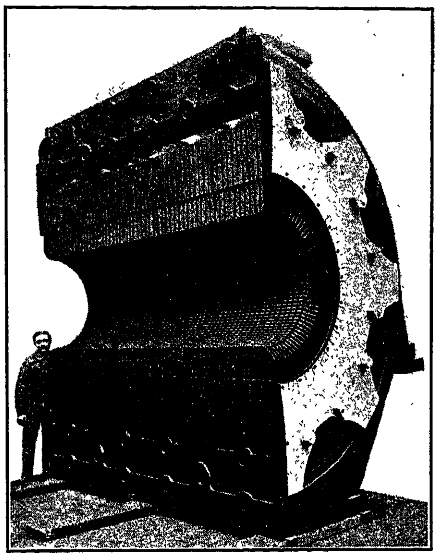


Fig. 4

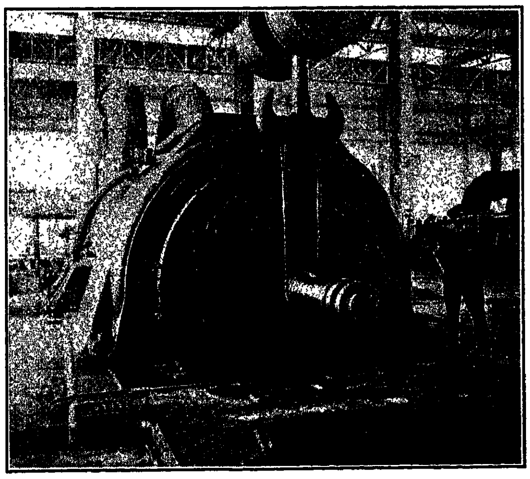


Fig. 5

The problem of the suppression of eddy currents in the bars has found a perfect solution in the use of the special bars of which a detailed description was given in my paper.

The swelling and wrinkling of the insulating tubes has also been for us an object of research, and I agree entirely with all that the authors say on this subject. I might add that it is obvious that the manufacture of the tubes is the first condition of success and that the quality of the raw material, which is the mica folium, is also very important, not only as to the kind of bond used, but also in respect to the constitution of the material itself. The mica folium must be thin and of a very regular thickness; it must be flexible, very homogeneous, and the quantity of bonds used must be *very small.* These properties can be given to the material only by a strictly mechanical manufacturing process, excluding any manual intervention; it is only in this way that perfect homogeneity can be assured. A material of this kind is produced quite automatically by a very ingenious machine, built by a manufacturer of insulating materials in France; this machine needs no other manual operation than that which is absolutely necessary for its general handling.

The problem of the elimination of corona was solved by us many years ago, in exactly the same manner as that given by the authors, namely by the application of a semi-conductive coating on the insulating tubes.

I should like to ask the authors whether they have ever found trouble in using transformer plates for building up the stator core, as this steel is very brittle, and whether an increase in radius of the fillets has given a complete remedy to these troubles.

I will not dilate upon the mechanical construction of the rotors. Up to the present time, we have always used, for four-pole rotors, the construction which is described in my paper; on a shaft, grooved to permit the entrance of the cooling air, are placed forged disks, with clearances between them, forming vents. This construction will still be used for a 75,000-kv-a. alternator, of 65 per cent power factor, 1500 rev. per min., which is now being designed. I may add that this alternator will be built very liberally, and would be capable, with normal heating, of an output of 90,000 kv-a.

The rotors for 3000 rev. per min., are made of solid steel forgings. The study which the authors make of the different stresses that may occur in rotors is very interesting. I understand very well that the problems which present themselves to American engineers on this subject are very difficult on account of the great length of the rotor; this is not only due to the large outputs of the machines, but also to the fact that the speed is 1800 rev. per min., and not 1500 as in Europe.

The critical speed of our 3000 rev. per min. machines is always under the normal speed. Up to the present this was not the case for the 1500 rev. per min. alternators, for which it is above the normal speed. For the alternator mentioned above, it will, however, be above the normal speed.

A. B. Field: One is perhaps disappointed that among references to so many investigations one finds no signs that the possibilities of an oil-immersed stator have received consideration or been investigated. We hear of the increased size having necessitated higher voltages, and of the difficulties thereby encountered; but, even so, do we reach a voltage which is really reasonable for the output? Emphatically not, at 22,000 volts. We hear of the huge volumes of cooling air to be handled; of the windage loss which seems to be leading to hydrogen cooling with all its complications, granted that the danger scare is largely a bogey, and so on.

Oil immersion is no recent suggestion. No doubt most designers and users have in a casual way considered its possibilities during the past 25 years. Its possibility first really appealed when bakelite-paper products had proved their worth, and the production of a thick insulating tube of mechanical construction suitable as a separator between rotor and stator came within the realm of practical politics, somewhat less than 20 years ago. In reviving the suggestion then, one realized that, even if it were practicable, the construction could not be justified by the requirements at that date. The position has been different for a good half-dozen years now, and the matter calls for a sufficient expenditure to determine definitely whether the difficulties and disadvantages justify a continuance of the cold shoulder. With a bakelite-paper tube between stator and rotor the rotor can be water, oil, or possibly steam cooled, and run in a partial vacuum, eliminating windage loss. The stator can be oil-cooled by a confined forced circulation, not the kind of casual circulation which a transformer puts up with. For a given temperature rise of the cooling medium the flow of oil across unit area of cross-section of the duct at say 300 ft. per min. deals with about 130 times as many watts as the flow of air at 4000 ft. per min. Thus, end-to-end axial ducting becomes practicable, or end-to-center; if desired, the cooling medium can be given direct access to the bare copper, or to metal walls almost in contact with such. The voltage limitations are pushed away to a figure with which we need not quarrel at the moment. The possibilities and difficulties of oil immersion cannot be determined without expenditure on an adequate scale; if the matter has not been investigated within the last 5 years or so, a serious study is due.

W. J. Merten: I want to point out in more detail two outstanding metallurgical developments included here and of great importance to the design engineer.

One is the full exploitation of the potential physical characteristics of plain carbon steel in the disk forgings described, by a selective heat treatment for uniform grain refinement, allowing thereby very much higher working stresses without a reduction of the factory of safety.

The second is the development of a new type of steel for retainer rings and bolts with properties not as yet equaled in any of the alloy steels which have been tried for this service. The nickel-molybdenum steel and its composition has been mentioned in this paper.

1. The problem of producing a high-strength disk was worked out from a standpoint of securing complete and uniform distribution of structural components, diffusion of metalloids, and complete destruction of segregation by the employment of high temperatures after the forging operations were completed, and before reheating for grain refinement and quenching which was then followed by strain-relieving thermal treatments. The successful and uniform structural refinement of large massive sections of steel forgings by high-temperature treatment has been described in a paper[1] before the A. S. S. T. September, 1927, at their Detroit Convention. Since then, a more complete refinement by quenching and tempering has been introduced, with corresponding increase in physical properties. This step to make use of the steel to the safe limit of its capacity is not surprising at all and a natural consequence of more complete knowledge of the potential properties of steel and its response to thermal treatments. It will be noted that advantage has also been taken of all the mass reduction that the design engineer would permit by hollow forging so as to facilitate the greatest possible rate of heat abstraction during quenching.

The microphotograph of the etched steel shows a sorbitic structure in which a combination of high elastic properties with a satisfactory degree of ductility can generally be assured.

In concluding this part of the discussion, I should like to state that I believe the exploitation of the inherent properties of carbon steels by thermal treatment for this class of service has been carried to the limit of safety and any further increase in stresses in rotor service must be met by judicious selection of alloy steels or adjustment of crystal structure by mechanical working.

2. The second metallurgical development—that of a new type of steel for retainer rings and bolts, is of even greater importance, since it indicates the direction towards successful alloy-steel application in parts of fairly large sections and actually obtains a corresponding return in physical values for the additional cost of the alloys.

The working stresses in rings and bolts are of a magnitude which exclude the low-percentage alloyed steels from receiving consideration. The 1 per cent chrome 3 per cent nickel-steel has been favored for this service. Every now and then, however, the inherent difficulties from temper brittleness as well as its sluggishness and resistance to complete structural refinement during forging and heat treatment and its tendency to develop hair cracks during heat treatment and probably during rough machining makes it a rather questionable material for a basis of composition for still greater performance. Experiments with modified percentage compositions of chrome and nickel confirmed this conclusion.

---

[1] "High-Temperature Treatment of Castings and Forgings as Evidenced by Core Drill Tests from Heavy Sections," by W. J. Merten, *A. S. S. T. Transactions*, January, 1928.

The molybdenum-nickel alloy steel for this application resulted from cooperative studies and experiments with the staff of the Molybdenum Corporation of America and the Midvale Steel Company and has consistently given on retainer rings an elastic limit of 95,000 lb. minimum with a yield point of 115,000, ultimate tensile of 130,000 and reduction of area of 55 per cent.

A 6-in. diameter, 16-ft.-long forged bolt showed 90,000 lb. elastic limit and equal values of ductility. The impact izod values of this steel are over twice those of the chrome-nickel type. While this characteristic may not add to the usefulness of this alloy in rotor service, I have always associated good or high impact values with a decidedly amplified crystal-recovery tendency when such steel is subjected to alternating stresses of great magnitude, suggesting good cold-working properties and possibilities to improve their physical characteristics thereby.

R. B. Williamson: The problem of ventilation in large turbo generators has been so well worked out that, for the present at least, the size of these machines is not limited by this feature. Also by the use of transposed stator conductors the difficulties due to eddy currents have been overcome. So far, therefore, as stators are concerned, the limit as to size and output may be very large because stators can, if necessary, be shipped in parts and built up and wound in the station.

The real limit as to the output of these large turbo generators lies in the rotor which must be shipped complete, as it is hardly practicable to ship one of these large rotors in parts and wind and balance it at the station.

Another limit lies in the materials available for the construction of the rotor. The paper illustrates a very interesting method of sectionalized construction whereby the use of very large forgings is avoided. However, it will be noted that even with this construction, quite large and massive forgings are required for the end shafts. These must have material in them of high grade as they are of the same diameter as the central disks and subjected to similar stresses.

Steel makers have made great progress in the last few years in the production of reliable forgings of large weight and dimensions. What the designer has most to fear is the presence of hair-line cracks in such forgings. A heavy forging may show excellent physical characteristics, yet if any cracks are present these may spread and cause failure. The danger of cracks is much reduced if the forging is made from an ingot that has not been allowed to cool. In other words, the forging should be made by the steel maker, and from the time the steel is poured until the forging is finished, the process should be continuous so that the steel is not subjected to the strains of cooling and reheating. All rotor forgings should be bored for inspection.

However, the main body of the rotor is only one item; suitable material for the coil-retaining rings is just as important. Failures have occurred in these rings; at least two cases having come under my observation. These were both due to cracks starting at one edge and working their way across the ring until it finally failed. Coil-retaining rings for turbo generators are generally made of alloy steel in order to secure the necessary tensile strength. Such alloy steels, if tempered too hard, are liable to develop cracks. While a relatively high tensile strength may look well, because it shows a high factor of safety as compared with the working stress, it is in fact more dangerous than a lower tensile strength combined with more ductility. Some years ago, the writer specified 100,000 lb. per sq. in. as the yield point for this material with an elongation of 16 per cent. This was later changed to a maximum of 85,000 lb. yield point with 22 per cent elongation. Also special precautions were taken in the machining of the end rings to secure smooth surfaces, free from tool marks and scratches that might start cracks, by carefully finishing and polishing all corners and fillets.

Robert Pohl: It may be of interest to American designers to compare the views and experience of a promiment European company, the Allgemeine Elektricitäts Gesellschaft. The points raised are, however, so numerous that I can only briefly touch upon some of them. The prevalence of 50-cycle systems and other causes have had the effect to direct the development in Europe chiefly upon two-pole (3000 rev. per min.) machines. These are constructed for far bigger outputs than the two-pole machines in the United States. The largest 3000 rev. per min. generator in operation is a 40,000 kv-a. A. E. G. unit. Even bigger outputs are at present contemplated. Four-polar designs are correspondingly less frequent. While they have been worked out up to 100,000 kv-a. at 1500 rev. per min. there has as yet been no chance here of building four-pole machines above 50,000 kv-a. Six-pole turbo alternators at 1000 rev. per min. have altogether ceased to play any part. The largest units of this type, unique at the time of their construction, were the 62,500 kv-a. A. E. G. machines of the Goldenberg Station, designed in 1916. Their system of ventilation is multiple radial, similar to Fig. 1 of the paper, with this difference, that the radial ducts are alternately inlet and outlet passages so that even more complete uniformity of ventilation over the whole active length was obtained. We find it advantageous, however, to let the air from the back pass from the inlet to the outlet ducts through axial holes in the core immediately above the winding and to cool the rotor exclusively with air passed through it and over its surface from either end of the rotor. Much depends, in this connection, on whether the stator or the rotor is the limiting member from the temperature point of view. The scheme of rotor ventilation of the A. E. G. rotor allows a greater quantity of air to pass through its vent ducts, and that is a particularly effective way. The typical A. E. G. rotor design—laminated teeth inserted in a solid core, with form-wound coils, baked, insulated, and assembled before assembly of the teeth—I suppose to be known. It may not be appreciated, however, that, like the Westinghouse plate rotor, it greatly simplifies the forging question, since the forged core has only about 70 per cent of the diameter and half the weight of a solid rotor forging. It also permits of very effective coil ventilation since the air can be made to pass axially along the copper through passages between teeth and coils. We found this direct cooling of the copper about three times as effective as indirect cooling.

The Westinghouse plate rotor design evoked great interest when it was discussed in London some twelve years ago. The investigation and development of its mechanics as disclosed in the paper, form perhaps its most interesting part. Perhaps the authors could add some further information as to the tests to make sure that the bolts are free from flaws and residual strain. Can they be tested by Röentgen rays?

While large two-pole forgings require special precaution, we find less difficulty over here in getting reliable forgings for four-pole machines of drum design with separate shaft ends, where the drum can be forged on a mandrel. We employ high-grade alloy steel, twice heated, quenched in oil at different temperatures and annealed. We test the finished forging for flaws by heating as well as by electro-magnetic methods. Test specimens are taken axially, tangentially, and radially. Apart from yield point, elongation and contraction of area, importance is attached to the span from yield point to ultimate strength and to the micro-photograph, but particularly to the impact test. For the latter, we employ not the Izod but the Mesnager specimen. Unfortunately, different impact tests are not comparable and it is to be hoped that some international agreement on the details of impact tests will be reached before long.

With regard to the mechanical factor of safety, the formula

$\frac{1}{n} = \frac{1}{n_e} + \frac{1}{n_v}$ appears to me somewhat arbitrary, especially considering that the alternating bending stress in the rotor is independent of the speed while the centrifugal stresses increase with the square of the speed. A more practical conception of the safety of a rotor would, I think, be conveyed by stating the overspeed at which ultimately failure will occur. The endurance limit, by the way, is not defined in the paper. Some define it as the amplitude of reversing stress which the material will stand indefinitely, others for 10,000,000 reversals. Again the method of test may be of importance. We employ high-frequency load variations, 500 periods per sec., so that an endurance test up to 10,000,000 reversals can be carried out in less than 3 hrs.

The bending stress does not appear to us, however, to play an important part. In our largest rotors, it only amounts to 3.5 per cent of the tangential stress at the 30 per cent overspeed. Of course, it will become much bigger in four-pole rotors when the critical speed of the shaft is dropped below the running speed.

Coming to the question of the critical speed, our experience does not confirm the observation that it actually occurs below the running speed even where it was meant to be above. On the contrary, we always find it in close agreement with the calculated values for the shafts alone. I suppose the difference is explained by our different practise in building machine foundations. While in the United States steel structures are usual, we invariably employ heavy concrete foundations together with rather heavy bedplates. So we feel justified in adhering to the old conception that the running speed is definitely either below or above the running speed. I may mention that our 100,000-kv-a., 1500 rev. per min. design still belongs to the former class, the large two-pole rotors to the latter.

Another difference, not of experience, but of opinion, concerns the generator voltage to be chosen. While the authors favor raising it as high as 22,000 volts for large sizes, I prefer to keep to 6600 volts as far as possible and to form a working unit of generator and step-up transformer, the latter placed near the generator and joined to it through copper bars without any intermediate switch gear. We thus gain an extraordinarily high degree of electrical safety together with a high space factor for the slot area, the necessary reactance without leakage slots or the like, and practical freedom from transients. Corona precautions can be dispensed with. By raising the potential of the neutral point, we obtain a simple and comprehensive protective system. This generator-transformer layout at 6600 volts was adapted for the 62,500-kv-a. Goldenberg machines and recently for the 90,000-kv-a. machines of the Klingenberg Station. It can be used for the largest units now contemplated without raising the voltage above 13,000 volts. It is very likely the cheapest, as well as the safest system wherever step-up transformers have to be employed in any case.

J. A. Kuyser: (communicated after adjournment) Concurrent with the development of large 1500 and 1800 rev. per min. sets in the U. S. A. has been the development of 3000 rev. per min. generators in Europe, culminating in alternators of 50,000 kv-a. 3000 rev. per min. which are now put forward commercially.

Several of the problems touched on by the authors have been encountered by other designers and different solutions have been proposed.

The problem of the relative movement of the coil-retaining rings and the rotor body arose on some 18,000-kv-a. 3000 rev. per min. Metropolitan Vickers alternators built about 8 years ago which were at that time the largest alternators at that speed in existence. In its most severe form the movement results in pieces breaking off the tip of the steel retaining ring and in breakage and open circuits in the top turns of the rotor winding. In a less severe form it is observed on smaller sets and causes powdering and disintegration of the mica insulation, and severe rust formation on the ring and rotor body due to the hammering at the spigot end.

The method adopted by the M. V. Company to overcome this trouble is illustrated in the accompanying Fig. 6 which is almost self-explanatory. A sleeve concentric with the shaft is provided either in one piece with the rotor body or bolted or shrunk into position. The rotor shaft ends are free to move inside this sleeve without communicating any movement to the rotor end plates and the coil-retaining rings. In practise all relative movement at the spigot is completely eliminated without the necessity of having heavy pressfits at this point.

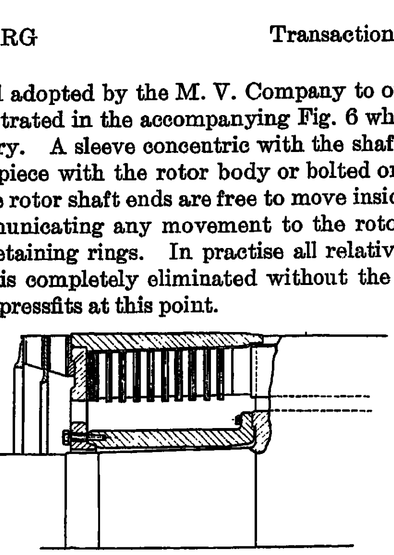


FIG. 6

With reference to the transposed slot conductor shown in Fig. 4 of the paper and the combination with transpositions at the coil ends, we have used this winding with highly satisfactory results for the past 12 years.

With regard to the question of stator insulation considerable speculation has arisen on the swelling of the stator slot insulation into the radial air ducts. A very obvious method of preventing this difficulty is illustrated in Fig. 7 herewith. In this construction the wall of the slot is continued across the air duct by metal spacers and a continuous slot is obtained presenting a smooth surface to the coil.

The authors emphasize the difficulties of obtaining large forgings having adequate elongation. In this connection it seems that the view taken is rather pessimistic. We have obtained many forgings up to 60 tons in weight having guaranteed properties as follows:

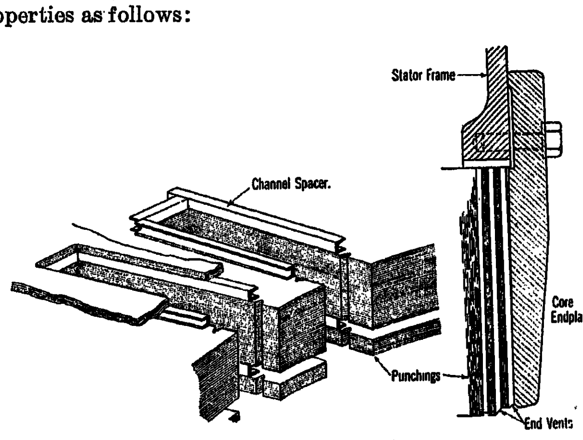


FIG. 7—STATOR VENT SPACERS

Ultimate tensile strength 80,000 lb. per sq. in.

Yield point 46,000 lb.; elongation, 20 per cent; radial, 24 per cent, tangential 24 per cent longitudinal.

The properties on a test piece taken from a core trepanned out of the center of these forgings are on an average: ultimate tensile strength, 75,000 to 80,000; yield point, 40,000 to 50,000; elongation, 17-20 per cent, although the makers do not give guarantees on the latter figures. These properties are very satisfactory for annealed forgings of such large dimensions.

The authors make some interesting remarks on the question of stability. Hitherto in discussions concerning stability the system characteristics have generally been neglected; as pointed out the percentage lighting load is of importance, but of great importance also is the amount of synchronous load on the system, since this load will contribute leading kv-a. in case of a voltage drop and tend to counteract the drop. Another point of importance is the magnitude of the load fluctuations and the rapidity of the load increase. We are now using in some cases a small permanent-magnet generator driven at about 800 rev. per min. from the governor shaft to supply current to the field system of the main exciter. By means of this system and a voltage regulator considerably greater stability can be obtained than that corresponding to constant excitation.

In case of a unit being suddenly tripped by the Merz Price gear it is doubtful whether a short-circuit ratio of 0.9 will be sufficient to ensure that the load will be picked up by the remaining sets if the faulty unit is large compared to the remaining units.

The difference with European practise as regards short-circuit ratio is not as great as the authors state. Large 3000 rev. per min. generators are generally designed for a ratio of 0.65; the lowest ratio used is 0.6.

G. A. Juhlin: (communicated after adjournment) The lines of development given by the authors are in general the same as those which European engineers have pursued. The conclusions to be drawn, it seems to me, must also be those we have arrived at, namely, that the limiting factors in design of large units are no longer electrical but mechanical ones.

The introduction of the multi-inlet ventilating system would enable the designer to increase the length of his machines almost indefinitely, were it not for the fact that other limitations, such as deflection of the rotor, mechanical stress in the shaft, etc., prevent him from increasing the length beyond certain limits.

In connection with the ventilation scheme shown by the authors, it is of interest to note that the whole air volume required is provided by fans mounted on the rotor.

In Great Britain separately driven fans only are used by some manufacturers, or a combination of, small fans mounted on the rotor which deal with a portion of the total air volume required, the amount depending on the length of the machines and a separately driven fan supplying the rest of the air.

Higher efficiency can be obtained from separately driven fans than from fans mounted direct, especially when dealing with large 3000-rev. per min. units.

For such units it would be necessary to adopt large-diameter rotors in order to provide sufficient inlet area for the fans. Such designs are undesirable from the point of view of mechanical stresses which become very high.

The windage losses increase very rapidly with the diameter with consequent reduction in efficiency of the unit. The efforts of British engineers have therefore been directed towards keeping the rotor diameters as small as possible, hence the use of the separately driven fan. In some recent machines complete turbo blowers have been mounted and driven direct from the alternator shaft.

Trouble with swelling of the stator-bar insulation was experienced as far back as 1916, and was diagnosed as being due to the solvent in the bond used in building the mica folium. This difficulty was reduced to a negligible amount by steam-curing the mica wrap after it had been applied by the wrapping machines.

Another problem which arose due to the necessity of subdividing the stator conductor into a large number of small strands was the difficulty of preventing looseness between the individual strands after prolonged operation.

Mica tape was used as insulation and after some years operation it was found that the individual strands were loose due to the bond having lost some of its adhesive properties. This difficulty was overcome by using very thin asbestos tape and treating the whole bar with a synthetic gum.

By this means a perfectly solid bar was obtained—uninfluenced by temperature as the gum used had to be treated at a much higher temperature than the copper would reach in operation, thus ensuring that the bar remained solid under operating conditions. Considerable credit is due to the manufacturer of this asbestos tape for producing a tape thin enough to enable us to employ it without losing too much space in the slot.

Non-magnetic finger plates and end plates have been used by British manufacturers for some years with good results. Better results still have been obtained by the use of copper end-plates having slots at the inner periphery corresponding to the stator slots.

The details of construction of the built-up plate rotor which the authors have given are extremely interesting. To have successfully overcome the difficulties mentioned in the paper is no mean achievement and reflects great credit on the engineers who have carried out the work. The construction is daring in conception and has proved itself sound in operation over a number of years. We in Europe are fortunate in being able to obtain large solid forgings with much better physical properties than those described in the paper.

I agree with the authors in their statements regarding the fundamental basis for the working stresses. It would seem, however, that the statement should be qualified to take care of special materials such as heat-treated nickel-chrome steels where the yield point and ultimate tensile strength are very close to each other. For such materials we have adopted a factor of safety on the mean of the yield point and the ultimate strength of the materials equal to the mean factor of safety of ordinary steels. With reference to tests and inspection of solid rotor forgings it has for many years been standard practise of the company with which I am associated, to insist on longitudinal, radial, and tangential tests on all our solid forgings.

All rotors above 25 in. in diameter have a hole bored through the center and a very stringent examination of the bore by means of a telescope and mirrors is made. If there is any doubt as to the soundness a special magnetic test is applied. In addition to this, sulphur prints are taken on the external surfaces for detection of segregations.

The authors' discussion on "Critical Speeds" is a very valuable addition to our knowledge on this important subject.

The influence of the flexibility of bearing supports and foundations is of special interest. Personally I have not come across any case where the critical speed of the set was influenced by the foundations, but this is undoubtedly due to the fact that all our machines are mounted on reinforced concrete foundations which, of course, are exceedingly stiff. Bearings are mounted in pedestals having low center and are therefore also very stiff.

I agree with the authors as to the necessity of the rotor winding being seasoned until stability is obtained and in this connection I would suggest that the baking and pressing of the coils before wedging is of great importance.

I also agree with the authors as to the necessity of balancing after a set has been put into operation no matter how carefully it has been balanced in the works. Minute changes in the rotor winding as well as in the alinement are bound to occur and must be corrected by re-balance in the field.

P. L. Alger: (communicated after adjournment) The authors have presented a most interesting and comprehensive survey of turbine-generator design, which strikingly exhibits the complexity of the modern designer's task. Their remarks in regard to armature coil insulation seem to me particularly significant of the progress being made. They state that the cause of insulation swelling and increase of dielectric loss with temperature is the boiling off of the alcohol in the shellac bond of the mica, and illustrate the point by Fig. 6, which shows a more than five-fold increase of insulation temperature rise when the ambient temperature is raised from 60 to 100 deg. cent. This idea was suggested in a discussion[2] of Professor Whitehead's article on Gaseous Ionization before the Institute in 1924.

All experience indicates that the black-asphalt-base insulating varnishes have superior characteristics to the yellow materials at elevated temperatures. The former retain their flexibility after prolonged heating, while shellac becomes brittle under like conditions. If the black bond described by the authors does not give off vapor when heated, and so does not swell, it will undoubtedly retain its flexibility also, and so will be suitable for use in external mica insulation on the end windings, as well as in the slots.

---

[2] A. I. E. E. TRANSACTIONS, 1924, p. 124.

Gaseous Ionization before the Institute in 1924.

Practise in regard to insulation of turbine-generator end windings has always varied greatly, some companies abroad using hardly any end insulation at all; others, like the authors, using different kinds of insulation on the ends and on the slots; and still others using the same insulation around the entire coil. Which practise is best may always remain a moot question, but I believe that such papers as the present are of great value in bringing our ideas into better accord.

S. L. Henderson: I think Mr. Foster may have left an impression that I am sure he did not mean to leave, and that was that we really did not have any satisfactory means for preventing corona. He was simply talking about the permanency of this film. We have tested the film for many hours' operation at 30,000 volts to ground, in open air, and there has been no deterioration of the film, so that at least for that type of protection we feel that it has a permanent value.

Mr. Roth asked one question as to the brittleness of the transformer grade of iron which is used in the stator punching. We have been using silicon steel in turbo-generators for over five years, and there has been no indication of any teeth failing through the vibration. We feel that, of course, with silicon steel the question of stress concentration is of more importance than it is in the old rolled annealed steel, but we have increased radii at the bottom corners of the slot to such an extent that we feel certain there is no danger from the amount of vibration that can occur in a stator.

We certainly do owe a note of apology to Mr. Field for not having mentioned the question of oil insulation on the stator. However, it is a problem that we have not actively attacked; consequently, we could not add anything to this information that was already out of the problem. It certainly is not one that is going to be passed over. It is of interest in the high-voltage machines, and it has occurred to us many times in the last two years during the period that we were considering high voltages. After all, oil insulation would be a solution of many difficulties of high-voltage stators.

There are many mechanical difficulties in applying oil to the stator. It is necessary to devise some means of keeping the oil from the rotor; otherwise, the friction would be enormous. Consequently, we must find some barrier that will contain the oil around the stator that will be tight and mechanically strong, and that is rather a big problem.

C. R. Soderberg: Referring to Mr. Behrend's discussion, I should like to point out that his descriptions of rotor explosions show very clearly the need of scientific analysis of the stresses in the rotor structure. The fact that a manufacturer of machines of this kind has been fortunate enough to avoid failures for a considerable period of time certainly is very comfortable, but I wish to point out that it is dangerous to assume that this is a conclusive evidence of infallible design and construction methods. The problem of failures always involves an element of chance, and the designer who has been consistently fortunate may be next in turn.

The change in our rotor-plate material has proceeded through two steps. The start of the development was made on account of certain more or less unsatisfactory properties of the stock material to which Mr. Behrend referred. It was not as uniform as might be desired, and it was impossible to raise the strength above a certain limit. The first step consisted in forging the plates from a cylindrical ingot and rolling it in a tire mill. By this method we obtained a plate without a hole in the center with considerable improvement in elastic properties, as compared to the old material. If I remember correctly, the maximum yield point, which we could obtain with satisfactory ductility, was between 40,000 and 50,000 lb. per sq. in., as compared to about 35,000 lb. per sq. in. in the old material. The material described in our paper was then developed, and here it was possible to obtain outstanding strength and ductility.

As pointed out by Mr. Merten, the only method of utilizing carbon steels is by improvement in the heat treatment. Fully satisfactory heat treatment is possible only with a shape of the disks which admits of a uniform temperature diffusion. Thus, the hole in the center is necessary in order to raise the elastic properties and the ductility of the material. Of course, it is quite true that the stress in the center is increased by cutting the hole, but the existence of a certain stress is not in itself objectionable if its magnitude is known with certainty, and if it can be controlled to correspond to the properties of the material. The advantage of the solid disk, having a stress at the center, which is theoretically one-half of that in a pierced disk, appears to me somewhat doubtful. If the uncertainty of the material at the center of a solid disk, and the risk of microscopic cracks, are taken into account, it would seem that the disk with a central hole is preferable.

Referring to the discussion by Mr. Roth, I should like to mention that non-magnetic materials for retaining rings have been under consideration. Such materials are in use and appear satisfactory from the designers' point of view except for the fact that they are very difficult to machine.

Referring to the discussion by Mr. Field I wish to repeat the statement made by Mr. Henderson, that the authors very deeply regret the omission of the oil-cooled stator. The possibilities of oil cooling are obvious, but the difficulties of the mechanical design certainly appear formidable in comparison with those encountered with hydrogen cooling. I wish to point out, however, that the opinion of a man of Mr. Field's experience should receive due consideration, and it may be that when the problem is studied somewhat more closely, these difficulties may be found less discouraging.

Mr. Merten's discussion contained a reference to microphotographs. The microphotographs in our paper, more than anything else show the great improvement in the grain structure of the rotor material that we are now using, as compared with the old materials.

I think that Mr. Williamson's point on the question of unexpected cracks or weaknesses is very well taken. If we had complete knowledge of the failures that have occurred, I believe we would find that their occurrence very rarely can be attributed to too high values of the calculated stresses, but nearly always to unexpected faults in the materials.

Mr. Williamson's remarks on retaining rings is also very important. There is a very natural tendency on the part of the designers to raise the strength (the yield point) of the material at a sacrifice of ductility. While the theoretical value of the factor of safety may be increased, we may actually obtain a structure which is less safe than one having a material of a lower yield point. At the same time I wish to point out that in the retaining-ring material in use at the present time by the Westinghouse Company, and by other manufacturers, the ductility is actually superior to that of previous materials having a lower yield point. Thus, we have improved both the strength and the ductility and this is, of course, a logical procedure.

Referring, finally, to the discussion of Dr. Pohl, I wish to add a few remarks on the formula for actual factor of safety in cases of variable stresses, which was given in our paper. This formula is arbitrary only in the sense that the somewhat erratic experimental results available on this subject have been interpreted in a manner giving, in some cases, a slight amount of added safety. I refer to the replacement of the Gerber parabola by a straight line, joining the endurance limit and the yield point. An explanatory footnote on this formula will be given in further publication of the paper. The endurance limit is considered as the limiting amplitude of reversing stress, which can be applied in straight tension and compression an infinite number of times without failure. The available investigations on this subject seem to indicate that, for steels, there is no difference in the endurance limit when the number of reversals considered is greater than 10×10^6.

Design and Application of Two-Pole
Synchronous Motors

BY D. W. McLENEGAN*
Associate, A. I. E. E.
and
IVAN H. SUMMERS†
Associate, A. I. E. E.

Synopsis.—The paper presents a brief résumé of the problems encountered in the design of two-pole high-speed synchronous motors together with a description of how they are met. A satisfactory design is worked out. The theory of the starting winding is given briefly in non-mathematical language, and curves are presented showing the results obtained in so far as current and torque are concerned. It also outlines the applications in which these motors have been used, pointing out both the advantages and the limitations of synchronous motors. This is followed by a description of the operating characteristics and of the method of controlling the motors.

INTRODUCTION

WHEN there came a demand for large 3600-rev. per min. synchronous motors, it was apparent that it emanated chiefly from manufacturers of centrifugal compressors. In this application high speed is essential in keeping down the number of stages and the weight in the compressor. Steam turbine drives are used to some extent, and on the smaller sizes, induction motors have been used. The steam turbines are very satisfactory where a suitable supply of cheap steam is available, but frequently the source of steam is too remote. In most cases, 60-cycle electric power is readily available.

It was natural, in designing the high-speed synchronous motors, to turn to the already existing two-pole turbine alternators, which have specially designed features to make them suitable for high speed operation. The essential mechanical features of this type of generator are determined by the high centrifugal stresses involved, which make a small diameter imperative for the rotating elements. In order to get the required volume of iron into the rotating element, it is thus necessary to use a relatively long machine. This makes a long shaft necessary, and, when the rotor is of laminated construction, the shaft cannot be of large diameter on account of the stress at the bore in the punchings. With this in mind, the designer must obtain a proper balance between the unit stresses in the various rotating parts, at the same time maintaining sufficient stiffness in the shaft to insure smooth operation. The electrical requirements are to maintain a proper balance between the various flux densities and current densities, so that heating within the guarantees will be assured, and to obtain as high an efficiency as is compatible with reasonable cost. The various compromises thus utilized are to a considerable extent matters of experience.

It can readily be seen that a large part of the problem had already been solved by using the existing high-speed turbine alternators as a starting point. The available turbine alternators were of the type which had laminations for the rotating elements pressed on a steel shaft. These laminations had radial slots in which the polar field windings were wound, and these in turn were held in place in the core portion by metal retaining wedges placed in suitable wedge slots just above the polar windings. The polar windings were held in place on the end portions by steel retaining rings. By employing these special structural features, coupled with appropriate starting windings and changes in the electrical design, synchronous motors can be made having starting characteristics comparable with those of induction motors.

Although the two-pole synchronous motors are usually built for operation at unity power factor, a few have been built for operation at leading power factor. The design of motors for 0.9 or 0.8 power factor rating does not involve difficulties because the construction of the amortisseur winding leaves sufficient space in the rotor slots for a field winding of adequate capacity.

The problem of designing suitable starting windings for two-pole synchronous motors is much more difficult than in the case of squirrel-cage induction motors, principally because the short-circuiting rings must not in any way interfere with the polar field windings. Various schemes were proposed; one consisted of making two crown rings of bronze or copper slotted from continuous cylinders, then passing these over the polar windings and uniting the ends by a special brazing process or by electric welding. Another scheme for the starting winding was to insert bars of bronze or hard copper as slot wedges above the polar windings and properly connect these to low-resistance end rings consisting of annular plates just outside the end portions of the polar winding. The first scheme was impractical, due to the great difficulty of making a successful joint in the middle of the field core. It made the repair of the polar windings impossible, except by destroying the starting winding, and, finally, it occupied valuable space needed for the polar windings. The second scheme was actually tried out successfully. But in this case, also, the starting windings occupied some of the space required for the polar windings, and, while the joints with the short-circuiting rings were simpler,

---

*Industrial Engg. Dept., General Electric Co., Schenectady, N. Y.

†Turbine Generator Eng., General Electric Co., West Lynn, Mass.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

they were difficult and expensive to make. The plan finally adopted was to utilize the space at the thick ends of the teeth between the rotor slots. In carrying out this scheme the teeth are sufficiently cut away, as shown in Fig. 1, to provide room for two thin but deep conductors at the sides of the slots. Each of these flat conductors is folded over and joined outside the core ends to a flat copper or bronze disk which constitutes a very satisfactory end ring. Not only are two of these flat conductors employed in each winding slot, but extra slots are provided in the polar section, thus making the starting winding continuous about the entire periphery of the rotor; a condition difficult to obtain on a salient pole machine. In assembling this form of starting winding, the ends of the flat bars are temporarily held in the end plate by screws, and then the joints are perfected by a very carefully made junction of silver solder which has a melting point of approximately 660 deg. C. This arrangement of bars and end rings, in conjunction with the specially arranged wedges, provides an excellent starting winding. Since the bars are inserted in the field core before the insulated polar winding is assembled, the latter can be repaired at any time without disturbing the former. Proceeding along these lines, it was found possible to get enough copper into the starting winding to give a d-c. resistance of approximately one-half of 1 per cent. The starting winding is shown in Fig. 2, which indicates clearly how the winding is connected to the end rings. Fig. 3 shows the field windings and the starting winding assembled in the rotor without the steel retaining rings.

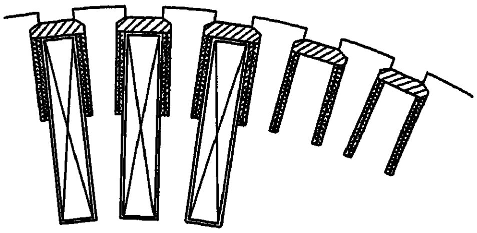


FIG. 1—ARRANGEMENT OF POLAR WINDING, STARTING WINDING AND WEDGES IN ROTOR OF TWO-POLE SYNCHRONOUS MOTOR

The first two two-pole 3600-rev. per min. motors, built with this form of starting winding, were rated 2550 hp., and were installed in a plant in the Andes Mountains at an elevation of 14,000 ft.

THEORY

It is not within the scope of this paper to develop a mathematical theory of the starting characteristics. Such a theory has been worked out and will be briefly described in non-mathematical terms. Curves of torque and current calculated by means of this theory have been found to check approximately with the test curves.

Referring again to Fig. 1, it will be seen that the squirrel-cage bars are relatively deep. With this construction, the effect of the leakage flux at 60 cycles is to cause an uneven distribution of current density, concentrating it at the top of the slot. This causes an increase in the effective resistance of the bar, and a decrease in the reactance. This effect can be calculated by well-known methods applied to rectangular bars in rectangular slots with open tops.[1,2,3] For these motors, it was found that the effective resistance of the bars at 60 cycles was approximately twice their d-c. resistance.

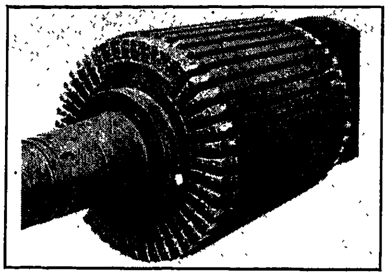


FIG. 2—VIEW OF UNWOUND ROTOR OF TWO-POLE, 3600-REV. PER MIN. SYNCHRONOUS MOTOR, SHOWING CONSTRUCTION OF STARTING WINDING

We come now to consideration of a particularly interesting feature of the starting winding. It was considered necessary to make the retaining wedges above the polar winding of short sections, each section being insulated from the other. This was necessary to prevent currents from flowing axially along the rotor. Such currents might cause dangerous burning unless proper connections were provided at the ends of the rotor to confine them, and it is difficult to do this without interfering with the accessibility of the polar windings. However, even though these wedges consist of short sections insulated one from another, they still possess the essential features of a dead conductor placed above a current carrying conductor in a slot. That is, the leakage flux from the starting winding can induce a voltage in the wedges which will cause a current to flow axially along the top of the wedge and return along the bottom, a complete circuit taking place in each wedge section. Theories of this action have been worked out by various authors.[2] It was found advisable to employ two kinds of wedges, some being of steel and some of an extruded metal. Various effects have been obtained by using alternately steel

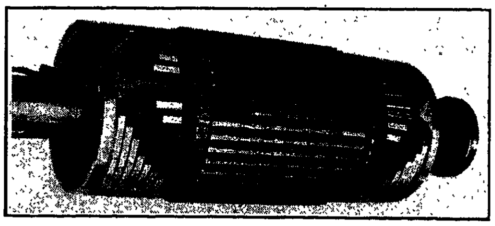


FIG. 3—COMPLETELY WOUND SYNCHRONOUS MOTOR FIELD BEFORE RETAINING RINGS ARE SHRUNK ON

and non-magnetic wedges in different proportions. By thus varying the proportions of the two materials, different effective resistances and reactances can be obtained. The effect of a steel wedge above the starting winding is to increase the reactance by providing a path of lower permeance for the leakage flux; and to increase the resistance by contributing a heat loss caused by the eddy currents induced in the wedge by the leakage flux. The action is like that of a tertiary winding in a transformer, except that saturation is present in the wedge-leakage paths to a large degree and modifies the results considerably.

The method of calculation is to ascribe the additional resistance and reactance to the starting winding, although the flux causing the additional reactance cuts the wedge, and the resistance loss actually takes place in the wedge. By thus fictitiously placing the reactance and resistance in the starting windings, the motor starting characteristics can be calculated in the usual way with little difficulty. The start of the calculation is made by assuming a current in the starting winding. This determines the m. m. f. in the leakage paths and thus determines their degree of saturation. When this is known, it is easy to calculate by known methods[3] the leakage flux and get the effective resistance and reactance of the starting winding caused by the wedge. The effective resistance of the starting winding itself, due to its own non-uniform current distribution, can be determined in the usual way. The effect of the end ring is usually small but it may be included and thus the total effective resistance and reactance of the rotor for that particular frequency and applied voltage may be determined. Using these, the current which would flow in the starting winding is found. If it does not check with the value assumed, it is necessary to make a new assumption and repeat the process until a sufficiently close check is obtained. Points for different voltages and frequencies must be obtained separately because the constants depend on both. Thus the torque is not necessarily proportional to the square of the applied voltage, nor is the current necessarily proportional to the applied voltage.

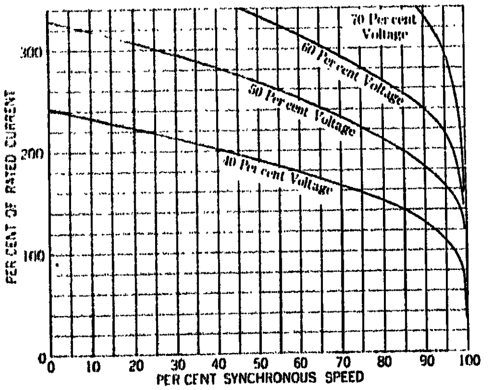


Fig. 4—Armature Current-Speed Characteristics of Two-Pole Synchronous Motor at Various Voltages

Starting Characteristic Curves

Torque and current curves which were taken by test on a 1500-hp. 3600-rev. per min. motor, are found in Figs. 4-5 for different applied voltages. These curves clearly show how the current and torque vary with voltage and speed. It will be noted that the torque is maintained up to a point very close to the synchronous speed. The slope of the torque curves, from this point to synchronism, is very great and a measurement of the slope near synchronism indicates that the d-c. rotor resistance is about one-half of 1 per cent, as calculated. The fact that the torque is maintained to a speed so close to synchronism makes it comparatively easy to synchronize the motor. A consideration of the theory, described above, indicates that these torque and current curves could be changed materially by changing the proportions of steel and non-magnetic material in the slot wedges. Calculations indicate that at standstill the rotor resistance and reactance are each of the order of ten times the value calculated for direct current flowing in the starting winding. This shows how greatly the characteristics depend on the retaining wedges during the first part of the starting cycle.

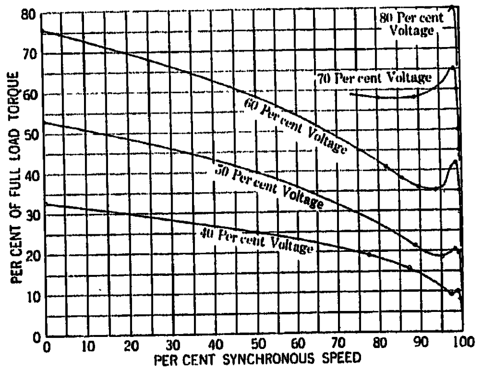


Fig. 5—Torque-Speed Characteristics of Two-Pole Synchronous Motor at Various Voltages

Heating

When a synchronous or induction motor is started by means of a squirrel-cage winding from zero speed to synchronous speed, it may be readily proved that the heat generated in the rotor over the total cycle is practically independent of the construction of the starting winding. This heat energy is equal to the stored energy of the rotating parts at synchronous speed, plus an amount represented by the area of a curve obtained by plotting the products of slip and load torque against time over the whole starting cycle. It was found in some actual cases that the kinetic energy of the rotating mass was equal to about 80 per cent of the total heat loss in the rotor. Thus, it is evident that a high amount of inertia inevitably causes a heating problem which cannot be ignored. Furthermore, it is impossible to reduce the amount of heat to be stored in the rotor by any change in the construction of the starting winding.

One of the ways in which this problem was considered was as follows:

The rate of heat flow into the rotor having been determined by the above means, Fourier's heat equation was integrated, considering the rotor to be a massive cylinder with heat applied to the surface. In such a case, it is theoretically possible to determine the temperature rise at the surface, at the end of any given time. This gives a result which is independent of any method of construction of the starting winding and serves to indicate the limiting condition. Such results, however, must be modified empirically, due to the actual construction being different from the assumption. It has been found possible to get a fairly good idea of the temperatures to be expected by this means, but some judgment is required in applying the results. Machines undergoing test have been put through severe starting cycles, and then opened up and examined carefully. Such examinations have shown that no damage of any kind has resulted to any of the motors as a result of tests considerably more severe than their usual starting requirements. If more than one starting cycle is to be considered, it is evident that careful consideration must be given to see that sufficient time is allowed for re-distribution of the heat generated, in order to avoid undue accumulation of heat. Special methods of ventilation would be of little value in reducing the temperature of the starting winding because the heat is generated at such a rapid rate that a flow of heat into the steel mass of the rotor must be depended on rather than convection to keep the winding cool. As a result of these studies, it has been found quite feasible to synchronize, without exceeding safe temperatures in the rotor, 3600-rev. per min. compressors requiring a starting torque of 30 per cent, and a pull-in torque of 50 per cent, and having a $W R^2$ of 2 lb-ft. squared per hp. of motor rating. While somewhat greater values of $W R^2$ per hp. may be successfully started, the heating difficulties are then more serious.

SYNCHRONOUS CHARACTERISTICS

The synchronous characteristics of the two-pole motors are similar to those of lower speed salient pole machines. Since the amount of excitation required by any synchronous motor is a function of the number of poles, the excitation required by two-pole synchronous motors is of course very small. A 2500-hp. 0.9-power factor 3600-rev. per min. motor, for example, requires only 13.5 kw. Efficiencies of about 94.5 per cent have been obtained for 1250-hp. rating, and about 95.5 per cent for 3000 hp. rating, while slightly higher figures apply to larger sizes. Windage losses, which are included in calculating the efficiency, account for these results not equalling those of lower speed machines.

APPLICATION

Several large two-pole synchronous motors have been built within the past five years, and these are operating successfully. However, the range of application has so far been quite limited. This is due in part to the constant speed limitation of the synchronous motor and possibly to the fact that very meager data have been published on the design, construction, and operating characteristics of motors of this type.

In sizes smaller than 800 to 1000 hp. at 3600 rev. per min., the synchronous motor is hardly competitive with the squirrel-cage or the wound rotor motor unless the need of power factor improvement justifies a higher first cost. With increasing size, however, the comparative cost of the synchronous motor improves, and this brings the synchronous motor into consideration for drives requiring greater power. Wound-rotor induction motors of several hundred hp. at 3600 rev. per min. have been built, and such motors up to 2000 or 3000 hp. have been considered. Their application, however, depends on the solution of the problem of handling large currents at the collector rings.

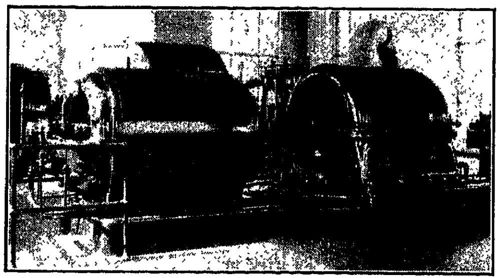


FIG. 6—1750-HP., 3600-REV. PER MIN. SYNCHRONOUS MOTOR DIRECT-CONNECTED TO CENTRIFUGAL COMPRESSOR

For large capacities, therefore, the choice is limited to the synchronous motor, the squirrel-cage induction motor, and the steam turbine. As the power factor of induction motors is always substantially below unity, large induction motors must be supplemented by corrective devices to make them comparable with synchronous motors.

At the present time, as nearly as we can ascertain, the largest two-pole, 60-cycle synchronous motor in operation is rated 2550 hp. This, however, does not represent the limit of sizes that can be built with the present construction. Designs have been made for a 7000-hp. 60-cycle motor and a 4000-hp., 1500-rev. per min., 25-cycle motor.

In most of the processes to which a two-pole motor could be applied, the suitability of a constant speed drive must be determined by the requirements of the individual installation. The two-pole synchronous motors now in service are used to drive centrifugal gas boosters or to drive centrifugal air compressors in connection with sewage disposal or copper refining processes.

The flotation process, for the recovery of copper from low-grade ore, requires a large volume of air at low and constant pressure and furnishes a steady load. For this service the constant speed of the synchronous motor is suitable. The high efficiency and favorable power factor of the synchronous motor may, therefore, be utilized to advantage. Copper converters also require air at approximately constant pressure and volume, except during the intervals required for emptying the converter. A number of two-pole synchronous motors are now in use for "blowing" these converters.

The "activated sludge" system of sewage disposal consists of blowing air through a porous medium into a sewage tank. The required pressure increases as the action proceeds, but the reduction in air flow offsets this pressure increase to such an extent that a constant speed motor may be applied.

Gas boosters, on the other hand, are frequently required to operate over a wide range of pressure and volume. In some cases, where the density of the gas varies, speed variation becomes a necessity in obtaining the required pressure. However, synchronous motor drive has been found satisfactory in several booster installations, compressing gas of fairly constant quality. Large compressors have been used as basic units, the variations in demand being cared for by smaller machines which may be operated at adjustable speeds. Another method involves the use of several two-pole synchronous motors, which are started and stopped in accordance with an output requirement which follows a fairly regular cycle through the day. If the explosion hazard is sufficient to warrent the use of special safeguards, the collector rings or the complete motor may be enclosed.

For high-pressure pumping the tendency has been to use higher speed to obtain the pressure that would otherwise require an excessive number of pump stages. Synchronous motors at 3600 rev. per min. have been considered for driving boiler-feed pumps in high-pressure steam plants, although the general practise calls for a motor which will allow 15 to 25 per cent speed reduction. However, two-pole constant-speed motors have been applied to this service. This would indicate that speed variation is not always essential for boiler-feed pumps. In such cases, a synchronous motor with suitable torque characteristics may be considered. For an essential drive, such as a boiler-feed pump, it is necessary to consider the reliability of the synchronous motor and its operating characteristics under unusual conditions, as well as its cost, efficiency, and power-factor. These points will be discussed later.

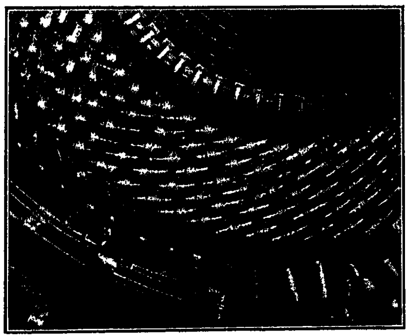


FIG. 7—STATOR OF TWO-POLE SYNCHRONOUS MOTOR, SHOWING METHOD OF BRACING END TURNS OF COILS

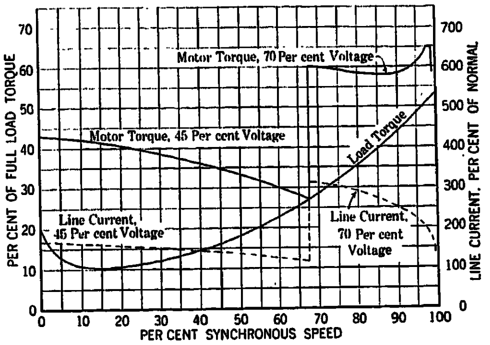


FIG. 8—TORQUE-SPEED AND LINE CURRENT-SPEED CHARACTERISTICS OF TWO-POLE SYNCHRONOUS MOTOR, WITH TWO-STEP COMPENSATOR STARTER; AND TORQUE-SPEED CHARACTERISTIC OF CENTRIFUGAL COMPRESSOR

OPERATING CHARACTERISTICS

Since the two-pole synchronous motors are of large ratings and necessarily have low reactance, it is important to design the control equipment to start and accelerate the motor with the lowest possible inrush. Considering only the motor, full voltage starting might be used, since these motors are capable of withstanding a direct short circuit. The long end turns of the stator coils (Fig. 7) are tightly blocked to prevent movement, and severe factory tests have failed to disclose any injury to the coils after repeated short circuit at full voltage. However, the limitations of the power supply usually require reduced voltage starting, and frequently make it advisable to use two reduced voltage steps, approximately 45 per cent and 70 per cent of rated voltage, before applying full voltage to the stator.

Fig. 8 shows the torque-speed and current-speed curves of a 1500 hp., 3600-rev. per min., 60-cycle motor, based on the use of a two-step reduced voltage starter, together with a typical torque-speed curve for a centrifugal compressor with the discharge gate closed. Although the shape of the torque-speed curve of the motor may be modified to give lower break-away torque and higher torque at intermediate speeds, this change would result in a greater starting current.

The motor torque curve is therefore made to correspond roughly with the load requirements by means of starting connections as shown in Fig. 9. The neutral point of the auto transformer is closed by switch C. After closing the line oil-circuit breaker F and the compensator breaker E, a voltage approximately 40 per cent of normal is applied to the motor through switch *A*, which connects the motor to the taps of the auto transformer. After accelerating to about 70 per cent of synchronous speed, breaker *A* is opened, and breaker *B* is closed, increasing the voltage to approximately 70 per cent of line voltage. If the blower load is not too great, the motor may be synchronized at this voltage by applying excitation to the field. The transfer to full line voltage is made with the Korndorfer connection, by which an interruption of power is avoided. The compensator neutral is opened, and the running breaker is closed before opening the compensator breaker. During the first transfer, in which power is momentarily interrupted, the transient inrush following the transfer is considerably reduced by the reactance of the auto transformer. However, when changing to full voltage, it is desirable to maintain the circuit. A high current might otherwise flow, limited only by the transient reactance of the motor. It is of interest that, when opening the high tap *B*, before closing the line breaker *D*, there is practically no change in the voltage across the motor terminals. The upper section of the auto-transformer, serving as a series reactor, limits the voltage to the same value or even a slightly lower value than that previously obtained with the neutral point of the compensator closed.

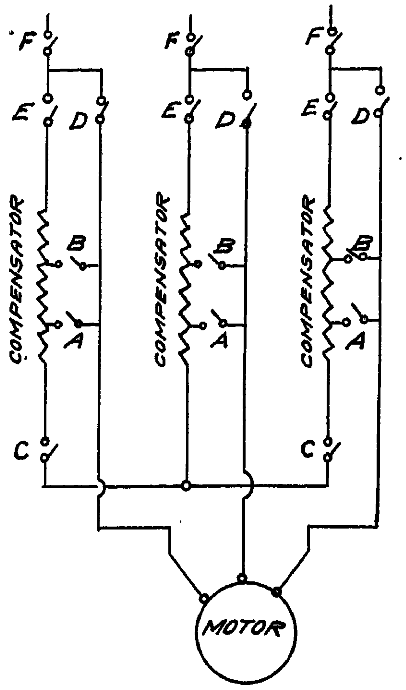


Fig. 9—Starting Connections for Two-Pole Synchronous Motor, using Two Compensator Taps, with Korndorfer Connection for the Transfer to Full Voltage

If the motor is synchronized before transferring to full voltage, it becomes essential to avoid opening the circuit when making this transfer. If power were interrupted, the motor speed would drop so quickly that the motor would probably be unable to re-synchronize.

By using two taps of the compensator, and the Korndorfer connection when changing to full voltage, the current drawn from the line can usually be limited to about 300 per cent of the rated current of the motor. In most installations of large two-pole motors, the limitations of the power supply require approximately this restriction of the motor current when starting.

Where such restrictions are not necessary, and a high starting current is allowable, the use of a series reactor provides a simpler and less expensive control, by reducing the number of oil-circuit breakers and eliminating many of the interlocks and relays required for the two-step auto-transformer starter. In Fig. 10 will be found a study of starting characteristics of the 1500-hp. motor with a two-step reactor starter. These curves are based on the use of 15 per cent external reactance, which is reduced to 8 per cent when the motor reaches 70 per cent speed. The torque-speed curve is generally similar to the curve of Fig. 8, representing starting characteristics with compensator starting. The line current, however, is considerably higher. Except in locations where an exceptionally large power supply is available, this fact would probably rule out the reactor-type starter.

For some installations, the use of a permanent series reactor has been proposed. This arrangement is practical, but will involve a greater current inrush than either of the starting arrangements just described. Referring again to the 1500-hp. motor curves (Fig. 10) the permanent reactance must be less than 10 per cent to insure keeping the motor-torque curve above the load requirement. The starting current would then be approximately 500 per cent of full-load current.

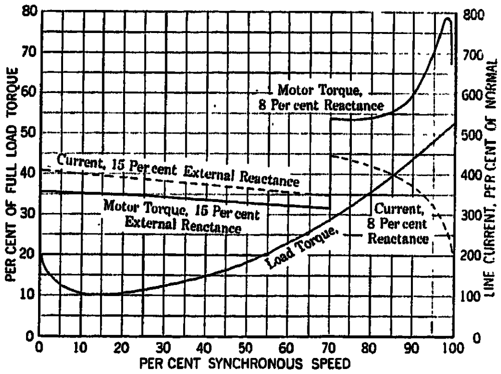


Fig. 10—Torque-Speed and Line Current-Speed Characteristics of Two-Pole Synchronous Motor with Two-Step Reactor Starter; and Torque-Speed Characteristic of Centrifugal Compressor

The accelerating duty required of a 3600-rev. per min. motor is severe, and requires careful study. The rotor of a two-pole motor, whether synchronous or squirrel-cage is physically smaller than that of a lower-speed motor of equal hp. With a given amount of heat to be dissipated in the rotor, the temperature rise is necessarily greater. Ordinarily, the rs are started only once a day, or even less fre- tly, but provision should be made for repeated ing under emergency conditions. Where the n machine is a centrifugal air compressor, or a ompressor of average design having approximately ft.^{2} inertia per hp. at 3600 rev. per min., motors be designed to start twice in succession without gerous heating of the rotor or the stator. The time val between starts is important. Obviously a er time interval allows more cooling as well as a er redistribution of heat between the two complete ing cycles.

ue to the unusual torque characteristics of the motor below synchronous speed, as previously described, synchronizing is easily accomplished in spite of the inertia of the average centrifugal compressor. rotating machine is to be pulled into step from a synchronous speed, its kinetic energy must be eased by an amount corresponding to the difference peed, and this energy must be supplied in a time esponding to less than one-half cycle at slip fre- ncy. It is therefore important to reduce this slip to inimum, to insure proper synchronizing and to t the current peak which the motor may draw nentarily. Since the two-pole synchronous motor develop approximately full load torque when ning as an induction motor at 99 per cent speed, the rifugal compressor load can be accelerated to a slip bout one-half of one per cent, and the change from ction-motor operation to synchronous operation ade with very low current inrush.

he use of a synchronous motor on any essential hine, where an outage of a few minutes may be ous, immediately raises a question regarding the ration of the motor under unusual conditions, such nomentary loss or dip of line voltage or failure of exci- on voltage. With suitable torque characteristics, the chronous motor can be arranged to continue opera- after a momentary loss of a-c. power, but the trol must be supplemented by additional devices to omplish this result.

n general, if a synchronous motor falls more than a per cent below synchronous speed, due to a momen- y voltage drop, it will not re-synchronize upon return voltage. But, if the d-c. excitation can be noved, and the oil-circuit breaker in the armature uit remains closed, the motor can re-synchronize vided the squirrel-cage torque at sub-synchronous eds just below synchronous is sufficient to carry the d. The characteristic curve of the 1500-hp. motor, lotted on the basis of 100 per cent voltage, would w approximately 150 per cent torque at 98 per cent ed, and 130 per cent torque at 75 per cent speed. us the torque near synchronous speed is sufficient enable the motor to accelerate, upon return of full tage, and re-synchronize with full load on the driven chine. With automatic starting it would be ad- able to allow full voltage to be applied immediately after a momentary voltage failure, rather than to go through the usual starting steps, with the resulting loss of time and reduction of speed. With a centrifugal blower load or a pump load, and an external $W R^2$ of 2 lb. ft.^{2} per hp., the motor would drop to 94 per cent speed in one sec., after failure of voltage. If the time delay of the undervoltage device is limited to one or one and one-half sec., the motor can still be re-started without an excessive current inrush. If continuous operation is essential, it is necessary to resort to this procedure in emergency cases, although the current inrush would be higher than that normally allowed for starting. The use of a time-delay undervoltage device on the line breaker is not in itself sufficient to insure continuous operation of the synchronous motor during voltage dips, even of short duration.

The problem of removing excitation and of re-synchronizing at the proper time requires additional relay equipment and careful consideration of the effect on the supply voltage which these operations will cause.

If the field excitation is lost, the two-pole synchronous motor can operate for a very limited time as an induction motor, without overheating. Continued operation without excitation is not possible with the present designs. The use of these motors for essential service consequently requires a dependable source of direct current for excitation, or an alternative source, to which a transfer can be made quickly.

Various methods are available for removing excitation if the synchronous motor falls out of step, and for re-synchronizing after short-time interruption. However, the devices required to accomplish these steps make the control more complicated than the control required for the same type of service with induction motor drive.

The design of the two-pole synchronous motor permits the use of a large air-gap without requiring a high excitation current. An air-gap clearance of one-half inch or more is used in the larger motors. The use of forced lubrication for the bearings is an added assurance of successful operation from the mechanical standpoint.

If the motors are designed to operate at leading power factor, then, in addition to the advantage of power factor improvement, an increase in maximum synchronous torque is obtained. The pull-out torque is increased from approximately 150 per cent of full-load torque, for unity power factor operation, to 200 per cent or more for motors operating at 0.8 leading power factor. With 200 per cent available torque at rated voltage, the motor can carry full load momentarily with line voltage as low as 50 per cent of normal.

CONCLUSION

The problems involved in the design and application of two-pole synchronous motors have been outlined so as to indicate the applications for which these motors are best adapted, and also to indicate their limitations. It is believed that there are other possible uses to which these motors have not yet been applied, and it is hoped that this paper will contribute to an understanding of the field of usefulness of the two-pole synchronous motor.

The authors wish to acknowledge their indebtedness to Mr. W. F. Dawson, under whose supervision these motors were developed, and who has contributed material for the introduction.

Bibliography

1. *Eddy Currents in Large Slot-Wound Conductors*, A. B. Field, Proc. A. I. E. E., Vol. 24, pp. 761-768, 1905.

2. "Eddy Currents in Stator Windings," H. W. Taylor, *Journal I. E. E.*, Vol. 58, pp. 279-300. (London) April 1920.

3. *Heat Losses in the Conductors of A-C. Machines*, Waldo V. Lyon, Trans. A. I. E. E., Vol. 40, pp. 1361.

4. *Starting Performances of Synchronous Motors*, H. V. Putnam, Trans. A. I. E. E., Vol. XLVI, 1927, p. 39.

Discussion

M. D. Ross: The paper by Messrs. McLenegan and Summers is the first adequate paper dealing with the two-pole synchronous motor. A great deal has been written in Europe on the subject of the synchronous induction motor which uses a non-salient pole rotor, but neither the proportions nor the performance of the synchronous induction motor are comparable to the machines described. The construction of the two-pole synchronous motors is not new. As early as 1910, the company with which I am associated built a 750-hp., 2-pole, 3600-rev. per min. motor to drive a centrifugal blower. As stated by the authors, the application of these motors has been limited, which probably accounts for the lack of information to be found in technical magazines.

It has been our practise to build two-pole turbine generator rotors using solid forgings into which radial slots are machined to take the windings. The solid forging of the rotor is capable of acting as a damper winding, and the standard turbine generator can be redesigned for operation as a self-starting synchronous motor. Eddy currents are set up in the surface of the steel forging during starting, which produce ample torque for starting. The flux linking the rotor is confined to a relatively small depth on the surface of the rotor due to the well known skin effect. This effect is shown by the low voltage induced in the field winding during starting, indicating that very little flux links the field winding, but rather crosses the tops of the rotor slots. As the slip frequency decreases, the flux and eddy currents penetrate deeper into the rotor, thus giving the desirable effect of a high resistance damper winding at standstill with a high starting torque and a low resistance damper winding near synchronous speed which will allow the rotor to accelerate to a low slip.

The synchronous motor rotor differs from that of the generator only in that the steel retaining rings over the rotor coil ends are insulated from the rotor body to prohibit heating at the contact surface between them due to eddy currents. A number of synchronous motors has been built with this rotor construction, which has given entire satisfaction and has the advantage of ease of construction. Since no special damper winding is required, the amount of rotor copper is not reduced in the motor design.

The number of successive starts a motor can satisfactorily make depends largely upon the heat storage capacity of the rotor as the proportion of the heat absorbed by the cooling air during the starting cycle is relatively small. In tests of two 2700-hp., 4-pole, 1800-rev. per min. motors in which the above construction was used, it was found that the heat generated during the start had distributed itself nearly uniformly through the rotor by the time the rotor came to rest due to the good heat conductivity of the solid rotor forging which allowed the losses to escape from the surface of the rotor to the center in a relatively short time. Dangerously high temperatures cannot therefore be expected to build up on the rotor surface during several successive starts, provided the total heat storage capacity of the rotor is large enough.

One interesting feature of these motors which the authors have not mentioned is the absence of pull-in torque with no field excitation. A salient pole synchronous motor will pull into step without excitation if it is not too heavily loaded due to the difference of reluctance of the magnetic paths through the polar and inter-polar parts of the rotor. The difference in reluctance between the slotted portion of the rotor and the solid pole center is quite small and the two-pole motor will not synchronize under load unless excitation is applied.

Due to the relatively low reactance of the two-pole motor and its rapid drop in speed when disconnected from the power circuit, the design of control equipment for these motors is more difficult than for slower speed motors. When auto-transformers are used, we have so designed them that the motor is connected to the auto-transformer taps through balance coils, so that taps can be changed during starting without opening the circuit. The motors are usually started on about 40 per cent voltage, transferred to 60 per cent voltage, and are synchronized after transferring to full voltage.

The reactor method of starting described by the authors has the advantage over auto-transformers of much lower control cost and somewhat smoother operation during starting and can be used to advantage where the source of supply will allow the use of the higher starting current involved.

It is to be hoped that a better understanding of the performance of the two-pole synchronous motor by engineers interested in motor drives as brought about by papers such as the one under discussion, will open up new fields of application.

A. B. Owen: As the authors have indicated, the demand for 3600-rev. per min. synchronous motors comes almost exclusively from builders of centrifugal blowers and compressors. For such service this speed is as a rule too slow, as turbine-driven blowers for such applications as gas boosters are run at speeds of 10,000 to 20,000 rev. per min. However, it is the highest speed at present available in 60-cycle motors and hence the blower manufacturer must design on that basis. In centrifugal blowers and compressors, the pressure is developed by using high tip velocities to impart high velocities to the gas and then converting this velocity into its equivalent pressure. The blower designer is always interested in the highest possible tip velocity without resorting to disproportionately larger diameters.

In the development of the first successful synchronous motors for driving blowers, about the year 1914, the problem of a sufficient starting torque was naturally prominent; in fact, it might fairly be said that this problem was and is today the only problem of a motor design to distinguish it from the design of a corresponding turbine generator. Fortunately, the starting requirements of a motor driven blower are simple and easy. Only sufficient torque is required to break the rotors away from rest and to accelerate them. The work done in moving the air at slow speed is negligible; consequently, at the start when the rotor frequency is a maximum, the required torque is a minimum and it is only as the rotor frequency drops due to increase of speed that the required torque increases—an ideal starting condition.

Even with this favorable condition the problem of the starting winding is difficult, and not the least of the difficulties is to handle the expansion in the axial conductors. High heat must be expected under starting conditions and only a part of this heat can be conducted away even in such an excellent arrangement as that shown by the authors of this paper. The older practise on Elliott motors of this type was to attach the conductors to end rings having quite a measure of axial flexibility. Expansion from heat can fairly be called an irresistible force and if provision is not made for necessary movement, joints will be broken.

A recent development that may be of interest consists in using the steel of the solid-forging type of rotor as the starting winding. Each tooth of the rotor is an axial conductor, connected at its ends to the solid body of the shaft. Its resistance is high and its heat storage capacity and also its heat conduction to the shaft are excellent.

This simple construction, entirely free from joints and special materials, makes it ideal for the particular application under consideration, and it has been used successfully for the last four or five years in motors from 750 to 2200 hp. There seems to be no reason why it should not be equally applicable to motors of larger sizes. These motors will break away and accelerate to about 30 per cent speed on the application of a starting voltage as low as 35 per cent of rated voltage, though 40 to 45 per cent is desirable on the first starting point.

I wish to endorse the method of starting such motors by increasing the applied voltage without opening the stator circuit. This is very desirable in order to avoid shock to the rotor and possible distortion of the end connection of the stator winding.

R. B. Williamson: The authors say: "compressors requiring a starting torque of 30 per cent, and a pull-in torque of 50 per cent, and having a WR^2 of 2 lb-ft.^{2} per hp. of motor rating." I want to ask if that is the combined compressor unit or just the compressor alone.

I. H. Summers: With reference to Mr. Ross' discussion, particularly in regard to building a synchronous motor with a solid-core rotor, I may say that we have also built a machine of this construction and tested it. We feel that a laminated construction with a properly designed squirrel-cage winding is probably a more flexible design because, by changing the material, or the dimensions of the windings and the retaining wedges above these windings, different characteristics can be obtained for different purposes.

With regard to his point that a round-rotor synchronous motor does not have pull-in torque, I wish to point out that in the motor we tested and for which curves are given in the paper, the torque is very large at speeds close to synchronism, so that the motor reaches a slip of probably 0.5 per cent at the time the field is applied, which makes it very easy to synchronize.

Mr. Ross evidently felt that the temperature rise during a cycle in which several starts are made is not a serious problem, provided the motor be made large enough, and we agree thoroughly with that. Of course the point is that the motor must be made large enough. In this connection I have prepared some curves showing the temperature distribution to be expected in a cylindrical rotor, assuming a homogeneous mass of iron with heat applied to the surface. These curves indicate in a general way what maximum temperatures would be obtained under this assumption.

Mr. Owen, in his discussion, pointed out that the problem of starting a centrifugal compressor is not serious. The main thing is to have sufficient torque to accelerate its inertia without overheating the motor and we agree thoroughly with this also. In these applications the inertia of the load is often very large and is what limits the size of the motor if repeated starts have to be made. With a two-pole motor, having a rotor of as small size as it normally would be in this design, the problem is more serious than it appears at first sight.

With regard to his point about the expansion of the conductors being a serious problem, I might mention that although the photograph in our paper does not show it, the squirrel-cage winding which we use is formed of copper ribbons and is so constructed that a wave is placed in the ribbons at the ends of the rotor to take up the expansion and this completely prevents all trouble which might otherwise result from restraint of the expansion.

In answer to Mr. Williamson's question as to whether the figure given in the paper of 2 lb-ft.^{2} per hp. rating applies only to the compressor or to the combined unit, I may say that that was intended to apply to the compressor only.

Consider a homogeneous cylinder with heat applied uniformly all over the cylindrical surface. Assume that this heat is known as a flow function of time in terms of power per unit area. The problem of determining the temperature distribution at any point in the cylinder at any time may be reduced to the solution of a partial differential equation.

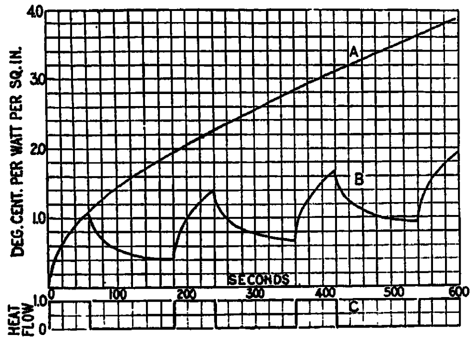


Fig. 1

The heat equations are

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) = \frac{p}{\alpha^2}\theta \qquad (1)$$

$$f = -c\frac{\partial \theta}{\partial r} \qquad (2)$$

where
r is the radius of the cylinder
p is time differentiating operator
f is heat flow
c is conductivity
θ is temperature
α^2 is the diffusivity

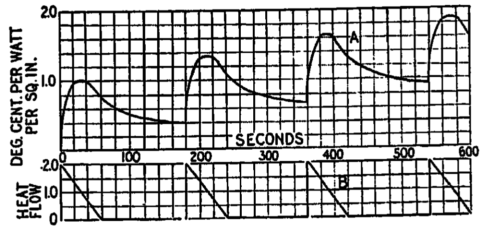


Fig. 2

Substituting the boundary conditions that the cylinder is not hollow and that $f = f_0$ at $r = r_0$ and solving, the temperature is found in operational terms

$$\theta = \frac{1}{n\,c}\,\frac{J_0(n\,r)}{J_1(n\,r_0)}\,f_0 \qquad (3)$$

where
$n = j\dfrac{\sqrt{p}}{\alpha}$
$j = \sqrt{-1}$
J_0, J_1 = Bessel functions

In order to get this solution in explicit form it is convenient to expand it in terms of partial fractions.[1] Owing to lack of space this development will be omitted and the result simply stated.

$$\theta = -\frac{2\,\alpha^2}{c\,r_0}\left[\frac{1}{p} + \sum \frac{J_0\,(y\,a_n)}{J_0\,(a_n)}\;\frac{1}{p + \left(\dfrac{a_n\,\alpha}{r_0}\right)^2}\right] f_0$$

where the summation is taken over all the positive roots of $J_1\ (x) = 0$, a_n representing these roots. In this expression $y = r/r_0$.

Now the flow function is taken as a constant flow directed inward, in the Heaviside notation.

$$f_0 = -F\,1 \tag{5}$$

When this is substituted and the operations performed according to the Heaviside rule, the final solution after simplification is:

$$\frac{\theta}{F} = \frac{2\,r_0}{c}\left[\left(\frac{\alpha}{r_0}\right)^2 t + \left(\frac{y^2}{4} - \frac{1}{8}\right) - \sum \frac{J_0\,(y\,a_n)}{a_n^2\,J_0\,(a_n)}\,\epsilon^{-\left(\frac{a_n\,\alpha}{r_0}\right)^2 t}\right] \tag{6}$$

where t is time.

At the surface of the cylinder where the temperatures are maximum this reduces to:

$$\frac{\theta}{F} = \frac{2\,r_0}{c}\left[\left(\frac{\alpha}{r_0}\right)^2 t + \frac{1}{8} - \sum \frac{1}{a_n^2}\,\epsilon^{-\left(\frac{a_n\,\alpha}{r_0}\right)^2 t}\right] \tag{7}$$

This curve is plotted in Fig. 1A herewith for an iron cylinder of 10 in. radius in which $\alpha^2 = 0.0188$ sq. in. per sec. and $c = 1.14$ watts per sq. in. per deg. cent. per in. Fig. 1B is the temperature when the flow function is intermittent as in Fig. 1C. Fig. 2A is the temperature when the flow function is as shown in Fig. 2B and the total heat applied is the same as in Fig. 1C. It will be noted that the maximum temperatures are practically the same in either case so that it is only necessary to know the total heat applied to get an approximate idea of the answer. The ordinates in either figure are to be multiplied by average watts per square inch for one heat cycle to get degrees centigrade.[2]

In the application of these curves to a practical problem in the starting of synchronous motors it should be remembered that herein no heat is assumed to escape from the surface. This would probably not modify the result radically, however, and to offset it there is the consideration that the rotor construction is not perfectly homogeneous, having slots and other irregularities. At best the curves are a rough approximation to a practical case but it is felt that they do serve to illustrate some interesting points in connection therewith.

D. W. McLenegan: Mr. Ross mentioned the smoother starting which is possible with the reactor method. If the motor could be designed so that its impedance increased rapidly at intermediate speeds, this type of starting would be ideal. However, as the motor accelerates, the impedance does not rise sufficiently to increase the voltage drop across the motor and provide a motor torque corresponding to the increasing load torque. It is therefore impossible to use a constant external reactance throughout the accelerating period except by making this reactance very small and accepting a high initial current. The reactor is cut out in steps, and this procedure involves abrupt changes in the motor current.

Mr. Owen mentioned the desirability of maintaining the circuit while increasing the applied voltage during the starting period. However, with the auto-transformer method described in the paper, the magnetizing current of the auto-transformer is not interrupted when changing taps, and consequently the inrush is materially reduced due to the reactance of the auto-transformer. It has been our experience that the coil bracing is adequate to prevent any distortion of the end turns of the stator winding.

---

[1] The writer is indebted to Mr. R. H. Park for showing him how to make this expansion and simplify the result.

[2] The author is indebted to Mr. S. Paglinca for assistance in the numerical work and plotting the curves in the figures.

Heat Losses in D-C. Armature Conductors

BY W. V. LYON[1] E. WAYNE,[2] and M. L. HENDERSON[3]
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—The pioneer paper on this subject of copper losses in armature conductors was presented by A. B. Field.[4] Since then Field's fundamental premises have been adopted by practically all who have contributed to this subject, both in this country and abroad, although the method of presentation has been modified as the individual thought best. Thus one of the present authors has favored a vector method of analysis that seems to him to have marked advantages. In this paper the vector method is extended to the case of the d-c. machine.

Without doubt, it is generally believed that there are extra losses in the armature coil of a d-c. machine when it is undergoing commutation, but that during the major portion of the time, since the current is steady, the loss in the coil is correctly computed by squaring the value of the current and multiplying by the true or ohmic resistance of the coil. This view, however, is not correct. The disturbance in the coil current produced by the process of commutation persists throughout the cycle so that at no time is the current uniformly distributed over the cross-section of the conductor. Although the disturbance and resulting extra loss factor are greatest during commutation, the extra losses are present at all times, even while the current in the coil is steady. The procedure for finding the extra losses is simple. First, the time variation of the coil current must be known and analyzed for its harmonic components. In the present analysis it is assumed to be trapezoidal because this is, at once, a simple and a desirable manner of variation. The extra loss is then computed for each harmonic component as described in "Heat Losses in the Conductors of A-c. Machines," by W. V. Lyon, and "Reduction of Armature Copper Losses," by I. H. Summers. The total extra loss is the sum of the component extra losses. In case there are several coils side by side, as is usual in a d-c. armature winding, the average extra loss in the coils for any harmonic component of current is computed as if the coils that are side by side were connected in parallel and carried the vector sum of the currents in these coils; assuming that the coils are alike and are symmetrically arranged. If there are more than two coils side by side, the losses in them are not the same.

It is important to realize that the armature copper loss in a d-c. generator is not correctly computed by squaring the armature current and multiplying by the measured armature resistance between brushes; and furthermore, that it is more important to laminate and properly arrange the armature conductors of a large d-c. generator than it is to do so in the case of an a-c. generator.

PART I

ALTHOUGH there have been excellent papers[5] on this problem of the armature copper loss in a d-c. machine, as far as the authors are aware it has received no serious consideration in this country and it is not generally realized that the time-honored method of computing the armature copper loss does not give a correct value of the loss. The error may range from 10 or 20 per cent in a small machine to several hundred per cent in a large and poorly-designed one. The determination of this loss and the factors which influence it are thus matters of prime importance. Due to the process of commutation the current in the armature coil of a d-c. machine is an alternating one which has a fundamental and various harmonic components. The problem of an armature coil that carries a sinusoidal current has been investigated by several writers since Field[4] first published his solution of the problem. The experimental results[6] which have since been obtained agree so closely with the computations that we are warranted in extending the application of the theory to this problem of the heat losses in the conductors of a d-c. armature when not only the fundamental but all of the harmonic components are present in the current. Due to the fact that the losses, produced by the higher harmonics in the current, increase about as the square root of the frequency, the extra loss factor for a d-c. armature is greater than the extra loss factor for an a-c. armature under the same conditions of speed, poles, size, and arrangement of the armature conductors. Laminated conductors are indispensable in the design of a-c. generators, but it is even more important to laminate and properly arrange the armature conductors of a d-c. machine.

One phase of the present investigation was to compare the observed and computed results when the current in an embedded coil (Fig. 1) is commutated so that it varies as the current does in the armature coil of a d-c. generator. A typical curve showing the time variation of the current is shown in Fig. 2. The coil shown in Fig. 1 consisted of 51 turns of copper ribbon one inch wide by 0.0062 inch thick. Five No. 40 wires were attached to but insulated from the ribbon at uniformly spaced intervals. One of these wires was 1/16 inch from the upper edge of the ribbon; one, 1/4 inch from the upper edge; one at the middle; one, 1/4 inch from the lower edge and one, 1/16 inch from the lower edge. The ribbon and the attached wires were then wound

---

[1] Associate Professor of Electrical Machinery, Massachusetts Institute of Technology.

[2] Swope Fellow in Electrical Engineering, 1926-27, Massachusetts Institute of Technology.

[3] Graduate Student in Electrical Engineering 1926-27, Massachusetts Institute of Technology.

[4] *Eddy Currents in Large Slot-Wound Conductors,* A. B. Field, A. I. E. E., 1905, p. 761.

[5] L. Dreyfus, "Zusätzliche Kommutierungsverluste bei Gleichstrommaschinen," *Elekt. und Masch.* XXXII, 281, 1914.

L. Dreyfus. "Die Theorie der zusätzlichen Kommutierungsverluste von Gleichstrommaschinen," *Arch. fur Elek.* III, 273, 1915.

L. Dreyfus. "Zusätzliche Kupferverluste durch Stromverdrangung bei Einankerumformern," *Arch. fur Elek.* IV, 42, 1915.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-17, 1928

[6] Trans. A. I. E. E., 1922 Vol. 41, p. 213, and present paper.

to form the coil, a cross-section of which is shown in Fig. 3. These exploring wires enable one to make a direct measurement of the current density at five different points in the ribbon. Inasmuch as an exploring wire and a filament of the ribbon in the same horizontal plane link the *same flux* the difference in their potentials is the resistance drop in this filament of the ribbon and is, therefore, proportional to the current density in this filament. The arrangement of the coil, commutator, and oscillograph vibrators is shown in Fig. 4. The current-density vibrator was calibrated by sending a known value of steady current through the coil. Five calibrations were made, one for each exploring wire. The commutator was driven so that the current in the coil was reversed 120 times a sec., giving a 60-cycle variation. The oscillograms obtained are shown in Fig. 5 (a. b. c. d and e). These oscillograms show the current in the coil and the current density at each of the five points. The scale of the current is such that the curve also shows the average current density and may thus be compared with the actual current density. The computed time variation of current density at each point is also shown. The close agreement between the observed and computed values shows that, under the conditions existing during this measurement, the premises on which the theory is built—*viz.,* those adopted by Field—are amply justified.

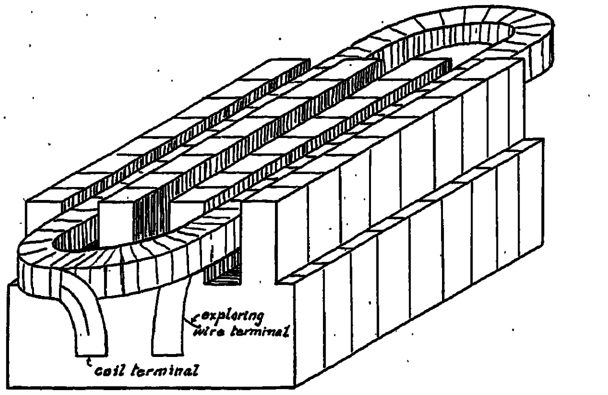


FIG. 1—GENERAL ARRANGEMENT OF COIL AND EXPLORING WIRE

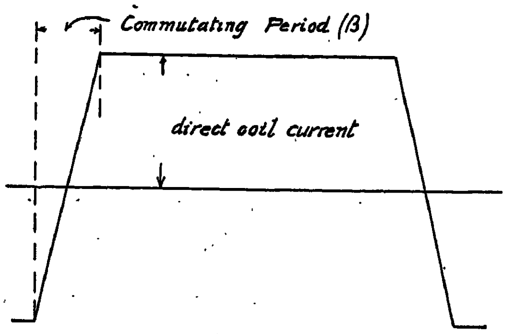


FIG. 2—CURVE SHOWING TIME VARIATION OF COIL CURRENT

The computed values of the current density were obtained in the following manner. The time variation of the current in the coil was first analyzed for its harmonic components. By means of the fundamental relation[7] between the current in the conductor and the current density at any point, the time variation of the current density at each point for each harmonic component of the current up to and including the eleventh harmonic, was determined. By combining the harmonic variations in current density at one point the resultant variation in the current density at the point was determined and plotted. The oscillograms show distinctly that the current density is greater in the upper portion of the conductor, as it is when the current is sinusoidal; that the higher harmonics are practically confined to the upper portion of the conductor; that during the commutating period the current is flowing in opposite directions in the conductor at the same time; that at the beginning of each half-cycle the current is most unevenly distributed, but toward the end of the half-cycle the distribution has become nearly uniform. In describing the phenomenon, it might be said that the continued reversal of the current, due to the process of commutation, produces a turbulence in the current flow of such a nature that the current distribution consists of a series of harmonic components, each one of which can be determined in the same manner in which the distribution is determined when the current variation is purely sinusoidal. This turbulence is not confined to the commutation period alone but continues throughout the cycle. The

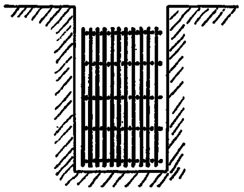


FIG. 3—CROSS-SECTION OF COIL AND EXPLORING WIRES

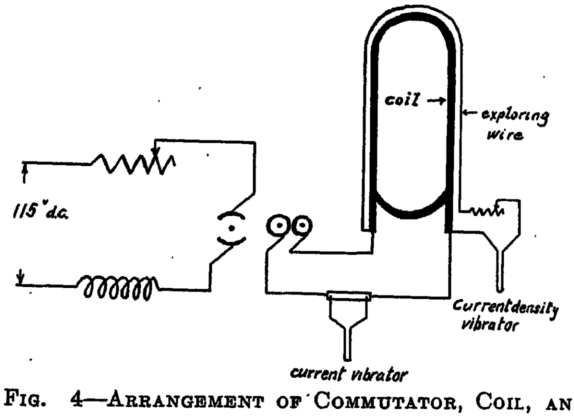


FIG. 4—ARRANGEMENT OF COMMUTATOR, COIL, AND OSCILLOGRAPH VIBRATORS

heat loss in the conductor is the sum of the losses due to the individual harmonic components of current. Each of these component losses can be determined by the same procedure that is used in computing the loss in an a-c. generator. Computation of the resistance ratio for a d-c. machine is thus a much more laborious process, than is the computation of the ratio for an a-c. generator. The resistance ratio may be defined in one of two ways, either as the ratio of the actual loss to the loss that would occur if the same coil current were uniformly distributed in the conductor, or as the ratio of the actual loss to that as computed according to the standardization rules of the A. I. E. E. These ratios would be somewhat different. In our case the measured loss in the coil was 14.8 watts and the calculated loss, 14.4 watts. In the calculation, only the harmonics up to and including the 13th were considered. The loss due to the fundamental, or 60-cycle, component is 12.9 watts, showing that the loss due to the harmonics is 10.4 per cent of the total. When this measurement of power loss was made a different commutator was used, and the wave form of the coil current is shown in Fig. 5 (f). The maximum flux density in the teeth during these power measurements was approximately 1000 gausses.

---

[7] W. V. Lyon, *Heat Losses in the Conductors of A-C. Machines*, TRANS. A. I. E. E., 1921 Vol. 40, p. 1361.

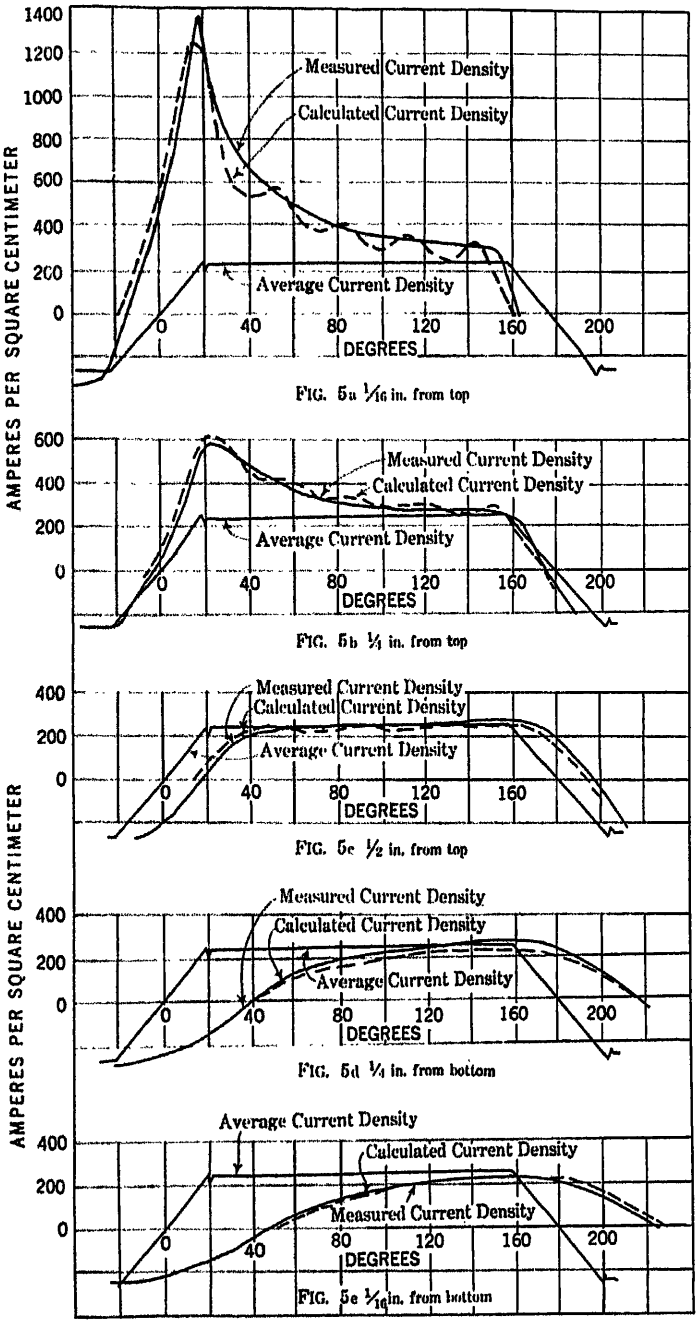


FIG. 5a 1/16 in. from top

FIG. 5b 1/4 in. from top

FIG. 5c 1/2 in. from top

FIG. 5d 1/4 in. from bottom

FIG. 5e 1/16 in. from bottom

FIG. 5—a, b, c, d, e, OSCILLOGRAMS AND CALCULATED CURVES OF CURRENT DENSITY AT DIFFERENT POINTS

There is another point of interest. It is the heat loss in the coil during the period of commutation. If it be assumed that the variation of the total current, and of the current density during the time of commutation is linear, an approximate value of the ratio of the actual heat developed, to that which would be developed if the current density were uniform, can be calculated. In our case this ratio is approximately 5, that is, during commutation, the conductor acts like one having a resistance approximately 5 times as great as it actually is.

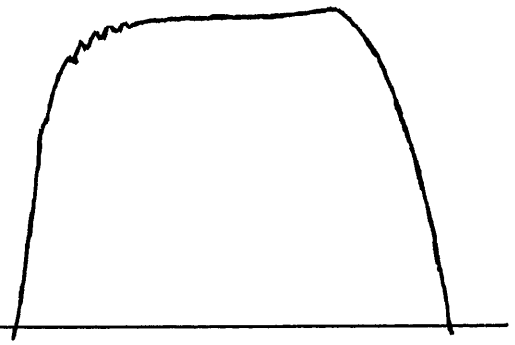


FIG. 5f—WAVE FORM OF COIL CURRENT WHEN LOSS MEASUREMENTS WERE MADE

With 60-cycle sinusoidal current in the conductor the measured and computed resistance ratios are 1.66 and 1.61 respectively. Figs. 6 and 7 show the observed and computed current densities and their phase-angles with respect to the total current in the conductor.

All of these results seem to show that the original premises as stated by Field are so nearly true, even when applied to this problem of the commutated current, that the errors in the computation are scarcely of engineering importance.

When these principles are applied to the determination of the armature copper loss in a d-c. machine it must not be expected that the accuracy of the result will equal that attained in the case of the a-c. generator. There are two principle reasons for this. First it is improbable that the variation of the current during the commutation period will be known with sufficient precision. Poor commutation may give rise to harmonics that will materially increase the copper loss. Secondly, in d-c. machines, coils which undergo commutation at slightly different times are placed side by side in the slot; producing a phase displacement between the components of the currents in these adjacent coil sides which is directly proportional to the order of the harmonic. Some of the higher harmonics will thus be approximately in opposition. This condition violates one of the Field premises, *viz.*, that the current density should be the same in any horizontal plane. Three approximate methods for computing the loss in this case are described in the Part II of the paper.

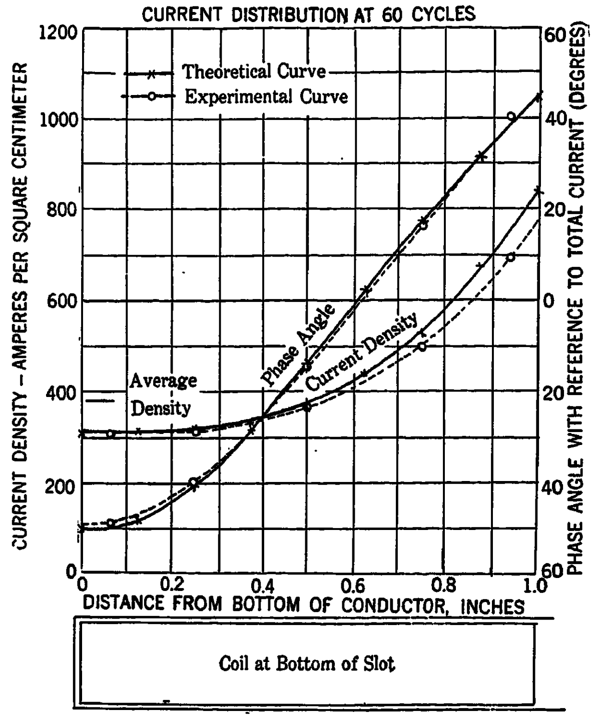


Fig. 6

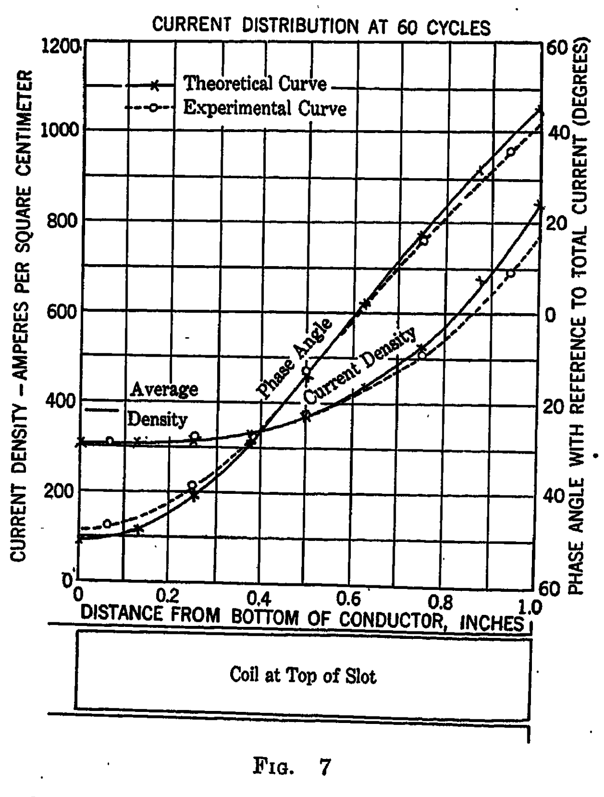


Fig. 7

Figs. 6 and 7—Comparison of Measured and Computed Current Densities

The general principles that have been discussed here can also be applied to the determination of the copper loss in a synchronous converter. If, for example, it be assumed that the current in the conductor is the sum of a sinusoidal component received from the a-c. side and a trapezoidal component, such as might flow in a d-c. generator, there will be a resultant fundamental component depending upon both the a-c. input and the d-c. output, and a series of harmonic components which depends only upon the d-c. output.

In conclusion it should be said that this paper is limited solely to the discussion of the so-called "skin-effect" losses that are due to the direct action of the armature current in heating the armature conductors.

PART II

Since the analysis of the problem in the case of the a-c. generator has been worked out in detail in previous articles,[8] it will only be shown here where the analysis needs to be modified to conform with the conditions met with in the problem of the d-c. machine.

Since the time variation of the coil current during commutation is seldom known, it is customary for writers on this subject to assume that it is linear. In this case, if I_{dc} represents the steady value of the conductor current and β is the angular period of commutation, the r. m. s. value of the nth harmonic component of the current is:

$$I_n = \frac{4}{\pi\sqrt{2}} \; \frac{\sin\dfrac{n\beta}{2}}{\dfrac{n\beta}{2}} \; \frac{1}{n} \; I_{dc}$$

Only the odd harmonics are present.

If the winding is similar to the standard type of a-c. winding having two coil sides per slot, one placed above the other, the resistance ratio for the nth harmonic K_n, can be determined by the same procedure that would be followed in the case of an a-c. generator. The resistance ratio, K_0, for the d-c. armature is then

---

[8] A. E. Gilman, *Eddy-current Losses in Armature Conductors*, Trans. A. I. E. E., Vol. 39, 1920, Part I, p. 997.

W. V. Lyon, *Heat Losses in the Conductors of A-C. Machines*, Trans. A. I. E. E., Vol. 40, 1921, p. 1361.

W. V. Lyon, *Heat Losses in Stranded Armature Conductors*, Trans. A. I. E. E., Vol. 41, 1922, p. 37.

I. H. Summers, *Reduction of Armature Copper Losses*, Trans. A. I. E. E., Vol. XLVI, 1927, p. 101.

For a brief bibliography see Transactions of A. I. E. E., Vol. 41, page 213. A paper by F. W. Carter has just come to out attention. "Parasitic Currents Losses in Bar-wound Direct-current Armatures." *Jl.* I. E. E., Vol. 65, p. 1002.

$$K_0 = \frac{\displaystyle\sum_{n=1}^{n=n}\left(\frac{\sin\frac{n\beta}{2}}{\frac{n\beta}{2}}\,\frac{1}{n}\right)^2 K_n}{\displaystyle\sum_{n=1}^{n=n}\left(\frac{\sin\frac{n\beta}{2}}{\frac{n\beta}{2}}\,\frac{1}{n}\right)^2} \quad \text{(First definition)}$$

$$\text{or } K_0' = \frac{\left(\frac{4}{\pi\sqrt{2}}\right)^2\displaystyle\sum_{n=1}^{n=n}\left(\frac{\sin\frac{n\beta}{2}}{\frac{n\beta}{2}}\,\frac{1}{n}\right)^2 K_n}{1-\frac{\beta}{\pi}} \quad \text{(Second definition)}$$

The upper limit of n should be as large a value as is considered necessary.

The latter definition assumes that the brushes short-circuit $\frac{\beta}{\pi}$ of the armature coils per pole when the d-c. measurement of armature resistance is being made.

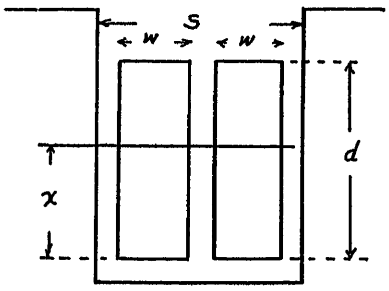


FIG. 8—CROSS-SECTION OF COIL AND SLOT

D-c. armatures are often wound so that coils which undergo commutation at slightly different moments are placed side by side in the slot. In this case three different approximations are suggested. There are no data which show their relative accuracy.

1. The difference in time of commutation can be ignored, and the resistance ratio computed as described above. This approximation is the simplest of the three. It neglects the phase relation of currents that are side by side in the slot, which in the case of some of the higher harmonics is approximately 180 deg. This method can be applied to any type of symmetrical winding whether of solid or of laminated conductors. The computed value of the loss would probably be too great.

2. It may be assumed that the current density in any conductor is uniform in any horizontal plane but is not necessarily the same in the different conductors that lie side by side in the slot, (Fig. 8). The current densities in the conductors are respectively c_1 and c_2 in the plane through X. A simple case of but two conductors has been chosen as an illustration.

The following differential equations for the current densities must be satisfied:

$$\frac{d^2 c_1}{d x^2} = j\,\frac{4\,\pi\,\omega}{\rho\, s}\left(c_1\frac{w}{2} + c_2\frac{w}{2}\right)$$

$$\text{and} \quad \frac{d^2 c_2}{d x^2} = j\,\frac{4\,\pi\,\omega}{\rho\, s}\left(c_1\frac{w}{2} + c_2\frac{w}{2}\right)$$

When the currents are equal in magnitude but differ in phase by a time angle θ, it follows that the copper loss in each conductor is

$$R\left[\left(I\sin\frac{\theta}{2}\right)^2 + \left(I\cos\frac{\theta}{2}\right)^2 M_r\right]$$

where: R is the true resistance of one conductor
I is the current in each conductor
M_r is the real part of $\alpha\, d \coth \alpha\, d$

$$\alpha\, d = 2\,\pi\, d\sqrt{\frac{j\,2\,f\,w}{\rho\, s}}$$

f is the frequency of the harmonic considered.

This shows that when the currents are in opposition the loss is the same as it would be with steady direct current. This method is readily extended to the case of any number of solid conductors side by side with other conductors carrying current below them.

In the case of laminated conductors there is no difficulty in obtaining a simple solution provided the armature coils are alike and are symmetrically arranged. The loss in such a winding is the sum of two component losses. The first is the loss that would be produced if there were in each coil a steady direct current equal in magnitude to the r. m. s. value of the commutated coil current. The second of these component losses is the sum of the extra[9] losses due to a series of harmonic components of current. Any one of these harmonic components of current is the vector sum of the actual harmonic components of the currents in any row[10] of conductors in the same slot. The extra loss due to any one of these harmonic components is then computed as if the coils that are side by side in the same slot were in parallel. This can be done by following the same procedure that would be followed in the case of an a-c. generator. Reduced to a formula the loss becomes

$$R\, I_{dc}^2\left[\left(1-\frac{2}{3}\,\frac{\beta}{\pi}\right) + \frac{8}{\pi^2}\sum_{n=1}^{n=n}\left(\frac{\sin\frac{n\beta}{2}}{\frac{n\beta}{2}}\,\frac{1}{n}\,\frac{\sin\frac{n\,q\,\Delta}{2}}{q\sin\frac{n\,\Delta}{2}}\right)^2 (K_n - 1)\right]$$

Where: R is the true or ohmic resistance of the armature coils all connected in series;

---

[9] In excess of the loss as computed with direct current.

[10] In a circumferential direction and *not* in a radial direction.

I_{dc} is the maximum value of the coil current;

p is the angular duration of commutation;

Δ is the time angle—on fundamental scale ($n = 1$)—between the moment at which one coil of a row in any slot is commutated and at which the next coil of the row is commutated;

q is the number of coils in a row;

K_n is the resistance ratio of the winding for the nth harmonic, computed as it would be for an a-c. generator which had the coils in any row connected in parallel.

The summation is taken for all odd values of n.

The form of the time variation of the coil current is assumed to be trapezoidal.

3. It may be conceived that the currents, I_1 and I_2, (Fig. 9) consist of the following components.

$$I_1 = \frac{1}{2}(I_1 + I_2) + \frac{1}{2}(I_1 - I_2)$$

$$I_2 = \frac{1}{2}(I_1 + I_2) - \frac{1}{2}(I_1 - I_2)$$

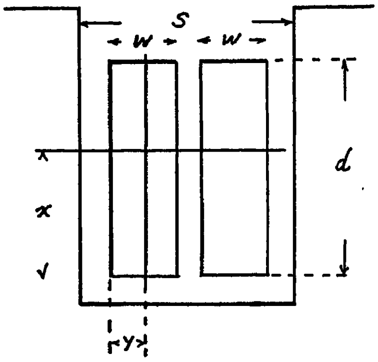


FIG. 9—CROSS-SECTION OF COIL AND SLOT

Let it now be assumed that these components are distributed over the cross-section of the conductors as if they existed separately. The first components, being equal, are distributed according to Field's theory. Let the current density in the plain through x be c_x. The second components are in time opposition in the conductors and it may reasonably be assumed that they are distributed as they would be in an infinite sheet lying close to a region of infinite permeability. The vector current density in the plane through y is then of the form $c_y = A \cosh \alpha_2 y$; where $\alpha_2 = 2\pi\sqrt{j\frac{2f}{\rho}}$.

The total vector current density at the point (x, y) is then $(c_x + c_y)$ and the loss in one conductor is

$$\int_0^{\frac{w}{2}}\int_0^{d} \rho\,(c_x + c_y)\,.\,(c_x + c_y)_{\text{conjugate}}\,d\,x\,d\,y$$

In the case of equal currents differing by a time-phase angle of θ this expression readily reduces to

$$R\left[\left(I\cos\frac{\theta}{2}\right)^2 M_{r1} + \left(I\sin\frac{\theta}{2}\right)^2 M_{r2}\right]$$

Where M_{r1} is the real part of $\alpha_1\,d\coth\alpha_1\,d$, and M_{r2} is the real part of $\alpha_2\frac{w}{2}\coth\alpha_2\frac{w}{2}$.

$$\alpha_1\,d = 2\,\pi\,d\sqrt{j\,\frac{2\,f\,2\,w}{\rho\,s}}\quad \alpha_2\,w = 2\,\pi\,w\sqrt{j\,\frac{2\,f}{\rho}}$$

R is the true resistance of one conductor and f is the frequency of the particular harmonic considered. This method can be used with any symmetrical type of winding that has not more than two conductors side by side in the slot.

The armature copper loss in a synchronous converter can readily be computed if it be assumed that the current in a coil consists of two components, one of which is the trapezoidal current that would flow if the machine were operated as a d-c. generator, and the other is the sinusoidal current that would flow if the machine were operated as a synchronous motor. This assumption may not be true during the commutation period. The current in an armature coil is

$$i = \sqrt{2}\,I_{ac}\sin(\omega\,t - \theta - y)$$

$$-\frac{4}{\pi}I_{dc}\sum_{n=1}^{n=\infty}\frac{\sin\frac{n\,\beta}{2}}{\frac{n\,\beta}{2}}\,\frac{1}{n}\sin n\,\omega\,t$$

Where θ is the power-factor angle of the alternating current and y is the position angle of the coil with respect to the middle of the phase.

It can readily be shown that the loss is

$$\mathrm{R}\left[I_{ac}^2 K_1 + \frac{8\,I_{dc}^2}{\pi^2}\sum_{n=1}^{n=n}\left(\frac{\sin\frac{n\,\beta}{2}}{\frac{n\,\beta}{2}}\,\frac{1}{n}\right)^2 K_n\right.$$

$$\left.-\frac{4\sqrt{2}}{\pi}I_{dc}\,I_{ac}\,\frac{\sin\frac{\beta}{2}}{\frac{\beta}{2}}\,k_b\,K_1\cos\theta\right]^{11}$$

Where: R is the series resistance of all of the armature coils;

K_1 is the resistance ratio of the winding for the fundamental;

K_n is the resistance ratio of the winding for the nth harmonic;

k_b is the breadth factor for the winding;

β is the angular time of commutation.

The higher harmonics in the conductor current produce such a rapid variation in the tooth flux density that it is no longer uniform, being greater at the surface

---

[11] This assumes that the armature coils are *not* arranged side by side in the slots.

than at the center of the laminations. There thus appears to be a possibility that the resulting density at the surface of the lamination, due to the leakage flux and that which enters the teeth from the air-gap, may be so great as to reduce the permeability to such a small value that one of the fundamental premises of the Field theory—*viz.*, that the permeability is very great—is seriously violated. This phenomenon of skin effect in iron laminations is readily solved if the permeability is assumed to be constant. Actually, the permeability goes through a cyclic series of values. Since, however, the skin effect is exaggerated by choosing a relatively high permeability, say 4000, we shall do so, although the average value may be much less. For a similar reason we will use a relatively low resistivity, 36 microhms per centimeter cube. The angular depth to be used in computing the skin effect in 14 mil iron at 900 cycles (15th harmonic for 60-cycle fundamental) is:

$$\alpha\, d = 2\,\pi \times \frac{0.014 \times 2.54}{2} \sqrt{\frac{2 \times 900 \times 4000}{36 \times 10^{-6} \times 10^{9}}}\,\cdot$$

For this value of $\alpha\, d$ the ratio of the density at the surface to the average density is but 1.4 and skin effect is thus playing no important part in the problem.

As an illustration of the method of calculating the loss factor for a given armature winding, the following example is taken.

The armature considered is that of a 6-pole, d-c. machine operating at 900 rev. per min. The winding is full pitch, simple lap wound. There are two coils per slot, each consisting of a single turn, and the total number of coils is 216. In the armature slot the conductors are arranged two in width and two in depth. The conductors are solid, and rectangular in section. Principal conductor and slot dimensions are as follows:

Conductor width, $\frac{w}{2}$	=	0.405 cm.
Total width of copper in slot, w	=	0.81 cm.
Conductor depth, d	=	1.40 cm.
Slot width, s	=	1.19 cm.

The fraction of a cycle during which a coil is short-circuited is $\frac{\beta}{\pi} = 0.0725$. On the assumption that the current—time curve for the coil is trapezoidal, the r. m. s. value of the nth harmonic component of coil current is:

$$I_n = \frac{4}{\pi\sqrt{2}}\;\frac{\sin\frac{n\beta}{2}}{\frac{n\beta}{2}}\;\frac{1}{n}\,I_{dc}$$

or

$$= \frac{4}{\pi\sqrt{2}}\;\frac{\sin\frac{n\beta}{2}}{\frac{n\beta}{2}}\;\frac{1}{n}\;\frac{I_e}{\sqrt{1-\frac{2}{3}\frac{\beta}{\pi}}}$$

where I_{dc} is the magnitude of the direct current in the coil and I_e is the r. m. s. value of this current. Table I shows the mean-square values of the harmonics up to the fifteenth. The mean-square value of the coil current is 95.17 per cent of the square of its maximum, or d-c. value.

When there is a row of conductors side by side in the slot, carrying currents equal in magnitude but different in phase, the loss due to any harmonic current is taken as the sum of the normal ohmic loss due to the currents actually flowing and the extra loss calculated as if each of the conductors in the row carried the *vector average* current for the row. This is the second approximate method. The nth harmonic extra loss factor for the embedded portion of the conductor is

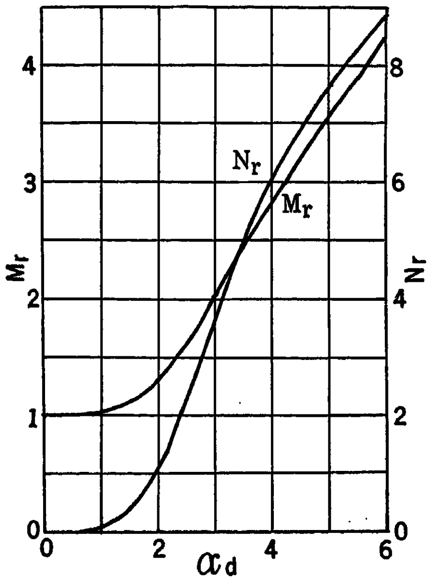


FIG. 10—VALUES OF M_r AND N_r

$$K_n - 1 = (M_r - 1) + \left[\left(\frac{I_b}{I_{av}}\right)^2 + \left(\frac{I_b}{I_{av}}\right)\cos\delta\right] N_r.$$

I_{av} is the magnitude of the vector average of the currents in conductors of the row and I_b is the magnitude of the vector current flowing in the slot below the row in question per conductor in that row. δ is the angular phase difference between I_{av} and I_b. M_r and N_r are taken at the value of $\alpha\, d$ for the harmonic considered.

In the example considered, successive armature coils are commutated at intervals of 1/216 revolution. The nth harmonic components of current in conductors side by side are out of phase by an angle $\theta = 5\,n$ degrees. With two conductors in a row across the slot the ratio of the vector average current to each current, is

$$\frac{I_{av}}{I} = \cos\frac{1}{2}\,n\,\theta = \cos n \times 2.5 \text{ deg.}$$

Table I shows the value of the square of this ratio for harmonics up to the fifteenth.

The winding scheme used in this armature causes the currents in conductors in the same radial plane to be commutated simultaneously so that the phase displacement between the harmonics in the top and bottom rows is zero, *i. e.*, δ is zero.

TABLE I.

HARMONIC ANALYSIS OF COIL CURRENT

Order of Harmonic	Mean-square values		Vector average factor
n	$\left(\frac{I_n}{I_{dc}}\right)^2$	$\left(\frac{I_n}{I_e}\right)^2$	$\left(\frac{I_{av}}{I}\right)^2_n$
1	0.811	0.852	0.998
3	0.087	0.091	0.983
5	0.029	0.030	0.953
7	0.013	0.014	0.911
9	0.007	0.007	0.855
11	0.004	0.004	0.787
13	0.002	0.002	0.712
15	0.001	0.001	0.630
	0.954	1.001	

The fundamental frequency of armature coil current is $f = 45$ cycles per second. For the nth harmonic

$$|\alpha d| = 2\pi d\sqrt{\frac{2wfn}{\rho s}}$$

Taking $\rho = 2270$ abohm-cm., corresponding to a temperature of 100 deg. cent., and substituting numerical values, $|\alpha d| = 1.45\sqrt{n}$.

For the top row of conductors $I_b = I_{av}$ and $\delta = 0$, and the extra loss factor for a given harmonic becomes

$$K_n - 1 = (M_r - 1 + 2N_r)$$

TABLE II

LOSS FACTOR FOR THE CONDUCTOR

n	αd	$M_r - 1$	N_r	$K_n - 1$	$(K_n - 1)\left(\frac{I_{av}}{I}\right)^2_n$
1	1.45	0.096	0.347	0.79	0.672
3	2.50	0.635	2.345	5.33	0.477
5	3.23	1.23	4.37	9.97	0.285
7	3.82	1.70	5.72	13.14	0.168
9	4.33	2.07	6.67	15.41	0.092
11	4.79	2.40	7.36	17.12	0.054
13	5.21	2.69	7.88	18.45	0.026
15	5.59	2.96	8.33	19.62	0.012
					1.786

Total extra-loss factor, K_o', for the embedded portion of the upper conductors is 1.79.

Since the length of the embedded portion is one-half of the half-coil length the extra-loss factor for the half coil is 0.89.

Table II summarizes the calculation of losses for harmonics up to the fifteenth. This gives a total extra loss of 1.79 times that due to a constant direct current equal in magnitude to the r. m. s. value of the coil current. For the bottom conductors $I_b = 0$, and the extra loss factor reduces to $K_n - 1 = (M_r - 1)$. The total extra loss factor calculated for these conductors is 0.18. The average extra loss factor for the embedded portion of the conductors is 0.93. The ratio of the copper loss in the coil to the loss which would be produced by the same coil current if uniformly distributed over the cross-section of the conductor is:

$$K_{dc} = \frac{\Sigma (K_n - 1)(I_{av})_n^2 L_1 + I_e^2 L_2}{I_e^2 L_2}$$

where:

$(K_n - 1)$ is the nth harmonic extra loss factor for the embedded portion of the coil.

L_1 is the length of the embedded portion of the coil.

L_2 is the length of the coil.

$(I_{av})_n$ and I_e are as previously defined.

In our case the numerical value of this ratio is 1.47. As the armature copper is about equally divided between the slots and the end connections, in which the loss is assumed to be normal, the total extra loss factor for an armature coil is 0.47. That is to say, the copper loss in the coil is 47 per cent greater than would be produced by the same coil current if uniformly distributed over the cross-section of the conductor. The ratio of the copper loss in the coil to that as computed by the standardization rules of the A. I. E. E. is

$$K_{dc}' = \frac{\Sigma (K_n - 1)(I_{av})_n^2 L_1 + I_e^2 L_2}{I_{dc}^2 \left(1 - \frac{\beta}{\pi}\right) L_2}$$

Where I_{dc} is the maximum value of the trapezoidal coil current and it is assumed that the brushes short circuit $\frac{\beta}{\pi}$ of the armature coils.

In our case the numerical value of this ratio is 1.51.

$$\text{Notice that } K_{dc}' = \frac{1 - \frac{2}{3}\frac{\beta}{\pi}}{1 - \frac{\beta}{\pi}} K_{dc}.$$

Thus it is seen that the armature copper loss for this machine, is more than 50 per cent greater than the value as computed in the time-honored manner. Obviously, the reduction in the extra losses of such a winding by laminating and transposing the conductors would result in a higher rating and efficiency.

The ratio of the heat generated in the embedded portion of the top conductors to that in the embedded portion of the bottom conductors is $2.79 \div 1.18 = 2.36$. This is probably a more important consideration than the loss factor for the entire winding, as it measures the non-uniformity of heat distribution between the conductors. If this winding were used in an a-c. generator the loss factor for the winding at the same fundamental frequency would be 1.22. Notice that, in this case, the extra loss factor for the commutated current is more than double the extra loss factor for the sinusoidal current.

Discussion

M. L. Henderson: It has been shown that the true copper losses in a d-c. armature with large conductors may be a great deal more than the losses as computed in the time-honored way. It was also brought out that the same principles are applicable to the calculation of losses in a synchronous converter. I should like to discuss this application further.

It has been demonstrated that the loss factor for the conductors of a d-c. armature is greater than for the same conductors if used in an a-c. armature on account of the less uniformity of distribution of the higher harmonic components of the current in the d-c. coil. In a synchronous converter the fundamental component of the direct current in the coil is nearly canceled by the alternating current, but the higher harmonics are still present. Nearly the whole loss, therefore, is due to the harmonics and as the loss factors for them are comparatively high, it follows that the combined total copper-loss factor for a synchronous converter is greater even than for a d-c. machine with the same frequency and conductor dimensions.

As an illustration of this, the copper-loss factors for the armature considered in the latter part of this paper have been computed on the assumption that it is to be used in a six-phase converter operating at unity power factor.

For the conductors in the center of a phase belt, where the fundamental components practically cancel, the loss in the embedded part of the top conductor is 8.6 times as large as it would be if the current distribution were uniform, while the loss in the bottom conductors is nearly doubled.

In the conductors at the edge of a phase belt, where maximum heating is expected, the net fundamental component is a larger proportion of the whole, since the alternating current is not exactly in opposition to the fundamental of the direct current in the coil, but the loss in the top conductor is still 4.4 times as large as if the current were uniformly distributed. Even in the bottom tap conductor there is 50 per cent extra loss. These loss factors are much higher than the ones given for the d-c. armature, which are 2.79 and 1.18 for the top and bottom conductors respectively.

This example is not given as representative of actual converter-design practise, but as a comparison of the loss factors for a-c. machine, d-c. machine, and synchronous converter when the same conductors are used. The relatively greater importance of the consideration of extra copper losses in the converter is evident.

Since a large proportion of the extra loss occurs during the commutating period it increases the effective resistance during that period and helps to produce good commutation. This improvement, however, comes only at the expense of machine rating and efficiency.

R. H. Park: A study of the effects of eddy currents in d-c. machines and converters was made by the organization with which I am associated, about four years ago. At that time I happened to be interested in the subject to some extent, and made some curves which show the distribution of current density from the top to the bottom of a top conductor in a d-c. machine. These curves, which are shown in Fig. 1 herewith, are arranged a little differently from the curves given in the paper.

In computing the data I didn't calculate the harmonics in the usual way but rather I used Heaviside's operational calculus, since it seemed to be well adapted to the problem. The curves show the current to be initially uniform all over the slot. After the commutation is 5 per cent of the way through, the current hasn't changed at all from the bottom up to ¾ of the way to the top of the slot, although it has already become positive at the top. Each succeeding curve shows the distribution at a later stage of the process of commutation. For this case, the current distribution is practically uniform after an interval of 4 or 5 times the period of commutation.

When the operational method of calculation is used it is possible, at least in some cases, to obtain a somewhat simpler formula for the losses than that given in the paper.

In the simple case of two conductors per slot, full pitch, and linear commutation, the extra loss factor as calculated by the operational method can be put in the form

$$K_u = K_o\,[9\,g\,(\theta_o) + h\,(\theta_o)]$$
$$K_l = K_o\,[g\,(\theta_o) + h\,(\theta_o)]$$

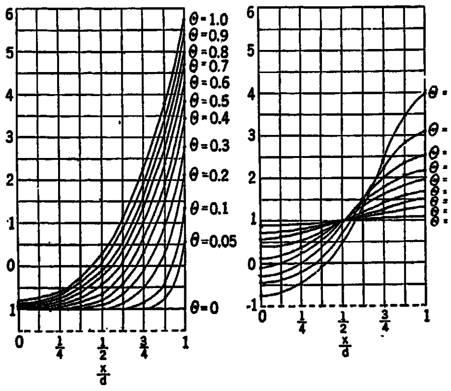


FIG. 1

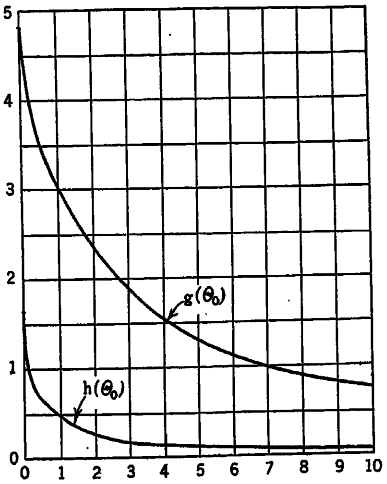


FIG. 2

where, K_u = ratio of extra loss in copper conductor to loss which would exist with a constant d-c. current equal to the maximum value of the trapezoidal coil current.

K_l = same factor for lower conductor.

$$K_o = \frac{8\,w\,d^2 f}{\rho\,s\,\pi}$$

$$\theta_o = \frac{1}{K_o}\,\frac{\beta}{\pi}$$

$g\,(\theta_o)$ and $h\,(\theta_o)$ are functions of (θ_o) as shown in Fig. 2 herewith.

R. F. Franklin: It has long been recognized that there are eddy current losses in the conductors of d-c. machines and synchronous converters similar to those in a-c. machines, and that due to the trapezoidal current wave introducing numerous harmonics, the relative loss should be greater. Several years ago I made such a mathematical investigation and found this to be the case.

The value in this paper lies not only in giving the method of calculation of this loss, taking into consideration the difference in time of commutation of conductors in the same slot, but also in the presentation of experimental data confirming this theory. Without this experimental confirmation the increase of the eddy-current loss due to the trapezoidal current wave might be questioned.

One of the startling results of this investigation is the large increase of coil resistance during the commutating period. It might be further added that this increase in resistance is accompanied by a decrease in inductance. Such a condition should be advantageous to the commutation of the coil, and if the conductor is laminated or transposed in order to reduce eddy-current losses, the gain in losses might be partially offset by poorer commutation. In my investigation of eddy-current losses, I also investigated this problem and concluded that in most cases the effect on commutation was slight. Experience since then has seemed to bear out this conclusion.

In studying the tests made by the authors, one question arose in my mind which I could not settle from the data given. The exploring wires which are fastened to the copper ribbon measure the resistance drop in the end portions of the coil as well as the imbedded portion. Now the non-uniform distribution of current is confined largely to the imbedded portion, so that the resistance drop measured is an average of the drop due to the non-uniformly distributed current in the imbedded portions and the more or less uniformly distributed current in the end portions. If the length of the end portion is not small compared to the imbedded portion, an appreciable error may be introduced. The close agreement between test and calculations would seem to indicate that either this error was small or else had been considered.

B. A. Behrend: The great value of this paper appears to me to lie in its bringing out the importance of current distribution in the conductors of direct current machines. This matter was first referred to in 1905 by A. B. Field and later in 1906 by M. B. Field, the brother of A. B. Field to whom a great debt is due for first presenting the investigation of the phenomena of eddy currents in deep slot-wound conductors. As Mr. Field was associated with me at the time he made his investigation I feel great satisfaction and pride in the value which his work has acquired as the art of designing large machines has progressed. Professor Lyon has done so much to elucidate and elaborate these theories that a paper like the present will once for all dissipate the conception, held by most designers, that the current distribution in the conductors of direct current machines or synchronous converters can be uniform. This is a great service besides the value of the paper with its instructive diagrams and its painstaking calculations including harmonics up to the thirteenth.

As Mr. Park has pointed out, it is possible to attack this subject with the powerful methods of the operational calculus. The electrical engineer must recognize that resistance operators and operational solutions as first propounded by Heaviside forty years ago cannot be ignored much longer. They have come to stay, greatly simplifying the mathematical work and bringing new problems under the scope of the new methods.

The subject is now ripe for a complete review and it is to be hoped that Professor Lyon will give us this and thus bring out the thread which runs so beautifully through all the problems of this nature. Eddy currents in solid slot-wound conductors exhibit in a marked degree the phenomena which occur in solid conductors in air at much higher frequencies. The magnetic field is under control by shaping the slots and, therefore, a great many experimental configurations can be subjected to analysis both experimental and mathematical. By the use of Heaviside resistance operators solutions can be obtained in little time and generalizations effected which are too cumbersome with other methods.

W. V. Lyon: In answer to Mr. Franklin's query, I will say that the length of the end turn was approximately 10 per cent of the length of the coil. All of the results have been corrected for this.

Effect of Transient Conditions
On Application of D-C. Compound Motors

BY LEON R. LUDWIG[1]
Non-member

Synopsis.—*When a d-c. compound motor is started, the rise of current in the series field induces a voltage in the shunt field counter to that applied, and the shunt field current may reach several times normal value in a negative direction. This current reversal gives, in effect, a differentially compounded motor during the starting period and causes the starting torque to be materially reduced. In many applications of compound motors, therefore, predetermined performance is not obtained. In particular cases where the motor is continually started and stopped, the effect may be so serious as to dictate the choice of another type of motor, or one of a special design to minimize the undesirable action.*

In case the compound motor is disconnected from the line while running, and reconnected with no external resistance, the delay in establishing a flux due to the damping action of the shunt field may permit excessive currents to flow.

In other d-c. motors, currents induced in the solid iron portions of the frame and poles exert a damping effect similar to that of the shunt field in the compound motor, which may at times be undesirable. A study of such transients leads to means by which they may be minimized and provides calculation methods for predetermining motor behavior.

* * * * *

INTRODUCTION

IN the compound motor there are two distinct electrical circuits linked by a common flux. The current changes during transient periods are therefore not inter-dependent, though in most motors the current in the shunt field is dependent largely on the current change in the series field. In the series circuit also, there is an induced voltage which depends on the flux linking the two coils as well as on the speed, which in turn is a function of the flux, series current, and time.

There is, in effect, a third closed electrical circuit consisting of the solid iron portions of the frame and poles, in which eddy currents flow exerting an influence on the main flux.

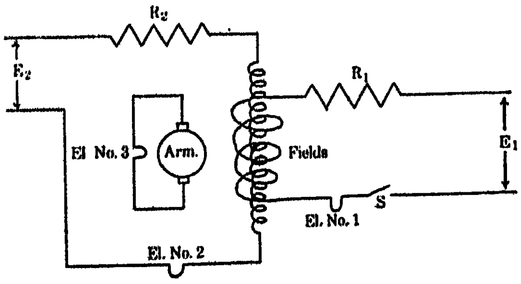


FIG. 1—DIAGRAM OF TEST CIRCUIT

The motor may be started with an initial current in the shunt field or by closing both field circuits simultaneously. In either case the rise of series-field current will induce in both the shunt field and the solid iron a current tending to oppose the establishment of a flux. The induced voltage in the shunt field will be opposite to the impressed voltage and usually will be greater on account of the high transformer ratio between the fields. The shunt field current will usually reverse and restrain the flux from increasing as rapidly as the series current. Under such conditions, the starting torque would be less than with an equivalent straight-series motor.

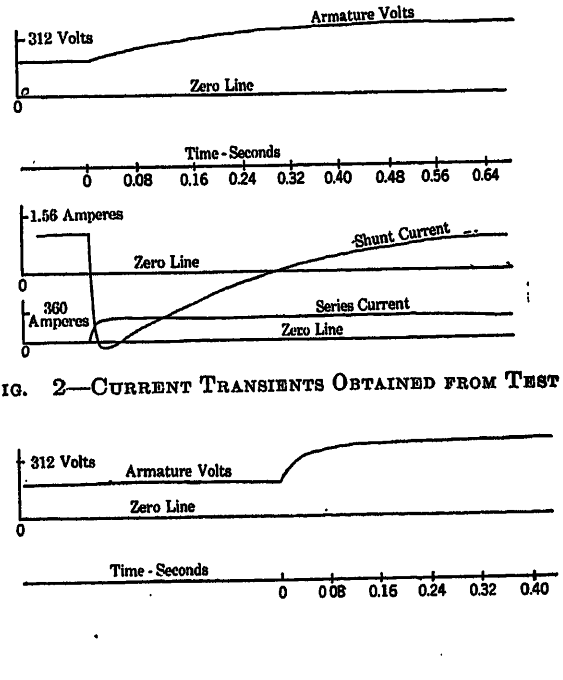


FIG. 2—CURRENT TRANSIENTS OBTAINED FROM TEST

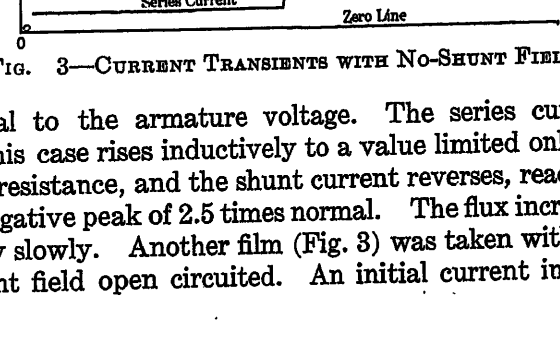


FIG. 3—CURRENT TRANSIENTS WITH NO-SHUNT FIELD

The oscillogram in Fig. 2 was taken with a compound motor connected as in Fig. 1. The machine was externally driven at constant speed in order to obtain the instantaneous field flux which is directly proportional to the armature voltage. The series current in this case rises inductively to a value limited only by the resistance, and the shunt current reverses, reaching a negative peak of 2.5 times normal. The flux increases very slowly. Another film (Fig. 3) was taken with the shunt field open circuited. An initial current in the series field gave the same initial flux as in the first test, and this current was increased to the same final value. It will be seen that the flux rise is much more rapid, though it does not increase directly in proportion to the series current because of the solid iron damping.

---

[1] Westinghouse Elec. & Mfg. Company, East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

The reduction of starting torque due to the shunt field damping is usually obviated by reducing the starting resistance in series with the motor. This procedure, however, causes the energy required for starting the machine to be increased, and also causes the motor current to reach a higher starting peak. The latter may cause sparking at the commutator and would be undesirable from a standpoint of contactor burning if the motor circuit were opened frequently very soon after the machine was started.

Transients in d-c. machines in general have been dealt with by K. L. Hansen in a paper presented at the Midwinter Convention of the A. I. E. E. in 1917, and though the reversal of the shunt field current was cited in this paper, there was no attempt made at that time to find the exact effect of this reversal on the transient motor characteristic.

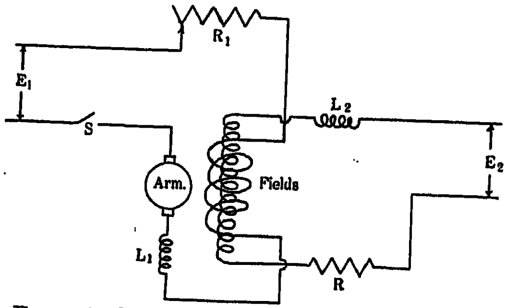


Fig. 4—Diagram of Motor Connections

Analysis of Transient Currents and Flux

In compound machines most of the damping is because of the shunt field. It is also possible to take into account the effect of eddy currents in the iron by properly choosing the circuit constants, and the third electrical circuit may therefore be neglected. If i_1 is the series current and i_2 is the shunt current, the flux will be, neglecting saturation, proportional to a fictitious current (i) which is expressed as

$$i = i_1 + a\,i_2$$

in which a is the ratio of the number of shunt field-turns to the series field-turns. The two fundamental equations, referring to Fig. 4, will be:

$$E_1 = M\frac{d\,i}{d\,t} + L_1\frac{d\,i_1}{d\,t} + R_1\,i_1 + e\,g$$

$$E_2 = a\,M\frac{d\,i}{d\,t} + L_2\frac{d\,i_2}{d\,t} + R_2\,i_2$$

The generated voltage $e\,g$ must be expressed in terms of instantaneous speed and flux. The constant M is the coefficient of mutual inductance. L is the inductance due to the leakage flux linking the series field, plus the armature inductance, and L_2 is the leakage inductance of the shunt field plus any external inductance.

The solution of these equations is given in the appendix (Equations I and II). In Fig. 5 the curves are plotted for the two currents, flux, and torque, assuming that the motor is started with an initial shunt field. As shown by the curves, four-tenths of a second elapses before the shunt field ceases to handicap the series field. It is interesting to note that as the series current decreases when a counter e. m. f. is established, it causes the shunt current to become greater than normal in a positive direction.

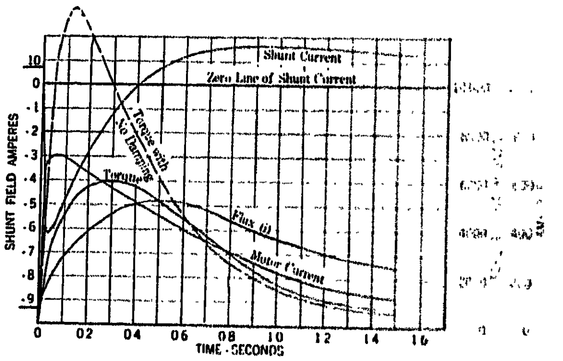


Fig. 5—Calculated Transient Curves with Motor Starting

From these curves it is evident that a very short time elapses before the currents reach their peak values. Since the magnitude of the damping is largely a function of these peak values, it is usually sufficient to obtain only the very early part of the transient in order to determine the effect of damping. The armature of the motor will not begin to rotate until after the current peaks are passed. It is, therefore, often convenient to assume that the motor is blocked and that $e\,g$ in the equation is zero. A solution under this condition is given in the appendix (Equations III and IV). The curves are plotted in Fig. 6, and compare with the test of Fig. 2, except that armature inductance

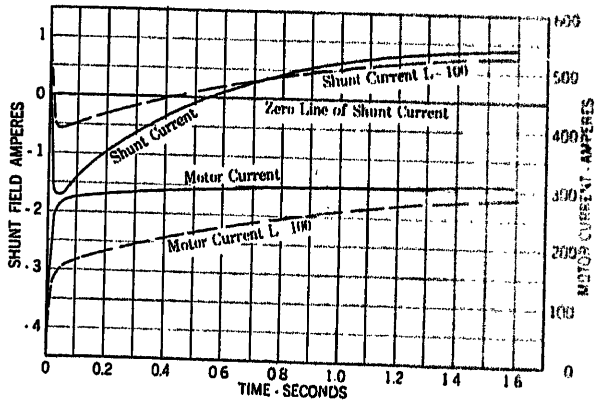


Fig. 6—Calculated Transient Curves with Motor Blocked

was not in the circuit during the test. Furthermore, the effect of saturation was neglected in the calculations.

In case the motor is running and the circuit is opened and then closed before the speed has dropped appreciably, the term $e\,g$ is a function of flux only. Equations (V and VI) in the appendix give this solution, and in Fig. 7 the curves are plotted. It was assumed that the shunt and motor circuits were both opened and closed, or that no initial flux was present. In previous solutions the transients were non-oscillatory; that is, there was no periodic pulsation of current. Under the present conditions, however, the constants must nearly always be such that the final solution contains sine and cosine terms. The phenomenon therefore resembles a condenser charge.

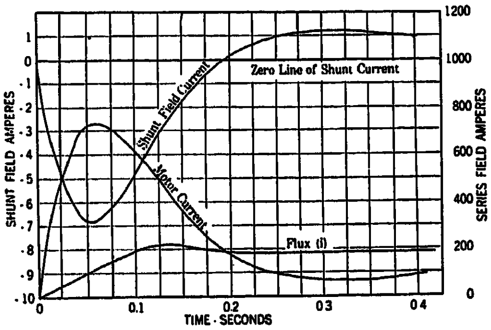


FIG. 7—CALCULATED TRANSIENT CURVES WITH MOTOR RUNNING AND CIRCUIT CLOSED

The load on the motor was assumed to be such that 100 amperes were required under steady-state conditions. The curve of series current during the oscillation decreases below 100 amperes, later regaining this value. The shunt current, also, oscillates past the steady-state value. The time for one cycle of the oscillation is about the same as that required for the transient to disappear.

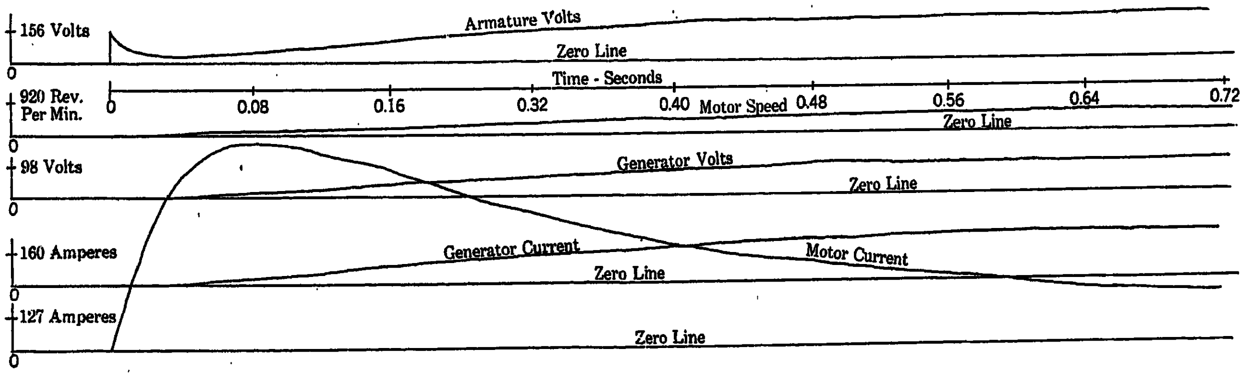


FIG. 8—OSCILLOGRAM OF MOTOR-STARTING TRANSIENTS

RESULTS OF CURRENT TRANSIENTS

In order to find the quantitative reduction in starting torque resulting from the transients described, the curves in Fig. 5 may be compared with curves obtained from an ideally perfect case. This ideal case may be represented by a compound motor in which it is assumed that there is no inter-action between the two fields. The shunt field current will remain constant at its initial value during the transient, if such an assumption is made. The fundamental voltage equation for the series circuit then becomes:

$$E_1 = L\frac{d\,i_1}{d\,t} + R_1\,i_1 + e\,g \qquad \text{(C)}$$

In this equation, L is the sum of L_1 and M. The curves of torque and current resulting from its solution (Equation (VII), Appendix) are also plotted in Fig. 5. The maximum torque is more than twice as great as that obtained with damping. The average value of torque during the starting period, with damping, is 3250 ft-lb., and without damping, is 4600 ft-lb., or the reversal of the shunt field current causes the torque to be decreased to 71 per cent of its value. The iron would saturate during the current peak, however, and the true torque would be less than 4600 ft-lb. if there were no field interaction.

If it were not necessary to increase starting torque by reducing field inter-action, but rather to decrease the current peak and starting energy, additional starting resistance could be inserted in the ideal motor circuit and the same average torque could be obtained with 71 per cent of the average starting current. The starting loss would be decreased as the first power of the current because additional resistance had been added, and the average starting power could be reduced to 71 per cent.

Fig. 8 is an oscillogram taken as a compound motor was started with a shunt generator for load. The curves in Fig. 9 are an analysis of this oscillogram showing how the total input power is divided in the motor. The accelerating power was calculated from the moment of inertia and instantaneous speed. The losses due directly to field inter-action are included in the load loss curve. Actually, the additional current necessary to provide a given starting torque with a weakened field, gives rise to additional and unnecessary copper loss. The curve of "essential input" is the sum of the actual required power, and the area between the two input curves is the loss which may be charged to field inter-action. This loss is within the motor itself and does not include the additional loss incurred in the starting resistance because of field interaction.

The peak of starting current is a function of generated voltage at the time of the peak. With no field inter-action, and consequently a higher flux, this peak will be reduced at the same time the starting torque is increased.

METHODS OF REDUCING FIELD INTER-ACTION

The damping action of the shunt field in magnitude is proportional to the area traced by the curve of shunt current during its decrease, and this in turn is somewhat proportional to its lowest peak. There are three methods of reducing this area:

1. Altering the time constant of the shunt circuit.
2. Placing resistance in the series circuit.
3. Changing design constants.

A study of the curve of shunt current in Fig. 6 shows that the current decreases very rapidly and increases slowly. The increase is a function of the time constant of the circuit itself, but the decrease may be very rapid because the series current is changing at the same time but in the opposite direction and, therefore, the flux need not change rapidly. In fact, if L_1 and L_2 are zero in the fundamental equation, the current may reach its peak in zero time. Such phenomenon has been observed in case of very small leakage between the windings. If the time constant were increased, as by placing the field coils in parallel rather than in series, the shunt current would regain normal value more rapidly, thereby increasing the flux more rapidly. At the same time, however, the current peak is increased in a negative direction and the traced area will be larger than before.

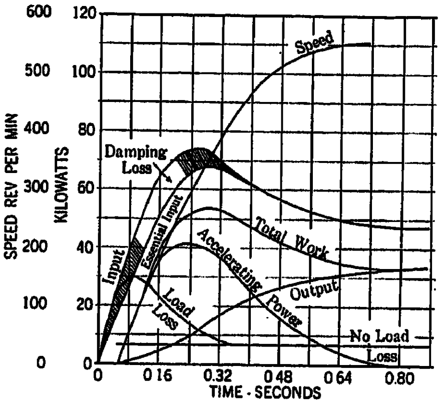


FIG. 9—CURVES PLOTTED FROM TEST OF MOTOR STARTING

If, on the contrary, inductance is added to the shunt field circuit, the negative peak is decreased and the area will be less. In Fig. 6, curves are plotted for the currents and flux, with a large inductance in the shunt circuit. These may be compared with the other curves in the same figure, in which L_2 was 10 henrys. It will be seen that the peak is reduced from 1.7 amperes to 0.5 ampere.

The oscillogram in Fig. 10 was taken with the field coils in parallel and may be compared with Fig. 2. It will be seen that the negative peak has been increased and the area increased even though the current regains steady state more rapidly.

Equation (VIII) in the appendix, which is a simplification of Equation III if the self-inductive coefficients are zero, states that the negative peak of shunt current depends directly on the resistance of the series circuit. Therefore, if the starting resistance is increased the field inter-action is decreased, and within limits the starting torque will remain materially unchanged. In Fig. 11 it will be seen that the series current is less than in Fig. 2 because of additional resistance, and that the negative peak is somewhat reduced.

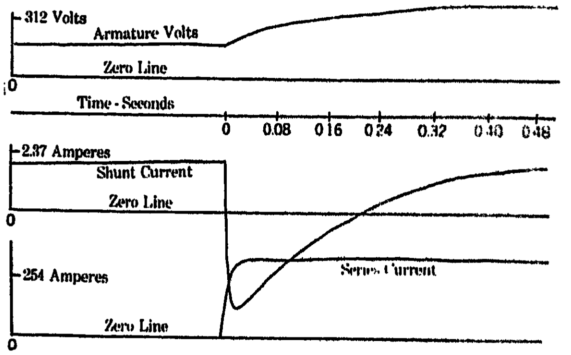


FIG. 10—CURRENT TRANSIENTS WITH SHUNT COILS IN PARALLEL

The design of the motor may be changed by altering the relative copper on the two fields. The worst condition is an equal division. Equation (8) also shows the inter-action to be a function of the turns ratio; and that if more turns were placed on the shunt field and a smaller current were carried by the coils, transient conditions may be improved. It is not always feasible to make such changes, however, and in such cases another type of motor is the best solution. This, though, would rarely ever be necessary.

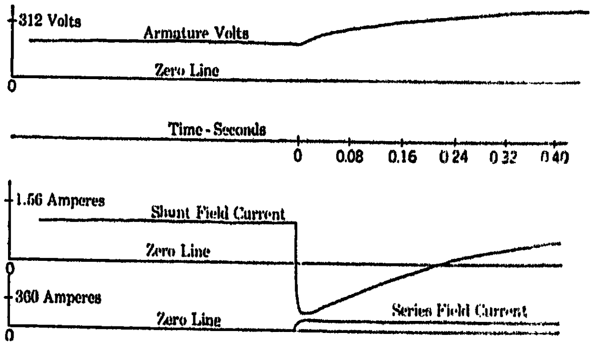


FIG. 11—CURRENT TRANSIENTS WITH INCREASED STARTING RESISTANCE

CONCLUSIONS

It is obvious from the preceding discussion that with many applications of compound motors, the action of the shunt field has no particular disadvantage. It is only in case the starting torque must be all that the motor could possibly supply, or in case the motor is constantly started and stopped, that such phenomena become important.

In case there is doubt as to whether or not the transient conditions will influence compound motor behavior Equations (3) and (4) in the appendix may be used with good accuracy to calculate the initial transient. The effect of saturation should be taken into account, and it may be obtained by first finding the relation of i and time, then plotting the flux against time by the use of a saturation curve of the particular motor involved and the curve of i. The resulting curve of flux and time will be somewhat in error because saturation is still neglected in finding the transients of i_1 and i_2.

In case the shunt field current reverses appreciably, and the flux rise is much slower then it would be neglecting field inter-action, (even though saturation is also taken into account in finding the flux rise in case there was no field inter-action), it is probable that the application would be best fulfilled by a straight series or shunt motor. In the event that a compound motor is desirable, however, for other reasons, the design may be altered by changing the relative amount of copper in the two fields until field inter-action as calculated by Equations (3) and (4) is sufficiently reduced to give satisfactory operation.

In drawing general conclusions from the above, it must be remembered that there are many types of motors so-called compound, including those which have only a small series or shunt stabilizing winding. Because field inter-action is most severe with an equal division of copper in the two fields, this latter group of motors is only slightly affected by the described phenomena. In case of true compound motors, which derive a large portion of their flux from both fields, the general conclusion is that on account of their peculiar transients, they must be applied with care.

Also, because of the variation in magnitude of starting resistance used with the motor, it is not possible to state definite conclusions with regard to a particular motor, without taking into account the exact application conditions. The contents of this paper may therefore serve to aid in checking doubtful applications and in making more clear the general phenomena of compound field inter-action.

Appendix

EQUATIONS FOR MOTOR STARTING

The fundamental equations for obtaining compound motor transients are (from Fig. 4):

$$E_1 = M \frac{d i}{d t} + L_1 \frac{d i_1}{d t} + R_1 i_1 + e g \qquad (1)$$

$$E_2 = a M \frac{d i}{d t} + L_2 \frac{d i_2}{d t} + R_2 i_2 \qquad (2)$$

$$i = i_1 + a i_2 \qquad (3)$$

If the motor is to be started, the speed and flux will vary with time, and therefore:

$$e g = k \phi s \qquad (4)$$

in which s and ϕ are instantaneous values of speed and flux. The instantaneous speed is

$$S = \int_0^t \alpha \, d t \qquad (5)$$

in which α is the angular acceleration.

Then $$\alpha = \frac{T a}{I} \qquad (6)$$

$(T a)$ is the accelerating torque and equals the total torque produced by the motor minus the counter or load torque. The accelerating torque is usually large as compared with the counter torque, and the latter may be neglected during the transient.

Then $$\alpha = \frac{k_1 \phi i_1}{I} \qquad (7)$$

in which I is the moment of inertia.

Combining Equations (4), (5), (6), and (7) gives:

$$e g = k \phi \int_0^t \frac{k_1 \phi i_1}{I} d t \qquad (8)$$

If saturation is not taken into account, ϕ is proportional to i and may be replaced by this fictitious current times a constant.

Equations (1) and (2) are very difficult to solve if $e g$ is replaced by Equation (8), but it may be assumed in Equation (8) that the flux, which does not vary a great deal if the motor is started with an initial field, is constant. The average value of the flux may be used. Equation (8) therefore becomes:

$$e g = K \int_0^t i_1 \, d t \qquad (9)$$

and (1) and (2) are then:

$$E_1 = M \frac{d i}{d t} + L_1 \frac{d i_1}{d t} + R_1 i_1 + K \int_0^t i_1 \, d t \qquad (10)$$

$$E_2 = a M \frac{d i}{d t} + L_2 \frac{d i_2}{d t} + R_2 i_2 \qquad (11)$$

The solution may be obtained easily by use of the Heaviside Expansion Theorem. (See "Heaviside's Operators in Engineering and Physics," by Ernst Julius Berg in the "*Journal* of the Franklin Institute," November, 1924.)

Letting p be the operator $\frac{d}{d t}$ and $\frac{1}{p}$ be the operator $\int d t$ under Heaviside conditions, and from the relation

$$\frac{d\,i}{d\,t} = \frac{d\,i_1}{d} + a\,\frac{d\,i_2}{d\,t}$$

Equations (10) and (11) may be rewritten:

$$E_1 = a\,M\,p\,i_2 + \left(M\,p + L_1\,p + \frac{k_1}{p} + R_1\right)i_1$$

$$= b\,i_2 + A_1\,i_1 \qquad (12)$$

$$E_2 = a\,M\,p\,i_1 + (a^2\,M\,p + L_2\,p + R_2)\,i_2$$

$$= b\,i_1 + A_2\,i_2 \qquad (13)$$

Let $E_2 = 0$ to find the equation for the transients of i_2, then the steady state term $\frac{E_2}{R_2}$ may be added for the complete solution.

From (13) and (12)

$$i_1 = -\frac{A_2}{b}\,i_2 \qquad E_1 = b\,i_2 - \frac{A_1\,A_2\,i_2}{b}$$

$$i_2 = \frac{E_1\,b}{b^2 - A_1\,A_2}$$

Replacing the three constants by their values and simplifying,

$$i_2 = -E_1\,\frac{a\,M\,p^2}{F\,p^3 + G\,p^2 + H\,p + K\,R_2} = \frac{Y\,(p)}{Z\,(p)} \quad (14)$$

where $F = (a^2\,M\,L_1 + L_2\,M + L_1\,L_2)$

$G = (a^2\,M\,R_1 + R_1\,L_2 + R_2\,M + R_2\,L_1)$

$H = (R_1\,R_2 + a^2\,M\,K + L_2\,K)$

Equation (14) is termed the operational solution, and it may be transformed into an algebraic solution by means of the Expansion Theorem, the form of which is:

$$i_2 = -E_1\left[\frac{Y_0}{Z_0} + \sum \frac{Y_p\,\epsilon^{p\,t}}{p\,\frac{d\,Z}{d\,p}} \quad . \quad . \quad . \quad . \right]$$

$$p = p_1, p_2 \ldots$$

The roots p_1, p_2, and p_3 may be obtained by solving $Z\,(p)$, the denominator of (14) for p.

The term $\frac{Y\,o}{Z\,o}$ is the steady state solution and in this case, with zero substituted for p in (14), is zero. The final result obtained by substituting in the Expansion Theorem is:

$$i_2 = \frac{E_2}{R_2} - E_1\left[\frac{a\,M\,p_1\,\epsilon^{p_1 t}}{3\,F\,p_1^2 + 2\,G\,p_1 + H} + \frac{a\,M\,p_2\,\epsilon^{p_2 t}}{3\,F\,p_2^2 + 2\,G\,p_2 + H} + \frac{a\,M\,p_3\,\epsilon^{p_3 t}}{3\,F\,p_3^2 + 2\,G\,p_3 + H}\right] \quad (I)$$

In this equation the steady state term has been added.

To find i_1, solve for i_2 in (13); substitute the term for i_2 in (12) and solve (12) for i_1. From this operational solution, the complete solution may be obtained as before, and is:

$$i_1 = E_1\left[\frac{[(a^2\,M + L_2)\,p_1 + R_2]\,\epsilon^{p_1 t}}{3\,F\,p_1^2 + 2\,G\,p_1 + H} + \frac{[(a^2\,M + L_2)\,p_2 + R_2]\,\epsilon^{p_2 t}}{3\,F\,p_2^2 + 2\,G\,p_2 + H} + \frac{[(a^2\,M + L_2)\,p_3 + R_2]\,\epsilon^{p_3 t}}{3\,F\,p_3^2 + 2\,G\,p_3 + H}\right] \quad (II)$$

The curves plotted in Fig. 5 were from the numerical evaluation of these equations in case of starting a 70-hp. compound motor with an external resistance of 0.32 ohm. The ratio of series field ampere-turns to shunt field ampere-turns is 2.5. The constants of the motor are:

$R_1 = 0.378$ $\qquad a = 88$

$R_2 = 0336.0$ $\qquad K = 0.5$

$E_1 = 310$

$E_2 = 230$ $\qquad$ Then $F = 0.635$

$M = 0.012$ $\qquad G = 44.5$

$L_1 = 0.005$ $\qquad H = 177.5$

$L_2 = 10$

The cubic equation for $Z\,(p)$ is

$$0.635p^3 + 44.5\,p^2 + 177.5\,p + 160 = 0$$

The three roots are:

$$p_1 = -65.7 \qquad p_2 = -2.7 \qquad p_3 = -1.45$$

Equations (1) and (2) become:

$$i_2 = 0.685 + 8.5\,\epsilon^{-65.7t} - 18\,\epsilon^{\;2.7t} + 9.5\,\epsilon^{-1.45t}$$

$$i_1 = -770\,\epsilon^{-65.7t} - 350\,\epsilon^{-2.7t} + 1120\,\epsilon^{1.45t}$$

EQUATIONS FOR MOTOR BLOCKED

In case the motor is blocked, $e\,g$ is zero and equations (12) and (13) become:

$$E_1 = i_1\,(R_1 + p\,L_1 + p\,M) + a\,M\,p\,i_2 \qquad (15)$$

$$E_2 = i\,(R_2 + p\,L_2 + a^2\,p\,M) + a\,M\,p\,i_1 \qquad (16)$$

E_2 may be called zero as before if the motor is started with an initial field.

The two operational solutions are:

$$i_2 = \frac{-E_1\,(a\,M\,p)}{F\,p^2 + G\,p + R_1\,R_2} \qquad (17)$$

$$i_1 = \frac{E_1\,(r + p\,L_2 + p\,a^2\,M)}{F\,p^2 + G\,p + R_1\,R_2} \qquad (18)$$

in which $F = (L_1\,L_2 + L_2\,M + a^2\,M\,L_1)$

$G = (R_2\,L_1 + R_2\,M + L_2\,R_1 + R_1\,a^2\,M)$

In this case $Z\,(p)$ is a quadratic and the algebraic roots may easily be found.

The complete solutions are then:

$$i_2 = \frac{E_2}{R_2} - \frac{E_1\,a\,M}{\sqrt{G^2 - 4\,F\,R_1\,R_2}}\left[\epsilon^{\sqrt{\frac{G^2}{4F^2} - \frac{R_1\,R_2}{F}}\,t}\right.$$

$$- \epsilon^{-\sqrt{\frac{G^2}{F^2F} - \frac{R_1 R_2}{4}}\,t} \Big] \epsilon^{-\frac{Gt}{2F}} \qquad \text{(III)}$$

$$i_1 = \frac{E_1}{R_1} + E_1 \Big[\Big(\frac{2\,F\,R_2}{G^2 - G\sqrt{G^2 - 4\,F\,R_1\,R_2} - 4\,F\,R_1\,R_2}$$

$$+ \frac{L_2 + a^2\,M}{\sqrt{G^2 - 4\,F\,R_1\,R_2}} \Big) \epsilon^{\sqrt{\frac{G^2}{4F^2} - \frac{R_1 R_2}{F}}\,t}$$

$$+ \Big(\frac{2\,F\,R_2}{G^2 + G\sqrt{G^2 - 4\,F\,R_1\,R_2} - 4\,F\,R_1\,R_2}$$

$$- \frac{L_2 + a^2\,M}{\sqrt{G^2 - 4\,F\,R_1\,R_2}} \Big) \epsilon^{-\sqrt{\frac{G^2}{4F^2} - \frac{R_1 R_2}{F}}\,t} \Big] \epsilon^{-\frac{Gt}{2F}} \qquad \text{(IV)}$$

These equations were evaluated for the starting of the same motor but with different voltages and resistances. The changes are:

$$E_1 = E_2 = 230$$
$$R_2 = 230$$
$$R_1 = 0.75$$

If $L_2 = 10$,

$$i_1 = 307 - 40\,\epsilon^{-2.1t} - 267\,\epsilon^{-102.7t}$$
$$i_2 = 1 - 3\,\epsilon^{-2.1t} + 3\,\epsilon^{-102.7t}$$

If $L_2 = 100$,

$$i_1 = 306 - 160\,\epsilon^{-1.2t} - 146\,\epsilon^{-130t}$$
$$i_2 = 1 - 1.65\,\epsilon^{-1.2t} + 1.65\,\epsilon^{-130t}$$

An approximate solution will be obtained with the motor blocked for the purpose of showing the dependence of field interaction on certain factors. The fundamental equations might be written, neglecting leakage inductance,

$$E_1 = M\,\frac{d\,i}{d\,t} + R_1\,i_1 \qquad (19)$$

$$E_2 = a\,M\,\frac{d\,i}{d\,t} + R_2\,i_2 \qquad (20)$$

These equations may be rewritten in terms of i.

$$\left(\frac{a^2\,R_1\,M + R_2\,M}{R_2}\right)\frac{d\,i}{d\,t} + R_1\,i$$

$$= \left(\frac{R_2\,E_1 + a\,R_1\,E_2}{r}\right) = K\,\frac{d\,i}{d\,t} + R_1\,i = b \qquad (21)$$

The solution is known to be:

$$i = \frac{b}{R_1}\left(1 - A\,\epsilon^{-\frac{R_1 t}{K}}\right) \qquad (22)$$

in which A is an arbitrary constant. If an initial field is present, when $t = 0$

$$i = \frac{a\,E_2}{R_2}$$

and (22) becomes

$$i = \frac{b}{R_1} - \left(\frac{b}{R_1} - \frac{a\,E_2}{R_2}\right)\epsilon^{-\frac{R_1 t}{K}} \qquad (23)$$

This equation may be differentiated and substituted in (20), from which

$$i_2 = \frac{E_2}{R_2} - \frac{a\,E_1}{a^2\,R_1 + R_2}\,\epsilon^{-\frac{R_1 t}{K}} \qquad (24)$$

The maximum negative peak occurs when $t = 0$, and is therefore

$$i_2\,(\text{max.}) = \frac{E_2}{R_2} - \frac{a\,E_1}{a^2\,R_1 + R_2} \qquad \text{(VIII)}$$

R_2 is small compared to $a^2\,R_1$, so that, approximately,

$$i_2\,(\text{max.}) = \frac{E_2}{R_2} - \frac{E_1}{a\,R_1} \qquad (25)$$

This equation shows that the negative current peak depends on the primary voltage and resistance and the turns ratio.

EQUATIONS FOR MOTOR RUNNING

In case the switch is closed with the motor running at normal speed, the expression for $e\,g$ becomes:

$$e\,g = K\,\phi$$

and it is not necessary to make any assumptions. The voltage equations become then:

$$E_1 = (p\,L_1 + p\,M + R_1 + K)\,i_1 + (M\,a\,p + K\,a)\,i_2 \qquad (26)$$

$$E_2 = a\,M\,p\,i_1 + (p\,L_2 + a^2\,M\,p + R_2)\,i_2 \qquad (27)$$

If the shunt field has no initial current but a voltage is impressed on both circuits simultaneously, the solution may be obtained by solving for i_1 and i_2 with $E_2 = 0$, repeating with $E_1 = 0$, and adding the two solutions to obtain the complete one. The operational solutions are:

$$i_2 = \frac{-\,E_1\,a\,M\,p}{F\,p^2 + G\,p + H} \qquad (28)$$

$$i_1 = E_1\,\frac{(p\,L_2 + a^2\,M\,p + R_2)}{F\,p^2 + G\,p + H} \qquad (29)$$

when $E_2 = 0$, and are

$$i_2 = E_2\left(\frac{p\,L_1 + p\,M + R_1 + K}{F\,p^2 + G\,p + H}\right) \qquad (30)$$

$$i_1 = -\,E\left(\frac{p\,a\,M + K\,a}{p^2\,F + G\,p + H}\right) \qquad (31)$$

when $E_1 = 0$

$$F = (L_1\,L_2 + L_2\,M + a^2\,L_1\,M)$$
$$G = (L_2\,R_1 + L_2\,K + a^2\,M\,R_1 + L_1\,R_2 + M\,R_2)$$
$$H = (R_1\,R_2 + R_2\,K)$$

Solving $Z\ (p)$ for p gives:

$$p = \frac{-\ F}{2\,G} \pm \sqrt{\frac{F^2}{4\,G^2} - \frac{H}{F}}$$

In most cases $\frac{H}{F} > \frac{F^2}{4\,G^2}$ and p_1 and p_2 must be expressed as

$$p_1 = -\ \alpha + j\,B$$
$$p_2 = -\ \alpha - j\,B$$

Equation (**28**) placed in the expansion becomes

$$i_2 = \frac{-\ 2\,E_1\,a\,M}{\sqrt{4\,H\,F - G^2}} \sin \sqrt{\frac{H}{F} - \frac{G^2}{4\,F^2}}\ t\ \epsilon^{-\frac{Gt}{2F}} \qquad (\mathbf{32})$$

This equation forms a part of the complete solution, the remaining part being obtained from (**30**).

The entire solution for i_2 is:

$$i_2 = \frac{E\ (R_1 + K)}{R_1\,R_2 + R_2\,K} - 2\,E\left[\frac{2\,F\,Q + G\ (L_1 + M)}{4\,F^2\,B^2 + G^2} \cos B\,t + \frac{a\,M}{\sqrt{4\,H\,F - G^2}} \sin B\,t + \frac{G\,Q - 2\,F\,B^2\ (L_1 + M)}{4\,F^2\,B^3 + G^2\,B} \sin B\,t\right] \epsilon^{-\frac{Gt}{2F}} \qquad (\mathrm{V})$$

in which $Q = (R_1 + K - L_1 - M)$ and $E_1 = E_2$.

The entire solution for i_1 obtained from (**29**) and (**31**) is likewise:

$$i_1 = \frac{E\ (R + K\,a)}{R_1\,R_2 + K\,R_2} - 2\,E\left(\frac{2\,F\,P + G\,S}{4\,F^2\,B^2 + G^2} \cos B\,t + \frac{G\,P - 2\,F\,B^2\,S}{4\,F^2\,B^3 + G^2\,B} \sin B\,t\right) \epsilon^{-\frac{Gt}{2F}} \qquad (\mathrm{VI})$$

in which

$$P = (R_2 + K\,a - L_2 - a^2\,M - M)$$
$$S = (a\,M + L_2 + a^2\,M)$$

Assuming that K is 1.2, the steady-state motor current will be 100 amperes. Also, $R_1 = 0.058$ $E_1 = E_2 = 230$ $R_2 = 230$. The above equations were evaluated by supplying the other constants for the same motor with the results:

$$i_2 = 1 - (\cos 13\,t + 24.5 \sin 13\,t)\ \epsilon^{-14.2\,t}$$
$$i_1 = 100 + (-\ 100 \cos 13\,t + 2223 \sin 13\,t)\ \epsilon^{-14.2\,t}$$

EQUATIONS WITH NO DAMPING

To obtain a solution for the ideally perfect case of no damping, the fundamental equation is:

$$E_1 = \frac{L\,d\,i_1}{d\,t} + R_1\,i_1 + K \int_0^t i_1\,d\,t$$

It will be assumed that the motor is starting with an initial field. The operational solution is:

$$E_1 = \left(R_1 + L\,p + \frac{K}{p}\right) i_1$$

and the complete solution is:

$$i_1 = E_1\left[\frac{2\,L\,\epsilon^{\sqrt{\frac{R_1^2}{4L^2} - \frac{K}{L}}\,t}}{\sqrt{R_1^2 - 4\,K\,L} - R_1} + \frac{2\,L\,\epsilon^{-\sqrt{\frac{R_1^2}{4L^2} - \frac{K}{L}}}}{-\sqrt{R_1^2 - 4\,K\,L} - R_1}\right] \epsilon^{-\frac{Rt}{2L}} \qquad (\mathrm{VII})$$

If the same constants are used as those in the first part of the appendix, except that $L = L_1 + M$,

$$i_1 = 950\ \epsilon^{-1.4\,t} - 950\ \epsilon^{-20.6\,t}$$
$$i_2 = 0.685$$

Throughout all the above equations, the action of the eddy currents in the solid iron may be taken into account approximately by using a fictitious value of M. The action of the eddy currents is to permit more rapid current changes than would otherwise be possible, which means that M must be reduced. The proper value may be found experimentally by taking with an oscillograph the rise of current in either field with the other field open circuited. The value of inductance calculated from a curve thus obtained will be low even if the iron does not saturate, and this value used in the equations will approximate the eddy current damping.

Discussion

R. F. Franklin: There is one important point which Mr. Ludwig has not discussed. The effect of the induced shunt-field current in retarding the flux build-up and reducing the motor starting torque has been ably treated; the effect on commutation, another important consideration, has been neglected. During transient conditions, commutation is affected by both the magnitude of the armature current and the rate of flux change in the motor. For various reasons, enumerated by the author, it is desirable to keep the starting current a minimum, and from a commutation point of view this is desirable also. However, with regard to the flux change, for high torque it is desirable to allow the flux to increase as rapidly as possible, but for best commutating conditions the flux should change only slowly. And so we have two operating conditions which require conflicting features of design and in the consideration of one the other cannot be neglected.

L. R. Ludwig: It must be remembered that there are two applications, in one sense of the word, for compound motors. The limitation to be met in one of these cases is the torque, and severe applications may dictate maximum possible starting output. It is in such cases as these that the elimination, as far as possible, of field interaction will lead to better compound-motor performance.

If, on the other hand, the application is such that commutation is a limit, then if it is possible to reduce field interaction, and, therefore, to insert additional starting resistance in series with the motor, and by this means decrease the peak value of current, commutation can certainly be aided. The changes in field flux in cases with which the paper deals, are so slow (see Fig. 2) that increasing this flux more rapidly will increase the voltage induced in the short-circuited coils by transformer action, to a very small extent. That is, the effect on commutation of more rapid acceleration can be neglected because the decrease of peak armature currents has a much greater beneficial effect on commutation than the more rapid flux change has harmful effect.

Loudspeakers of High Efficiency
and Load Capacity

C. R. HANNA[*]
Associate, A. I. E. E.

Synopsis.—The design of high quality horn-type loudspeakers with moving coil drivers is considered. Efficiency and maximum output capacity obtainable from this type of loudspeaker are calculated. Methods of providing load capacity greater than that possible with a single loudspeaker are described.

* * * * *

I. INTRODUCTION

TWO very desirable properties in loudspeakers besides fidelity of reproduction, are high efficiency and high output capacity. Efficiency is desirable not because power from the source (light socket or batteries) is costly, but because it means that a smaller power amplifier may be used. This greatly decreases the first cost and also the cost of maintenance. In the case of auditorium loudspeakers, high output capacity from the amplifier is obtained usually by the use of high plate potentials. For this reason, it is desirable to make it possible to use amplifiers of smaller rating with consequent lower voltages so that inexperienced operators may readily maintain them. In the case of a home loudspeaker, this is even more desirable.

High output capacity, of course, is more necessary for auditorium loudspeakers than for home loudspeakers. The average small loudspeaker, however, is not capable of radiating sufficient sound without overload when certain kinds of music are being reproduced even for the home.

In the early days of loudspeaker development, loudness and fidelity in reproduction were thought to be incompatible. As time went on, some workers began to reason that perhaps by sacrificing efficiency, the fidelity of reproduction at reasonable volumes could be improved. So, by heavy damping or by the use of vibrating systems in which inertia played the principal role, they made devices which were more faithful in reproducing sounds, but were less efficient than the preceding highly resonant loudspeakers. The success of these developments led many to believe that high efficiency and high quality were not to be found together in loudspeaking devices.

The purpose of this paper is to describe a high quality loudspeaker, the efficiency of which is many times that of the conventional type and of very large load capacity. The author is indebted to many other workers in this field for the great wealth of information that has been made available in recent years[†] through publications.

The new device combines the excellent radiating properties of the slowly expanding exponential horn having a large mouth[1] with the good electromechanical coupling of the electrodynamic or moving coil motor element. Both of these make for high efficiency. They also make for great load capacity, for with a good radiating system, the diaphragm may be small and the motion of the diaphragm slight, making it fairly easy to prevent rattling; and with a moving coil drive, the force does not vary with displacement, eliminating a source of non-linearity common to moving iron types of drivers. It is likely that no other combination of radiating system and motor element has such possibilities of efficiency and load capacity.

II. DESCRIPTION AND DESIGN OF THE LOUDSPEAKER

a. The Horn. Before considering efficiency and load capacity, the design of a system which responds uniformly to a broad band of frequencies must be dealt with. It is well known that the exponential horn is a uniform radiator of sound for a given air velocity at its throat for all frequencies, from the highest down to a certain critical or cut-off frequency determined by the rate of expansion of the horn section.[1] If the equation of the horn is

$$A = A_o \epsilon^{Bx} \tag{1}$$

the cut-off frequency is given by the relation

$$f = \frac{B\,a}{4\,\pi} \tag{2}$$

where a is the velocity of sound in air. If cm. units are used for x, a should be in centimeters per sec. or 34,400. Above this critical frequency, the radiation resistance at the throat of the horn rapidly approaches a constant value of

$$r = \rho\, a\, A_o \tag{3}$$

where ρ is the density of air. The value of $\rho\, a$ is 41.2, this figure representing the mechanical impedance density in dynes/(cm./sec.), (or mechanical ohms) per sq. cm. of area at the throat of the horn. To attach some physical significance to this figure, a throat of one sq. cm. section, having for its mechanical impedance 41.2 ohms, will radiate 41.2 ergs per second of sound power when the velocity of air at its throat is one cm. per second. This relation, of course, is exactly analogous to the electric case of a resistance of 41.2 ohms.

---

[*] Research Engineer, Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

[†] For all references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-17, 1928.

which would dissipate the same number of watts for a current of one ampere.

The relation given above assumes that the horn is extended to a large area at the mouth so as to transmit the sound energy effectively to the atmosphere. In the paper to which reference is made, it was suggested that the diameter of the mouth (if of circular section) be

$$D = \frac{4}{B} \tag{4}$$

In terms of the wavelength corresponding to the cut-off frequency,

$$D = \frac{\lambda}{\pi} \tag{5}$$

In practise, the mouth may be somewhat smaller without causing serious resonance effects. Horns of sections other than circular should have mouth areas equivalent to the above circle.

The exponential horn is now generally recognized as the superior shape. A recent paper by Ballantine[2] shows that the so-called Bessel horns of the type $A = b\, x^n$ are similar to the exponential horn in their radiation characteristic if n is chosen very large, and a statement is made that, in view of this, the exponential horn loses a great deal of its significance. As a matter of fact, the Bessel horn approaches the exponential horn not only as to radiation characteristic but as to shape as well when n is increased without limit. This fact was shown by the present writer in his discussion of Ballantine's paper.[3] Thus, Ballantine's work in reality gives further evidence of the superiority of the exponential horn.

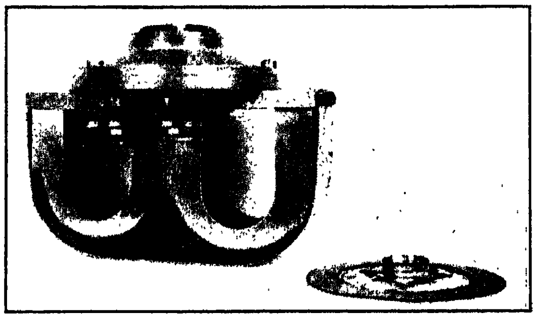


FIG. 1—LOUDSPEAKER ELEMENT

b. Mechanical Vibrating Systems. Fig. 1 shows one form of the new loudspeaker elements with the diaphragm placed alongside. Permanent magnets of cobalt steel supply the magnetic field. A piston or a plunger acting diaphragm with a flexible mounting is chosen because it can be made to have only one resonance in the acoustic range of frequencies; *i. e.*, a lumped mass in combination with a lumped compliance has only one resonance. Flat diaphragms as used in the past have a number of resonance frequencies within the acoustic range because such diaphragms cannot act as plungers. To obtain this plunger motion, the central portion of the diaphragm is stiffened by any of a number of methods of corrugation while the edge is made very flexible, either by mounting on an annular ring of rubber or by suitable corrugation of the metal at the periphery. In the diaphragm shown in Fig. 1, the shape is that of seven intersecting cones with a circular bead at the periphery for additional stiffening. Rubber is used for the flexible mounting.

The driving coil may be attached rigidly to the diaphragm, but it is better if an intervening resilient member of the correct stiffness is placed between the coil and diaphragm. In mechanical systems made up principally of masses and a mechanical resistance, better transmission of motion results if spring members are interposed between the separate masses in the chain.[4] This is similar to the electrical case of a telephone cable in which capacity and resistance predominate and in which the transmission characteristic is improved by inserting loading coils at intervals along the line. In the diaphragm shown, six cantilever arms of a metallic spider are inserted between the coil and the diaphragm.

The diaphragm is made, of course, as thin as is consistent with internal rigidity. With most sizes and shapes of corrugated aluminum diaphragms, the thickness must be at least

$$t = 0.00035\, A_d \tag{6}$$

(A_d = area of the diaphragm) to insure plunger action and adequate strength. The justification of the constant proportionality between thickness and area is that the fundamental resonance frequency of such diaphragms, considering the edge at rest, will be the same, assuming similar contours for all. This internal resonance must of necessity be above the highest frequency it is desired to reproduce, if the motion of the diaphragm is to be like that of the plunger. A fixed lower limit to this resonance frequency requires that the thickness shall not be less than a certain constant times the area.

If an air chamber transformer is used to step-up the velocity of the air on entering the horn, the mechanical impedance imposed on the diaphragm by the horn will be increased by the square of the transformation ratio

$$r = 41.2\, A_o \left(\frac{A_d}{A_o}\right)^2 = 41.2\, \frac{A_d^2}{A_o} \tag{7}$$

Thus, for a given area of diaphragm, the damping may be varied by varying A_o, and the correct value of damping is secured by proper choice of A_o[1]. The transformation from the large area of the diaphragm to the small area of the throat cannot be obtained physically without an air space immediately above the diaphragm. As pointed out previously,[1] this space must be small if the air displaced by the diaphragm is to be forced into the horn at the higher frequencies. Some of the volumetric displacement of air by the diaphragm must go to increase the pressure of air in the chamber and therefore this chamber is the equivalent of a mechanical capacity.[4] The stiffness of the air chamber referred to the diaphragm is obtained by considering its reaction on the diaphragm when the throat of the horn is closed. This stiffness, assuming adiabatic changes, is

$$S_2 = \frac{1.4\,A_d^2}{V_o}\,p_o \text{ dynes per cm.}^4 \qquad (8)$$

where V_o is the air chamber volume and $p_o = 1.01 \times 10^6$ dynes per cm.^{2} or atmospheric pressure.

If the relation between the values of mass of diaphragm, mass of coil, the spring stiffness, air chamber stiffness and radiation damping is properly chosen, it is possible to obtain a device which responds uniformly to all frequencies from the lower cut-off of the horn to an upper cut-off frequency which is determined by the mechanical system itself. The following relations hold for the design if this cut-off frequency is f_c

$$s_1 = \pi^2 f_c^2\, m \qquad (9)$$

$$s_2 = 2\,\pi^2 f_c^2\, m \qquad (10)$$

$$r = \pi\, f_c\, m \qquad (11)$$

where m = mass of diaphragm = mass of coil
s_1 = stiffness of spider
s_2 = stiffness of air chamber
r = radiation damping due to horn.

Having chosen these quantities in this way, the response curve for $f_c = 5000$ will be as in Fig. 2, for a constant driving force F due to current in the moving coil. The equation of this curve is:

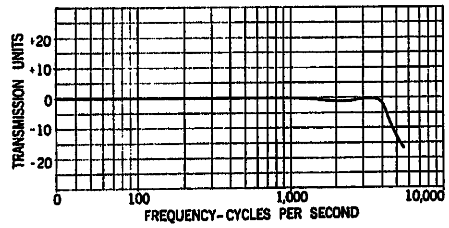


Fig. 2—Calculated Frequency Response

$$W = 10^{-7}\,\frac{F^2}{r}$$

$$\frac{1}{1 + 16\left(\frac{f}{f_c}\right)^4 - 64\left(\frac{f}{f_c}\right)^6 + 64\left(\frac{f}{f_c}\right)^8} \text{ watts} \qquad (12)$$

Over most of the range $W = 10^{-7}\,\frac{F^2}{r}$. This relation does not take into account the falling off effect at low frequencies caused by the stiffness of the diaphragm supporting member or the cut-off of the horn. Because of the fact that the moving coil-type of drive requires no stabilizing restoring force as does the moving iron type, this edge stiffness may be made as small as desired. In like manner if space permits the use of a large horn, the cut-off frequency of the horn may be made as low as desired.

The application of the foregoing will be made clear if an actual design is considered. Suppose we choose a diaphragm of aluminum 1.7 in. in diameter.

$$\therefore\ A_d = 14.7 \text{ cm.}^2$$

The minimum thickness is

$$t = 0.00035 \text{ by } 14.7 = 0.00508 \text{ cm.} = 0.002 \text{ in.}$$

The mass of this diaphragm is

$$m = 2.7 \text{ by } 14.7 \text{ by } 0.00508 = 0.202 \text{ gms.}$$

Considering that the mass of the prongs of the spider resilient member, through which the coil drives the diaphragm, adds to the effective mass of the diaphragm itself, this mass will be increased to approximately

$$m = 0.230 \text{ gms.}$$

The mass of the coil and coil form together should also be approximately 0.230 grams. By careful design, the mass of the form can be made small, so that a good percentage of the 0.230 grams will be useful conducting material.

The stiffness of the resilient member between the driving coil and the diaphragm for $f_c = 5000$ cycles is $s_1 = \pi^2$ by 5000^2 by $0.230 = 56.7$ by 10^6 dynes per cm. This member may be provided by six spokes of a spider, the ends of which are attached to the diaphragm. The spider shown in Fig. 1 is integral with the coil form, the spokes being a continuation of a thin aluminum cylinder. The eddy-current losses in the coil form are negligible for thicknesses of the order of 0.003 in. Each spoke should have 1/6 of the above stiffness or 9.3×10^6 dynes/cm., the dimensions being determined by the formula

$$s = \frac{E\,w\,t^3}{l^3} \qquad (11)$$

where E is the modulus of elasticity (7 by 10 in. for aluminum), and w, t, and l are the width, thickness, and length of the spokes. The formula given is for a cantilever free at one end. In the present case, the width and length of the spokes are chosen and the thickness calculated.

$$w = 3/64 \text{ in.} = 0.119 \text{ cm.}$$

$$l = 3/16 \text{ in.} = 0.476 \text{ cm.}$$

$$\text{From (12) } t = 0.476\sqrt[3]{\frac{9.3 \text{ by } 10^6}{7 \times 10^{11} \text{ by } 0.119}}$$

$$= 0.0224 \text{ cm.} = 0.009 \text{ in.}$$

Next in order is the determination of the volume of the air cavity above the diaphragm. Its stiffness is $s_2 = 2\,s_1 = 113.4 \times 10^6$. The volume is determined by equation (8)

$$113.4 \text{ by } 10^6 = \frac{1.4 \text{ by } 14.7^2}{V_o} \text{ by } 1.01 \text{ by } 10^6$$

$$V_o = 2.7 \text{ cm.}^3$$

To allow for the annular rubber supporting ring, the diameter of the cavity will be ⅛ in. larger than the metallic part of the diaphragm or approximately 1.825 in. The height of the air cavity is obtained by dividing the volume of the cavity by the area corresponding to this diameter.

$$h = 1.60 \text{ mm., or } 0.063 \text{ in.}$$

A uniform height of cavity is provided, because all parts of the diaphragm move through the same displacement.

The required damping and throat area are determined from equation (**11**)

$$r = \pi \times 5000 \text{ by } 0.230 = 3620 \text{ dynes per cm. per sec.}$$

Substituting in equation (**7**) $3620 = 41.2 \times \frac{14.7^2}{A_o}$ from

which $A_o = 2.46 \text{ cm.}^2$

A circle whose diameter is 0.7 in. has this area.

All of the principal dimensions of the loudspeaker element have now been determined. The horn should have an orifice 0.7 in. in diameter and expand slowly enough and have a mouth area large enough to transmit the desired low frequencies. Suppose the required lower cut--off frequency is 100 cycles. From equation (**2**)

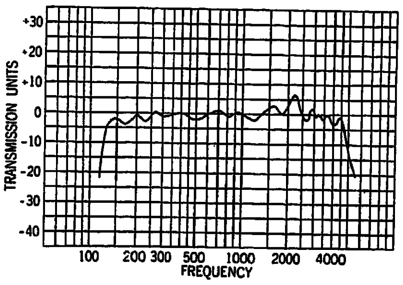


Fig. 3—Experimental Response Curve

$$100 = B \text{ by } \frac{34400}{4\,\pi}$$

$$B = 0.037$$

The equation of the horn is $A = A_o\,\epsilon^{0.037x}$ for cm. units. The mouth of the horn should have a diameter at least equal to one fourth wave length of the lowest frequency or

$$D = \tfrac{1}{4} \text{ by } \frac{34400}{100} = 86 \text{ cm.} = 34 \text{ in.}$$

If the horn is of square section, the mouth area should be equivalent to a circle having the above diameter.

Fig. 3 shows an experimental curve of response against frequencies for the loudspeaker of Fig. 1 when placed on a horn having a 100-cycle cut-off.

III. Calculation of Efficiency

By efficiency we shall mean the ratio of sound output to electrical input to the moving coil. The power required for field excitation if electromagnets are used will not be considered. The real advantage of high efficiency, as above defined, is that a great saving in amplifier capacity is effected. Although the field excitation power usually amounts to as much or more than the electrical input to the moving coil, it is conveniently obtained. Sufficient power for the voice coil however, is not so easy to obtain.

The calculation of efficiency for the design just given will first be made to illustrate the method, and then the general consideration of efficiency will be taken up. Suppose that the mass of conducting material in the coil may be 0.110 grams, without causing the mass of the coil and coil form to exceed 0.230 grams. A coil consisting of 56 turns of No. 40 copper wire, on a form 0.540 in. in diameter, has this mass, and its resistance is approximately 8 ohms. With small coils, the reactance is not sufficiently great to cause the impedance to be much greater than this for frequencies up to 5000. When a voltage is applied to the coil of such a structure, part of it is consumed in resistance drop and part will be expended in overcoming a generated voltage due to motion of the coil in the magnetic field. A convenient quantity which will be used throughout this discussion of efficiency is known as motional impedance.[8] This impedance is obtained by dividing the generated back voltage by the corresponding current in the coil. In the case of this type of loudspeaker, the velocity of the coil is in phase with the current because the driving force is in phase with the current and the velocity over most of the acoustic range of frequencies is in phase with the force. Likewise the generated voltage is in phase with the velocity. Therefore the generated voltage and the current will be in phase and their ratio will be a pure resistance.

This is called motional resistance and is determined as follows. The force per unit current, or force factor, is

$$K = 1/10\,B\,l \quad \text{dynes per ampere} \qquad (\mathbf{13})$$

where B = the flux density in gausses.

l = the length of the moving conductor.

The generated back voltage per unit velocity is

$$E_o/v = B \text{ by } l \text{ by } 10^{-8}$$
$$= 10^{-7} \text{ by } K. \quad \text{volts per cm. per sec.} \qquad (\mathbf{14})$$

The velocity

$$v = \frac{F}{r} = \frac{K\,I}{r} \text{ cm. per sec.} \qquad (\mathbf{15})$$

The back voltage

$$E_o = 10^{-7}\,K\,v = \frac{10^{-7}\,K^2\,I}{r} \text{ volts} \qquad (\mathbf{16})$$

from which

$$R_m = \frac{E}{I} = \frac{10^{-7}\,K^2}{r} \text{ ohms} \qquad (\mathbf{17})$$

Having determined this motional impedance, the efficiency is obtained from the equation

$$N = \frac{R_m}{R_m + R} \qquad \textbf{(18)}$$

In the case at hand, the length of the wire in the coil is 246 cm., and if the magnetic field has a density of 12,000 gausses

$K = 1/10$ by 12,000 by 246

$= 0.295$ by 10^6 dynes per ampere

$$R_m = \frac{10^{-7} K^2}{r} = \frac{10^{-7} \text{ by } (0.295 \text{ by } 10^6)^2}{3620} = 2.41 \text{ ohms}$$

The efficiency

$$N = \frac{2.41}{8 + 2.41} = 0.23$$

It might be expected that the efficiency in the general case will depend upon the flux density, the cut-off frequency, and the conductivity of the material used for the moving coil. An expression for efficiency in general terms will now be developed. Suppose that the coil form has negligible mass and that all the mass of the coil may be considered as useful conducting material. Let

m = mass in grams.
l = total length of conductor, cm.
A = area of one conductor, cm.^{2}
δ = density, gms. per cm.^{3}
ρ = resistivity, ohms per cm.^{3}
B = flux density in gap, gausses.
K = force factor, dynes per ampere in conductor.
f_c = cut-off frequency.
r = required damping, dynes per cm. per sec.
v = velocity of vibration, cm. per sec.
I = current in winding, amperes.

As before, the motional resistance

$$R_m = \frac{10^{-7} K^2}{r} \text{ ohms}$$

But $K = 1/10\, B\, l$

and $r = \pi f_c m = \pi f_c \delta A l$

Therefore $R_m = \dfrac{10^{-9} B^2 l}{\pi f_c \delta A}$

The resistance of the winding $R = \dfrac{\rho l}{A}$

The efficiency $N = \dfrac{R_m}{R + R_m}$

$$= \frac{\dfrac{10^{-9} B^2 l}{\pi f_c \delta A}}{\dfrac{\rho l}{A} + \dfrac{10^{-9} B^2 l}{\pi f_c \delta A}}$$

$$= \frac{10^{-9} B^2}{\pi f_c \rho \delta + 10^{-9} B^2}$$

$$= \frac{1}{1 + 10^9 \dfrac{\pi f_c \rho \delta}{B^2}} \qquad \textbf{(19)}$$

The efficiency is seen to be independent of the size of the unit, if the upper cut-off frequency, the flux density, and the mass conductivity of the coil are the same for all. High flux density and high mass conductivity (low $\rho\,\delta$) are needed. A field of 12,000 gausses is about as great as can be easily obtained. The value of $\rho\,\delta$ is smallest for aluminum being 7 by 10^{-6} as against 14 by 10^{-6} for copper. Curves of Fig. 4 show the efficiency of a 5000-cycle loudspeaker at different flux densities for both aluminum wire and copper wire coils. At 12,000 gausses the ultimate efficiencies are 57 and 40 per cent respectively.

In practical cases the coil form will not have a negligible mass; hence, a smaller amount of wire than mentioned above must be used. This is equivalent to an increase of $\rho\,\delta$ in equation (**19**) with a consequent reduction of N; efficiencies, however, up to 40 per cent for aluminum wire and 30 per cent for copper wire are attainable.

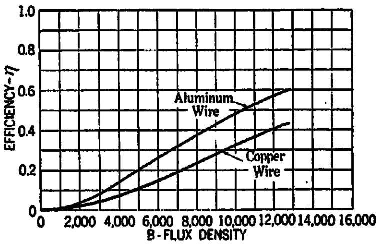


Fig. 4—Ultimate Efficiency of 5000-Cycle Loudspeaker for Different Densities

It is interesting to compare these efficiencies with that of the best hornless loudspeakers of today. A calculation shows the latter to be less than 5 per cent in the range of frequencies up to about 1000 cycles. The gain in efficiency by using the horn results in a great saving in amplifier capacity.

IV. Load Capacity Determination

It might be expected that a loudspeaker of the type here described could be made to have a load capacity as great as desired by increasing its physical dimensions. If the unit is to respond to the high frequencies, however, there is a very definite limit to the size of the unit, and therefore to the power which it may radiate without overload. In all cases this maximum power is less for low frequencies because the maximum diaphragm velocity is smaller. This is due to the limited diaphragm displacement as determined by the clearance in the air cavity. Thus, the figure for maximum power must always be accompanied by a statement of the frequency at which this limit is reached. Since for a given maximum diaphragm displacement, the velocity is proportional to the frequency, and since the power radiated by the diaphragm is proportional to the square of the velocity, the maximum power output is proportional to the square of the frequency. Hence, if one watt is the load capacity of a given loudspeaker at 100 cycles, its capacity at 50 cycles will be only ¼ of a watt while at 200 cycles, it will be 4 watts. The above reasoning falls down in cases where mechanical weakness of the moving parts is the limiting factor.

Before dealing with load capacity in general terms, the maximum output of the particular design of loudspeaker previously given will be determined. Suppose that the maximum displacement of the diaphragm is arbitrarily fixed as that corresponding to a volumetric displacement of air equal to one-half the air chamber volume. In this case, this displacement will be 0.036 in. or 0.092 cms. The power output

$$W = \tfrac{1}{2} v^2 r \text{ by } 10^{-7} = \tfrac{1}{2} (2 \pi f x)^2 r \text{ by } 10^{-7}$$

$$= \tfrac{1}{2} (2 \pi f \text{ by } 0.092)^2 \times 3620 \text{ by } 10^{-7} = 0.605 \text{ by } 10^{-4} f^2$$

When f is 100 cycles, W is 0.605 watts.

A general expression for the maximum load capacity obtainable from a loudspeaker having a given upper cut-off frequency f_c will be derived. It has been shown that the correct damping for a given diaphragm is obtained by a proper choice of throat area for the horn. It will now be proved that all diaphragms which require equal throat area will also require equal diaphragm chamber volumes if the cut-off frequency is fixed. The correct air chamber stiffness

$$s_2 = 2 \pi^2 \text{ by } f_c^2 m = 2 \pi f_c r \tag{20}$$

Also

$$s_2 = \frac{1.4 A_d^2 \text{ by } 1.01 \text{ by } 10^6}{V_o} \tag{21}$$

and

$$r = 41.2 \frac{A_d^2}{A_o} \tag{22}$$

Substituting in equation (20)

$$\frac{1.4 A_d^2 \text{ by } 1.01 \text{ by } 10^6}{V_o} = \frac{2 \pi f_c \text{ by } 41.2 \text{ by } A_d^2}{A_o}$$

from which

$$V_o = \frac{5470}{f_c} A_o \tag{23}$$

Thus, for a given cut-off frequency and a given throat area, the volume of the air cavity is constant.

The volumetric displacement of air in and out of the throat of the horn is definitely limited by this volume, and should preferably not exceed ½ V_o. The power is determined by the frequency, the volumetric displacement of air in and out of the orifice, and the orifice area in the following way;

$$W = \tfrac{1}{2} 41.2 A_o \left[\frac{2 \pi f (\tfrac{1}{2} V_o)}{A_o} \right]^2 \times 10^{-7} \text{ watts}$$

Substituting for V_o

$$W = \tfrac{1}{2} 41.2 A_o \left[\frac{\pi f \text{ by } 5470}{f_c} \right]^2 \text{ by } 10^{-7}$$

$$= 609 \left(\frac{f}{f_c} \right)^2 A_o \tag{24}$$

From this it is seen that a large orifice is desirable to give large amounts of sound output. With available materials for the diaphragm, however, sufficient damping for obtaining a uniform response over the audio frequency range is secured only by the use of fairly small orifice areas. With an upper limit of A_o, therefore, there is a very definite upper limit of load capacity in a unit which responds uniformly up to f_c.

This upper limit of A_o may be shown as follows: It has been stated previously that, with existing materials for the diaphragms, the thickness should not be less than a certain constant times its area.

$$t = 0.00035 A_d$$

The mass of an aluminum diaphragm for this assumption is

$$m = t A_d \text{ by } 2.7 = 0.00035 A_d^2 \text{ by } 2.7 \tag{25}$$

The required damping

$$r = \pi f_c m = \pi f_c \text{ by } 0.00035 A_d^2 \times 2.7 \tag{26}$$

But this value of r is obtained by properly choosing A_o

$$r = \frac{41.2 A_d^2}{A_o} \tag{27}$$

From equations (26) and (27)

$$A_o = \frac{41.2}{\pi f_c \text{ by } 0.00035 \text{ by } 2.7} = \frac{13900}{f_c} \tag{28}$$

Taking $f_c = 5000$

$$A_o = 2.78 \text{ cm.}^2$$

It is seen that A_o is definitely limited to this size no matter what diaphragm area may be chosen.

From equation (24), the maximum power for $f_c = 5000$ is definitely limited to

$$W = 0.68 \text{ by } 10^{-4} f^2 \tag{29}$$

In view of the requirement of great power in the very low register for organ and drum reproduction, this limit is somewhat unfortunate. A knowledge of the limitation, however, prevents one from seeking in the dark for a single unit loudspeaker which will give larger outputs than the above at the low frequencies and still be responsive to the very high frequencies.

V. METHODS OF OBTAINING HIGHER LOAD CAPACITY

There are two avenues of approach to the problem of obtaining greater load capacity. The first and most evident is to use a multiplicity of similar loudspeaker units as above designed, either on separate horns, or on a single horn having a divided or multiple orifice. By this means, the load could be divided equally between the several elements and thus prevent overloading of any one of them. A second and more practical approach problem is based on the following reasoning:

It was noticed that the load capacity of a loudspeaker which responds to 5000 cycles was greater at higher frequencies, and it is possible that such a single-element loudspeaker would handle without overload the upper register of music and speech for very large amounts of power. If a supplementary loudspeaker, (or a group of loudspeakers), could be made to take care of large volumes in the lower register, the different bands of frequencies could be divided properly between the two by means of a suitable electrical network, and thus, a great increase in load capacity could be effected. From the relations already given (equation (**24**)), it is seen that loudspeaker elements which do not have to respond to the higher frequencies can be made to have greater load capacity. Hence, by making sturdier units with heavier diaphragms and larger air chamber volumes, the necessary load capacity for the low-frequency range can be secured.

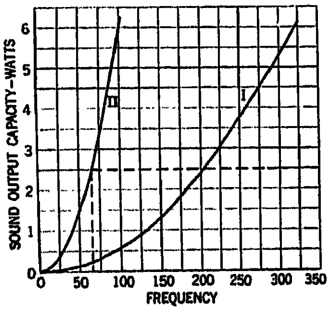


Fig. 5—Output Capacity

This is illustrated by the curves of Fig. 5, wherein curve 1 shows the load capacity of a small unit at different frequencies, this unit being capable of responding to 5000 cycles with uniformity; and curve 2 is for a loudspeaker with a lower cut-off frequency and, while similar in shape, it rises to higher values in the low frequency register. If the power level required is that corresponding to the dotted horizontal line, this low-frequency unit will be capable of supplementing the high-frequency unit down to a frequency corresponding to the dotted ordinate. The increase in power output capacity over what it would have been with higher-frequency unit alone is seen from a comparison of the powers where this vertical line cuts curves 1 and 2. In this case, the increase in power is about ten, whereas, if two similar small units had been used, the power output capacity would have only been doubled.

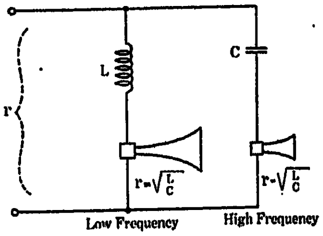


Fig. 6—Circuit Connection for Two-Element Loudspeaker

To make use of an arrangement of this kind, it is necessary to devise an electrical network which will divide the frequency bands properly between the two loudspeaker elements. Of the many circuits which might be employed, the circuit shown in Fig. 6 was chosen because (a) it has constant impedance over the band of frequencies for which the loudspeakers are purely resistive in their impedance, and (b) because for a given electrical input, the sum of the sound outputs of the two loudspeakers in the overlapping range is uniform. It is seen that the condenser in series with a small unit protects it from the damaging effect of large power at low frequencies, while the inductance in series with the low-frequency element prevents the higher frequencies from passing through this element which is incapable of responding to them.

In the case of a demand for greater capacity than is possible with two loudspeakers of different size, a third and larger element may be added. A simple circuit for three is shown in Fig. 7. This is seen to be an extension of the circuit of Fig. 6 by the insertion of a capacity in series with the square, paralleled with a series combination of large unit and inductance. This circuit may also be made to have constant impedance over the band of acoustic frequencies and, by proper choice of the inductances and condensers, it may be made to divide the frequencies between the several elements so as to secure the maximum overall output capacity, as well as a uniform over-all response to a wide band of frequencies. Dr. J. Slepian of the Westinghouse Research Laboratories is responsible for the suggestion of this circuit.

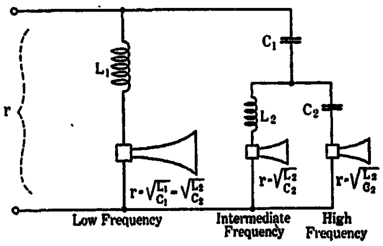

C_1
L_1
r
L_2
C_2
$r=\sqrt{\frac{L_1}{C_1}}=\sqrt{\frac{L_2}{C_2}}$
$r=\sqrt{\frac{L_2}{C_2}}$
$r=\sqrt{\frac{L_2}{C_2}}$
Low Frequency
Intermediate Frequency
High Frequency

Fig. 7—Circuit Connection for Three-Element Loudspeaker

It is evident that the use of two or more loudspeakers in this way is for an entirely different purpose from that of earlier attempts to secure a more uniform response to a wide band of frequencies by the use of several loudspeakers covering different ranges. In the present case, the small unit alone is capable of uniform response to a sufficiently wide band of acoustic frequencies for high-quality reproduction, but is found limited as to its output capacity for large power applications. The use of one or two supplementary loudspeakers with proper associated electrical network is made not to improve the quality of the reproduction, but to increase the output capacity. Each loudspeaker with its particular horn has a flat frequency response characteristic over a certain range, and in good designs, these ranges overlap considerably. The electrical network is what causes the tapering off of response of the individual loudspeakers in the overlapping range, so as to make the combined outputs uniform.

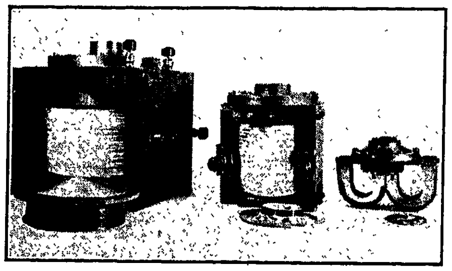


FIG. 8—THREE ELEMENTS FOR MULTIPLE LOUDSPEAKER

It is necessary, of course, to have a large horn on the low-frequency element in order to make it responsive down to very low frequencies. The smaller elements may be used in conjunction with smaller horns, however, since the electrical network allows very little low-frequency current to pass through them with the result that they do not have to radiate low-frequency sounds.

FIG. 9—THREE-ELEMENT LOUDSPEAKER

Fig. 8 shows the three elements of a loudspeaker of this type, with their corresponding moving elements placed in front of each. Fig. 9 shows the appearance of the loudspeaker mounted in cabinet form. This particular one responds from 60 to 5000 cycles with a load capacity of a 10-watt output at 60 cycles for the bass unit.

The details of design of such a combination of loudspeakers have been reserved for a later publication, but the foregoing will serve to illustrate in a general way the method of solving the problem of great load capacity.

VI. SUMMARY

The methods of designing high-quality horn-type loudspeakers, having high efficiency and maximum load capacity, are shown. The limit of efficiency is determined by three factors: the cut-off frequency, the mass conductivity of the material used for the moving coil, and the flux density of the field in which the coil moves. Efficiencies up to 40 per cent are attainable in practise for loudspeakers which are uniformly responsive up to 5000 cycles.

The output capacity of a single element loudspeaker is determined by the upper cut-off frequency and the mechanical properties of the best known shapes and materials for diaphragms. This load capacity is proportional to the square of the frequency and at 100 cycles is approximately 0.6 watt.

A method of providing increased load capacity with greater simplicity than by using a great many similar units is to employ two or more loudspeakers covering different ranges of frequencies, but each having sufficient load capacity for its own range. With such an arrangement, fewer elements are required to give the desired increase in output capacity than if several small elements of similar design are employed.

The writer desires to thank Dr. J. Slepian for valuable suggestions, and Mr. W. O. Osbon for carrying through much of the design work and for his contributions to the development.

Bibliography

1. C. R. Hanna and J. Slepian, *Function and Design of Horns for Loud Speakers*, TRANSACTIONS A. I. E. E., 1924, Vol. 43, p. 393.

2. S. Ballantine, "On the Propagation of Sound in the General Bessel Horn of Infinite Length," *Jour. Franklin Institute*, January 1927.

3. C. R. Hanna, Discussion of Ballantine's paper. *Jour. Franklin Institute*, June 1927.

4. J. P. Maxfield and H. C. Harrison, *High Quality Recording and Reproducing of Speech and Music*, TRANS. A. I. E. E., 1926, Vol. 45, p. 334.

5. C. W. Rice and E. W. Kellog, *Notes on the Development of a New Type of Hornless Loud Speaker*, TRANSACTIONS A. I. E. E., 1925, Vol. 44, p. 461.

6. I. Crandall, "Theory of Vibrating Systems and Sound." D. Van Nostrand.

7. C. R. Hanna, "Design of Telephone Receivers for Loud-speaking Purposes," *I. R. E.*, Aug. 1925.

8. A. E. Kennelly, "Electrical Vibration Instruments," Macmillan.

Discussion

Harvey Fletcher: There are some interesting and novel features connected with Mr. Hanna's paper, particularly the suggested use of three loud speakers for increasing the load capacity. His numerical calculations help to make clear the various factors affecting the efficiency of loud speakers.

It is well, however, to call attention to the fact that loud speakers are now being commercially used which have efficiencies greater than that described in his paper and which have practically a uniform response through a greater range of frequencies than indicated by the curves in his paper. The loud speaker used in connection with the public address system which is used in the Engineering Auditorium is such a one. A paper by E. C. Wente and A. L. Thuras entitled "A High-Efficiency Receiver for a Horn-Type Loud Speaker of Large Power Capacity" which appeared in the last issue of the *Bell System Technical Journal*, describes the design and construction of this type of loud speaker. The driving receiver is one of the moving-coil type and is connected to an exponential horn. The essential difference between this one and the one Mr. Hanna described is in the method of connecting the coil to the diaphragm and in the form of the air chamber connecting the diaphragm to the horn. In the design by Wente and Thuras the coil is connected directly to the diaphragm instead of by means of a spider. Also with the form of air chamber which they used, it is possible to use a larger diaphragm without sacrificing efficiency at the high frequencies, thus making it possible to radiate more energy at the low frequencies. In this paper Wente and Thuras also describe three methods of making the measurement of the absolute efficiency of the receiver, (1) by calculation based upon a theory similar to that given in Mr. Hanna's paper, (2) by experimental measurements of the motional impedance of the receiver, and (3) by measuring the acoustic pressure and the mechanical impedance near the diaphragm. The close agreement between the three methods shows the reliability of the measurements. It seems to me that it would be desirable if some measurements of this sort were made upon the type of receiver described by Mr. Hanna.

The type of loud speaker having such a high efficiency is in commercial use in connection with the Vitaphone and Movietone types of talking motion pictures. As commercially produced in quantities numbering several thousand, an average efficiency of the order of 30 per cent has been realized.

I should like to ask Mr. Hanna how the response curve shown in Fig. 3 of this paper was obtained and also what the zero line represented, and what sort of argument he used for deducing Eq. 6. In the Vitaphone loud speaker which was described by Wente and Thuras a diaphragm of about one-half the thickness as called for by this formula was used and it has been found to be free from any such resonance as described in his paper.

A. E. Kennelly: The paper relates to conditions in an electric circuit and an associated "acoustic circuit." While it is not strictly accurate to speak of "acoustic circuit," yet there are so many points of resemblance between an acoustic system and an associated electric system, that the inaccuracy may be overlooked for the sake of the convenience. The power input of the acoustic system is on the electric side I^2R, and on the acoustic side $v^2\,r$, where v is the velocity and r the acoustic resistance, both in appropriate units. Equating the two,

$$I^2R = v^2\,r \quad . \quad . \quad . \quad . \quad . \quad \text{(power units)}$$

It is necessary to employ the same unit of power for both sides. It seems that, at the present time, the c. g. s. electro-magnetic system, the parent of our practical, volt-ampere-ohm system, is the most convenient in which to select the unit of power on both sides of the equation. In that case, I will come out in abamperes, R in abohms, v in cm. per sec., or kines, and r in acoustic abohms. Every one knows what these units mean, and they do not have to be learned afresh. If we do not use the c. g. s. system, we have to "create new unit" names, in order to complete the equation. The mechanical or "acoustic ohm" is not the same as the mechanical or "acoustic abohm," because it belongs to the practical system, and has one of several possible different magnitudes.

I think we are much indebted to the author for giving us a paper in which the quantities involved are expressed in simple arithmetical equations, which can be readily understood, if the proper units are employed.

W. C. Jones: There are two or three points to which I should like to call attention.

The first of these has to do with load capacity. After making certain assumptions relative to the mechanical properties of available diaphragms and the permissible volumetric displacement in the cavity which forms the coupling stiffness between the diaphragm and the horn load, the author concludes that the output from a single loud speaker is limited to approximately 0.6 watt, if it is to respond well at high frequencies. The so-called lumped-constant system has therefore been abandoned in favor of a system in which the various masses instead of being rigidly coupled are joined by members of moderate stiffness. In this connection it is of interest to note that the experience gained in connection with the production of talking motion pictures, has shown that a single loud speaker of the lumped-constant electrodynamic type can be operated at output levels three to four times higher than the limiting level set by this paper without serious distortion or sacrifice in high-frequency response. The limiting factors appear to be heating and mechanical strength of the moving parts rather than distortion due to air space phenomena.

I should like to ask in connection with Fig. 3 whether the efficiency based on motional impedance measurements and computed in accordance with equation 18 is substantially uniform from 150 cycles to 4000 cycles. It is my impression that, if the acoustic output from the motor element to the throat of the horn bears the same relation to the electrical input at all frequencies, the response curve measured at a point in front of the loud speaker will not be flat but will rise at the higher frequencies.

One other point is the matter of field density. All who have worked with electrodynamic structures appreciate the importance of securing high flux densities in the air-gap, and will, I believe, agree with Mr. Hanna that an air-gap density of 12,000 is about as high as is obtained commercially in circuits involving a good grade of magnetic iron. In this connection it is of interest to note that certain of the cobalt alloys have higher saturation densities than magnetic iron, and make possible air-gap densities of the order of 20,000 without excessive dissipation of power in the field winding.

C. R. Hanna: The paper by Messrs. Wente and Thuras, mentioned by Mr. Fletcher in his discussion, was published subsequent to the time my paper was submitted to the Institute. This paper describes the design of loud speakers which correspond most nearly to the smallest of the three elements described in my paper because the response is uniform up to about 5000 cycles. Since only one type of loud speaker is described, I judge that the method of obtaining sufficient load capacity for auditorium use is to employ a great many of these units coupled to a single horn with a divided orifice or to as many horns. It is interesting to note that in the lower range of frequencies, say around 60 cycles, at least 10 to 15 of these units would be required to provide the same load capacity as is obtained by the use of three dissimilar units as described in my paper.

Mr. Fletcher asks for the reason underlying Eq. 6, in which the thickness of the diaphragm is equal to or greater than a certain constant times the area of the diaphragm. The constant in this equation was determined from experience as one which would ensure piston action of the diaphragm up to 5000 cycles, as well as adequate mechanical strength. Mr. Fletcher states further that the diaphragm as described by Wente and Thuras is approximately half as thick as would be required by the relation given in Eq. 6 and is found to be free from resonance within the acoustic range of frequencies. I note, however, from the curves of the paper by Wente and Thuras, that some resonance does occur in the range of frequencies from 3000 to 5000, and it is my belief that this resonance is caused by breaking up of the diaphragm. In answer to the question regarding the method of taking the response curve which was given in the paper, this was done in the usual way by measurement of radiation pressure with the condenser transmitter at different frequencies in a damped room. The zero line in the graph is arbitrarily chosen.

Dr. Kennelly urges the adoption of a more convenient nomenclature for mechanical and acoustical impedance. There is no doubt that standardization along this line will come about as the number of workers in this special branch of engineering increases.

Mr. Jones asks if the spring method of mounting the driving coil upon the diaphragm was used to give greater load capacity than is possible with the unit structure. This is one of the reasons, the greater load capacity resulting from the greater air cavity possible with the divided structure. He points out that it has been his experience that heating of the moving coil limits the output of loud speakers more than air-cavity clearances. Our experience has been that if the full range of frequencies in music and speech is applied to the loud speaker, limitation in output will occur in the low-frequency range because of cavity clearances. Of course if the low frequencies are not applied to the loud speaker in their correct relative proportion, it is quite possible that greater amounts of power may be applied to the loud speaker or speakers, in which case the coil may become overheated.

Another advantage of the compliance method of mounting the coil upon the diaphragm is that for a given field density the efficiency will be higher than in the unit structure. In the paper by Wente and Thuras, efficiencies of the order of 50 per cent are obtainable only by the use of fields as strong as 20,000 gausses. If such fields were used in the divided structure of coil and diaphragm assembly, efficiencies of the order of 75 per cent would be obtainable for aluminum coils and 60 per cent for copper coils. These figures are based on a weight factor of 90 per cent.

In the loud speakers described in my paper, no attempt has been made to go the limit in obtaining these high efficiencies. With copper-wire coils and field densities of the order of 12,000 the efficiency for the high-frequency units will be of the order of 25 to 30 per cent, which is of the same order as the commercial efficiencies obtained in loud speakers described by Wente and Thuras.

The field density required to obtain equal efficiency in the lower-frequency elements is less than the above because a greater mass of conductor may be used in the moving member. In the bass unit, for example, the field strength is about 5000 gausses, and the coil which drives the diaphragm has a mass of nearly ½ lb.

Certain Topics in Telegraph Transmission Theory

BY H. NYQUIST[1]

Member, A. I. E. E.

Synopsis.—*The most obvious method for determining the distortion of telegraph signals is to calculate the transients of the telegraph system. This method has been treated by various writers, and solutions are available for telegraph lines with simple terminal conditions. It is well known that the extension of the same methods to more complicated terminal conditions, which represent the usual terminal apparatus, leads to great difficulties.*

The present paper attacks the same problem from the alternative standpoint of the steady-state characteristics of the system. This method has the advantage over the method of transients that the complication of the circuit which results from the use of terminal apparatus does not complicate the calculations materially. This method of treatment necessitates expressing the criteria of distortionless transmission in terms of the steady-state characteristics. Accordingly, a considerable portion of the paper describes and illustrates a method for making this translation.

A discussion is given of the minimum frequency range required for transmission at a given speed of signaling. In the case of carrier telegraphy, this discussion includes a comparison of single-sideband and double-sideband transmission. A number of incidental topics is also discussed.

* * * * *

SCOPE

THE purpose of this paper is to set forth the results of theoretical studies of telegraph systems which have been made from time to time. These results are naturally disconnected and in order to make a connected story it has been necessary to include a certain amount of material which is already well known to telegraph engineers. The following topics are discussed:

1. The required frequency band is directly proportional to the signaling speed.

2. A repeated telegraph signal (of any length) may be considered as being made up of sinusoidal components. When the amplitude and phase, or real and imaginary parts, of these components are plotted as ordinates with their frequencies as abscissas, and when the frequency axis is divided into parts each being a frequency band of width numerically equal to the speed of signaling, it is found that the information conveyed in any band is substantially identical with that conveyed in any other; and the bands may be said to be mutually redundant.

3. The minimum band width required for unambiguous interpretation is substantially equal, numerically, to the speed of signaling and is substantially independent of the number of current values employed.

4. A criterion of perfect transmission is selected; and a discussion is given of the characteristics which the received wave must have to be non-distorting with the requirement that the frequency range should not be greater than necessary.

5. Directions are sketched for specifying systems to meet this requirement.

6. Several alternative criteria of distortionless transmission are considered and a method for computing the corresponding transmission characteristics of the circuit is explained and illustrated.

7. An analysis is given of the carrier wave, and it is shown that the usual carrier telegraph requires twice as much frequency range as the corresponding d-c. telegraph, other things being equal.

8. A discussion is given of two alternative methods for overcoming this inefficiency of carrier telegraphy, namely, the use of phase discrimination and of a single sideband.

9. After the d-c. and carrier waves have thus been analyzed a corresponding analysis is given of an arbitrary wave shape, including these two as special cases. Calculations are given on the shaping of the transmitted wave so as to make the received wave perfect.

10. A discussion is given of the dual aspect of the telegraph wave. The wave may be looked on either as a function of ω, requiring the so-called steady-state method of treatment, or as a function of t requiring the so-called method of transients. It is shown that the steady-state theory can be made to yield the information necessary to specify the characteristics of an ideal system.

11. A discussion is given of the effect of interference and departures from ideal conditions.

The economy in frequency range, indicated under 2 and 3, should be considered as a theoretical limit which cannot be attained in practise but which can be approached, the closeness of approach depending on the degree with which the requisite conditions are fulfilled. In practise, it is essential to limit the cost of terminal apparatus and this, in turn, may lead to imperfect utilization of the frequency range.

In certain portions of the paper the discussion is limited to ideal telegraph systems and it is the characteristics of such systems which are referred to above (4, 5, and 6). These ideal systems have certain ideal transmission characteristics; while the reader is given sufficient information to specify suitable equilizers to produce an ideal system, there is no information given as to how to proceed to build equalizers to meet the requirements. It is not within the scope of the paper to enter into the practical questions of cost and detailed construction of signal shaping devices. While these subjects are of great importance, it is thought best to confine the paper to theory. Lest the reader should think this lack of equalizer theory a fatal weakness in the whole theory presented, attention is called to the discussion of generalized wave shape; where directions are given for approaching the ideal case as closely as desired, by shaping the sent wave to take care of any residual imperfections in the transmission characteristic. Finally, under the discussion of interference, mention is made of the effect of departures from the ideal in the matter of signaling speed, transmission characteristic, etc.

---

[1] American Telephone and Telegraph Co., New York, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

SYMBOLS

$A\ (t)$	Indicial admittance.
A_n, A	The coefficient of the nth cosine term in a Fourier series.
a_h, a	Magnitude factor of the hth signal element.
a	General quantity in formulas.
B_n, B	The coefficient of the nth sine term in a Fourier series.
b_h, b	Magnitude of the hth step of a rectangular stepped signal element.
b	General quantity in formulas.
b, b_0, b_1, b_2	Susceptance (coefficient of imaginary part of transfer admittance).
C_n	Real part of discrimination factor.
C'_n, C''_n	Defined in Appendix IV.
γ	Numerical factor near unity.
δ	An arbitrary increment of frequency.
$E\ (t)$	Wave (usually voltage wave) as a function of t.
e	Base of natural logarithm.
$F\ (\omega), F$	Shape factor.
$F'\ (\omega), F''\ (\omega)$	Defined in Appendix IV.
$F_0\, F_\Delta$	Specific shape factors.
$f\ (t)$	Wave form.
g, g_0, g_1, g_2	Conductance (real part of transfer admittance).
$g\ (t)$	Envelope of a carrier wave.
h	Any positive integer, indicates order of signal element.
$H\ (\omega)$	An abbreviation for $\frac{\sin\ \omega/4\,s}{\omega} Y(\omega)\, e^{-i\omega}$
θ	Phase angle of carrier wave.
$I\ (t), I$	Wave as a function of t.
I_h	Wave due to hth signal element.
i	Unit of imaginary quantity, $i^2 = -1$.
k	An integer or zero.
λ	Variable under integration sign.
m	An integer or zero.
N	Number of signal elements in one repetition of a signal.
n	Any integer or zero, indicates order of sinusoidal component.
p	$2\,\pi$ times frequency of repetition of a signal.
q	An integer or zero.
R	A constant.
r	A subscript referring to receiving end.
S_n, S	The coefficient of the imaginary part of the discrimination factor.
S'_n, S''_n	Defined in Appendix IV.
s	Speed of signaling.
s	A subscript referring to sending end.
T	Duration of one signal repetition.
t, t'	Time.
τ	An arbitrary interval of time.
$Y\ (\omega), Y, Y_0$	Transfer admittance.
Y_1 and Y_2	Components of Y.
$\overline{Y}$	The conjugate of Y.
ϕ	Phase shift of a given circuit.
ω	When positive, equals $+\,2\,\pi$ times the frequency; when negative equals $-\,2\,\pi$ times the frequency.
ω_c	$2\,\pi$ times carrier frequency.

PRELIMINARY DISCUSSION

A simple telegraph system is obtained by connecting a telegraph key and battery in series with a line at one end, the other end of the line being connected to a sounder, both ends of the system being grounded. Signals are transmitted by opening and closing the key, which causes corresponding movements of the sounder at the receiving end if the system is properly designed and adjusted. Suppose it is desired to transmit the Morse letter *a* over the system. This may be done by closing the key during one unit of time, opening it for one unit of time, closing it for three units of time and finally opening it for a sufficient period to assure the receiving operator that the signal is completed. This simple telegraph system illustrates two characteristics of telegraph communication as the term is here used which will serve, for the purposes of the present paper, to distinguish telegraphy from other forms of electrical communication. These characteristics are:

1. The time is divided into approximately equal units which will be called time units.
2. There is a finite number of conditions, and each time unit is characterized by a single one of these conditions.

In the simple illustration given, where the first time unit is characterized by closing, the result is the "dot" of the letter *a*. The second time unit is characterized by opening the key. Thus, the total number of conditions is two, open and close. This is in sharp distinction to the case of telephony where there are neither simple numerical relations between the various time intervals, nor a finite number of possible current values.

In this simple illustration the rate with which the new condition establishes itself throughout the system is not generally the same on opening and closing. This is because there is a change in the terminal impedance as well as in the voltage applied. The system would be simpler, mathematically, if the keys were arranged to substitute for the battery an impedance equal to that of the battery. It will be assumed in this paper that the impedance is not altered when one condition is substituted for another. This is substantially true for polar telegraphy but not for single-line telegraphy.

In the system under discussion there are two conditions, battery and no battery. In the usual forms of polar telegraphy, there are also two conditions, corresponding to positive and negative battery potentials impressed at the sending end. In well-known forms of submarine telegraphy there are three conditions corresponding to positive battery, negative battery, and ground or zero voltage. Throughout the paper, unless otherwise stated, the discussion will apply to any number of conditions and will, therefore, include the cases enumerated as special cases.

Due to the distorting effect of the telegraph circuit the part of the total signaling wave which is originally associated with a given time unit does not remain confined to one time unit throughout. There is overlapping between the wave portions originating in successive time units. It is, of course, important to follow the history of a given contribution to the signal wave regardless of whether it is prolonged to occupy more time than it did originally. To this end the term *signal element* is introduced and is defined as the contribution to the signal ascribed to a given time unit. Whereas a time unit is of a definite duration uniquely determined by the speed of signaling, the signal element ascribed to a specific time unit may, by the use of signal shaping, be made to extend into adjacent time units.

The concept which has been discussed under the term "conditions" is an important one and it is desirable to formulate it more precisely. It will be assumed, when not otherwise stated, that the shapes of the successive signal elements are the same, so that they differ only by a factor which may differ from element to element. For illustration: in the case of the submarine cable system the wave shape of the impressed voltage is the same in all three conditions, being rectangular, and there are three proportionality factors $+a$, 0, and $-a$. In the general case any signal element may be expressed by the product

$$a_h f(t)$$

where a_h is a real factor which may vary from one signal element to another and where $f(t)$ is a function of time. The origin of t is, of course, a fixed instant with respect to the signal element. The function $f(t)$ will be called the *wave form*. It is determined by the wave form at the sending end, and by the transmission characteristics of the system between the sending end and the point under consideration. It is not affected by the particular signal or form of intelligence being transmitted over the system. The factor a_h will be called the *magnitude factor*. It differs from one signal element to another but is the same in all parts of the system and, in fact, in all systems transmitting the same signals.

Speed of signaling, s, is usually specified in dots per second and is defined as the number of signal elements per second divided by two.

D-c. telegraph is characterized by a rectangular signal element; the voltage is held constant during the whole time unit.

A telegraph circuit may be considered as a network upon which a signaling wave is impressed at one point, called the sending end, and from which a signaling wave is derived at another point, called the receiving end. In the illustrations given, the sent waves were voltage waves. This is not necessarily the case in order for the discussion to apply. The sent wave may be a current wave, or it need not even be electrical. By providing suitable coupling the sent wave may be mechanical or, say, in the form of variations in a light beam. All that is required is that the system should be linear, including the coupling between mechanical and electrical circuit elements. Similarly with the receiving end it is permissible to consider either the voltage across the receiving device, the current through it, or the motion of the receiving device to be the received wave; provided the device is linear. An important exclusion is the ordinary telegraph relay which is equipped with stops and which, therefore, is a non-linear device. The siphon recorder on the other hand is probably substantially linear. The theory of mechanical impedances and of coupling between mechanical and electrical impedance elements will not be dealt with in this paper.

Proportionality Between Speed of Signaling and Transmitted Frequency Band

It will aid the discussion to assume that the signal considered is repeated indefinitely at equal intervals. While this is convenient, it constitutes no real limitation on the generality of the analysis, because the intervals of repetition may be made as large as desirable. There is nothing to prevent us from making the interval very great, say, an hour or a year.

Take first the case of any arbitrary signal made up of any number of elements and any finite number of different magnitude factors (current values) and repeated an indefinitely great number of times. Such a periodic wave can be resolved into direct current and a series of sinusoidal components. The lowest frequency component has a period equal to the period of repetition of the signal. The next component is of double the frequency or one-half the period of the first. The third component is of triple frequency or one-third period, and so forth. Certain components may, of course, be of zero amplitude, *i. e.*, entirely lacking. While there always is a definite lowest frequency component, there is generally no highest component; in other words, the total number of components is usually infinite.

The transmission medium may alter the magnitude and phase of the sinusoidal components. Some of them may be substantially completely suppressed. Further, the medium may introduce components either because it is non-linear or because interfering waves exist. The net result is a deformation of the original wave.

Now consider another signal which is just like the one discussed except for the distinction that the signal elements of the sent wave are of half the duration, that is to say, everything happens twice as fast and the signals are repeated twice as frequently. It will be obvious that the analysis into sinusoidal terms corresponds, term for term, with the case just considered, the difference being that corresponding terms are of exactly twice the frequency. Now if the second telegraph system transmits currents up to twice the frequency of the first and if, further, the transmitted currents are treated the same in respect to attenuation, phase shift, interference, and other modifying factors as the corresponding currents in the first system (*i. e.*, currents of half the frequency), then the received wave in the second case will be the exact counterpart of the wave in the first case; that is, its deformation will be the same. Generalizing, it may be concluded that for any given deformation of the received signal the transmitted frequency range must be increased in direct proportion to the signaling speed, and the effect of the system at any corresponding frequencies must be the same. The conclusion is that the frequency band is directly proportional to the speed.

It is apparent on examination of existing telegraph systems that the factor relating band width and line speed does, in fact, vary from system to system. The reason for this is that different systems utilize the available range with different effectiveness. In the first place, the various components suffer different attenuation and phase changes from system to system. Secondly, interference varies from system to system; and there are also secondary imperfections. The question then arises: What is the limiting value for this factor under ideal condtions? It is one of the purposes of the following analysis to answer this question.

ANALYSIS OF D-C. WAVE

Initially we shall assume a system which is ideal in all respects of interest. Later on we shall indicate how these ideal conditions can be approximated and what will be the effect of small departures from the ideal conditions.

Let us consider again an arbitrary signal repeated at regular intervals. As has been brought out above, such a signal has an infinite number of components and can be represented by a Fourier series. Suppose that each component is measured at the receiving end, and suppose that proper allowance is made for the known effect of the circuit on each component, it is then possible to construct each component as it was impressed on the sending end. If all the components were received and measured, it would be possible to reconstruct the original wave without deformation. Now for a given wave form, (which in this case is rectangular) the magnitude and phase-angle of the individual sinusoidal components of the impressed wave are determined by the values of the quantities a_h of the signal elements, and equations can be set for computing each component. Conversely these equations can be solved if a sufficient number of them is taken so that the a's are completely determined. Now the number of a's is finite, whereas the number of sinusoidal components is infinite. It follows from simple equation theory that all of the equations cannot be independent; and, in fact, since each component (with rare exceptions)[2] yields two constants, it follows that a number of components approximately equal to one-half the number of signal elements should be sufficient to determine the a's completely, provided the components are chosen to be mutually independent. It is concluded that full knowledge of $N/2$ sinusoidal components is necessary to determine the wave completely. It will be shown below that this number is also sufficient.

The mathematical work of analyzing the wave into its sinusoidal components is carried through in Appendix I. It is there shown that if the magnitude factors of N successive signal elements are given the values a_1, a_2, a_3, etc., up to a_N, and if the signal is then repeated,[3] the expression for the applied voltage can be written in the following form:

$$E\,(t) = A_0/2 + A_1 \cos pt + A_2 \cos 2\,pt + \text{etc.}$$
$$+ B_1 \sin pt + B_2 \sin 2\,pt + \text{etc.}$$

where $p/2\,\pi$ is the frequency with which the signal is repeated and where the A's and B's can be computed from the formula

$$(A_n - i\,B_n) = F\,(\omega)\,(C_n - i\,S_n)$$

in which

$$F\,(\omega) = \frac{8 \sin \omega/4\,s}{\omega/s}$$

$$C_n = (1/N)\,(a_1 \cos n\,\pi/N + a_2 \cos 3\,n\,\pi/N + \ldots + a_N \cos\,(2\,N - 1)\,n\,\pi/N)$$

$$S_n = (1/N)\,(a_1 \sin n\,\pi/N + a_2 \sin 3\,n\,\pi/N + \ldots + a_N \sin\,(2\,N - 1)\,n\,\pi/N)$$

s is the speed of signaling, and

ω is $2\,\pi$ times the frequency of the component.

$F\,(\omega)$ is independent of the values of the a's and depends only on n and the wave form within one signal element, which in the present case is rectangular. Its value is altered if, for instance, curbed signals are used. It may, for convenience, be called the *shape factor*. It should be emphasized that $F\,(\omega)$ is independent of the intelligence transmitted. $C_n - i\,S_n$ on the other hand is independent of the type of wave employed, but depends only on the particular signal which is considered. $C_n - i\,S_n$ is thus determined by the particular intelligence being transmitted and, in turn, determines that intelligence completely. This factor may be called the *discrimination factor*. An examination of the expressions for C_n and S_n discloses the dependence predicted above and will enable us to select components which are independent. Both C_n and S_n are made up of periodic terms, namely, cosines and sines. If the value of n is increased by the value $2N$ each one of these periodic functions has its argument increased by an integral multiple of 2π and is, therefore, unaltered. Similarly if n is increased by N the arguments are increased by an odd multiple of π and all the terms are simply changed in sign. Further, if the expressions for C_n and S_n are written down for the values $N - n$, and if it is remembered that $\cos(\pi - x) = -\cos x$ and $\sin(\pi - x) = +\sin x$, it will be found that

---

[2] B_0 and $A_{N/2}$ are identically zero.

[3] A convenient but not a necessary restriction.

$$C_{N-n} = -C_n \text{ and } S_{N-n} = S_n$$

The results may be written

$$C_{N+n} = -C_n, S_{N+n} = -S_n$$

$$C_{2N+n} = C_n, \text{ and } S_{2N+n} = S_n$$

Successive applications of the last formula give

$$C_{2kN+n} = C_n, \text{ and } S_{2kN+n} = S_n$$

where k is any integer.

These relations are illustrated in Fig. 1, in which the discrimination factor is plotted as a function of n. The signal which is illustrated in this figure may be referred to as the c signal. It is made of 10 signal elements whose successive magnitude factors are 1, 0, 1, 0, 0, 1, 0, 0, 0, 0. An inspection of the figure will show the symmetrical relation between points in adjacent bands of width $N/2$, or 5 in this specific example. Suppose that the c signal is sent in T seconds. Then the frequency corresponding to the fifth harmonic is $5/T$ (generally $N/2\,T$) which is also the speed of signaling.

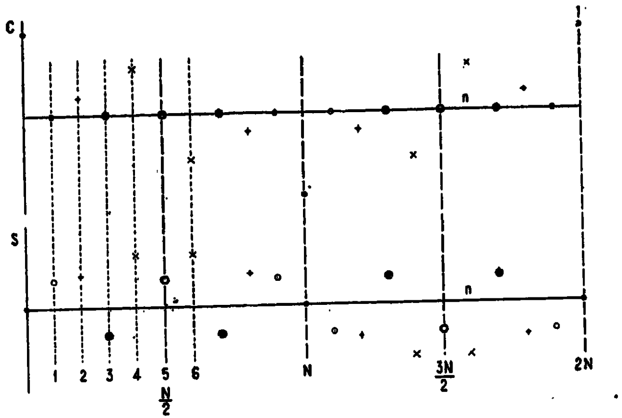


Fig. 1—Discrimination Factor of C Signal

Showing the symmetry and redundancy of successive bands. The corresponding points in the various bands are indicated by the same symbol

If C_n and S_n have been determined throughout any one of the bands, their values (and, $F(\omega)$ being known, also the values of A_n and B_n) are immediately known for the whole range from zero to infinity. Each such band contains all the information about the signal that the totality of bands from zero to infinity contains. One such band (or its equivalent) is necessary and sufficient to determine the original signal. It will be understood that information about $F(\omega)$ is not intelligence in the sense here considered. It is not necessary that the receiving end should have any information about $F(\omega)$ at the sending end and consequently, no frequency range or line time need be set aside for transmitting such information. The frequency range which must be transmitted to specify one band is numerically equal to the speed of signaling.

A special case occurs if the transmitted band is chosen such that the shape factor is zero at some point within it. Then A_n and B_n corresponding to this term, will be zero and the values of C_n and S_n are indeterminate. The mathematical treatment of this case would be long and difficult and it is, moreover, thought that these cases are of little practical interest. For these reasons no further discussion will be given to them.

Distortionless Transmission

It is obvious that with a finite transmitted frequency range and with a rectangular sent wave, there will be deformation of the wave form. It is, however, possible to have a deformed wave and have a receiving device such that a perfect signal is received. Suppose, for instance, that the receiving device records the value of the wave at the mid-instant of the time unit corresponding to each signal element. Then all that matters is the value of the wave at that instant; it does not matter how much it is deformed in intervening intervals. For convenience the term *non-distorting wave*, will be introduced and will be defined as a wave which produces perfect signals. A non-distorting wave may or may not be deformed. The criterion for a non-distorting wave will vary with the manner of receiving the signal. In this paper, when not otherwise stated, the illustration just indicated will be taken as the criterion, *i. e.*, a wave will be said to be non-distorting when the value at the mid-instant of any time unit is proportional to the magnitude factor for the corresponding element. In Appendix II-A, the shape factor for this kind of wave is computed. When the transmission characteristic of the system is such that the received wave has this shape factor, the received wave is non-distorting regardless of the number of distinct magnitude factors employed. It will be understood that this criterion is illustrative only. Later, alternative criteria will be illustrated and discussed sufficiently to enable the reader to construct his own criterion proper for the system in which he is interested.

Before proceeding to determine the characteristics of the telegraph system which insure the ideal shape factor just discussed it is worth while to broaden the subject somewhat by considering what other shape factors are capable of producing the same results. Appendix II-A gives the deduction of more general shape factors which meet the requirements of being ideal. In Fig. 2, *a* is a diagram of the shape factor deduced as ideal, above. It is shown in Appendix II-A that shape factors symmetrical about $2\pi s$, after the manner illustrated at *b* in Fig. 2, do not contribute anything to the current value at the middle of the signal elements. Hence it is deduced that shape factors such as shown at *c* are also ideal in the same sense. There are obviously an infinite number of these shape factors. In addition, there are an infinite number of purely imaginary shape factors having the type of symmetry about $2\pi s$ which do not contribute to the value of the wave at the middle of the signal element. Each one of this infinity of imaginary shape factors may, of course, be combined with any one of the infinity of real shape factors spoken of above.

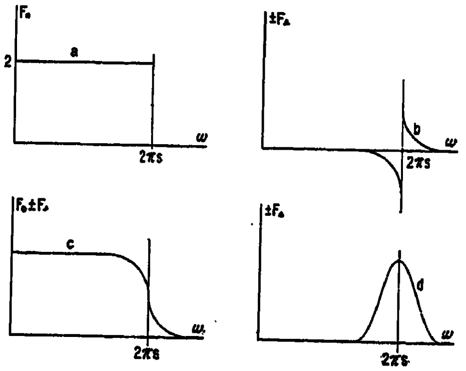


FIG. 2—IDEAL SHAPE FACTORS

Here the criterion for distortionless transmission is that the height of the middle of each signal element should be undistorted

a and *c* represent real shape factors which produce a non-distorting wave,—*b* and *d* shape factors which may be added without producing distortion, the former representing a real and the latter an imaginary value

In order to visualize better the theoretical results obtained so far, these results will be stated in terms of the design of a telegraph circuit. What must be the characteristics of a telegraph circuit in order that the application of the rectangular voltage wave whose shape factor is F_s shall result in the non-distorting received wave just discussed, whose shape factor may be taken as F_r? One answer is that the transfer admittance[4] of the circuit Y should be made such that $F_r = Y F_s$. It will be understood that the quantities Y and F_r may be complex. If the condition just stated were met, transmission would be instantaneous. This imposes an unnecessary restriction on the transfer admittance Y. The signal will still be distortionless if all the components suffer a constant lag τ. If all the components are given a phase lag, which, when specified in angular measure, is directly proportional to the frequency, the received wave is not changed in shape but merely displaced in time and this is, of course, permissible. This leads to the following expression:

$$Y = \frac{F_r}{F_s} (\cos \omega \tau - i \sin \omega \tau)$$

for the transfer admittance where τ is the delay. Once the desired value of Y has been found by this method, the problem is reduced to measuring the phase and attenuation characteristics of the circuit and designing suitable attenuation and phase correctors to make the total transfer admittance equal to Y. The actual design of attenuation and phase correctors does not come within the scope of the present paper.

Appendix II-B has been added for the purpose of making a concrete application. It may be omitted without loss of continuity in the theory.

The criterion of perfect transmission in what precedes has been that the current value at the midpoint of the time unit should bear a constant ratio to the impressed voltage. It will be noted that this is considerably different from the case of ordinary two-condition telegraphy, where the criterion for perfect transmission is that the interval between the instants when the current passes through the mean value (or some other specific value) shall be the same as the corresponding interval at the transmitting end. The principal reason why this criterion has not been used above, is that the corresponding method of reception is not so readily available in the general case where more than two conditions are used. However, it is of considerable practical importance and for this reason, computations are given, (Appendix III), of the shape factor for a wave which has the property in question, *i. e.*, of giving a non-distorting wave in ordinary land-line telegraphy. While the mathematics in Appendix III is carried out for an arbitrary number of magnitude factors, it is suggested that it will simplify the reader's picture to think of two magnitude factors only. Then the discussion applies to ordinary land-line telegraphy. It is there found that a shape factor for the received wave which satisfies this condition is

$$F_r = 2 \cos \omega/4\,s, \text{ when } 0 < \omega < 2\pi s$$
$$F_r = 0, \text{ when } 2\pi s < \omega$$

For d-c. telegraphy (rectangular sent wave) this corresponds to a transmission characteristic given by the equation

$$Y = \frac{F_r}{F_s} (\cos \omega \tau' - i \sin \omega \tau')$$

$$= \frac{\omega}{4\,s} \cot \frac{\omega}{4\,s} (\cos \omega \tau' - i \sin \omega \tau'),$$

$$\text{when } 0 < \omega < 2\pi s$$

$$\text{and } Y = 0, \qquad \text{when } \omega \geq 2\pi s$$

This particular shape factor has the disadvantage that the amplitude is a function of the signal combination; in particular, a long continuation of reversals finally approaches zero in amplitude. To overcome this, it is possible to add shape factors which are symmetrical about the signaling frequency, such as are shown in Fig. 3, and discussed more fully in Appendix III.

---

[4] The term admittance and the notation Y is used to denote the ratio of the numerical value of two waves regardless of their nature. This is a convenient generalization of the customary usage, where this terminology is confined to the case where the numerator of the ratio is a current, and the denominator a voltage.

By way of further illustrating the method, the shape factor is worked out for a received wave having the property that the *area* under the wave for a time unit is proportional to the current at the transmitting end. This case may be of interest in some methods of picture transmission where the integrated exposure over a small interval of time may be important. Other criteria for distortionless transmission suggest themselves. For example, the signal may be taken distortionless when the area under the wave for the center fifth of the time unit is proportional to the sent current. Such cases may be of interest in certain types of printer reception where the current flowing during the center fifth of each element might be the quantity which is of interest. As another example, it may be that the slope of the received current as well as the current value determines the operation of the receiving device, in which case some such function as $a\,I + b\,\frac{d\,I}{d\,t}$ would probably be a suitable quantity in terms of which to define distortionless transmission. It is thought that the reader who is interested in any specific receiving device will have no difficulty either in formulating a suitable criterion or in computing the corresponding shape factor in accordance with the method illustrated. In the meantime, the current at the center of the signal element will be taken as the criterion.

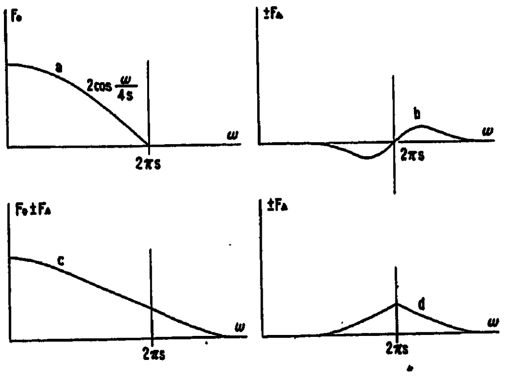


FIG. 3—IDEAL SHAPE FACTORS

In this figure, the criterion of distortionless transmission is that the width or duration of each signal element at the mean-value point should be uddistorted

a and *c* represent real shape factors which produce a non-distorting wave,—*b* and *d* shape factors which may be added without producing distortion, the former representing an imaginary and the latter a real value

ANALYSIS OF CARRIER WAVE

The analysis for the carrier-current wave, analogous to the d-c. wave discussed above, is carried out in Appendix IV. There it is shown that when no care is taken to relate the speed of signaling and the carrier frequency there is no longer a simple separation into a shape factor and a discrimination factor, such as was observed in the case of d-c. telegraphy. The expression for the sinusoidal components is:

$$A_n - i\,B_n = F'(\omega)\,(C_n' - i\,S_n') + F''(\omega)\,(C_n'' - i\,S_n'')$$

If, however, the carrier frequency is taken as an even multiple of the speed of signaling, so that the carrier current is proportional to $\cos\,(2\,\pi\,q\,s\,t - \theta)$, where q is even, it is found that

$$C''_n - i\,S''_n = e^{i2\theta}\,(C'_n - i\,S'_n) = e^{i(\pi q/2+\theta)}\,(C_n - i\,S_n)$$

so that

$$A_n - i\,B_n$$
$$= [e^{-i(\pi q/2+\theta)}\,F'(\omega) + e^{i(\pi q/2+\theta)}\,F''(\omega)]\,(C_n - i\,S_n)$$

which may be written

$$A_n - i\,B_n = F(\omega)\,(C_n - i\,S_n)$$

where C_n and $i\,S_n$ have the same values as in the direct-current case.

$F(\omega)$ is, of course, different in the present case and depends among other things on the phase of the carrier wave at the beginning of the signal element. More specifically the value of the shape factor is given by the expressions:

$$F(\omega) = e^{-i(\pi q/2+\theta)}\,F'(\omega) + e^{i(\pi q/2+\theta)}\,F''(\omega)$$

$$F'(\omega) = \frac{4\,s\sin\,(\omega_c - \omega)/4\,s}{\omega_c - \omega}$$

$$F''(\omega) = \frac{4\,s\sin\,(\omega_c + \omega)/4\,s}{\omega_c + \omega}$$

It is shown in Appendix IV that when the carrier frequency is an odd multiple of the speed of signaling, $(A_n - i\,B_n)$ takes on a slightly different form. However, the following discussion applies equally well whether q is odd or even.

$F'(\omega)$ represents two bands symmetrical about the carrier frequency, each being identical except for sign with the band determined for the direct-current cases. $F''(\omega)$ is a non-symmetrical band and interferes with the symmetry between the upper and lower sidebands. It is shown in Appendix IV that this factor is due to the presence of components higher than the carrier frequency in the modulating wave. Hence, one method of suppressing this factor, and keeping the sidebands symmetrical, is to remove these components from the modulating wave before modulation takes place. A second method of making the shape factor symmetrical is to use an equalizing device, which is so designed as to multiply each component by the ratio

$$\frac{F'(\omega)}{F'(\omega) + e^{i2\theta}\,F''(\omega)}$$

Finally it should be noted that if the band width is small in comparison with the carrier frequency, $F''(\omega)$ is small in comparison with $F'(\omega)$ and, therefore, substantial symmetry is obtained without any special precautions.

If the two sidebands are symmetrical, corresponding frequencies equidistant from the carrier frequency combine to form a wave whose frequency is equal to the carrier frequency and whose envelope is a sine wave corresponding to a component in the direct-current case. The frequency of the envelope equals the difference in frequency between the carrier frequency and that of each of the components forming the pair. On account of the mutual redundance of the two sidebands, the total frequency range required in the symmetrical carrier case (*i. e.*, in the case where the carrier frequency is located in the middle of the transmitted range) for a given speed of signaling is just twice that required in the direct-current case. Except for this, the results as to band width required, and ideal shape factors for the received wave, are the same as obtained above for the direct-current case and it is not necessary to repeat them.

It is obvious that this type of carrier telegraph is relatively inefficient in its utilization of the available frequency range, and it becomes of great interest to inquire how this condition may be improved. An obvious thought is to move the carrier frequency toward one edge of the available band and to increase the speed of signaling in proportion to the interval between the carrier frequency and the far edge of the transmitted range. If an experiment of this kind is undertaken the outcome will be disappointing, unless certain stringent requirements, (explained below), are met. Far from increasing the speed of signaling, it will usually be found that the relocation of the carrier frequency decreases it. This is more fully explained bleow and in Appendix V.

Examining the symmetrical, carrier wave more closely we find that the redundancy residing in having two symmetrical sidebands gives rise to an important property of the received wave. Its frequency is constant, in the sense that the instants of zero value are spaced at constant intervals. This property will be found to restore the intelligence carrying capacity of a given frequency range to the same point as an equal range used in d-c. telegraphy. Let the received signal be $g(t) \sin \omega t$, where the factor $\sin \omega t$ expresses the fact that the frequency is constant. Consider a device which has two sets of input terminals and whose response is proportional to the product of the two waves impressed. Let the wave applied to one set be $g(t) \sin \omega t$, and let the other applied wave be $\sin \omega t$. Then the average response over a cyle is well known to be proportional to $g(t)$, *i. e.*, the envelope. If the second applied wave is $\cos \omega t$, the average response over a cycle is zero. There are various devices whose response is proportional to the product of two inputs. For the purposes of this paper it will be convenient to think of a dynamometer with one fixed and one movable coil.

Phase Discrimination. Systems of telegraphy based on these principles, which may be called phase discrimination systems, have been proposed. Two carrier currents are provided, one of which may be represented by $\sin \omega t$ and the other by $\cos \omega t$. Each of these is then modulated in accordance with a separate signal, and the mixture of the signals is impressed on the line. At the receiving end the composite signal wave is impressed on each of two dynamometers whose other windings are energized by currents of the values $\sin \omega t$ and $\cos \omega t$, respectively. Thus one dynamometer responds to each signal and there is substantially no mutual interference.

It is clear that in this system the total amount of intelligence transmitted for a given band width is twice as great as for the symmetrical carrier case which does not utilize phase discrimination and is, in fact, the same as for the direct-current case.

Single Sideband Transmission. We are now in position to take up the single sideband case. In Appendix V, it is shown that when the carrier frequency is removed from the center of the transmitted band the received wave may be considered to be made up of three component waves. The first of these is an in-phase wave and builds up in the same manner as does a d-c. wave in a low-pass system having a band width equal to the distance from the carrier frequency to the far edge of the transmitted band. This is obviously the wave which we are trying to obtain; and if it existed by itself, the problem would be solved and the experiment referred to above would be successful. The second wave is also in-phase but builds up at a slower rate, namely, that corresponding to the distance from the carrier frequency to the nearer edge of the transmitting band. Since these components are in phase they add algebraically at any given time and it is obvious that the time of building up will tend to be determined by the slower component. Hence, the normal result of moving the carrier from the middle is a slowing up rather than the reverse. Besides, there is the third component wave. This is a quadrature component which is determined in magnitude and rate of building by the frequency band, by which the separation of the carrier and the far sideband exceeds the separation between the carrier and the near edge. This component, moreover, does not build up to the signal wave, but rather to something approximating the first derivative of the signal envelope. It is clear that the second and third components must both be eliminated. In addition, there is an important condition as to phase correction. Ordinary filters are subject to considerable phase distortion in the vicinity of the edges of the transmitted band. It is of importance that this phase distortion be corrected by suitable means. The problem of single sideband transmission may now be separated into three parts.

1. Phase correction.
2. Elimination of the quadrature component, being the third component discussed.
3. Elimination of the sluggish in-phase component, being the second component discussed.

The first of these problems, the design of phase-correcting networks, does not come within the scope of the present paper. The second part of the problem, the elimination of the quadrature component, is solved by the methods of phase-discrimination telegraphy described above. The third part of the problem, the elimination of the sluggish in-phase component, is solved by moving the carrier so far toward the edge that it disappears. The exact requirements to be met are set forth in the appendix.

Generalized Wave Form

Two kinds of impressed wave forms have been discussed, the rectangular, used in d-c. telegraphy, and the sinusoidal, used in carrier telegraphy. It is of interest to generalize the treatment of impressed waves to cover any other wave form which may be considered, and to include these two as special cases. The restriction will be imposed that the successive signal elements are alike, except for a constant factor which is given the values, a_1, a_2, etc., for the various signal elements. There is no need to confine the elements to a single time unit. It may overlap into neighboring time units. To facilitate visualizing the wave, Fig. 4 is drawn, which indicates how an impressed wave may overlap both antecedent and subsequent elements. Mathematically it may be expressed by saying that the contribution made by the hth signal element to the impressed wave is given by the expression

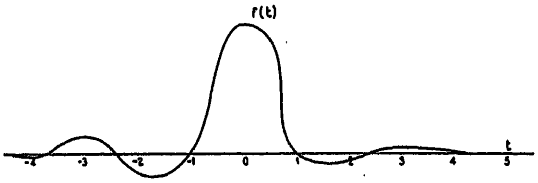


Fig. 4—Specimen of General Wave Form

$$I_h = a_h f(t)$$

where $f(t)$ is an arbitrary function of the time t, whose origin is conventionally taken at the center of the signal element in question. It will be remembered that a_h is called the *magnitude factor* and $f(t)$ the *wave form.* Incidentally, the assumption that the wave form is the same from element to element, excludes the carrier waves where the carrier frequency is not an even multiple of the speed of signaling. We may, therefore, expect to avoid some of the complications which arose in the analysis of those waves. The total wave at any given time is, of course, the algebraic sum of all the components existing at that time.

$$I = I_1 + I_2 + I_3 +, \text{etc.}$$

The Fourier analysis is given in Appendix VI. It is there shown that the sinusoidal components are given by the expression

$$A - i B = F \cdot (C - i S)$$

In other words, there is still the simple separation into two factors, the shape factor and the discrimination factor. The latter is identical with that obtained in the previous cases. The shape factor is determined from the known value of $f(t)$ by the expression

$$F(\omega) = 4 s \int_{-\infty}^{+\infty} f(t)\, e^{-i\omega t}\, d t$$

Now it will be recalled that the shape factor of the received wave is given by the expression

$$F_r = Y F_s$$

and in our previous work, we have evaluated Y, the transmission characteristic of the circuit, so as to obtain a desired value of F_r for a given F_s. In other words, a desired received current is obtained with a given sent wave by means of circuit design. The present results suggest that the desired F_r can be partly obtained by modifying F_s as well; in other words, the desired received current can be obtained in part by circuit design and in part by signal shaping at the transmitting end.

While signal shaping and equalizing are equivalent it does not follow that they are equally practicable. It may be said that under usual conditions the use of networks is the simpler when only moderate accuracy is required; whereas, the use of signal shaping provides the greater accuracy but is complicated.

There is an infinite number of sent waves which result in non-distorting received waves. Thus it is, for instance, permissible to use a sent signal element consisting of a succession of rectangular waves such as shown in Fig. 9; provided the intervals between changes are not made too great. The calculations for this case are given in Appendix VII.

In the introductory part of the paper mention was made of the fact that the design of equalizers would not be covered. It was also stated that a method of signal shaping would be discussed, which would make up for the resulting incompleteness of the theory. The discussion of the present section, together with Appendix VII, in the matter there referred to.

Dual Aspect of Signaling Wave

It will be apparent from what has preceded that there are two distinct methods of specifying the wave. One method is based on the time t as the independent variable. This aspect, which may be referred to as *progressive,* contemplates the signaling wave as a succession of signal elements following one another at constant intervals of time from the first to the Nth. The other aspect which may be called the *cyclical* aspect, is based on ω as the independent variable. This contemplates the signaling wave as the sum of direct current and a succession of sinusoidal components beginning with the fundamental and including harmonics at least up to the $N/2$th. In the cyclical aspect, it is necessary to know only the magnitude and phase of the successive components. It does not matter in specifying the signal what time it took to transmit the signal. It may be of some economic interest to know how rapidly the signal is transmitted, but from a technical standpoint, t is very much in the background. Analogously, with the *progressive* aspect of the wave the frequency is very much in the background. It is not required to specify the wave, but is of economic interest, in the sense that it is of interest to know the frequency range required for transmission.

An enumeration will now be made of analogous processes and quantities under the two aspects. The *signal element* is the unit out of which the signal is made up when considered as a progressive wave; when considered as a cyclical wave the corresponding elementary unit is the *sinusoidal component*. The signal element has been denoted by I_h and the whole wave by $I = \Sigma\, I_h$: The sinusoidal component is specified by A_n and B_n and the whole wave by $I = \Sigma\,(A_n \cos npt + B_n \sin npt)$. The intelligence transmitted is specified in the cyclical aspect by the *discrimination factor,* $C_n - i\,S_n$; in the progressive aspect this office is taken by the *magnitude factor* a_h.

The *wave form* $f\,(t)$ specifies the shape, or form, of the signal element and includes the degree of deformation. The analog in the cyclical case is the *shape factor* $F\,(\omega)$. The alteration in the wave, which is caused by the circuit or any circuit part, is specified in the cyclical aspect by the phase and magnitude changes; and to be definite, it may be taken as the *transfer admittance,* $Y\,(\omega)$, which is a function of ω, and which may be defined as the wave (specified as to magnitude and phase) resulting at the driven point due to impressing a unit sinusoidal wave at the driving point. The analog in the progressive case is the *indicial* admittance, $A\,(t)$, which may be defined to be the wave as a function of t, resulting at the driven point from suddenly impressing a unit wave (constant) at the driving point.

In order to change from one method of consideration to the other, it is desirable to have available equations for changing from the set of quantities suitable for one method to the set suitable for the other. These equations fall into three groups. The first relates the wave form and the shape factor, the quantities which define the type of wave used. The second relates the discrimination factor and the magnitude factor, the quantities which define the particular signal. The third relates the transfer admittance and the indicial admittance, the quantities which define the effect of the circuit on the signaling wave.

The equations are:

$$F\,(\omega) = 4\,s \int_{-\infty}^{+\infty} f\,(t)\, e^{-i\omega t}\, d\,t$$

$$f\,(t) = \frac{1}{8\,\pi\,s} \int_{-\infty}^{+\infty} F\,(\omega)\, e^{-i\omega t}\, d\,\omega$$

$$C_n - i\,S_n = \frac{1}{N} \sum_{h=1}^{\mathrm{N}} a_h\, e^{-i(2\pi n/\mathrm{N})\;(h-1/2)}$$

$$a_h = \sum_{n=-\mathrm{N}/2}^{\mathrm{N}/2-1} (C_n - i\,S_n)\, e^{i2\pi n\;(h-1/2)/\mathrm{N}}$$

$$Y\,(\omega) = -\int_{-\infty}^{+\infty} i\,\omega\, e^{-i\omega t} A\,(t)\, d\,t$$

$$A\,(t) = \frac{Y\,(0)}{2} + \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{Y\,(\omega)\, e^{i\omega t}}{i\,\omega}\, d\,\omega$$

Negative values of the argument ω do not require the conception of negative frequencies. The frequency may be thought of as signless and as represented by positive and negative values of ω or both.

With either set of quantities, given, or found by the equations, it is possible to make corresponding computations in the two systems. For instance, when the shape factor of the sent wave and the transfer admittance are known, the shape factor of the received wave can be computed from the expression

$$F_r\,(\omega) = F_s\,(\omega)\; Y\,(\omega)$$

Analogously, if the sent wave form and the indicial admittance are known, the received-wave form can be computed from the expression

$$f_r\,(t) = \frac{d}{d\,t} \int_{-\infty}^{+\infty} A\,(\lambda)\, f_s\,(t - \lambda)\, d\,\lambda$$

or others which are equivalent. Further, if the transmission characteristics of two circuits connected in tandem are known, their combined characteristic can be computed from

$$Y\,(\omega) = R\, Y_1\,(\omega)\; Y_2\,(\omega)$$

in the one system, and from

$$A\,(t) = R\,\frac{d}{d\,t} \int_{-\infty}^{+\infty} A_1\,(\lambda)\, A_2\,(t - \lambda)\, d\,\lambda$$

or an equivalent. R is a constant in these formulas. It is not the purpose of the present paper to go into circuit relations to any greater extent than is necessary to bring out, clearly, the parallelism and respective self-sufficiency of the two systems. For a fuller exposition of the relations between the various functions of ω, the reader is referred to K. S. Johnson's book, "Transmission Circuits for Telephonic Communication" and for a fuller treatment of the functions of t to J. R. Carson's book, "Electric Circuit Theory and the Operational Calculus."

The formulas shown in the last paragraph indicate that the fundamental parallelism between the two kinds of functions extends to calculations of the received wave. There is a difference, which is not of great significance from the standpoint of pure theory, but which is important from the standpoint of practical computation. It is seen that in the case of functions of ω the operations consist of the multiplication of two characteristics. In fact, by expressing the characteristic in terms of a logarithmic function of Y in the usual manner, it is possible to reduce these operations to additions. In the case of the functions of t the computations are much more complex involving differentiation, the multiplication of two functions, and the the evaluation of an integral. The processes of differentiation and integration, are, in general, essentially graphical operations which cannot be carried out with the same precision as processes which are essentially arithmetical. It is, therefore, as a practical matter, greatly advantageous to arrange the calculations so they can be carried out with functions of ω.

The important fact is that for telegraph purposes the two methods are substantially equivalent in result, though different in processes and labor involved. The fundamental reason for this difference is that, on the theory as developed in the paper, it is not necessary to compute the received wave as a function of t.[5]

The method of analysis used in this paper is that of the Fourier series. It should be mentioned at this point that there is an alternative method, known as that of the Fourier integral. The Fourier integral can be made to yield essentially the same mathematical formulas as have been obtained in this paper. Also, if care is taken, it is possible to obtain essentially the interpretations here given. Appendix IX sets forth the relation between the series and the integral, and points out the problems where special care is needed in applying the integral.

INTERFERENCE

So far it has been assumed that the circuit is free from external interference. In any actual case there is always some interference present, and it is necessary to take it into account. The effect of the interfering wave is to superpose itself on the singaling wave and give the resultant received wave a different value, which otherwise it would not have. If the interference is great enough, the received wave will be interpreted as different from that intended. If the interference is non-predictable, (as likely to be positive, as negative), its arithmetical value must be less than one-half the difference between any two current values employed in signaling. In fact, the number of distinct magnitude factors, hence, the amount of intelligence that can be transmitted over a circuit, are definitely limited by the interference. It is, therefore, important to consider the nature of the interference and to determine to what extent, if at all, it may be overcome.

Interference may be of many kinds and sources, including such telegraph currents as cross-fire and duplex unbalance. We shall go a step further and consider as interference such portions of the telegraph wave as result from departure from the ideal of the telegraph current. Thus the difference between the actual wave and the desired ideal wave is interference. It is realized that this convention is not in agreement with common usage but, for the purposes of this paper, it is justified by the simplification which results. In order to distinguish this interference from other forms it may be called the *characteristic* interference. The term *intersymbol* interference has also been used for this effect. It is closely related to characteristic distortion.[6]

Interference may usually be reduced by suitable means: Duplex unbalance currents may be reduced by improving the balance; characteristic interference may be reduced by improving the transmission characteristic; and interference from other circuits reduced by decreasing the coupling. But when everything practicable has been done to reduce the interference, there is, in general, a residual left which produces distortion in the signals. The total amount of interference sets a limit to the number of distinct magnitude factors which may be employed and, therefore, to the rate with which intelligence may be transmitted over the circuit. The interference which can be tolerated depends on the amount of energy in the signaling current. This is again limited by the power-carrying capacity of the line, either because certain apparatus overloads or because interference into other circuits becomes too great.

It is of considerable interest to determine the spectral distribution of power in the signal which is most suitable for overriding a given interference. The assumption will be made that the maximum power is definitely limited. It is shown in Appendix VIII that when the shape factor is a constant from zero to the frequency s and zero for other frequencies, the mean total power is proportional to

$$\frac{1}{N}\sum_{h=1}^{N} a_h^2$$

and it follows that for this case all signals which are made up of the extreme values of the magnitude factors, (it is assumed that the positive and negative values are numerically equal), load up the system to the same extent. If some other shape factor were used it would, in general, result in the maximum power only for the most favored signal combination; for others, the power would be less. It is concluded that for the purpose of overriding interference, when the power carrying capacity of the line is the limiting factor, the shape factor just considered is the most suitable at the point where the interference is introduced.

---

[5] *Submarine Cable Telegraphy*, J. W. Milnor, A. I. E. E. TRANSACTIONS, Vol. 41, p. 20; in particular the suggestion made in the closing paragraph on p. 38.

[6] *Measurement of Telegraph Distortion*, Nyquist, Shanck and Cory, A. I. E. E. JL., Vol. 46, p. 231, 1927.

In discussing the interference above, it has been assumed that the circuit is linear. If some of the circuit elements are non-linear, the result may be looked on as interfering currents generated in the elements. This interference is predictable and it is theoretically possible to eliminate or reduce it by introducing elements which produce a compensating distortion.

The assumptions have been made that the signal elements are all of equal duration and that the system is designed especially to handle signal elements of that specific duration. In other words, the system is designed for a given speed of signaling in preference to other speeds, both slower and faster. It is of obvious interest to inquire what the effect is when the speed is somewhat different from that for which the system has been designed, the signal elements being of equal length.

Suppose a system is designed to be distortionless at a speed of signaling s, and suppose that it is used to transmit signals of the speed γs, where γ is a factor which does not differ greatly from unity. Let the system be such that the admittance is given by

$$Y(\omega)\ (\cos \omega \tau - i \sin \omega \tau)$$

An admittance which would give distortionless transmission at the speed of γs is then given by the expression

$$Y(\omega/\gamma)\ (\cos \omega \tau - i \sin \omega \tau)$$

The difference between the actual admittance given by the first expression and the ideal admittance given by the second expression represents an admittance giving rise to interfering currents. Neglecting the constant delay, this admittance is given by the expression

$$Y(\omega) - Y(\omega/\gamma)$$

and the current corresponding to this admittance is the portion of the characteristic interference which arises from the alteration in speed.

Formulas for the interfering wave may be deduced from this expression, but it will be sufficient for our purposes to note the ratio of the interfering wave to the signaling wave at a specific frequency. This ratio is given by the expression

$$\frac{Y(\omega) - Y(\omega/\gamma)}{Y(\omega)}$$

or approximately

$$\frac{(\omega - \omega/\gamma)}{Y(\omega)} \cdot \frac{dY}{d\omega}.$$

It will be seen that for a small discrepancy between the ideal speed and the actual speed the amount of distortion is largely determined by the slope of the transfer admittance curve. The more abruptly the transfer admittance changes, the greater will be the interference accompanying a small change in signaling speed.

It has been assumed that the change of speed is uniform, *i. e.*, that all signal elements are lengthened or shortened in the same proportion. It is of considerable practical interest to inquire what happens when there is no such regularity in the change of speed, in other words, when some signal elements are shortened and some are lengthened and when the various elements are not changed in the same proportion. Consider first a signal consisting of a single element of finite current value, the current values of the remaining signal elements being zero. It is clear that it makes no difference where the ends of the signal elements are taken when the current value is zero, and consequently this particular signal may be considered to come under the case already discussed. In other words, the interference due to any one element is as discussed above. Further, the total interfering current at any instant is the sum of the interference caused by all the elements. It follows that to a first approximation, the interference is the same as that deduced above for the regular lengthening or shortening.

It has been indicated that if interference from foreign sources is kept low, the transfer admittance made to comply with certain requirements and the speed of signaling kept constant, it is possible to transmit signals with very little distortion, utilizing a frequency range which is only slightly greater than the speed of signaling. One advantage of keeping the distortion small is that a large number of current values may be employed, thus increasing the intelligence-carrying capacity of the line. As explained above, the total interfering current should be less than one-half the difference between adjacent current values. In actual practise it will probably be found desirable to keep considerably within these limits, so as to have a definite margin. On the other hand, there would be no object in reducing the interference beyond the point where the signal is decipherable with certainty and ease.

Appendix I.

Analysis of D-C. Wave. It is required to find the sinusoidal components of a telegraph wave of T sec. duration composed of N rectangular signal elements and repeated an indefinitely great number of times. Let the value of the wave at any instant be denoted by $E(t)$, where

$E(t) = a_1$ for $0 < t < T/N$, (time unit 1),
$E(t) = a_2$ for $T/N < t < 2\,T/N$, (time unit 2),
$E(t) = a_h$ for $(h-1)\,T/N < t < h\,T/N$, (time unit h).

Represented as a Fourier series the wave is

$$E(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos npt + B_n \sin npt), \qquad (1)$$

where $p/2\pi = 1/T$ is the frequency of the fundamental wave. The problem is to find expressions for the A's and B's, so they can be computed from the known values of the a's.

Multiply both sides of the equation by $\cos kpt\,dt$ and integrate from zero to T. In this expression k is a positive integer.

$$\int_0^T E(t)\cos kpt\,dt = \frac{A_0}{2}\int_0^T \cos kpt\,dt$$

$$+\sum_{n=1}^{\infty} A_n \int_0^T \cos kpt \cos npt\,dt$$

$$+\sum_{n=1}^{\infty} B_n \int_0^T \cos kpt \sin npt\,dt, \qquad (2)$$

From a well-known trigonometric formula

$$\cos kpt \cos npt = \frac{\cos(k+n)\,pt + \cos(k-n)\,pt}{2}.$$

The integration is carried out over an integral number of cycles and the result is, therefore, equal to zero with the single exception where $n = k$ when the integrand has the average value $1/2$ and the integral is $T/2$. Likewise,

$$\cos k\,p\,t \sin n\,p\,t = \frac{\sin(k+n)\,p\,t - \sin(k-n)\,p\,t}{2}$$

The integrals of these obviously are also zero. The right member of equation (2) reduces, therefore, to the single term

$$A_k\,T/2.$$

The left member of equation (2) may be written

$$a_1\int_0^{T/N}\cos kpt\,dt + a_2\int_{T/N}^{2T/N}\cos kpt\,dt + \text{etc.}$$

or

$$\sum_{h=1}^{N} a_h \int_{(h-1)T/N}^{hT/N} \cos kpt\,dt.$$

Changing the origin of t to the center of the time unit to which each integral refers, *i. e.*, putting

$$t = (h - 1/2)\,T/N + t'$$

and dropping the prime, we have

$$\sum_{h=1}^{N} a_h \int_{-T/2N}^{T/2N} \cos\left[k\left(h-\frac{1}{2}\right)\frac{T}{N}\,p + kpt\right]dt$$

$$= \sum_{h=1}^{N} a_h\left[\cos k\left(h-\frac{1}{2}\right)\frac{T}{N}\,p\int_{-T/2N}^{T/2N}\cos kpt\,dt\right.$$

$$\left. - \sin k\left(h-\frac{1}{2}\right)\frac{T}{N}\,p\int_{-T/2N}^{+T/2N}\sin kpt\,dt\right].$$

The integral in the second term is equal to zero. Rearranging and substituting n for k, this expression becomes:

$$\int_{-T/2N}^{T/2N}\cos npt\,dt\sum_{h=1}^{N} a_h \cos n\left(h-\frac{1}{2}\right)\frac{T\,p}{N} = \frac{A_n\,T}{2}. \qquad (3)$$

The value of the integral is:

$$\frac{2\sin npT/2\,N}{np}.$$

Remembering that $T\,p = 2\,\pi$, $N/2\,T = s$, the speed of signaling, and $np = 2\,\pi\,n/T = \omega$, the result becomes

$$\frac{A_n\,T}{2} = \frac{2\,\sin\,\omega/4\,s}{\omega}\sum_{h=1}^{N} a_h \cos\frac{2\,\pi\,n}{N}\left(h-\frac{1}{2}\right)$$

or

$$A_n = \frac{8\,\sin\,\omega/4\,s}{\omega/s}\,\frac{1}{N}\sum_{h=1}^{N} a_h \cos\frac{2\,\pi\,n}{N}\left(h-\frac{1}{2}\right). \qquad (4)$$

This has now been established for all values of n except zero. If the same procedure be gone through with equation (1) without multiplying by $\cos kpt$ it will be found that this formula does in fact hold for A_0 as well. Further, if equation (1) be multiplied by $\sin kpt$ and the same operations are performed it will be found that

$$B_n = \frac{8\,\sin\,\omega/4\,s}{\omega/s}\,\frac{1}{N}\sum_{h=1}^{N} a_h \sin\frac{2\,\pi\,n}{N}\left(h-\frac{1}{2}\right). \qquad (5)$$

Put

$$F(\omega) = \frac{8\sin\,\omega/4\,s}{\omega/s}, \qquad (6)$$

$$C_n = \frac{1}{N}\sum_{h=1}^{N} a_h \cos\frac{2\,\pi\,n}{N}\left(h-\frac{1}{2}\right) \qquad (7)$$

$$S_n = \frac{1}{N}\sum_{h=1}^{N} a_h \sin\frac{2\,\pi\,n}{N}\left(h-\frac{1}{2}\right), \qquad (8)$$

The expressions (4) and (5) may now be written

$$A_n - i\,B_n = F(\omega)\,(C_n - i\,S_n) \qquad (9)$$

The function F is determined by the wave form and $(C_n - i\,S_n)$ by the intelligence being transmitted. The former will be called the shape factor, the latter the discrimination factor.

It is a matter of convention whether a plus or minus sign is chosen in equation (9). The choice made is consistent with the more usual convention.

Appendix II-A

Specific Criterion of Distortionless Transmission. It is of interest to investigate how intelligence may be accurately transmitted by signals employing rectangular wave shape at the sending end with a minimum frequency band width. In Fig. 1 it is indicated that this is usually possible when a band width equal to s is transmitted. A case of practical importance is where frequencies are transmitted from zero up to the speed of signaling. When such a limited frequency range is employed the received wave is, of course, different from the sent wave. In order that the received wave convey intelligence, it is sufficient that something pertaining to the wave in each of its time units be proportional or equal to the magnitude factor of each signal element at the sending end. First, let it be assumed that the received wave at the middle of each time unit, measured by an ideal receiving device, be representative of the signal element sent during that time unit. No notice will be taken of the wave at any other point in the time unit, and consequently deformations of the wave at other points will not matter. It will be computed below what shape factor the received wave must have in order for its value at the middle of each time unit to equal the corresponding amplitude of the sent wave.

Let the current at the receiving end at the middle of the time unit, h, be equal to a_h. Now $t = (h - 1/2)T/N$ at the middle of the time unit, and the Fourier series becomes:

$$a_h = \frac{A_0}{2} + \sum_{n=1}^{N/2} \left[A_n \cos \frac{2\pi n}{N}\left(h - \frac{1}{2}\right) + B_n \sin \frac{2\pi n}{N}\left(h - \frac{1}{2}\right)\right]. \tag{1}$$

There are N such equations, since h may successively take the values 1, 2 . . . N. There are also N unknown values of A's and B's. (Note, $B_0 = 0$ and $A_{N/2} = 0$.) These unknowns may be determined by the usual methods of solving simultaneous equations, or more simply in the following manner: Multiply all equations from $h = 1$ to $h = N$ by $\cos \frac{2\pi k}{N} h - \left(\frac{1}{2}\right)$ and add them:

$$\sum_{h=1}^{N} a_h \cos \frac{2\pi k}{N}\left(h - \frac{1}{2}\right)$$

(a)

$$= \frac{A_0}{2} \sum_{h=1}^{N} \cos \frac{2\pi k}{N}\left(h - \frac{1}{2}\right)$$

(b)

$$+\frac{1}{2} \sum_{h=1}^{N} \sum_{n=1}^{N/2} \left\{ A_n \left[\cos (n+k) \frac{2\pi}{N}\left(h - \frac{1}{2}\right) \right.\right.$$

(c)

$$\left. + \cos (k-n) \frac{2\pi}{N}\left(h - \frac{1}{2}\right)\right]$$

(d)

$$+ B_n \left[\sin (n+k) \frac{2\pi}{N}\left(h - \frac{1}{2}\right) \right.$$

(e)

$$\left.\left. - \sin (k-n) \frac{2\pi}{N}\left(h - \frac{1}{2}\right)\right]\right\}. \tag{2}$$

In equation (2), use has been made of the trigonometric formulas:

$$\cos a \cos b = \frac{\cos (a+b) + \cos (a-b)}{2}$$

$$\cos a \sin b = \frac{\sin (a+b) - \sin (a-b)}{2}$$

In order to simplify equation (2) we will employ the identities:

$$\sum_{h=1}^{N} \cos x \left(h - \frac{1}{2}\right) = \frac{\cos Nx/2 \sin Nx/2}{\sin x/2} \tag{3}$$

$$\sum_{h=1}^{N} \sin x \left(h - \frac{1}{2}\right) = \frac{\sin^2 Nx/2}{\sin x/2}, \tag{4}$$

To prove these identities, multiply the left member of (4) by i and add to the left member of (3):

$$\sum_{h=1}^{N} \cos\left(h - \frac{1}{2}\right) x + i \sin\left(h - \frac{1}{2}\right) x = \sum_{h=1}^{N} e^{i(h-1/2)x}$$

$$= e^{-\frac{ix}{2}} \sum_{h=1}^{N} e^{ihx} = e^{\frac{ix}{2}}(1 + e^{ix} + e^{i2x} + \ldots e^{i(N-1)x})$$

$$= \frac{e^{\frac{ix}{2}}(1 - e^{iNx})}{1 - e^{ix}} = \frac{e^{\frac{iNx}{2}}(e^{-\frac{iNx}{2}} - e^{\frac{iNx}{2}})}{e^{-ix/2} - e^{ix/2}} = \frac{e^{\frac{iNx}{2}} \sin Nx/2}{\sin x/2}$$

$$= \frac{\cos Nx/2 \sin Nx/2}{\sin x/2} + i \frac{\sin^2 Nx/2}{\sin x/2}. \tag{5}$$

Equating real and imaginary parts gives equations (3) and (4), respectively.

By applying formula (3) to terms (a), (b), and (c), of equation (2), and applying formula (4) to terms (d) and (e),

$$\sum_{h=1}^{N} a_h \cos \frac{2\pi k}{N}\left(h - \frac{1}{2}\right) = \frac{A_0}{2} \frac{\cos k\pi \sin k\pi}{\sin k\pi/N}$$

$$+\frac{1}{2}\sum_{n=1}^{N/2}\Bigg[A_n\left(\frac{\cos(n+k)\,\pi\sin(n+k)\,\pi}{\sin(n+k)\,\pi/N}\right.$$

$$\left.+\frac{\cos(k-n)\,\pi\sin(k-n)\,\pi}{\sin(k-n)\pi/N}\right)$$

$$+B_n\left(\frac{\sin^2(n+k)\,\pi}{\sin(n+k)\,\pi/N}-\frac{\sin^2(k-n)\,\pi}{\sin(k-n)\,\pi/N}\right)\Bigg].\quad(6)$$

As k approaches any integral value of n, for instance m, from 0 to $N/2-1$ inclusive, all the numerators of the terms in the right-hand member of equation (6) approach zero. Consequently, these terms approach zero except one term

$$\frac{A_m}{2}\;\frac{\cos(k-m)\,\pi\sin(k-m)\,\pi}{\sin(k-m)\,\pi/N}$$

whose denominator also approaches zero. This term becomes $N\,A_m/2$. Setting $k=m$, solve equation (6) for A_m:

$$A_m=\frac{2}{N}\sum_{h=1}^{N}a_h\cos\frac{2\,\pi\,m}{N}\left(h-\frac{1}{2}\right)=2\,C_m.\quad(7)$$

Changing subscripts:

$$A_n=2\,C_n.\quad(8)$$

Now if, instead, we multiply equation (1) by $\sin\frac{2\,\pi\,k}{N}\left(h-\frac{1}{2}\right)$, a similar process gives:

$$B_n=2\,S_n,\qquad(n<N/2).\quad(9)$$

But when $n=N/2$ this process gives:

$$B_{N/2}=S_{N/2}.\quad(10)$$

From equations (8) to (10) it is seen that the shape factor which will be denoted by $F_0(\omega)$, is equal to 2 from zero frequency up to the speed of signaling where it suddenly drops to half that value. Above this frequency $F_0(\omega)$ is equal to zero, as shown in curve *a*, Fig. 2.

It will next be shown that there is an infinite number of shape factors which may be added to curve *a* which produce no change in the current at the middle of the time units. For example, curve *b* shows a specimen of such a shape factor, which may be any single valued curve so long as it is symmetrical about the speed of signaling except for a change in sign. When this shape factor is added to that of curve *a*, a curve such as *c* may result for the total shape factor which is also ideal. Denote the shape factor of curve *b* by $F_\Delta(\omega)$. The wave, due to such a shape factor, is:

$$\sum_{n=N/2-m}^{N/2+m}F_\Delta(\omega)\,(C_n\cos npt+S_n\sin npt)$$

$$=\sum_{n=0}^{m}F_\Delta(\omega)\,(C_{N/2-n}\cos(2\,\pi\,s-n\,p)\,t$$

$$+S_{N/2-n}\sin(2\,\pi\,s-n\,p)\,t)$$

$$+\sum_{n=1}^{m}F_\Delta(\omega)\,(C_{N/2+n}\cos(2\,\pi\,s+n\,p)\,t$$

$$+S_{N/2+n}\sin(2\,\pi\,s+n\,p)\,t)\quad(11)$$

where m is any positive integer less than $N/2$. It follows from equations (7) and (8) of Appendix I that:

$$-C_{N/2-n}=C_{N/2+n}\quad(12)$$

$$S_{N/2-n}=S_{N/2+n}.\quad(13)$$

Substitute these relations in equation (11) making use of the trigonometric formulas:

$$\cos(a-b)+\cos(a+b)=2\cos a\cos b\quad(14)$$

$$-\sin(a-b)+\sin(a+b)=2\cos a\sin b\quad(15)$$

and put $F_\Delta(2\,\pi\,s)=0$ and $F_\Delta(2\,\pi\,s+n\,p)=-F_\Delta(2\,\pi\,s-n\,p)$ so that the formula for the wave becomes:

$$\sum_{n=1}^{m}2\,F_\Delta(2\,\pi\,s+n\,p)\,(\cos 2\,\pi\,s\,t)\,(C_{N/2+n}\cos npt$$

$$+S_{N/2+n}\sin npt).\quad(16)$$

Since this expression contains the factor $\cos 2\,\pi\,s\,t$, which is zero at the middle of the time unit, the wave does not have any effect on the receiving device when the latter records current values at the middle of the time units only. The curve *b* is any real shape factor, symmetrical about the dot frequency except for a change in sign. Therefore, every such real shape factor when added to curve *a* gives a resultant shape factor which is ideal.

It is next desired to show that the addition of an imaginary shape factor which is any single valued function whatever, provided it is symmetrical about the dot frequency in both magnitude and sign, does not contribute to the value of the wave at the middle of the time units. Consider the curve *d* of Fig. 2, which represents the magnitude of any symmetrical imaginary shape factor, $i\,F_\Delta(\omega)$.

$$A_n-i\,B_n=i\,F_\Delta(\omega)\,(C_n-i\,S_n)=F_\Delta(\omega)\,(S_n+i\,C_n).\quad(17)$$

Equating real and imaginary parts of equation (17) the values of A_n and B_n are determined, so that the wave may be expressed as follows:

$$\sum_{n=N/2-m}^{N/2+m}F_\Delta(\omega)\,(S_n\cos npt+C_n\sin npt)$$

$$= \sum_{n=0}^{m} F_\Delta(\omega)\,(S_{N/2-n}\cos(2\pi s - n p)\,t$$

$$+ C_{N/2-n}\sin(2\pi s - n p)\,t\,)$$

$$+ \sum_{n=1}^{m} F_\Delta(\omega)\,(S_{N/2+n}\cos(2\pi s + n p)\,t$$

$$+ C_{N/2+n}\sin(2\pi s + n p)\,t\,)\,. \qquad (18)$$

Using formulas (12) to (15) and putting $F_\Delta(2\pi s + n p) = F_\Delta(2\pi s - n p)$ the wave becomes:

$$\sum_{n=0}^{m} 2F_\Delta(2\pi s + n p)\,(\cos 2\pi s t)\;(S_{N/2+n}\cos npt$$

$$+ C_{N/2+n}\sin npt), \qquad (19)$$

which is zero at the middle of the time units on account of the factor $\cos 2\pi s t$. Since this is true for any symmetrical function, $F_\Delta(\omega)$, there is an infinite number of imaginary shape factors which do not affect the values of the wave at the middle of the time units.

Appendix II-B

Network for Distortionless Transmission. The purpose of this appendix is to illustrate, concretely, the matter which has been discussed in Appendix II-A, and the portions of the main text associated therewith. This appendix does not form any part of the chain of reasoning in the sense that anything which is here discussed or deduced will be required subsequently. It can, therefore, be eliminated by those who are primarily interested in the deductions of the main theory.

It has been shown in Appendix II-A that a certain ideal shape factor can be modified by the addition of certain other shape factors which possess a specific kind of symmetry about a frequency equal to the speed of signaling. It is obvious, on consideration, that similar shape factors possessing symmetry about the frequencies 2 *s*, 3 *s*, etc., can also be added in a similar manner without affecting the principal characteristic of the shape factor. These considerations are not of great practical importance in the cases considered because it has been assumed that the important telegraph problem is to transmit maximum intelligence in minimum frequency range. However, these considerations will aid in understanding the subject matter of the present appendix.

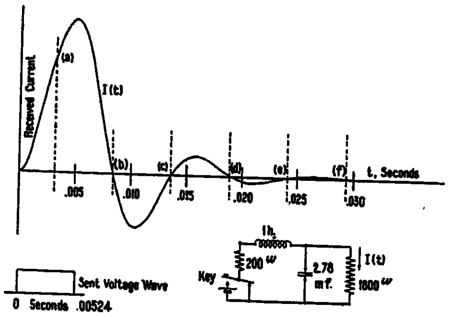


Fig. 5—Instance of Non-Distorting Wave

Fig. 5 shows a very simple circuit which has been designed for the purpose of illustrating a non-distorting wave. This circuit, containing only two reactance elements, has a very simple form of transient, namely, a uniformly damped sine wave. Now an important property of the sine wave is that its zeros are located at equal intervals, which is precisely the property required of a non-distorting wave under the criterion discussed in Appendix II-A. The interval between successive zeros in the illustration chosen is 0.00524 sec.; and this is also made the duration of the time unit of the sent signal, as illustrated in the figure. It will be remembered that on the criterion under discussion a specific point of the signal element is picked out and taken to represent the element. In the case illustrated, this point should be taken at an interval 0.00524 sec. previous to the first zero. It is obvious that if this is done the interference due to all previous signal elements is zero because the interference due to each one of them is zero. It follows that any receiving device which is made to function by the current at that point will be distortionless as far as interference from adjacent signal elements is concerned.

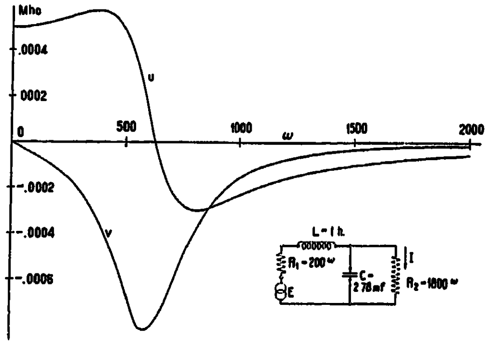


Fig. 6—Transfer Admittance, $u + i v$, Corresponding to the Non-Distorting Wave Shown in Fig. 5

Fig. 6 shows the transfer admittance of the same network. The speed of signaling corresponds to the point for which the abscissa is 600. It will be observed that these curves do not possess the simple symmetrical characteristics discussed in Appendix II-A, but this is because Appendix II-A was confined to a narrow frequency range not much exceeding the signaling range. If consideration be given to this fact, together with facts discussed in the second paragraph of the present appendix, it will be found that this simple network illustrates the theory.

Both the transfer admittance and the value of the current curve are easily computed in the present case. The transfer admittance is given by the expression:

$$Y = \frac{1}{R_1 + R_2 + i\,\omega\,(L + R_1 R_2 C) - \omega^2 L C R_2}.$$

The current, from the time $t = 0$ up to the time $t = 0.00524$ sec., is given by an expression which takes the following form for the numerical values given:

$$A\,(t) = \frac{1}{2000}\left[1 - e^{-200t}\left(\cos 600\,t + \frac{1}{3}\sin 600\,t\right)\right].$$

After the time $t = 0.00524$ sec., the current is given by the expression:

$$A\,(t) - A(t - 0.00524)$$
$$= \frac{-\,e^{-200t}}{2000}\left(1 + e^{1\cdot 048}\right)\left(\cos 600\,t + \frac{1}{3}\sin 600\,t\right).$$

Appendix III

Alternative Criteria of Distortionless Transmission. An alternative criterion of distortionless transmission is that the interval, between the instants when the received wave passes through the mean value, shall be the same as the corresponding interval at the transmitting end. A receiving device which responds to the values of the wave at the ends of the time units, instead of at the middle, will give distortionless transmission provided the wave at the end of the element, h, is proportional to $(a_h + a_{h+1})/2$. This is the criterion in ordinary land-line telegraphy. Strictly speaking, we should also require that the wave does not pass through this value at points located within the time unit. It is thought that the complication involved by introducing this condition is not warranted. We will first determine the coefficients of the Fourier expansion of this received wave for the case where the frequency range is limited to the interval between zero and s. We have:

$$\frac{a_h + a_{h+1}}{2} = \frac{A_0}{2} + \sum_{n=1}^{N/2}\left(A_n \cos\frac{2\,\pi\,n\,h}{N} + B_n \sin\frac{2\,\pi\,n\,h}{N}\right), \tag{1}$$
$$(h = 1, 2, \ldots N),$$

since $t = h\,T/N$ at the end of the hth time unit.

As in Appendix II-A, we have N simultaneous equations and N unknowns, the A's and the B's. The solution is of the same nature as before, except that the equations are multiplied by $\cos\dfrac{2\,\pi\,k\,h}{N}$ instead of $\cos\dfrac{2\,\pi\,k}{N}\left(h - \dfrac{1}{2}\right)$, before being summed. For values of $n < N/2$ we find by similar reasoning:

$$A_n = \frac{2}{N}\sum_{h=1}^{N}\frac{a_h + a_{h+1}}{2}\cos\frac{2\,\pi\,n\,h}{N}$$

$$= \frac{1}{N}\sum_{h=1}^{N} a_h\left[\cos\frac{2\,\pi\,n\,h}{N} + \cos\frac{2\,\pi\,n\,(h-1)}{N}\right]. \tag{2}$$

By means of the trigonometric formula

$$\cos a + \cos b = 2\cos\frac{1}{2}\,(a + b)\cos\frac{1}{2}\,(a - b)$$

we find that equation (2) may be written:

$$A_n = \frac{2}{N}\left(\cos\frac{n\,\pi}{N}\right)\sum_{h=1}^{N} a_h \cos\frac{2\,\pi\,n}{N}\left(h - \frac{1}{2}\right)$$

$$= 2\cos\frac{\omega}{4\,s}\,C_n \tag{3}$$

An analogous line of reasoning gives:

$$B_n = 2\cos\frac{\omega}{4\,s}\,S_n. \tag{4}$$

The shape factor is $2\cos\omega/4\,s$ (Fig. 3, curve a), for frequencies up to s, and remains zero for higher frequencies. A transfer admittance which will produce a wave of this form when the sent wave is rectangular is:

$$Y = \frac{\omega}{4\,s}\cot\frac{\omega}{4\,s},\ \text{when } \omega < 2\,\pi\,s \tag{5a}$$

$$Y = 0, \qquad \text{when } \omega \geq 2\,\pi\,s. \tag{5b}$$

This is obtained from the ratio of the shape factor at the receiving end, divided by the shape factor at the sending end.

Such a system would be incapable of transmitting ordinary reversals, since zero current is transmitted for frequencies equal to or greater than the dot speed. This defect may be remedied by the addition of shape factors which are symmetrical about the signaling frequency, and do not contribute anything to the wave at the ends of the time units. By reasoning, analogous to that given at the close of Appendix II-A, it follows that the wave due to any real shape factor, symmetrical about the speed of signaling in magnitude and sign, (Fig. 3, curve d), contains the factor $\sin 2\,\pi\,s\,t$, and, therefore becomes zero at the ends of the time units. In a similar manner it can be shown that the wave due to any imaginary shape factor, symmetrical about the dot frequency in magnitude but not in sign, (Fig. 3, curve b), also becomes zero at the ends of the time units. The addition of curve d to curve a gives curve c, which is also an ideal shape factor.

A third criterion for distortionless transmission is that the area under the received wave shall have the same value as that under the sent wave during each time unit. Thus during the time unit h, the area under the received wave must be:

$$a_h\,T/N = \int_{t=(h-1)T/N}^{hT/N}\left[\frac{A_0}{2} + \sum_{n=1}^{N/2}(A_n \cos npt\right.$$

$$+ B_n \sin npt) \Big] dt. \qquad (6)$$

This equation may be multiplied by N/T, integrated, and by means of the trigonometric formulas

$$\sin a - \sin b = 2 \cos \frac{1}{2} (a + b) \sin \frac{1}{2} (a - b)$$

$$- \cos a + \cos b = 2 \sin \frac{1}{2} (a + b) \sin \frac{1}{2} (a - b)$$

it may be reduced to the following form:

$$a_h = \frac{A_0}{2} + \sum_{n=1}^{N/2} \left(\frac{N}{\pi n} \sin \frac{\pi n}{N} \right) \times$$

$$\left[A_n \cos \frac{2 \pi n}{N} \left(h - \frac{1}{2} \right) + B_n \sin \frac{2 \pi n}{N} \left(h - \frac{1}{2} \right) \right] \qquad (7)$$

This equation is the same as equation (1) of Appendix II-A, except for the factor:

$$\left(\frac{N}{\pi n} \sin \frac{\pi n}{N} \right) = \left(\frac{4 s}{\omega} \sin \frac{\omega}{4 s} \right)$$

Consequently the shape factor for this wave is equal to the shape factor of curve a, Fig. 2, divided by $4 s/\omega \sin \omega/4 s$ which reduces to:

$$F_0 (\omega) = \frac{\omega/2 s}{\sin \omega/4 s}, \quad \text{for } 0 \leq \omega < 2 \pi s, \qquad (8)$$

$$F_0 (\omega) = \pi/2, \quad \text{for } \omega = 2 \pi s, \qquad (9)$$

$$F_0 (\omega) = 0, \quad \text{for } 2 \pi s < \omega, \qquad (10)$$

which, if the sent wave is rectangular, corresponds to a transfer admittance:

$$Y (\omega) = \left(\frac{\omega/4 s}{\sin \omega/4 s} \right)^2, \quad \text{for } 0 \leq \omega < 2 \pi s, \qquad (11)$$

$$Y (\omega) = \frac{\pi^2}{8}, \quad \text{for } \omega = 2 \pi s, \qquad (12)$$

$$Y (\omega) = 0, \quad \text{for } 2 \pi s < \omega. \qquad (13)$$

In order to determine shape factors which may be added to $F_0 (\omega)$ without changing the received signal, the following method may be employed. Let $F_\Delta (\omega)$ represent the shape factor to be added. The wave due to this shape factor comprises frequency components which lie above and below the dot speed. Let δ equal any convenient frequency increment not greater than s. Then consider any pair of components at the frequencies $(s + \delta)$ and $(s - \delta)$, respectively, and find what values of $F_\Delta (\omega)$ at these frequencies will produce components which do not change the received signal. The portion of the wave due to these components is:

$$\begin{aligned} I = {} & F_\Delta (2 \pi (s - \delta)) (C_{N/2-n} \cos 2 \pi (s - \delta) t \\ & \quad + S_{N/2-n} \sin 2 \pi (s - \delta) t) \\ & + F_\Delta (2 \pi (s + \delta)) (C_{N/2+n} \cos 2 \pi (s + \delta) t \\ & \quad + S_{N/2+n} \sin 2 \pi (s + \delta) t) \end{aligned} \qquad (14)$$

In order that this portion of the wave add nothing to the received signal, $\int I \, dt$ must be zero between $h/2 s$ and $(h - 1)/2 s$, which is the time of the unit h. Performing the integration and simplifying the result gives an expression which reduces to zero when

$$F_\Delta (2 \pi (s - \delta))/(s - \delta) = - F_\Delta (2 \pi (s + \delta))/(s + \delta) \qquad (15)$$

Thus $F_\Delta (\omega)$ may have any real value, provided the conditions of symmetry are fulfilled as expressed in equation (15). This is true for any value of $\delta < s$, and, therefore, is true for all values of $\delta < s$. A similar line of reasoning for imaginary shape factors, $i F_\Delta (\omega)$, which do not change the signal, gives:

$$i F_\Delta (2 \pi (s - \delta))/(s - \delta) = i F_\Delta (2 \pi (s + \delta))/(s + \delta). \qquad (16)$$

Insufficiency of Ranges Smaller than the Signaling Speed. It has been shown that a frequency range numerically equal to the speed of signaling is necessary for the case where there is no limitation on the number of distinct magnitude factors. It might be questioned whether the proposition is true when this number is small. Consider the case where there are just two, distinct magnitude factors, + 1 and − 1, and where the current at the middle of the time units is taken as the criterion of transmission. Since there are only two magnitude factors involved it is to be considered necessary and sufficient that the wave at the mid-point of each time unit should have the proper sign, regardless of magnitude. We will now show that this cannot be insured when the transmitted range is less than the speed of signaling.

Suppose first that the transmitted range extends from zero to a point lower than the speed of signaling, so that the missing portion is at the upper end of the range $0 - s$. Let $N/2$ be an odd number and let the first $N/2$ signal elements be $+ - + - \ldots\ldots - +$. Let the next $N/2$ signal elements be a repetition of this and let the signal be repeated indefinitely. By far the most important harmonic of this signal below the frequency s is the $(N/2 - 1)$th which corresponds to a frequency of $s (1 - 2/N)$. By taking N sufficiently large we can insure that this harmonic is suppressed. If all frequencies up to s are transmitted and equalized for distortionless transmission, and then the frequency $s (1 - 2/N)$ removed, it is found that, due to the great magnitude of this component, the signal is reversed in sign at some of the critical points (mid-points). The question is, whether by alteration of the remaining components this can be corrected. We next show that this cannot be done.

Consider the function $\cos 2 \pi s (1 - 2/N) t$. (This function has the same sign as the sent signal at all the critical points.) Next consider the sum obtained by adding the products of this function and the received wave value at the critical points. If the wave is correct in sign each one of these products is positive and therefore the sum is positive. Now, it is easy to show that for any component of the received wave of lower frequency than $s\,(1 - 2/N)$, the contribution to this sum is zero, the negative products balancing the positive. Therefore the assumption, that the received wave has the correct sign at all the critical points, and the assumption that it is made up entirely of frequencies lower than $s\,(1 - 2/N)$, are inconsistent.

In case the suppressed range is not at the upper end of the range $0 - s$ the procedure is similar. First find a frequency, prime with respect to s, and which lies in the suppressed range, and is an even harmonic of the period T. Let its frequency be s'. Construct the function $\cos 2\,\pi\, s'\, t$. Next construct a signal made up of $+\,1$ and $-\,1$ elements whose sign at each critical point is the same as that of $\cos 2\,\pi\, s'\, t$. The argument then proceeds as before.

Similar proofs can be produced for other criteria.

Appendix IV

Analysis of Carrier Wave. It is required to find the sinusoidal components of a carrier-telegraph wave of T sec. duration, composed of N time units, and repeated an indefinitely great number of times. The carrier current is modulated by a rectangular envelope during each of the time units, so that the wave has the following values:

$E\,(t) = a_1 \cos\,(\omega_c\, t - \theta)$, for $0 < t < T/N$, (time unit 1),
$E\,(t) = a_2 \cos\,(\omega_c\, t - \theta)$, for $T/N < t < 2\,T/N$, (time unit 2),
. . . .
$E\,(t) = a_h \cos\,(\omega_c\, t - \theta)$, for $(h - 1)\,T/N < t < h\,T/N$, (time unit h), etc.

Let the total wave be represented by the Fourier series:

$$\frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos npt + B_n \sin npt)\,. \qquad (1)$$

By reasoning similar to that of Appendix I:

$$A_n = \frac{2}{T} \sum_{h=1}^{N} \int_{(h-1)T/N}^{hT/N} a_h \cos npt \cos\,(\omega_c\, t - \theta)\, dt$$

$$= \frac{1}{T} \sum_{h=1}^{N} \int_{(h-1)T/N}^{hT/N} a_h\,(\cos\,[\,(\omega_c - n\,p)\,t - \theta\,] + \cos\,[\,(\omega_c + n\,p)\,t - \theta\,]\,)\,dt$$

$$= \frac{1}{T} \sum_{h=1}^{N} a_h \Bigg[\frac{\sin\,[\,(\omega_c - n\,p)\,h\,T/N - \theta] - \sin\,[\,(\omega_c - n\,p)\,(h - 1)\,T/N - \theta]}{\omega_c - n\,p}$$
$$+ \frac{\sin\,[\,(\omega_c + n\,p)\,h\,T/N - \theta] - \sin\,[\,(\omega_c + n\,p)\,(h - 1)\,T/N - \theta]}{\omega_c + n\,p} \Bigg]$$

$$= \frac{2}{T} \sum_{h=1}^{N} a_h \Bigg[\frac{\sin\,(\omega_c - n\,p)\,(T/2N)\cos\,[\,(\omega_c - n\,p)\,(h - 1/2)\,T/N - \theta\,]}{\omega_c - n\,p}$$
$$+ \frac{\sin\,(\omega_c + n\,p)\,(T/2N)\cos\,[\,(\omega_c + n\,p)\,(h - 1/2)\,T/N - \theta\,]}{\omega_c + n\,p} \Bigg]$$

$$= \frac{2 \sin\,(\omega_c - n\,p)\,(T/2\,N)}{T\,(\omega_c - n\,p)} \sum_{h=1}^{N} a_h \cos\,[\,(\omega_c - n\,p)\,(\,h - 1/2\,)\,T/N - \theta\,]$$
$$+ \frac{2 \sin\,(\omega_c + n\,p)\,(T/2\,N)}{T\,(\omega_c + n\,p)} \sum_{h=1}^{N} a_h \cos\,[\,(\omega_c + n\,p)\,(\,h - 1/2\,)\,T/N - \theta\,]\,. \qquad (2)$$

$$\therefore A_n = F'\,(\omega)\,C_n' + F''\,(\omega)\,C_n'', \qquad (3)$$

where: $F'\,(\omega) = \dfrac{2\,N \sin\,(\omega_c - n\,p)\,T/2\,N}{T\,(\omega_c - n\,p)}$

$$= \frac{4\,s \sin\,(\omega_c - \omega)\,\frac{1}{4\,s}}{\omega_c - \omega},$$

$$F''(\omega) = \frac{2N \sin(\omega_c + np)\,T/2N}{T(\omega_c + np)}$$

$$= \frac{4\,s \sin(\omega_c + \omega)\,\frac{1}{4\,s}}{\omega_c + \omega},$$

$$C_n' = 1/N \sum_{h=1}^{N} a_h \cos\left[(\omega_c - np)\left(h - \frac{1}{2}\right) T/N - \theta\right],$$

$$C_n'' = 1/N \sum_{h=1}^{N} a_h \cos\left[(\omega_c + np)\left(h - \frac{1}{2}\right) T/N - \theta\right].$$

In a similar manner it can be deduced that

$$B_n = F'(\omega)\,S_n' + F''(\omega)\,S_n'', \qquad (4)$$

where:

$$S_n' = -\,1/N \sum_{h=1}^{N} a_h \sin\left[(\omega_c - np)\left(h - \frac{1}{2}\right) T/N - \theta\right],$$

$$S_n'' = 1/N \sum_{h=1}^{N} a_h \sin\left[(\omega_c + np)\left(h - \frac{1}{2}\right) T/N - \theta\right].$$

Multiplying equation (4) by i and subtracting it from (3) gives:

$$A_n - i B_n = F'(\omega)\,(C_n' - i S_n') + F''(\omega)\,(C_n'' - i S_n''). \qquad (5)$$

It is evident that a special relation exists between $(C_n' - i\,S_n')$ and $(C_n'' - i\,S_n'')$ when the carrier frequency is an integral multiple of the dot speed, so that $\omega_c = 2\,\pi\,q\,s$. When q is even, we see by inspection that:

$$C_n' - i\,S_n' = \frac{e^{-i\left(\frac{\pi q}{2} + \theta\right)}}{N}$$

$$\sum_{h=1}^{N} a_h\, e^{-inp(h-1/2)T/N} = e^{-i\left(\frac{\pi q}{2} + \theta\right)}\,(C_n - i\,S_n), \qquad (6)$$

where $(C_n - i\,S_n)$ is the same as for the direct current case. Also:

$$C_n'' - i\,S_n'' = \frac{e^{-i\left(\frac{\pi q}{2} + \theta\right)}}{N} \sum_{h=1}^{N} a_h\, e^{-inp(h-1/2)T/N}, \qquad (7)$$

whence:

$$C_n'' - i\,S_n'' = e^{i(\pi q + 2\theta)}\,(C_n' - i\,S_n') = e^{i2\theta}\,(C_n' - i\,S_n')$$
$$= e^{i\left(\frac{\pi q}{2} + \theta\right)}\,(C_n - i\,S_n)\,. \qquad (8)$$

Substituting (6) and (8) in (5) gives:

$$A_n - i\,B_n = \left[e^{-i\left(\frac{\pi q}{2} + \theta\right)} F'(\omega) + e^{i\left(\frac{\pi q}{2} + \theta\right)} F''(\omega)\right]$$
$$\times\,(C_n - i\,S_n)\,. \qquad (9)$$

When q is odd, a similar line of reasoning gives:

$$A_n - i\,B_n = \left[e^{-i\left(\frac{\pi(q-1)}{2} + \theta\right)} F'(\omega)\right.$$
$$\left. + \, e^{i\left(\frac{\pi(q-1)}{2} + \theta\right)} F''(\omega)\right](C_{N/2-n} + i\,S_{N/2-n}). \qquad \textbf{(10)}$$

$F'(\omega)$ is symmetrical about the carrier frequency in magnitude, but $F''(\omega)$ is unsymmetrical. It will next be shown that $(A_n - i\,B_n)$ is symmetrical about the carrier frequency when the modulating wave contains no components higher than the carrier frequency. From this it may be inferred that $F''(\omega)$ is zero when these components have been removed from the modulating wave before modulation takes place. For example, let the modulating wave be:

$$\frac{A_0}{2} + \sum_{n=1}^{qN/2} (A_n \cos npt + B_n \sin npt)\,. \qquad \textbf{(11)}$$

Multiplying **(11)** by $\cos(\omega_c t - \theta)$ and making use of the trigonometric formulas:

$\cos a \cos b = (1/2)\cos(a + b) + (1/2)\cos(a - b)$

$\cos a \sin b = (1/2)\sin(a + b) - (1/2)\sin(a - b)$,

gives the carrier wave:

$$\frac{A_0}{2}\cos(\omega_c t - \theta) + \frac{1}{2}\sum_{n=1}^{qN/2} \{\, [A_n \cos(\omega_c t + npt - \theta)$$
$$+ B_n \sin(\omega_c t + npt - \theta)\,] + [A_n \cos(\omega_c t - npt - \theta)$$
$$- B_n \sin(\omega_c t - npt - \theta)\,]\,\}\,. \qquad \textbf{(12)}$$

We see by inspection that components of this wave have the same amplitudes, $\sqrt{A_n^2 + B_n^2}$, at equal distances $\pm\ np$ from ω, where np is never greater than ω_c. Hence an analysis of this wave by a method analogous to that given in the earlier part of this appendix, could yield no unsymmetrical shape factor such as $F''(\omega)$.

Appendix V

Single Sideband Transmission. The principle which will be employed in the following discussion is, that when the received wave through any complicated admittance is to be determined, it is permissible to subdivide the admittance into component parts to determine the received wave for each component, and to add the resulting received waves for all the components to obtain the complete wave. To make clear the use of this principle a beginning will be made with the transfer admittance shown at a in Fig. 7. Strictly speaking, this admittance is a hypothetical one only, but it illustrates the principles and is a first approximation to the actual case of band-pass filter.

The admittances are to be taken as real quantities. That is to say, either there is no delay or, if there is any delay, it is equalized and a new time origin is chosen so that the delay disappears from the expression for the admittance. The carrier frequency is assumed to be located at ω_c, and the transfer admittance of the system extends unsymmetrically in the two directions from this frequency as indicated in the figure.

There are infinitely many ways in which this area may be subdivided into component transfer admittances, but the one most suited for our discussion is shown in the remainder of Fig. 7. It will be observed that if the three admittances, (*b*, *c*, and *d*), are added together the result is the admittance shown at *a*. In accordance with the principle set forth above we conclude, therefore, that the received wave, in the case of the admittance shown at *a*, is equal to the sum of the received waves deduced from the other three admittances.

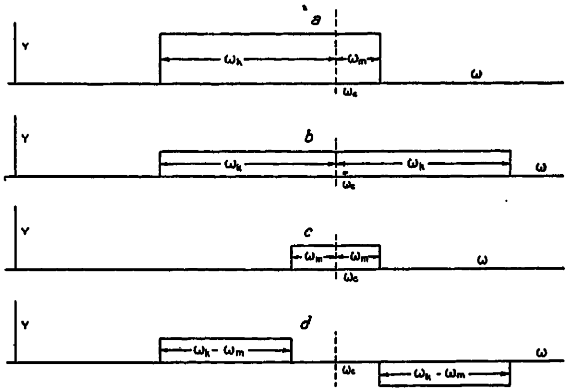


FIG. 7—GRAPHICAL ANALYSIS OF A TRANSMISSION CHARACTERISTIC

The sum of the transfer admittances shown at *b*, *c*, and *d* equals the transfer admittance shown at *a*. The components, *b*, *c*, and *d*, are chosen so as to possess symmetry about the carrier frequency

Considering first the admittance shown at *b*, we have here complete symmetry with respect to the carrier frequency; and it follows from the discussion of the symmetrical carrier case that the received wave is of the same frequency and phase as the carrier frequency and is modulated by a low frequency wave, which is identical with that obtained in d-c. telegraphy when a transfer admittance extending from 0 to ω_k is employed. It is assumed, for simplicity, that the wave used originally to modulate the carrier wave contains no components of as high frequency as the carrier frequency. The discussion of the admittance, shown at *c*, is identical with that of *b*. Comparing these two components, it is concluded that they are in phase relative to each other, and that the component due to *b* builds up more rapidly than that due to *c*. It is obvious that the time it takes the result, due to these two components, to reach a steady state is determined by the slower of the two. We can conclude, tentatively, that, when the carrier frequency is located near one end of the band, it is the frequency range from the carrier frequency to the nearer edge of the transmitted band which determines the rate of building up.

The admittance shown at *d* is also symmetrical about the carrier frequency, but it has a different type of symmetry. To contrast the effect of this kind of symmetry with that of the symmetry previously discussed, let us assume that the carrier wave is represented by $\cos \omega_c t$, and that the modulating wave is represented by $A_n \cos npt$. The modulated wave is then $A_n \cos npt \cos \omega_c t$. This can be divided in the usual way into two components located symmetrically with respect to the carrier frequency. The two components are

$$(A_n/2) \cos (\omega_c - n p) t \text{ and } (A_n/2) \cos (\omega_c + n p) t. \quad (1)$$

It is obvious that, in the case shown at *b*, these components are transmitted without any relative change and thus combine to form the original modulated wave without any modification other than a possible multiplication by a constant. In the case of the admittance shown at *d*, however, there is a relative change in these components. If one of them is left unchanged, the other is multiplied by -1 and the two components become

$$(A_n/2) \cos (\omega_c - n p) t \text{ and } - (A_n/2) \cos (\omega_c + n p) t. \quad (2)$$

The resultant wave is $A_n \sin npt \sin \omega_c t$. The important change produced by admittances, of the type shown in *d*, is that the carrier frequency has suffered a phase shift of 90 deg. An incidental change is that the modulating frequency, which represents the signal, has also suffered a change of 90 deg. This reasoning can be carried through for all the components making up the modulating wave, as long as their frequency is less than the carrier frequency. It follows that the zeros of the total wave, due to the admittance shown at *d*, are displaced one-quarter period of the carrier wave with respect to the zeros of the corresponding waves obtained with *b* and *c*.

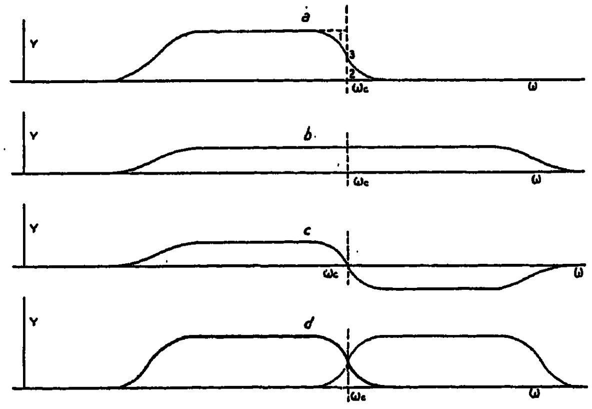


FIG. 8—GRAPHICAL ANALYSIS OF A TRANSMISSION CHARACTERISTIC

The sum of the symmetrical transfer admittances shown at *b* and *c* equals that shown at *a*. The heavy line in *d* is identical with the curve in *a*; the thin curve is the mirror image of the heavy one. The curve shown at *b* equals one-half the sum, and that shown at *c* equals one-half the difference of the two curves shown at *d*

It will be obvious that if the components due to *c* and *d* could be eliminated, the resultant wave would be determined by the separation between the carrier frequency and the far edge of the band transmitted. The wave, due to admittance shown at *d*, can be eliminated in the reception of the wave by the method discussed under "Phase Discrimination." The wave, due to *c*, is not disposed of so easily. It might be thought that by moving the carrier frequency nearer to the edge of the band the admittance shown at *c* could be made to extend over a very narrow frequency range, and thus be made negligible. It should be noted, however, that the steady state value of the received wave is the same, regardless of the width of the band, and that as long as any portion of *c* remains there will always be a slow, interfering wave. The only satisfactory answer is to locate the carrier frequency exactly on the edge of the band, so that there is no separation, however small, between the edge and the carrier frequency. This peculiar condition is due to the fact that we have assumed a transfer admittance, having an abrupt cut-off.

In the practical case the cut-off is gradual, not abrupt. The transfer admittance shown at *a*, (Fig. 8), represents a transfer admittance (real) which is suitable for single-sideband transmission. It can be separated into two parts as shown, respectively, at *b* and *c*: *c* possesses the kind of symmetry which leads to a phase-quadrature component; and *b* possesses the kind of symmetry which is desirable.

Strictly speaking, it is somewhat inaccurate to apply the term "single sideband transmission" to the case shown in Fig. 8, or, in fact, to any case where portions of both sidebands are transmitted. Since it would seem to be out of the question to transmit a single sideband, in a strict sense, in the case of telegraph waves, there should be no objection to this designation.

The results obtained in connection with Fig. 8 may be described in slightly different terms. Referring to *a*, the dotted line may be said to represent the ideal of single-sideband transmission. The numeral 1 marks the area which is removed from the ideal by the gradual cut-off of the admittance curve. The removal of this area results in an impairment of the wave due to the principal sideband. The area indicated by the numeral 2 represents the vestigial sideband, and results in an impairment. The result obtained, briefly stated is: If areas 1 and 2 are equal, in the sense that when either is rotated through point 3 it can be made to coincide with the other, the impairment due to a missing portion of the principal sideband is exactly compensated for by the wave due to the vestigial sideband.

The discussion above has been limited to the special case of real admittances. In what follows this restriction will be eliminated and general complex admittances will be considered. These do not lend themselves so readily to graphical representation, and for that reason the treatment will be made algebraical instead. There is, however, a close parallelism between the analysis below and that given in Fig. 8, and it is thought that reference to the figure will be useful.

Let the transfer admittance of the given system be denoted by $Y(\omega - \omega_c)$. Consider the admittance represented by $Y(\omega_c - \omega)$. These two characteristics are the mirror images of each other with respect to the carrier frequency. They are illustrated at *d*, in Fig. 8. Next let $Y(\omega - \omega_c)$ be resolved into two components after the manner discussed above

$$Y(\omega - \omega_c) = Y_1(\omega - \omega_c) + Y_2(\omega - \omega_c) \qquad (3)$$

where

$$Y_1(\omega - \omega_c) = \frac{1}{2}\Big[Y(\omega - \omega_c) + Y(\omega_c - \omega)\Big] \qquad (4)$$

and

$$Y_2(\omega - \omega_c) = \frac{1}{2}\Big[Y(\omega - \omega_c) - Y(\omega_c - \omega)\Big] \qquad (5)$$

These two components correspond to the graphical resolution shown at *b* and *c*, Y_1 being symmetrical about the carrier frequency both as to magnitude and sign and Y_2 being symmetrical with respect to magnitude but reversed with respect to sign.

Let $\cos npt$ be a component of an impressed wave, and let it modulate the carrier wave $\cos \omega_c t$. The resulting wave is

$$\cos npt \cos \omega_c t$$
$$= \frac{1}{2}\cos(\omega_c - np)\,t + \frac{1}{2}\cos(\omega_c + np)\,t \qquad (6)$$

Consider first the wave resulting from impressing the first of these (the lower sideband) on the admittance $Y_1(\omega - \omega_c)$. Let this admittance be represented by $g_1(\omega - \omega_c) + i\,b_1(\omega - \omega_c)$, the quantities g_1 and b_1 being real. Impressing the wave $\frac{1}{2}\cos(\omega_c - np)\,t$ on g_1 the resulting wave is

$$\frac{1}{2}\,g_1(\omega - \omega_c)\cos(\omega_c - np)\,t$$

or

$$\frac{1}{2}\,g_1(np)\cos(\omega_c - np)\,t \qquad (7)$$

Similarly, $i\,b_1$ gives rise to the received wave

$$-\frac{1}{2}\,b_1(np)\sin(\omega_c - np)\,t \qquad (8)$$

Carrying out similar operations for the upper sideband, and remembering the symmetry of Y_1, the total wave transmitted by Y_1 is

$$\frac{1}{2}\,g_1(np)\,[\cos(\omega_c - np)\,t + \cos(\omega_c + np)\,t]$$
$$-\frac{1}{2}\,b_1(np)\,[\sin(\omega_c - np)\,t + \sin(\omega_c + np)\,t] \qquad (9)$$

which may also be written

$$g_1(np)\cos npt \cos \omega_c t - b_1(np)\cos npt \sin \omega_c t \qquad (10)$$

In a similar manner the wave received through $Y_2(\omega - \omega_c)$ is

$$-g_2(np)\sin npt \sin \omega_c t - b_2(np)\sin npt \cos \omega_c t \quad (11)$$

Now if the wave is demodulated at the receiving end, by multiplying it by the factor $2\cos(\omega_c t - \phi)$, and if high frequencies are neglected, we have for the received wave after demodulation

$$g_1(np)\cos\phi\cos npt - b_1(np)\sin\phi\cos npt$$
$$- g_2(np)\sin\phi\sin npt - b_2(np)\cos\phi\sin npt \quad (12)$$

It now follows that the three operations, of modulation, passage through the filter, and demodulation supplemented by the suppression of high frequencies, are equivalent to the passage through a filter having the characteristic

$$Y_0(n\,p) = g_0(n\,p) + i\,b_0(n\,p) \quad (13)$$

where $g_0(n\,p) = g_1(n\,p)\cos\phi - b_1(n\,p)\sin\phi$ (14)

$$b_0(n\,p) = g_2(n\,p)\sin\phi + b_2(n\,p)\cos\phi \quad (15)$$

This formula then permits the direct computation of the received wave. It will be noted that ϕ may be given an arbitrary value. As a simplifying assumption, we may make ϕ equal to the phase shift of the carrier frequency, *i. e.*,

$$\tan\phi = -\frac{b_1(0)}{g_1(0)} \quad (16)$$

Appendix VI

Analysis for Generalized Wave Form. Let it be required to find the sinusoidal components of a periodic telegraph wave whose signal elements have the value:

$$I_h = a_h f(t - (h - 1/2)\,T/N) \quad (1)$$

where a_h is the magnitude factor and

$$f(t - (h - 1/2)\,T/N)$$

is an arbitrary function of time, and may overlap both antecedent and subsequent time units. The origin of t is taken at the beginning of the wave.

The total current, at the time t, is:

$$I(t) = \sum_{h=1}^{N} I_h = \sum_{h=1}^{N} a_h f(t - (h - 1/2)\,T/N \quad (2)$$

provided the period T is of greater duration than the wave form.

Let $I(t)$ be represented as a Fourier series; then, by a method similar to that followed in Appendix I

$$A_n = \frac{2}{T}\int_0^T \sum_{h=1}^{N} a_h f(t - (h - 1/2)\,T/N)\cos npt\,dt. \quad (3)$$

Changing the origin of t to the center of the signal element, to which each term in the summation refers, *i. e.*, putting

$$t = (h - 1/2)\,T/N + t'$$

and dropping the prime, we have:

$$A_n = \frac{2}{T}\sum_{h=1}^{N} a_h \int_{-T/2}^{T/2} f(t)\cos\left(npt + \frac{2\pi n}{N}\left(h - \frac{1}{2}\right)\right)dt$$

$$= \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos npt\,dt \sum_{h=1}^{N} a_h \cos\frac{2\pi n}{N}\left(h - \frac{1}{2}\right)$$

$$- \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin npt\,dt \sum_{h=1}^{N} a_h \sin\frac{2\pi n}{N}\left(h - \frac{1}{2}\right)$$

$$= 4s\left[C_n\int_{-T/2}^{T/2} f(t)\cos npt\,dt - S_n\int_{-T/2}^{T/2} f(t)\sin npt\,dt\right]. \quad (4)$$

In a similar manner it can be shown that:

$$B_n = 4s\left[C_n\int_{-T/2}^{T/2} f(t)\sin npt\,dt + S_n\int_{-T/2}^{T/2} f(t)\cos npt\,dt\right]. \quad (5)$$

Multiplying equation (5) by i, and subtracting from (4):

$$A_n - i\,B_n = F(\omega)\,(C_n - i\,S_n) \quad (6)$$

Where

$$F(\omega) = 4s\int_{-\infty}^{\infty} f(t)\,e^{-inp}\,dt = 4s\int_{-\infty}^{+\infty} f(t)\,e^{-i\omega t}\,dt. \quad (7)$$

To solve the reverse problem of finding $f(t)$ in terms of $F(\omega)$, put

$$f(t - T/2N) = \frac{A_0}{2} + \sum_{n=1}^{\infty}(A_n\cos npt + B_n\sin npt) \quad (8)$$

which is the Fourier series representing a signal, consisting of a single dot, whose $a_1 = 1$ and other values of a_h are equal to zero.

According to the definitions of A_n and B_n in Appendix I, it is permissible to arrange this expression as follows:

$$f(t - T/2N) = \frac{1}{2}\sum_{n=-\infty}^{\infty}(C_n - i\,S_n)\,F(\omega)\,e^{i\,npt}. \quad (9)$$

Substituting:

$$C_n = \frac{1}{N}\cos\frac{n\pi}{N} \quad (10)$$

$$S_n = \frac{1}{N}\sin\frac{n\pi}{N} \quad (11)$$

we get

$$f\,(t - T/2\,N) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \frac{F\,(\omega)}{N}\, e^{i\left(npt - \frac{n\,\pi}{N}\right)}\,. \tag{12}$$

Putting $t = T/2\,N + t'$ and dropping the prime

$$f\,(t) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \frac{F\,(\omega)}{N}\, e^{\prime\, npt} \tag{13}$$

As the period approaches infinity, the wave form approaches

$$f\,(t) = \frac{1}{8\,\pi\,s} \int_{-\infty}^{\infty} F\,(\omega)\, e^{i\omega t}\, d\,\omega\,. \tag{14}$$

Now, in order to observe the effect on the shape factor, let us consider a special wave form, for example, a wave obtained by curbing. For this type of signal the rectangular wave is cut short for a small portion of the time of the value x at the beginning and at the end of each time unit, so that

$f\,(t) = 1$, for $(-\,T/2\,N + x) < t < (T/2\,N - x)$
and

$$f\,(t) = 0$$

for
$-\,T/2\,N < t < (-\,T/2\,N + x)$
and $(T/2\,N - x) < t < T/2\,N$.

For this wave form, equation (7) reduces to

$$F\,(\omega) = 4\,s \int_{-T/2N+x}^{T/2N-x} [\cos npt - i \sin npt]\, dt$$

$$= \frac{2 \sin\,[\,n\,\pi/N - 2\,\pi\,n\,x/T\,]}{n\,\pi/N}\,. \tag{15}$$

Put $x = T/8\,N$, which corresponds to a shortening of the time during which the battery is applied to the cable to ¾ of the length of the total time unit, and apply ground during the remaining portion. For this case

$$F\,(\omega) = \frac{8\,s \sin\,(3\,\omega/16\,s)}{\omega}\,. \tag{16}$$

Comparing this shape factor with the corresponding one for the non-curbed wave, (given in Appendix I), equation (6), it is seen that the effect of curbing is to multiply the shape factor by $\sin \dfrac{3\,\omega}{16\,s} \Big/ \sin \dfrac{\omega}{4\,s}$. A network having this characteristic in the transmitted range would be equivalent to the curbing.

Appendix VII

Distortion Correction by Signal Shaping. The problem is: Given an arbitrary transfer admittance, *i. e.*, one which is not ideal, how should the sent wave be shaped in order that the received wave may be non-distorting? It is proposed to derive the solution for the case, where the value of the received wave at the middle of the time unit is taken as criterion for distortionless transmission. There are many solutions to this problem, and we will restrict ourselves, at the outset, to a particular type of wave form for the signal elements of the sent wave; namely, the type wherein each signal element is made up of a series of rectangular steps, each of one time unit duration (Fig. 9). The problem, then, consists in finding the height of each step.

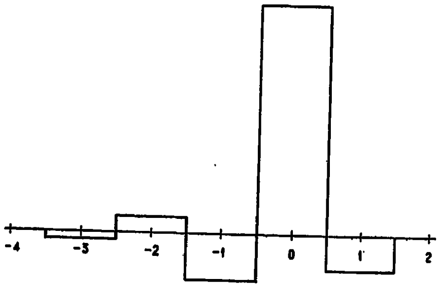


FIG 9—EXAMPLE OF SIGNAL SHAPING TO PRODUCE DISTORTIONLESS RECEIVED SIGNALS

By changing the point of view somewhat, it is possible to make use of some previously obtained results. Let us temporarily look on the unknown signal element, not as a signal element but as a signal made up of a succession of rectangular elements. The wave form of the unknown signal element is $f_s\,(t)$, and the magnitude factors of the individual rectangular elements are the unknown quantities $b_1, b_2, \ldots b_N$. We have

$$f_s\,(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} (A_n \cos npt + B_n \sin npt)\,. \tag{1}$$

The shape factor of the rectangular signal element is

$$\frac{8 \sin\,\omega/4\,s}{\omega/s}$$

and the discrimination factor obtained by looking at the sought signal element, as a signal, may be taken to be $(C_n - i\,S_n)$. The problem is now reduced to finding C_n and S_n for all values of n from 0 to $N/2$, since with these values known the b's are determined uniquely. Equation (1) may be arranged as follows:

$$f_s\,(t) = 4\,s \sum_{n=-\infty}^{\infty} (C_n - i\,S_n)\, \frac{\sin\,\omega/4\,s}{\omega}\, e^{inpt} \tag{2}$$

The transfer admittance of any network may be represented by $Y\,(\omega)$. In order to be more general, multiply $Y\,(\omega)$ by $e^{-i\omega\tau}$; where τ is an arbitrary constant, and is of the nature of a constant delay. Let $Y\,(-\,\omega) = \overline{Y}\,(\omega)$, where the bar denotes the conjugate of $Y\,(\omega)$.

The wave received over a circuit having this transfer admittance is:

$$f_r(t) = 4s \sum_{n=-\infty}^{\infty} (C_n - i\,S_n) \frac{\sin \omega/4s}{\omega}\, Y(\omega)\, e^{-i\omega\tau}\, e^{inpt} \tag{3}$$

Let $$H(\omega) = \frac{\sin \omega/4s}{\omega}\, Y(\omega)\, e^{-i\omega\tau} \tag{4}$$

and rearrange terms

$$f_r(t) = 4s \sum_{n=-N/2}^{(N/2)-1} \sum_{q=-\infty}^{\infty} (C_{qN+n} - i\,S_{qN+n}) \times H(4\pi s[q + n/N])\, e^{i(qN+n)pt} \tag{5}$$

It may be deduced from equations (7) and (8) of Appendix I, that

$$C_{qN+n} - i\,S_{qN+n} = (-1)^q (C_n - i\,S_n) \tag{6}$$

The receiving mechanism has been assumed to respond to the ordinate of the received wave at the middle of each time unit, (as representative of the signal for that unit), at which time

$$p\,t = \frac{\pi}{N}(2h - 1) \tag{7}$$

where h has values 1, 2, 3 . . . N for successive time units. We note that

$$e^{i(qN+n)pt} = e^{i(qN+n)(2h-1)\pi/N} = (-1)^q\, e^{i2\pi n(h-1/2)/N}. \tag{8}$$

Substituting equations (6) and (8), in (5), gives:

$$f_r(t) = 4s \sum_{n=-N/2}^{(N/2)-1} (C_n - i\,S_n)\, e_i^{2\pi n(h-1/2)/N} \sum_{q=-\infty}^{\infty} H(4\pi s[q + n/N]) \tag{9}$$

at the middle of the time units.

If we put $f_r(t) = 1$ for $h = 1$, and $f_r(t) = 0$ for $h = 2, 3$, etc., then the received wave has the effect of a single dot standing by itself.

Our next step is to solve the N equations, such as (9), for the N unknown values of $(C_n - i\,S_n)$, in order that we may be able to use the results in computing the sent wave. In a manner similar to that in Appendix II, multiply each of the simultaneous equations by $\cos \frac{2\pi k}{N}(h - 1/2)$, and add all the equations:

$$\cos \pi k/N = 2s \sum_{n=-N/2}^{(N/2)-1} (C_n - i\,S_n) \times \left(\frac{\cos(n+k)\pi \sin(n+k)\pi}{\sin(n+k)\pi/N} + \frac{\cos(k-n)\pi \sin(k-n)\pi}{\sin(k-n)\pi/N} + i\,\frac{\sin^2(n+k)\pi}{\sin(n+k)\pi/N} - i\,\frac{\sin^2(k-n)\pi}{\sin(k-n)\pi/N} \right) \sum_{q=-\infty}^{\infty} H(4\pi s[q + n/N]) \tag{10}$$

as k approaches any integral value, m, from $-N/2$ to $N/2$ inclusive, the numerators in the middle factor of each of the terms of the summation of equation (10) approach zero. Consequently, the factors are zero excepting one whose $n = +m$ and one whose $n = -m$, wherein certain denominators also approach zero, with the result that equation (10) reduces to

$$\cos \pi m/N = 2Ns(C_m - i\,S_m) \sum_{q=-\infty}^{\infty} H(4\pi s[q + n/N]) + 2Ns(C_{-m} - i\,S_{-m}) \sum_{q=-\infty}^{\infty} H(4\pi s[q - n/N]). \tag{11}$$

When each of the N equations such as (9) is multiplied by $\sin \frac{2\pi k}{N}\left(h - \frac{1}{2}\right)$, a similar line of reasoning gives

$$\sin \pi m/N = i\,2Ns(C_m - i\,S_m) \sum_{q=-\infty}^{\infty} H(4\pi s[q + n/N]) - i\,2Ns(C_{-m} - i\,S_{-m}) \sum_{q=-\infty}^{\infty} H(4\pi s[q - n/N]) \tag{12}$$

Multiplying (12) by i, subtracting from (11), and putting $m = n$ gives

$$\cos \pi n/N - i \sin \pi n/N = 4Ns(C_n - i\,S_n) \sum_{q=-\infty}^{\infty} H(4\pi s[q + n/N]) \tag{13}$$

$$\therefore C_n - i\,S_n = \frac{e^{-in\pi/N}}{4Ns \sum_{q=-\infty}^{\infty} H(4\pi s[q + n/N])} \tag{14}$$

At the sending end, the height of the hth step, for any convenient value of t during that time interval, is given by formula (2). Where the steps are rectangular, it is more convenient to use the formula

$$b_h = C_0 + \text{real part of } 2\sum_{n-1}^{N\,2-1} (C_n - i\,S_n)\, e^{i\frac{2\pi n}{N}(h-1/2)}$$

$$+ S_{N/2} \sin \pi \, (h - 1/2) \, . \qquad (15)$$

Substituting from (14) and (4):

$$b_h = \frac{1}{N} \left(\frac{1}{Y(0)} + \text{real part of} \right.$$

$$\left[2 \sum_{n=1}^{N/2-1} \frac{e^{i\frac{2\pi n}{N}(h-1+2s\tau)}}{\sin \frac{\pi n}{N} \sum_{q=-\infty}^{+\infty} \frac{Y(4\pi s[q + n/N])}{\pi [q + n/N]} e^{-i(4s\tau+1)q\pi}} \right.$$

$$\left. \left. + \frac{e^{i\pi(h-1+2s\tau)}}{\sum_{q=-\infty}^{+\infty} \frac{Y(4\pi s[q+1/2])}{4\pi s[q+1/2]} e^{-i(4s\tau+1)q\pi}} \right] \right) \qquad (16)$$

In using for.nula (16) for computing b_h, it will be understood that Y and s are the given quantities of the problem. The quantities N and τ which occur in addition to Y and s require comment. In the first place, τ is an arbitrary quantity and may be given, for instance, the value zero. By giving it, in succession, all the values between zero and $1/2\,s$ an infinite number of distinct solutions are obtained. The solution which is preferable, from a practical standpoint, may be selected after computations are completed for a number of values of τ. As for N, it should be chosen large in proportion to the precision required. It should be an even number, and preferably not less than twice the number of steps to be used.

The sum which occurs in the denominator is an infinite series, but it converges rapidly and in most practical cases it should be sufficient to use three or four terms. The function Y occurs with negative arguments in this expression. This does not imply negative frequencies but is merely to be taken as a convenient notation. The expression $Y(-\omega)$ is merely to be interpreted as the conjugate of $Y(\omega)$ where ω is positive.

Appendix VIII

It is required to compute the mean power of a wave characterized by the shape factor,

$$F(\omega) = 2, \text{for } 0 < \omega < 2\pi s,$$

$$F(\omega) = \sqrt{2}, \text{for } \omega = 2\pi s,$$

$$F(\omega) = 0, \text{for } \omega > 2\pi s.$$

Such a wave is

$$A_0/2 + \sum_{n=1}^{N/2} (A_n \cos npt + B_n \sin npt) \, . \qquad (1)$$

It will be sufficient to add the squares of the sinusoidal amplitudes divided by 2, to the square of the d-c. component. This gives for the mean power

$$A_0^2/4 + \left(\sum_{n=1}^{N/2-1} (A_n^2 + B_n^2)/2 \right) + \frac{B_{N/2}^2}{2} \, . \qquad (2)$$

Since

$$A_n = 2\,C_n, \qquad \text{for } n < N/2,$$

$$B_n = 2\,S_n, \qquad \text{for } n < N/2,$$

$$B_{N/2} = \sqrt{2}\,S_{N/2},$$

the power is:

$$C_0^2 + 2 \sum_{n=1}^{(N/2)-1} (C_n^2 + S_n^2) + S_{N/2}^2 \, . \qquad (3)$$

Now

$$C_n^2 = \frac{1}{N^2} \sum_{h=1}^{N} \sum_{q=1}^{N} a_h a_q \cos \frac{2\pi n}{N} \times$$

$$(h - 1/2) \cos \frac{2\pi n}{N} (q - 1/2) \, , \qquad (4)$$

$$S_n^2 = \frac{1}{N^2} \sum_{h=1}^{N} \sum_{q=1}^{N} a_h a_q \sin \frac{2\pi n}{N}$$

$$(h - 1/2) \sin \frac{2\pi n}{N} (q - 1/2) \, . \qquad (5)$$

Adding

$$C_n^2 + S_n^2 = \frac{1}{N^2} \sum_{h=1}^{N} \sum_{q=1}^{N} a_h a_q \cos \frac{2\pi n}{N} (h - q) \, . \qquad (6)$$

Substituting equation (6) in (3) and remembering that $C_{N/2}$ and S_0 are identically equal to zero, the total power becomes:

$$\frac{1}{N^2} \sum_{h=1}^{N} \sum_{q=1}^{N} a_h a_q \left(1 + 2 \sum_{n=1}^{(N/2)-1} \cos \frac{2\pi n}{N} (h - q) \right.$$

$$\left. + \cos \pi (h - q) \right)$$

$$= \frac{1}{N^2} \sum_{h=1}^{N} \sum_{q=1}^{N} a_h a_q \left[\sum_{n=0}^{(N/2)-1} \cos \frac{2\pi n}{N} (h - a) \right.$$

$$\left. + \sum_{n=1}^{N/2} \cos \frac{2\pi n}{N} (h - q) \right]$$

$$= \frac{1}{N^2} \sum_{h=1}^{N} \sum_{q=1}^{N} a_h a_q \left(\frac{\cos \frac{\pi}{N} (h-q) \sin \pi (h-q)}{\sin \pi (h - q)/N} \right) . \qquad (7)$$

The expression in parenthesis is equal to zero unless $h = q$, in which case it equals N. Hence the power is:

$$\frac{1}{N}\sum_{h=1}^{N} a_n^2 . \qquad (8)$$

Appendix IX

The Fourier Integral. Up to the present the Fourier series theorem has been used in the analysis to the exclusion of the Fourier integral theorem which is closely related to it, and it seems desirable to give a brief discussion of the Fourier integral at this point. In order to keep the discussion brief it will be necessary to make it sketchy and incomplete in respect to details. The complex notation will be used because of its compactness.

The Fourier integral theorem states that

$$f(t) = \frac{1}{\pi}\int_{0}^{\infty} du \int_{-\infty}^{+\infty} f(\lambda)\cos u\,(t-\lambda)\,d\lambda, \qquad (1)$$

regardless of the form of the function $f(t)$, provided it meets certain requirements. The requirement which is of importance in telegraph theory is that

$$\int_{-\infty}^{+\infty} |\,f(t)\,|\,dt = \text{a finite number.} \qquad (2)$$

The quantity u is so far a mathematical quantity which serves only as a variable under the integral sign, in the same sense that λ does. In studying the integrand it will be assumed that u is identical with the quantity ω which occurs in expressions for impedances and admittances. With this assumption the theorem expressed by equation (1) can be written, in complex notation:

$$I(t) = \frac{1}{2}\int_{-\infty}^{+\infty} J(\omega)\,e^{i\omega t}\,d\,\omega, \qquad (3)$$

$$J(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} I(t)\,e^{-i\omega t}\,dt. \qquad (4)$$

$I(t)$ expresses the wave as a function of time and $J(\omega)$ expresses it as a spectrum or a function of frequency.

For comparison we will now write down the Fourier series theorem in an analogous form:

$$I(t) = \frac{1}{2}\sum_{-\infty}^{+\infty} K(\omega)\,e^{i\omega t}\,\Delta\,\omega, \qquad (5)$$

$$K(\omega) = \frac{1}{\pi}\int_{-\pi/\Delta\omega}^{+\pi/\Delta\omega} I(t)\,e^{-i\omega t}\,dt\,. \qquad (6)$$

where ω takes the values 0, $\pm\,\Delta\,\omega$, $\pm\,2\,\Delta\,\omega$, etc., and $\Delta\,\omega$ is $2\,\pi$ times the reciprocal of the fundamental period of the series. It will be apparent that there is a close, formal similarity between equations (3) and (4) on the one hand, and equations (5) and (6) on the other.

Now let the wave $I(t)$ represent an arbitrary telegraph signal made up of N signal elements. Formulas (4) and (6) give alternative ways for expressing the wave spectrally. The latter represents the wave as a series of discrete frequency components; the former represents it as a continuous spectrum. Both representations are exact when properly interpreted, although they differ in form; for when substituted in formulas (5) and (3) respectively they yield the original wave precisely. Both representations are limited to a finite number of signal elements,—the series because its period must be finite, and the integral because of equation (2).

While these analyses are true for any succession of N signal elements which may be selected, their utility depends upon severe restrictions on the signal elements which precede and follow the signal under analysis. To be of value in circuit theory the series demands that the succeeding and preceding elements consist of an indefinitely great number of repetitions of the signal under analysis. Similarly, the integral depends for its utility on the assumption that it is followed and succeeded by infinite sequences of elements having the magnitude zero. These restrictions are normally not serious because there is no limitation on N other than that it should be finite. In addition to being equally exact, the two methods then appear to be substantially equally general (or restricted) in respect to the complexity of telegraph signals which they are capable of representing.

As might be surmised, most of the results obtained in the paper follow from either method of treatment. First, if the d-c. wave is analyzed by the integral method it will be found that $J(\omega)$ separates into two factors, a discrimination factor and a shape factor. The discrimination factor has the property of falling into equivalent bands, each of width equal to the speed of signaling. From this it is concluded that the necessary frequency range does not exceed the speed of signaling. The various ideal form factors corresponding to various criteria follow with somewhat greater difficulty. Whereas the series treatment led to a system of linear algebraic solutions, the integral treatment leads to a system of integral equations.

The only outstanding difficulty with the integral analysis appears in connection with establishing the fact that the minimum frequency band equals the speed of signaling. In other words, while it is easy to show that the information contained in one frequency band of width (s) is sufficient for the transmission of intelligence, it is difficult to show that it is also necessary. On the series analysis as used in the paper, this result followed very easily from the well-known principle that the number of unknowns which can be found does not exceed the number of given equations. When the frequency components are not discrete but form a continuous spectrum no such simple argument appears to be available. A somewhat full discussion will be given.

On considering the matter it is apparent that the difficulty arises from the lack of a precise formulation of what is meant by transmitting intelligence. No formal criterion was necessary in the case of the Fourier series treatment, but it will be necessary to formulate one to make satisfactory progress with the integral treatment. Stated in general terms the requirement of the received wave is that it should be possible to make measurements on it extending over a finite interval, and from these measurements (together with the known properties of the admittance) to determine the sent wave. More specifically, it should be possible to express the magnitude factors of the sent wave as a linear sum (or integral) of the measured values of the received wave. If the sent and received waves are expressed as functions of ω, and the computations modified accordingly, it is obvious that the operations can be carried out on the sent wave in which case the received wave will be non-distorting. We may then lay down as a criterion for a telegraph system's ability to transmit intelligence, that it should be possible to shape the sent signals so as to make the received wave non-distorting. In the remainder of the appendix this criterion will be used. It will be apparent that the introduction of this criterion constitutes, in a sense, a fresh start.

Taking for the criterion of a non-distorting wave that it should have correct magnitude at the central point of the time unit, and limiting the discussion to the case where the frequency is limited to the range $0 - s$, the shape factor of the received wave is uniquely determined and is 2 throughout that range. The shape factor of the transmitted wave is $2/Y$ and the wave form at the sending end is

$$f_s(t) = \int_{-2\pi s}^{+2\pi s} \frac{2\, e^{i\omega t}\, d\,\omega}{Y(\omega)}. \tag{7}$$

If $1/Y$ is finite throughout this range, this integral has a definite value. If Y has a simple zero at $\omega = \omega_a$ the integral may be written

$$f_s(t) = e^{i\omega_a t} \int_{-2\pi s}^{+2\pi s} \frac{2\, e^{i(\omega - \omega_a)t}\, d\,\omega}{Y(\omega)}. \tag{8}$$

The first derivative with respect to t of the definite integral may now be evaluated. The derivative is determinate and by integrating it with respect to t a suitable value for $f(t)$ may be obtained.

If Y has a finite number of simple zeros, the function $1/Y$ may be broken up into a number of terms each of which has not more than one simple zero; and the individual terms can be treated as above.

When the function Y has double zeros, or zeros of any finite order of multiplicity, suitable values of the function $f(t)$ can be found by an extension of the same method.

The significant results for a simple case will now be stated. Suppose that the transfer admittance has a simple zero at the origin (as in telegraphing through a transformer); the sent wave, which gives a distortionless received wave, does not approach zero as t approaches $+\infty$ but approaches a constant a. If the zero at the origin is double (telegraphing through a high-pass filter, made up of one series condenser and one shunted coil) the sent wave should approach the value $b\,t$ as t approaches ∞. For a triple zero the corresponding asymptotic value is of the form $c\,t^2$, etc. It is obvious that as the multiplicity of the zero increases the function representing the sent wave rapidly becomes more and more unmanageable. Now, if Y becomes zero throughout a finite interval at the origin, we may think of it as having a zero of an infinite order of multiplicity. It is to be expected then that there should be no finite function capable of satisfying the requirements.

Automatic Control of Edison Systems

As Applied in the St. Charles Street Substation of the Union Electric Light and Power Company

BY O. J. ROTTY* and E. L. HOUGH†
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—A brief description of the Edison System at St. Louis is given, and power supply, service restoration, and system balance are discussed. The history of the application of automatic control equipment to the system and a brief résumé of operating experiences are also given.

The relation of the St. Charles St. Substation to the remainder of the system is pointed out and certain features of its design explained. The scheme of loading the machines and the method of load shifting between machines is given in detail, and a table is given to show the "set-up" for any load. The distinctive features of the various types of machines at this station are discussed, particular attention being given to the control of the converter with transformer taps and high-voltage revolving field booster. The scheme of automatically changing taps under load is described fully. A brief summary of the protective features on this machine is also given.

The operating experience with the automatic equipment at the St. Charles St. Station is related.

* * * * *

INTRODUCTION

EDISON d-c. systems in large cities have been the result of natural growth and in many cases have reached such a size that their replacement would be a tremendous undertaking even though a careful study might indicate that such a step might be desirable from an engineering standpoint. Since the operation of Edison systems is a big problem to many companies, it was felt that a description of the manner of revamping the Edison system in St. Louis and the results obtained would be of interest. In revamping this system the use of automatically controlled machines was thoroughly studied and tried out. In this paper particular emphasis is given to the application of automatic control to the St. Charles Street Substation, Fig. 1, which is the largest Edison station and represents the latest design of equipments.

THE EDISON SYSTEM

The 250-volt Edison district of the Union Electric Light and Power Company covers an area approximately one mile wide by 1¾ mi. long and at present attains a peak load of approximately 32,000 kw. The district is supplied from eight substations located as shown on Fig. 2.

The a-c. power supply to the stations is derived from two separate and distinct systems, the system supplying each station being given in Table I. Five of the stations are supplied at 60 cycles and three at 25 cycles with approximately 50 per cent of the total capacity at each frequency. The 25-cycle power is generated at the Keokuk hydroelectric plant and at the Ashley St. steam plant. The 60-cycle power is supplied by the Cahokia and Ashley St. steam plants. In other words, the Edison stations receive power at two frequencies and from two plants at each frequency, so that trouble on either system or at any plant will only partly affect the Edison load.

No stand-by batteries are maintained on the 250-volt Edison system, the last ones having been disconnected from the system about four years ago. Although this type of load demands a high degree of service continuity, it was felt that the improved reliability of the a-c. supply, together with the use of automatic control equipment for service restoration, made the batteries unnecessary. Motor-generator sets are relied upon for service restoration, all of them having stable operation at all voltages and all are a-c., starting with the exception of two of the manual motor-generators at Twentieth Street. The rotary-converters are capable of operation through a considerable range of d-c. voltage and all are a-c. starting. The three 9000-ampere, 60-cycle converters have a lower limit at 50 per cent voltage while the others have lower limits varying from 60 to 90 per cent.

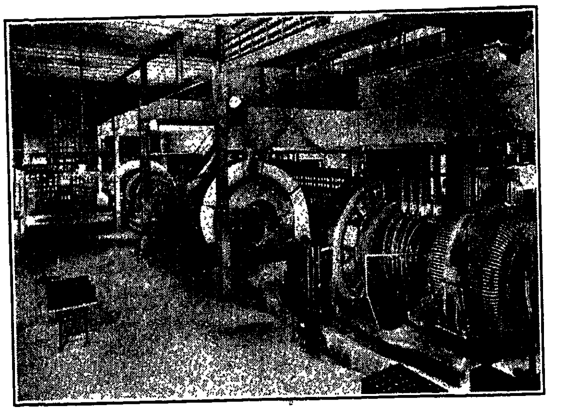


FIG. 1—GENERAL VIEW OF ST. CHARLES ST. STATION INTERIOR

It will be noted from Table I, that somewhat in excess of 50 per cent of the capacity is in motor-generators. This means that in the event of a complete Edison system outage, the motor-generators will "come in" first and will be of sufficient capacity to raise the voltage

---

*Substation Division, Union Elec. Lt. & Pr. Co., St. Louis, Mo.

†Engineering Dept., General Electric Co., St. Louis, Mo.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-18, 1928.

to something above 50 per cent. At this point the first converters will "come in" and further raise the voltage, being followed by the other converters as the system reaches their lower operating limit. As added insurance that the motor-generators will produce enough voltage for the converters to "come in," should some machines be out of service, most of the motor-generators have an emergency capacity of 120 per cent load for a limited period during times of low voltage.

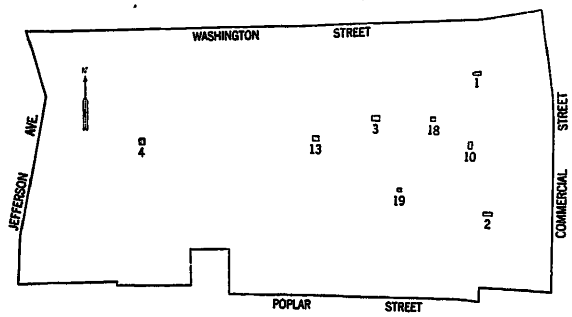


FIG. 2—OUTLINE OF ST. LOUIS EDISON DISTRICT SHOWING LOCATION OF SUBSTATIONS

This district is about 1¾ by 1 mi. in area.

TABLE I
SUBSTATION DATA

Substation Name	No.	Supply frequency	Control	Apparatus Motor-gen. No.	Apparatus Motor-gen. Capacity d-c. amperes	Apparatus Converter No.	Apparatus Converter Capacity d-c. amperes	Station total capacity d-c. amperes	Station load at time of system peak 12-22-26 d-c. amperes
Lucas Ave.	1	60	Auto.	1	7,500	1	9,000	16,500	16,400
Walnut St.	2	60	Auto.	1	7,200	1	9,000	16,200	16,200
Twelfth Blvd.	13	60	Auto.	1	7,200	..		7,200	6,800
Railway Exchange	18	60	Auto.	1	15,000	..		15,000	14,800
Eighth St.	19	60	Auto.	1	7,500	1	9,000	16,500	14,700
Total 60 cycle					44,400		27,000	71,400	68,900
St. Charles St.	3	25	Auto.	2	14,400	2	16,400	30,800	30,100
Twentieth St.	4	25	Auto. Man.	1 3	7,200 12,000	.. 1	9,000	28,200	22,700
Boatmen's Bank	10	25	Man.	..		2	16,000	16,000	7,500
Total 25 cycle					33,600		41,400	75,000	60,300
Total Edison System					78,000		68,400	146,400	129,200

NOTE. All current values at 250 volts.

The system neutral is supplied by the rotary converters, the motor-generators being two-wire machines in every case. The converters are fairly well distributed over the system and a sufficient number are kept running during the night to properly maintain the neutral. However, to protect against destructive voltages on one side of the system, due possibly to a shortage of three-wire machines at times of trouble, the motor-generators are provided with relays to prevent more than 55 per cent of normal voltage between either side of the system and neutral. In other words, the control is arranged so that with the system unbalanced, the voltage between the highest side and the neutral will not greatly exceed one-half of the normal maximum across it, the total voltage being whatever it may.

AUTOMATIC DEVELOPMENT

The company's first automatically operated d-c. machine was installed in 1922 at the Twelfth Boulevard Substation. The automatic features were considered somewhat of an experiment by the operating engineers but their installation was felt to be justified as the station was very close, electrically, to the St. Charles Street Substation and as it was located in the basement of the company's office building, it was possible to give it close observation.

Many difficulties were encountered in putting the equipment into service initially, but they were principally unforeseen operating problems or troubles with individual devices and not faults of the fundamental scheme of control. In practically every case, these difficulties were remedied by making certain alterations or additions either to details of the general scheme or to some particular device. During several system disturbances, the automatic control gave a good account of itself and functioned exactly as intended. It was apparent that automatic control was a practical thing and well worth further application.

In 1923 another 1800 kw. motor-generator, automatically operated, was purchased and installed at the Walnut St. Station. This station was then manually operated and the automatic machine was intended to relieve the off peak shift of operators, as well as to replace certain depreciated machines.

The scheme of control was somewhat different from the Twelfth Boulevard Station and included some previously untried device applications. Again many difficulties arose but in most cases the remedies were comparatively simple. It was felt that cooperation between the manufacturer and the operating company had solved many problems and that further experience and cooperation would result in additional improvements.

The third installation was made in 1924 and was a 3750-kw. motor-generator set installed in the very heart of the Edison load at a new station known as the Railway Exchange Substation. This was a three-unit set, one motor driving two generators in parallel with provisions for balancing the load on the generators. At that time it was reputed to be the largest automatically-controlled motor-generator set in Edison service.

At this time almost every case of trouble had its lesson, for it usually indicated a weak point in a particular scheme or device or a peculiar operating condition which had not been foreseen. A case of trouble on one automatic equipment usually meant the possibility of the same thing occurring on the others and consequently the remedy had to be applied to all.

The next addition was at the Eighth Street Substation. This was a new station in which was installed a 2250-kw. rotary converter and an 1875-kw. motor-generator set operating in parallel on both the a-c. and d-c. sides. The scheme of automatically operating machines of these characteristics in parallel was entirely new and several problems arose on that account.

Immediately after the Eighth Street installation was made, a duplicate 2250-kw. converter was installed at the Walnut St. Substation and interlocked so as to operate in parallel with the motor generator set installed there in 1923. This released all operators from Walnut St. and made the station fully automatic.

By the middle of 1925 all of the machines had had opportunities to perform on service restoration under various conditions. On almost every occasion they functioned properly and were carrying the load before the manual stations. Although troubles were still occurring, they were generally of a minor nature and most of them were being detected during regular inspections and before damage resulted. It was felt that the practicability of automatically controlled machines on a large system had been definitely proved.

THE ST. CHARLES STREET SUBSTATION

The St. Charles Street Substation has always been the largest Edison station and being very close to the load center, it has always been considered a base load station. Its d-c. feeders interlace with those of five other stations and it can affect more of the system than any other station. In 1925 all of the manual machines at St. Charles Street were, with one exception, reaching the end of their life and it was necessary that they be replaced. There was no question of whether the new machines should be manually or automatically controlled for aside from the apparent saving in operating expenses, the experience with automatic substations had demonstrated their ability to withstand system disturbances, their greater speed in service restoration, and their superior voltage regulation.

In planning the station it was found that no additional load was to be provided for at this time, the station capacity remaining at approximately 7500 kw. In accordance with the practise of having half the capacity in motor generators and half in converters, there were to be installed about 3750 kw. in each type of machine. Two 1800-kw. motor-generators were decided upon, as they matched the sizes of others in service and were standard with the manufacturers. It was felt that the condition of the 2000-kw. booster converter already in the station, although 8 years old, warranted the expense of automatic control. This left about 1900 kw. for new converter capacity. A 2250-kw. converter was selected, it being a standard size and of the same capacity as others in service.

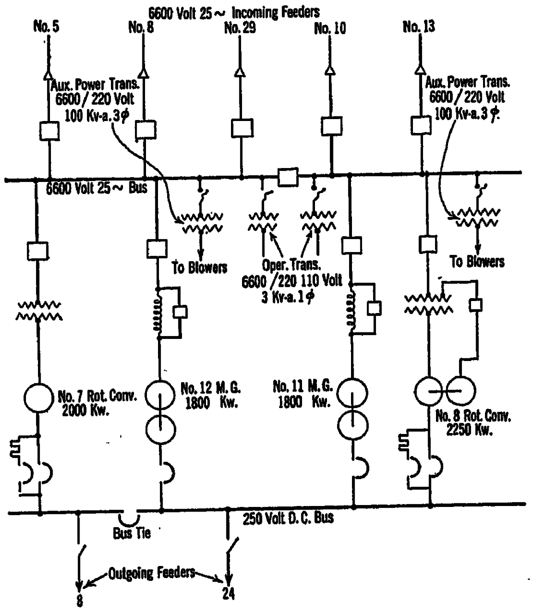

6600 Volt 25~ Incoming Feeders
No. 5
No. 8
No. 29
No. 10
No. 13
Aux. Power Trans. 6600/220 Volt 100 Kv-a. 3φ
Aux. Power Trans. 6600/220 Volt 100 Kv-a. 3φ
6600 Volt 25~ Bus
To Blowers
Oper. Trans. 6600/220 110 Volt 3 Kv-a. 1φ
To Blowers
No. 7 Rot. Conv. 2000 Kw.
No. 12 M. G. 1800 Kw.
No. 11 M. G. 1800 Kw.
No. 8 Rot. Conv. 2250 Kw.
Bus Tie
250 Volt D. C. Bus
Outgoing Feeders
8
24

FIG. 3—ONE-LINE DIAGRAM OF ST. CHARLES ST. SUBSTATION

Fig. 3 shows a one-line diagram of the station. It will be noted that power is received at 6600 volts, 25 cycles, over five underground feeders. The a-c. structure is insulated for 13,200 volts and all machines are provided for reconnection to that voltage, in order that the station can be changed over to the higher voltage at some future time with a minimum amount of work. The outgoing feeders are of 1,500,000 cm. and 2,000,000 cm. cross-section, about one-third being of the larger size.

The ventilating equipment is probably of interest There are two blowers, each rated at 57,000 cu. ft. per min. at 3½ in. pressure and driven by 50-hp. a-c motors. One blower is "preferred" and runs when

TABLE II
LOADING SEQUENCE CHART

Load on machine				Total load on station	Operating condition
No. 8	No. 12	No. 11	No. 7		
1000	0	0	0	1,000	Minimum load
8400	0	0	0	8,400	Load increases No. 8 gives signal to start No. 12
7000	1400	0	0	8,400	After No. 12 is on
7000	6300	0	0	13,300	Load increases Before No. 12 recalibrates No. 8
9000	4300	0	0	13,300	After No. 12 recalibrates No. 8
9000	6800	0	0	15,800	No. 12 gives signal to start No. 11
9000	5200	1600	0	15,800	After No. 11 is on
9000	5200	6300	0	20,500	Load increases Before No. 11 recalibrates No. 12
9000	7200	4300	0	20,500	After No. 11 recalibrates No. 12
9000	7200	6800	0	23,000	Load increases No. 11 gives signal to start No. 7
9000	5200	1400	7400	23,000	*After No. 7 is on
9000	5200	6300	7400	27,900	Load increases Before No. 11 rocalibrates No. 12
9000	7200	4300	7400	27,900	After No. 11 recalibrates No. 12
9000	7200	7200	7400	30,800	Load increases Maximum load
9000	7200	3000	7400	26,600	Load decreases Before No. 11 decalibrates No. 12
9000	5200	5000	7400	26,600	After No. 11 decalibrates No. 12
9000	5200	1000	7400	22,600	Load decreases *No. 11 gives signal to shut down No. 7
9000	7200	5400	1000	22,600	No. 7 reduces load, preparatory to timing off
9000	7200	6400	0	22,600	After No. 7 is off
9000	7200	3000	0	19,200	Load decreases Before No. 11 decalibrates No. 12
9000	5200	5000	0	19,200	After No. 11 decalibrates No. 12
9000	5200	1000	0	15,200	Load decreases No. 11 gets signal to shut down
9000	6200	0	0	15,200	After No. 11 is off
9000	3000	0	0	12,000	Load decreases Before No. 12 decalibrates No. 8
7000	5000	0	0	12,000	After No. 12 decalibrates. No. 8
7000	1000	0	0	8,000	Load decreases No. 12 gets signal to shut down
8000	0	0	0	8,000	After No. 12 is off
1000	0	0	0	1,000	Load decreases Minimum load

NOTE: All loads are in amperes at 250 volts.

*Loads given are with maximum load on No. 7 and represent the worst possible distribution of load.

ever any machine is running while the second "comes on" when the general station air or the discharge air reach certain predetermined temperatures. A throw-over switch is provided to make either blower "preferred," the other operating on temperature.

The flow of air through the station is opposite to natural circulation. The air enters the station from a shaft through the roof and is drawn into the machines which have their end bells left off. The machines discharge into the basement beneath them from which point the blowers draw the air to discharge it through a stack in a corner of the station. This results in the general station being in the "intake" and at a slight vacuum so that any leakage around doors or windows will let cool air enter the station rather than cause air to be wasted, as is the case in systems where the general station is under pressure. This also results in more bearable summer temperatures than in systems where the machines discharge directly into the general station.

The automatic control equipment for the station contains a number of features which are unique and new in this installation and will, for this reason, be of particular interest. Some of these schemes of control were made necessary by the peculiar problems met with in the design of this station such as the use of a comparatively large number of automatically controlled machines, the use of a rotary converter with a new type of control for picking up the bus at low voltages, and the necessity for unusually close voltage regulation due to the heavy net-work and load in the area surrounding the station. Other features are new developments in the art and will probably find further use in other installations. In so far as possible, this description will be confined to those features believed to be of particular interest and omitting such as have previously been described in the technical press.

The scheme of load division between machines of different characteristics operating in parallel, as employed at this station, was conceived and originally applied at the Eighth Street Substation by the company's operating engineers. The application in this case was simply an extension of the original scheme.

STARTING, STOPPING AND LOADING SEQUENCE

The entire sequence of operations with operating values is outlined in Table II. During normal light-load conditions the tap changing booster-type rotary converter, machine No. 8, supplies the entire station load. The station voltage is regulated by variation of the motor-operated booster field rheostat in response to the voltage regulating relay No. 58. This voltage regulating relay is controlled by potential from a pressure wire bus so that it tends to hold the net-work potential constant within its area. The bus voltage therefore varies in accordance with load conditions.

When the load on converter No. 8 reaches 95 per cent of full load, the load-responsive-starting relay No. 1A on No. 8 machine operates through a time-delay starting relay to bring onto the bus the second machine in the sequence, No. 12 motor-generator set. Since the current-regulating relays, No. 57, are set to limit the load on the converter to 100 per cent of full load, it can be seen that the lower setting on the starting relay permits bringing the second machine on before the system voltage commences to fall.

When machine No. 12 closes its d-c. breakers, it takes the voltage control away from converter No. 8 and assumes load. The converter voltage is then controlled by its current regulating relays No. 57. With no increase of station load, motor-generator No. 12 would pick up a relatively small load which would not be sufficient to keep it on. To prevent this its current limit recalibrating relay, No. 95, is arranged to decalibrate the current regulating relays on converter No. 8. This causes the rotary converter to reduce its load approximately 2000 amperes, or 25 per cent. This load, together with additional load which may come on, is picked up by motor-generator No. 12 until at 6300 amperes, its current limit recalibrating relay picks up to recalibrate the current regulating relays on converter No. 8 again. The converter then carries 100 per cent load and No. 12 motor-generator continues to absorb the oncoming load until it reaches 95 per cent of full load.

At this point, the load responsive starting relay No. 1A on No. 12 motor-generator functions to start the third machine in the sequence, No. 11, after a time delay. As soon as motor-generator No. 11 gets on the bus, it assumes control of station voltage and takes this control away from machine No. 12. Thus, rotary converter No. 8 and motor-generator No. 12 are being controlled by their respective current regulating relays and motor-generator No. 11 has its voltage controlled by voltage regulating relay No. 58. To insure that No. 11 machine will pick up enough load to stay on, its current limit recalibrating relay, No. 95, functions to decalibrate the current regulating relay on No. 12 machine by approximately 2000 amperes or 25 per cent. As the load builds up further, No. 12 machine is again permitted to assume full load as limited by its current regulating relay.

When the load on No. 11 machine exceeds 95 per cent of full value, its load responsive starting relay, No. 1A, causes rotary converter No. 7 to be started after the expiration of a short time delay. This machine is not paralleled with the others in the station but, instead, feeds the network through a separate group of feeders on a section of the main bus. This section is automatically split off by opening of the bus tie breakers when the converter is closed onto the bus.

With No. 7 converter on, voltage control is left with No. 11 machine as before and the voltage balancing relay No. 60 of converter No. 7 balances the voltage of its section of bus against that of the main bus and holds the voltage approximately two volts higher than the main bus. This results in converter No. 7 taking a little more than its share of the load and insures its taking enough load to stay on.

If for any reason the average feeder end voltage should become more than two volts low, the voltage regulating relay, No. 58 on No. 7 converter, will take control away from No. 60 balancing relay and will maintain, within the limit of No. 7 machine capacity, the average feeder end voltage at two volts below normal.

With decreasing load the machines are shut down in the reverse order from coming on. Machine No. 11 will reach its light load point before converter No. 7 and is arranged to give No. 7 the signal to shut down. No. 7 first closes the bus tie breakers and then reduces its load down to the setting of its own light load relays, that load being assumed by machines No. 11 and No. 12. Converter No. 7 then "times off" in the usual manner.

Further reduction of load shuts down machine No. 11 and still further reduction shuts down machine No. 12. Rotary converter No. 8 is not arranged with load-responsive starting and stopping since it is required on the bus at all times to maintain the net-work voltage and the system neutral at this point.

Provision is also made for starting the next machine in the sequence if the last machine called for by the load is held off by protective devices or manual control switches. The next machine then does whatever the disabled machine would have done in the way of load shifting and voltage control.

In coming back on the bus after a system outage, it is not permissible to allow converter No. 8 to be connected to the bus below 140 volts which is its lowest regulating point. In this case the bus undervoltage relay, although permitting No. 8 machine to complete its starting sequence to the point where it is ready to be connected to the bus, functions to start both motor-generator sets as fast as power supply conditions permit. As soon as the motor-generators, together with other stations, have raised the bus voltage to 140 volts or more, rotary converter No. 8 is connected to the bus and boosts the system voltage to the limit of its ability. At approximately 210 volts converter No. 7 will also be brought on the bus. This should raise the net-work voltage to approximately normal provided the other units on the system have started properly.

By means of pilot-wire remote-control, the system load dispatcher at Twelfth and Locust Streets may start or stop any machine. Since he has an indication of the load being carried by each machine and average feeder end voltage, this feature is of great value. It permits him to adjust, within limits, the Edison network load between the 25-cycle and 60-cycle systems in accordance with general system conditions.

MOTOR-GENERATORS

Except for the load-responsive starting, and voltage and load-regulating schemes described above, the control for both motor-generator sets is similar to installations previously described before the Institute. The synchronous motors are started by throwing onto the line through reactors, the reactors being shunted out when the machines have reached approximately synchronous speed. The field is applied at the time of transfer and upon equalizing the bus and machine voltages, load is applied to the d-c. generators. The sets are of the differential compound type, normally operated with the series field shunted by an automatically controlled circuit breaker. The control is arranged and the sets are suitable for re-synchronizing under load in the event of a temporary power interruption.

CONVERTER NO. 7

The booster-type synchronous converter No. 7 is arranged for *Y*-delta, starting by oil circuit breakers on the high-voltage side of the transformer. Correct polarity is established by temporary reversal of the field. This machine is connected to the bus through cushioning resistors which are arranged to give two-step operation. When the voltage of No. 7 machine and the station bus have been properly equalized the resistor breakers No. 73 are closed. After a short time delay, line breaker No. 72 positive is closed and after a further time delay, No. 72 negative is also closed. Thus only one step of resistance is shunted at a time. The closing of No. 72 negative is followed immediately by the closing of the neutral breaker No. 71 and the opening of the bus tie breakers No. 24. This sequence, in addition to giving two-step resistor operation, prevents lowering of the net-work voltage at the ends of the feeders connected to this section of the bus and allows ample time for the converter voltage to be adjusted properly.

CONVERTER NO. 8

Synchronous converter No. 8 is a shunt-wound, 2250-kw., three-wire machine with high-voltage, revolving-field booster; the scheme of connections being shown in Fig. 4. The converter has the customary series and shunt commutating fields. The booster is provided with a double armature winding so connected that with a given direction of excitation on the field, the voltage of one winding will be in the "boost," and the other in the "buck" direction. The winding in use under any particular condition will depend upon the position of the tap switches. The high-voltage neutral is formed at the common point of the booster windings. The transformer is of the air-blast type and is provided with an extended winding having four ten per cent taps. Connections between the transformer taps and the booster windings are made by means of tap switches which are triple-pole oil circuit breakers.

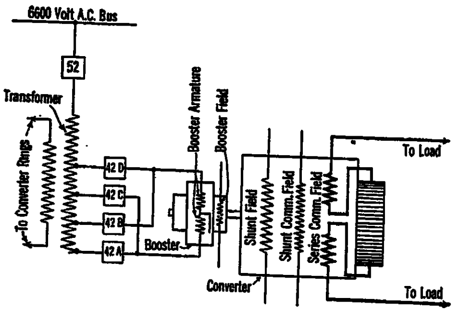


FIG. 4—SCHEME OF CONNECTIONS OF WINDINGS, NO. 8 CONVERTER

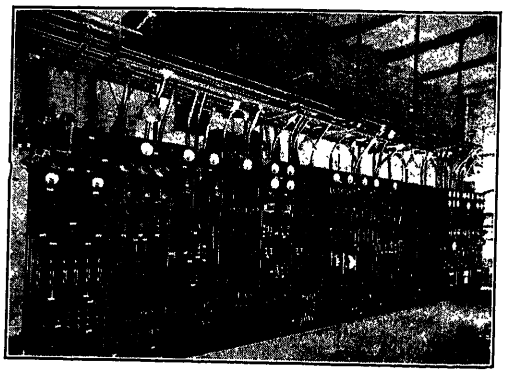


FIG. 5—CONTROL BOARD FOR NO. 11 AND NO. 8 MACHINES, BLOWERS, INCOMING FEEDERS AND COMMON MACHINE CONTROL

The control boards for machines No. 8 and No. 11 are shown in Fig. 5.

The control scheme used on converter No. 8 is in many respects new and novel and for this reason a rather full description will be given.

If we assume system conditions to be normal with bus voltage at normal value and machine No. 8 to be held off by the opening of master switch No. 5 the starting operation will be initiated by the closing of this switch which acts to energize the time-delay starting relay. After the expiration of the time interval for which this relay is set, the master contactor No. 4 will be closed provided line and machine conditions are proper as indicated by the protective devices. The oil circuit breaker closes immediately to energize the converter transformer and the motor-driven sequence switch No. 34 starts to revolve. This permits No 42A tap switch to close thereby closing the *Y* of the transformer through the booster windings and impressing half voltage on the converter.

While the converter is coming up to speed No. 34 sequence switch is stopped but restarts when synchronous speed is reached, as indicated by synchronous speed relay No. 13. This permits field-flashing contactor No. 31 to close and energize the field from a small motor-generator set to properly polarize the converter. When correct polarity has been established as indicated by field-changing relay No. 93, the field flashing contactor is opened and the shunt field contactor No. 41 is closed to make the converter self-excited.

When full field is reached as indicated by field relay No. 40 further movement of No. 34 sequence switch is permitted which opens the low-voltage tap switch

No. 42A and closes the highest-voltage tap switch No. 42D. The booster field is then applied by the closing of the booster field contactor, No. 41C, and the auxiliary commutating field is excited by the closing of auxiliary commutating field contactor, No. 41A.

The voltage of the converter is then equalized with the bus voltage by the operation of voltage equalizing relay, No. 91, which has control of the booster field rheostat at this time. When the voltages are properly equalized, voltage balance relay No. 60 permits the load limiting resistor breakers, No. 73, to close. The closing of No. 73 breakers is followed immediately by the closing of the line breakers, No. 72. Upon the closure of circuit breakers, No. 73, the control of the motor-operated booster field rheostat, No. 70, is given to the voltage and current regulating relays. These relays operate to hold the voltage of No. 8 machine within 1¼ volts of normal value on the pressure wire bus until the converter is loaded to approximately 8700 amperes. At this point the current regulating relays No. 57 take control and hold the current on this machine at an average value of 9000 amperes.

TAP CHANGING

When voltage reduction is necessary, the booster rheostat is run in the "buck" direction. When maximum buck is reached as indicated by limit switches on the booster rheostat and relay No. 60A, which balances the booster voltage with a-c. line voltage, a circuit is made to close tap switch No. 42C. It will be noted from the diagram, Fig. 4, that this connects the two ends of the booster across high-voltage taps on the transformer. As soon as No. 42C closes, No. 42D is opened and the converter is operating on the second transformer tap with the booster field rheostat in the maximum boost position. The transition from buck to boost is accomplished by the use of a double winding in the booster. When the tap change is made a shunt field contactor No. 41D is closed to shunt a section of the resistance in the shunt field of the converter in order to increase the shunt field current sufficiently to correct for the change from buck to boost on the booster with the attendant change from generator to motor action on the converter. This action is simultaneous with the reversal of the auxiliary commutating field of the converter. These operations tend to raise the voltage of the converter slightly so that the booster rheostat tends to run out of the maximum boost position due to the action of the current regulating relays. This prevents "pumping" of the transfer equipment at the tap point.

If further voltage reduction is necessary, the booster rheostat moves out of the boost position and finally reaches approximately maximum buck. At this point, as indicated by rheostat position and No. 60A relay, tap switch No. 42B is closed and No. 42C opened to connect the converter to the next lower tap. The reversal of the booster, the reversal of the auxiliary commutating field, the action of shunt field contactor No. 41D and the backing off of the booster rheostat are accomplished in identically the same manner as previously described.

The necessity for further voltage reduction will force the converter to the lowest tap by an operating sequence similar to that above which closes tap switch No. 42A and opens No. 42B. Demand for still lower voltage will force the booster rheostat into the maximum buck position and if this is not sufficient to reduce the load on the converter to a proper value the station d-c. undervoltage relay, No. 80A, will function to disconnect the machine from the bus. The converter will continue to run, in readiness to again assume load as soon as voltage conditions permit.

When the bus voltage rises above the setting of undervoltage relay No. 80A this relay will function to again connect the converter to the bus. The current regulating relays, which have control until the bus voltage is normal, which will be with tap switch No. 42D closed, function to run the booster rheostat to the maximum boost position. At this point, as indicated by rheostat position and booster voltage relay No. 60A, tap switch No. 42B will be closed and No. 42A opened. Simultaneously the auxiliary commutating field on the converter is reversed and shunt field contactor No. 41D is dropped out to insert resistance in the shunt field to again compensate the converter for the booster change, this time from boost to buck rather than from buck to boost as previously described. This compensation is sufficient to lower the converter voltage slightly and thereby cause the booster rheostat to run toward the boost position which, it will be noted, is the reverse of the condition when "tapping down." Further decrease of converter load or increase of bus voltage will cause the tap changing equipment to "tap up" until the highest tap is reached, each tap change following the sequence outlined above. When the highest tap is reached, the converter is regulated to hold proper voltage or current in accordance with the indications of the voltage and current regulating relays.

PROTECTIVE FEATURES

The size and importance of this converter unit warrant extensive protective equipment and an endeavor has been made to protect against all operating contingencies. Most of these protective features are of a standard type and, for this reason, the descriptions of these features will not be detailed.

1. Starting on single-phase or incorrect phase rotation is prevented by single- and reverse-phase voltage relay No. 47.

2. Starting on low a-c. voltage is prevented by undervoltage relay No. 27. If low voltage occurs while the machine is running, undervoltage relays No. 27A shut the machine down.

3. Correct position of the brushes is assured by

interlocks on the brush-operating mechanism which prevent starting with the brushes down or closing of the d-c. breakers with the brushes up.

4. If the starting sequence is not completed within a predetermined time interval, starting protective relay No. 48 will shut down and lock out the unit.

5. The failure of field excitation will be indicated by field relay No. 40 which is in series with the converter shunt field or by field relay No. 40A which is in series with the booster field. Either of these devices will operate through an auxiliary relay, No. 40X, to shut down the unit and hold it off awaiting an inspection.

6. Reverse power as indicated by reverse-current relay No. 56 opens the d-c. line breakers No. 72 to insert the load-limiting resistors in the circuit.

7. Overheating of any bearing will trip its bearing-temperature relay, No. 38, to shut down and lock-out the unit.

8. Overheating of the load-limiting resistors operates resistor-temperature relays No. 28 to shut the machine down. When the resistors have cooled sufficiently No. 28 relays permit restart.

9. Flashover or grounded windings will operate grounding protective relay No. 64 to shut down the converter and prevent its restart.

10. Overspeed causes the speed-limit switch No. 12 to operate, shutting down the machine and tripping the neutral circuit breaker. This guards against the possible failure of one side of the line to open. The unit must be inspected before a restart can be made.

11. A-c. power failure is indicated by undercurrent relay No. 37 and reverse-power relay No. 56 which operate to shut down the converter. Restart is permitted on restoration of power as indicated by undervoltage relay No. 27.

12. A-c. overload operates the overload relays No. 51 which trip the line oil circuit breaker No. 52. At the same time, locking-out relay No. 86 is tripped and prevents restart.

13. Correct polarity is insured by flashing of the field from field flashing generator No. 34E and possible connection to the bus with reversed polarity is prevented by voltage directional relay No. 91.

14. Overheated d-c. windings will operate d-c. thermal relay No. 68 to reduce the load to half of full load value by decalibrating the current regulating relays No. 57. At the same time a circuit is made to start the next machine in the sequence. If the converter should continue to heat thermal relay No. 68A will trip lockout relay No. 86 to shut down the machine so that an inspection is necessary before a restart can be made.

15. Excessive load unbalance as indicated by overheated neutral copper operates thermal relay No. 68N to lock out the converter.

16. Protection against overheating of the a-c. windings is obtained in two ways. Two a-c. thermal relays, No. 49, which operate on current input to the unit protect against possible overheating from moderate overload or single-phase operation by opening master contactor No. 4 and shutting down the unit until it has cooled sufficiently to permit restart.

Thermostats located in the air discharge paths from the booster and transformer operate to reduce the load to 50 per cent of full load. If the unit continues to overheat, additional thermostats shut it down and lock it out.

CONCLUSION

This equipment has now been operating fully automatic for somewhat over a year and its operation has been very successful. During the initial starting up period there were many difficulties encountered, and subsequently others arose, but they were all carefully studied and remedied. Here again, as previously, both the operating company and the manufacturers benefited by the experience gained through solving the problems which arose.

There have been quite a few system disturbances with this equipment in service and in practically every case the automatic control appeared to function correctly. In most cases, of course, there was no one at the station at the time of the trouble and the performance of the apparatus had to be determined from the chart records. However, during the disturbances when, by chance, men were at the station, many observations were made and a great deal of confidence gained. In one case in particular, when the Keokuk transmission circuits were completely disabled by a tornado and the load thus released was suddenly thrown on the steam plants, this control gear operated admirably over a wide range of frequency and a-c. voltage and through severe d-c. load fluctuations until the system was again stabilized. At the time of the disastrous tornado of September 29th, 1927, none of the machines at St. Charles Street even dropped their load, although the a-c. system was subjected to many severe short circuits and grounds.

With improvements being made rapidly in automatic control apparatus, it is felt that increasingly complicated operations will be controlled automatically and still greater reliability obtained in the future.

Bibliography

1. Wyatt, F. D., *Operating Experience with Automatic Equipment on an Edison System*, A. I. E. E. TRANSACTIONS, 1924, p. 768.

2. Shirley, O. E., "Resynchronizing Characteristics of Synchronous Motors," *G. E. Review*, June, 1923.

3. Fischer, F. P., "Bourbon Automatic Substation of the New Orleans Public Service, Inc.," *G. E. Review*, Feb. 1926.

4. Bany, H., *Automatic Edison Substation of the Indianapolis Light and Heat Co.*, A. I. E. E. TRANSACTIONS, 1924, p. 760.

5. Bany, H. and McDonald, G. R., "Automatic Control of Edison Lighting Motor-Generator Sets." *G. E. Review*, July 1926.

Discussion

Caesar Antoniono: I have been enthusiastic about automatic operation since the early days and I still am.

We have 18 automatic equipments on interurban service totaling 20,000 kw., leaving at present 5300 kw. of manually-operated equipment, in 4 stations.

I have been in contact with the operation of several other of our affiliated companies and have no reason to change my mind. It has reduced the operating cost and made it possible for us to better our distribution of power and voltage, without a prohibitive capital investment.

H. Bany: (by letter) It is interesting to note that no stand-by batteries are used on the Union Electric Light and Power Company's 250-volt Edison system. Although this is undoubtedly to a large extent due to the fact that this company has been able to avail itself of the increased reliability of its a-c. supply, due to the use of two separate supply sources, still the elimination of the stand-by battery must further be regarded as an expression of confidence in the automatic station for this exacting service.

In the St. Charles St. Substation no effort or expense was spared in the design of the automatic switching equipment to give the station the reliability and the voltage and load-control features it required as a base-load station. It is felt, in this case, that this has been well worth while in view of the results desired and obtained. It should be realized, however, that the number of special features incorporated in an automatic switching equipment should be a function of the requirements which must be met, and if the requirements are such that a simpler equipment will fulfill them, the simpler equipment should be used. This not only results in a lower first cost, but also eliminates the extra maintenance.

It is also of interest to note the creditable performance of the synchronous converter equipment during the low a-c. voltage and frequency period when the Keokuk transmission line was disabled by a tornado. Ordinarily it would be expected that the motor-generator sets alone would be of value at such a time but here the synchronous converter exceeded its expectations.

H. R. Summerhayes: It may be of interest to discuss briefly the various types of machines and schemes of control for use on Edison Systems. Under normal conditions, of course, the voltage range obtainable with the standard type of machine is sufficient to take care of normal operating conditions.

It is important, however, that the apparatus on the Edison System be arranged so that the load can be picked up without difficulty in the case of a partial or complete outage. In order to accomplish this, it is necessary either to arrange the machines so that stable operation can be obtained at any point from zero to full voltage or to arrange the circuit so that the current output from the machine will be limited when it is operating at or about normal voltage.

With the motor-generator sets this can be accomplished by the use of separately excited fields on the d-c. generator, the voltage and current output from the machine being limited by regulating devices operating on the field circuit. An alternative arrangement to the straight separate excitation is found in the scheme which uses a small generator in the field circuit, the voltage of which opposes the applied voltage. Thus, building up voltage on this counter e. m. f. generator will reduce the field current to the desired value. The voltage of the counter e. m.f. generator is controlled by current and voltage regulators which limit the current or voltage output from the main d-c. generator. The third method for motor-generator sets is found in the differential series field generator. The fields on this machine are wound so that the series field opposes the shunt field which makes the machine inherently load-limiting. The excitation applied to the shunt field is obtained from the generator terminals in series with a separate source of power which is usually obtained from a small direct-connected exciter, this exciter also being used to excite the fields of the synchronous motor. This combination of fields results in a design which will permit the d-c. generator to operate at any voltage from zero to full voltage and which is inherently load-limiting, thus giving a machine which will require a minimum of control to give the desired results. Contact-making ammeters and voltmeters may be used with these machines to limit the current or voltage to definite values.

Synchronous converters may be arranged to operate in the starting or Y connection which, of course, will give practically 57 per cent of normal voltage. In this case, of course, it is necessary to disconnect the latter from the line before changing over to running or delta connection. Arrangements may be made also for partial voltage reduction by transformer taps with or without arrangement for changing taps under load. Where transformer taps extend over any appreciable voltage range and the changing of taps under load is required it will be necessary to use a scheme involving the use of a booster or regulator in order to obtain a smooth transition.

An alternative method consists of using a standard converter operating at full voltage using a current-limiting resistor in series with the machine, the resistor being designed so that current output from the machine will be limited to a safe value. This resistor is divided into a number of steps, each of which can be short-circuited by a circuit breaker, the steps being designed so that the maximum safe value of operating current will not be exceeded when one block of resistance is short-circuited.

Where the load-limiting resistor is used it is, of course, possible to connect the converter to an entirely dead system, whereas with the other schemes mentioned for the converter, these machines, of course, cannot be connected to the system until the system voltage has been raised to this amount by means of other machines which may be connected to the system at a lower voltage.

I should like to ask a question with regard to operation of the system as supplied from 60-cycle converters and 25-cycle converters. In some cases it has been feared that such operation might result in trouble with burning of brush-rigging in case of a short-circuit on one of the a-c. systems; that the rotary converters would feed back through the d-c. system from the other a-c. system. It has been feared that in larger systems such feed-backs would be disastrous and, in fact, I have known of brush-holders being burned off on a number of converters. I should like to ask what precautions are taken against occurrences of this sort.

H. L. Wallau: In Cleveland we have a d-c. district similar to that in St. Louis, being slightly smaller in area, with a demand of about 35,000 kw. The number of stations is 4 instead of 8. One of these stations is an old generating plant and in winter supplies steam heat through geared turbine-generators with direct current as a by-product. There are also 3 motor-generator sets, and a 400-kw., 60-cycle rotary converter. Of the other 3 substations, the first put in service contains motor-generator sets and rotaries, all manually controlled, the newer and larger machines being all a-c. start. In our Public Square Station we have six 2000-kw. (nominal) rotary converters of the booster type, also manually controlled. Our latest station includes three nominal 3250-kw. units of the automatic type. Operators are maintained here because in addition to supplying direct current there is some important a-c. distribution from this station, and a large battery is installed as well. The operator selects the d-c. bus on which he desires to operate the converter and then through his control switch he gives the starting impulse and the rest of the operation is automatic. These machines are protected with load-limiting devices and tests indicate that they function properly, although no severe disturbances have occurred which have required their functioning for any long period of time.

The automatic equipment has operated satisfactorily. There have been very minor mechanical troubles which have been found on inspection and corrected, but nothing that would tend to discredit the equipment in any way.

George Cree: (by letter) A more complete description of the operation of the machine described as Converter No. 8 might give a better understanding of the control scheme which is used.

As shown in the diagram, the main converter transformer is equipped with an extended winding with four taps, each approximately 10 per cent, so that when the total primary winding is used the voltage applied to the converter is decreased by 40 per cent which is equivalent to 160 volts on the d-c. side of the converter. In addition to this, approximately 7½ per cent further decrease in d-c. voltage can be obtained by operating the booster with the field excited in the full buck direction. This will give a minimum full-load voltage of approximately 150 volts.

Therefore, in case of low voltage on the Edison System this converter can be connected to the system at approximately this voltage.

When connected to the system in this manner it is of course necessary to pass through the various taps on the transformer keeping a smooth voltage range so that the machine will not tend to pick up or drop any appreciable increment of its load. The booster winding is designed so that the voltage generated in the winding at full buck or full boost will be equal to the voltage across the transformer taps. This winding is also divided in two halves, the middle point of the booster winding forming the *Y* point for the primary of the power transformers. For instance, if the converter is operating at full boost on tap *A* and it is desired to increase the voltage, the switches which connect the other end of the booster winding to tap *B* may be closed, as the voltage generated in the total booster winding will be equal to the voltage across the transformer tap. Then if the switches connecting the booster to tap *A* are opened the current will flow through the other half of the booster winding in the opposite direction to the previous flow of current, thus giving a full buck condition instead of the previous condition of full boost. Inasmuch as the transformer voltage on tap *A* plus the boosting effect of the converter will be the same as the transformer voltage on tap *B*, minus the bucking effect of the booster, the a-c. voltage of the machine will not be changed.

The change in power-factor conditions resulting from changing the booster from boost to buck position makes it necessary to readjust the shunt field of the converter as mentioned in the paper.

E. L. Hough: Mr. Summerhayes has brought up the question of the operation of 60-cycle and 25-cycle systems when tied together through a d-c. network. The Union Electric Company does not use 25- and 60-cycle machines in the same station. The stations are all tied together through the feeder networks and the shortest distance between the two systems is approximately three blocks, so that no difficulties whatever have been experienced in operating the 25- and 60-cycle systems tied together through the network. In fact, it might be brought out that we really have more than the two supply frequencies and two stations at each frequency. The Cahokia plant is arranged so that bus sections may be isolated in case of trouble, and during such time we may have the equivalent of about three plants at Cahokia, since the bus may be split.

O. J. Rotty: I should like to point out just a few things with regard to Table I. I wish to bring out the way in which the capacities are balanced. It will be seen that the capacity in motor-generator sets totals 78,000 amperes, whereas the converter capacity totals 68,400, making them almost equally divided. Also note that the total of 60-cycle machines is 71,400 amperes and the total of 25-cycle machines is 75,000. Here again there is practically an equal balance of machines, but in this case the division is between the 25- and the 60-cycle systems.

Our Edison system peak of 1926 amounted to 129,000 amperes, while last December we exceeded that by approximately 6500 amperes; so it will be seen that we are approaching the capacity of the present machines. However, some of the old machines which were supplanted by the automatics are still in condition to run in emergencies and are considered as stand-by equipment. These machines are not included in the data of Table I.

Protection of Supervisory Control Lines Against Over-Voltage

BY EDWARD F. W. BECK[1]

Associate, A. I. E. E.

Synopsis.—Supervisory control lines used in the remote or automatic control of electric plants are subject to overvoltages dangerous to insulation and to operators. These overvoltages may be caused by lightning, crosses with the power lines, or induction from them. Open wire control lines are influenced by all of these, cables with grounded sheaths are immune to disturbances caused by lightning or other electrostatic induction, and are more or less safe from crosses. The ordinary cable, however, is still subject to high voltages by electromagnetic induction when a fault occurs on the power line.

These voltages may be dangerous not only to apparatus, but also to the cable insulation. Protection by means of lightning arresters is therefore necessary. Calculations show that even if the apparatus locations are protected, high voltages may occur along the line if the transmission line fault is between stations. This may cause a cable failure. It may, therefore, be advisable to protect the cable at certain intervals.

Supervisory control protectors may be called upon to discharge heavy currents of appreciable duration. This requires extremely sturdy arresters. The requirements are met by a spark-gap of special design in argon at a reduced pressure.

Specially made cables will shield the line against extraneous disturbances. In large installations, the use of such cables may be worthy of consideration.

SUPERVISORY control and remote metering for substations and small hydroelectric plants are widely used. In order to prevent insulation failures and consequent interruptions of the control, it is necessary to provide the control lines and apparatus with adequate protection against overvoltage. The wire lines over which the control is exercised are usually of considerable length. They are subject to overvoltage from several sources; hence protection in the form of lightning arresters is required to insure the continuous functioning of the system and to safeguard attendants from shocks.

The supervisory control line is usually of the same nature as a telephone lead, consisting of the necessary number of pairs of wires either strung in open wire lines or in cable. Complete metallic circuits are employed, the use of the earth as a conductor being avoided. The preferred mode of running the line where the number of circuits warrants it, is in a lead sheathed cable. Such cables with the sheath grounded are exempt from certain kinds of overvoltage disturbances, and the grounded metallic sheath mitigates the bad effects of others.

For the time being, consider an open wire line as it is exposed to more severe disturbances than is a cable. Overvoltages in such a line may occur from the following causes:

1. Atmospheric electricity and lightning discharges.
2. Direct crosses with high-voltage lines.
3. Potentials above ground caused by electrostatic or electromagnetic induction from neighboring power lines.

The hazardous voltages caused by atmospheric electricity will not be discussed here. This is a subject which has received considerable attention in the literature in connection with transmission lines. The reaction of an open wire line to lightning is similar to that of any transmission line. This phase of the problem is discussed in the first two references of the appended bibliography.

Direct crosses with high-voltage lines may occur since the control line is usually strung on the same poles with the power line. The best safeguard against damage in that event is the use of a high-voltage fuse in the control line together with a protector of sufficient discharge capacity to rupture the fuse and clear the apparatus from the charged control line. If the control line is strung in cable, the lead sheath, unless it is fused by the short-circuit power current, will keep the high voltage from the line.

This discussion relates principally to overvoltages due to induction. These are of considerable importance in connection with supervisory control lines, or others of a similar nature. The control lines run between power stations and therefore usually parallel the transmission lines which connect the stations, sometimes strung on the same poles with them, sometimes on separate pole lines, but never very far away. They are thus continually under the influence of the voltage and current conditions existing in the transmission line.

In the following discussion of induction, approximations will be generally resorted to, neglecting such things as the effect of the size of the wire, the configuration of the lines, or the shielding effects of nearby earthed conductors. Exact calculations will not be attempted. Hazardous voltages of the order of hundreds or more volts are the objects of interest. It is therefore not necessary to be as particular in the assumptions and calculations as would be the case were telephone or telegraph lines concerned in which very small disturbing voltages might cause false signals or interruptions of the service.

---

[1] Electrical Engr. Supply Div., Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-18, 1928.

If the line is open wire, disturbances may be caused either electrostatically or electromagnetically. Electrostatic induction will occur in the following way:

In Fig. 1, A is a conductor charged to a certain voltage E_A, and B is a nearby conductor, such as a supervisory line entirely insulated from ground. Between A and earth, G, there will be an electrostatic field due to the potential E_A. Some of the electrostatic lines between A and G will terminate on B, imparting to it a certain charge and therefore a potential above ground.

For an approximate solution, with the total number of lines of force between A and G known, a parallel plate condenser may be substituted for line and ground, this condenser having the same number lines of force between its plates as pass from A to G, when the plates are at a difference of potential E_A. In the same manner condensers may be inserted between A and B, and B and G. The capacities of such hypothetical condensers are known as the direct capacity of A to ground, A to B, etc. Thus, Fig. 1 may be replaced by Fig. 2. This facilitates the derivation of the induced potentials. Calculating the impedances of the circuit involved, the charging currents and the potentials may be found.

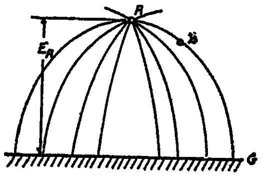


FIG. 1—DIAGRAM SHOWING HOW VOLTAGE IS INDUCED ELECTROSTATICALLY ON SUPERVISORY LINE

The potential to ground which B takes up is:

$$V_B = E_A \frac{C_{AB}}{C_{AB} + C_{BG}}$$

The capacities involved may be calculated if the spacing between conductors to the earth is known. This subject is treated by Schrottke, Eggeling, and Jaeger, (see bibliography). An approximate solution given by Jaeger is,

$$C_{AB} = 2.42 \left(\frac{a\,b}{a^2 + b^2 + c^2} \right) 10^{-3}\,\mu\text{ f. per mile.}$$

If $a = 40$ ft., $b = 20$ ft., and $c = 55$ ft.

$C_{AB} = 0.00038\ \mu$ f. per mile.

The capacity C_{BG} of the ordinary open wire line will be of the order of 0.0097 μ f. per mile.

Then $V_B = 0.0375\,E_A$.

If A is charged with an a-c. voltage of 22,000 above ground, $V_B = 22{,}000 \times 0.0375 = 825$ volts r. m. s. or 1160 volt peak.

As the spacing between A and B is increased, the value of C_{AB} and, hence, the value of V_B decreases rapidly, approaching a small constant value for spacings of about 150 ft.

If B is solidly grounded, V_B will be zero, the disappearance of V_B being accompanied by an increase in the charging current from A. On the other hand, if B is grounded through a very high impedance, such as the body of an attendant, the voltage V_B may be high. If the total impedance of the circuit A, C_{AB}, B, attendant, ground is sufficiently low, as it may be if C_{AB} and C_{AG} are large, which will be the case if the parallel is long, enough current may flow through the attendant to be dangerous. It is also possible that punctures of apparatus insulation will occur. For this reason, overvoltage protectors are necessary on a line subject to electrostatic induction, neglecting for the moment the additional requisite of safeguarding against electromagnetic induction. Low-voltage lightning arresters will hold this induced voltage down to the discharge voltage of the device.

The above discussion deals with what is in effect a single-phase ground return power line, such as a railway trolley, for instance. On a three-phase power system, the conditions are similar provided the phase voltages are unbalanced. Ordinarily, the configuration of the wires of the transmission line and the location of the supervisory line will be such that the electrostatic field resultant of the three phases, when the voltage is balanced, is small; in that case, there is no appreciable overvoltage on the control line. Fig. 3 shows the spacings between transmission and supervisory lines for which the normal electrostatically induced voltage is below 300 volts. The curves are plotted for a horizontal configuration of the transmission line. If a triangular configuration is used, the conditions will be better; if a vertical configuration is used, the induction will be about the same as for a horizontal configuration unless the control line is directly below the power line, in which case the influence of the lowest transmission line conductor will be quite marked. For the purpose of this discussion, it may be assumed that the normal electrostatic field is negligible. If the relative positions of the power and control lines are such that high voltages are steadily induced in the latter during the normal operation of the power line, these must be removed from the control line by drainage devices other than lighting arresters because the continuous discharge of arresters would make the control line inoperative.

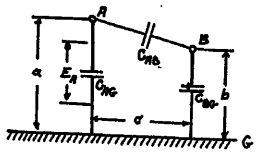


FIG. 2—SCHEMATIC DIAGRAM OF CONDITIONS IN FIG. 1

If a fault occurs on the transmission line so that the voltages are no longer balanced, there will be a pronounced electrostatic field causing an induced voltage to appear on the control line. If the substitution of condensers is made as outlined in Fig. 2, the distribution of charging currents and the induced voltages set up in the control line may be derived. The maximum induction takes place when a ground fault occurs on one phase of the transmission line in which case the electrostatic field between the ungrounded phases and the earth is most intense.

Considering a three-phase power line with ungrounded neutral, assuming one phase dead grounded through a fault, and assuming that the voltage of the grounded phase to earth is zero, the calculation by means of equivalent condensers leads to the conclusion that the disturbing field is approximately proportional to the vector sum of the line voltages between the sound and grounded wires, in the case of either a delta or a Y system. So the disturbing system may be replaced by a single wire having a potential above ground equal to the above mentioned vector sum of voltages and the case is then as pictured in Fig. 2.

Let us assume a three-phase delta or Y system with ungrounded neutral and a line to line voltage of 22,000 with one phase accidentally grounded—a case paralleling Fig. 2.

$\vec{E}_A = \vec{E}_{1-2} + \vec{E}_{1-3} = \sqrt{3} \times E_{line} = \sqrt{3} \times 22{,}000 = 38{,}200$, from which the crest value of the induced voltage is then: $V_B = 0.0375 \times 38{,}200 \times 1.41 = 2020$ volts.

A ground on an ungrounded system may stay on for some time before it is cleared, increasing the hazard. If the ground is a high resistance one, the transmission line voltage is not so badly unbalanced and the disturbance is less severe.

If simultaneous grounds occur on two phases, the induction is also less severe. The electrostatic field is then that due to one line wire only,—the sound one. The disturbance is then only $\frac{1}{\sqrt{3}}$ times as great as in the above mentioned case. A dead ground on one phase is the limiting case and therefore the one to be provided for.

On a system operating with the neutral grounded an accidental ground will not cause so high an electrostatic voltage on the control line as on an ungrounded system. In this case, the field will be proportional to the vector sum of the voltages to neutral of the sound phases:

$$\text{Thus } \vec{E}_A = \vec{E}_{line} \frac{1}{\sqrt{3}} + \vec{E}_{line} \frac{1}{\sqrt{3}} = \frac{E_{line}}{\sqrt{3}} \times \sqrt{3} = E_{line}.$$

Considering again a 22,000-volt system, $V_B = 0.0375 \times 22{,}000 \times 1.41 = 1170$ volts crest compared to 2020 volts for the system operating with an ungrounded neutral.

For simultaneous grounds on two phases, the field is proportional to the voltage from line to neutral, *i. e.*,

$$V_{AB} = 0.0375 \frac{22000}{\sqrt{3}} \times 1.41 = 675 \text{ volts peak.}$$

On a system operating with the neutral grounded, a third harmonic or multiples thereof will cause a steady induction. The 3 n harmonics are in phase between the line conductors and the neutral; hence they will originate an electrostatic field between line and ground proportional to the harmonic voltage to neutral, neglecting the increased capacity due to the three line conductors being in parallel. The 3 n harmonics should be avoided. In a poorly designed system the third harmonic can be 30 per cent of the fundamental, in which case there would be a steady voltage on the control line for the above-considered 22,000-volt system, of $2020 \times 0.3 \times 3 = 3800$ volts. A third harmonic of 3 per cent only will induce 380 volts above ground on the control line and this potential will be on the line as long as the transmission line is energized, probably continually discharging the protectors and interrupting the control. Therefore, 3 n harmonics should be drained out of the system or else the control line must be moved sufficiently far away from the transmission line to reduce the induced voltage below the breakdown voltage of the protectors. If this is not possible, special protective measures must be taken, such as the use of drainage devices already mentioned.

High voltages may be induced in the control line by transients in the transmission system. These voltages will be proportional to the transient voltage; so they may be exceedingly high, especially if the transients are due to lightning. When a fault occurs on the transmission line, surges are generated before a steady state is reached. On an ungrounded system, a solid ground may cause a surge of twice the normal voltage; an arcing ground may cause repeated surges of three and one-half times the normal voltage. On a grounded system conditions are not so bad, as the surges caused by accidental grounds will usually not be more than twice the normal voltage.

The presence of grounded conductors near the control line will materially reduce the induction due to the increase in the capacity to ground of the control line, C_{BG} in Fig. 2. These conductors may be in the form of continuous rows of trees, a ground wire, or the like. By far the most effective manner of shielding the control line against electrostatic induction due either to the transmission line or lightning is the running of the line in metallic sheathed cable with the sheath grounded. The line is then immune to such influences. If cable is used, the electrostatic lines of force terminate on the cable sheath and not on the control line conductors. Hence, considering only protection against overvoltage, regardless of other advantages derived from this method of installation, the use of cable for control lines is to be recommended. When the number of conductors in the line is so small that an open wire line is preferable to cable, other protection in the form of lightning arresters is essential to protect against electrostatic induction.

If there were not the possibility of electromagnetic disturbance and crosses with the transmission line, the use of sheathed cable would probably make it unnecessary to use protectors, provided the sheath were grounded at many points. The lead sheath of a cable, unfortunately, does not provide a perfect shield against electromagnetic induction, although it may reduce its effect appreciably.

Consider a single-phase transmission line with ground return and a neighboring control line consisting of a single isolated wire paralleling the transmission line. Suppose, also, that the transmission line is grounded at the far end so that a current flows around the loop consisting of transmission line and earth. This current will be attended by a magnetic field, part of which will cut the area enclosed by the control line and ground, inducing in the line an e. m. f. If the control line is grounded at both ends, a closed loop is formed in which a current will flow. The two lines together may be likened to a transformer of which the transmission line is the primary and the control line the secondary. The induced e. m. f. in the control line is dependent upon the total magnetic flux cutting the area between it and the ground, this magnetic flux, in turn, being a function of the area enclosed between the transmission line and ground return current. In order to calculate the induction, it is necessary to determine what these areas are. The length of the exposure being known, this resolves itself into a determination of the ground current, and the distance between the lines and the path of the returning earth current. For this purpose, the conception of the so-called equivalent ground plane is necessary.

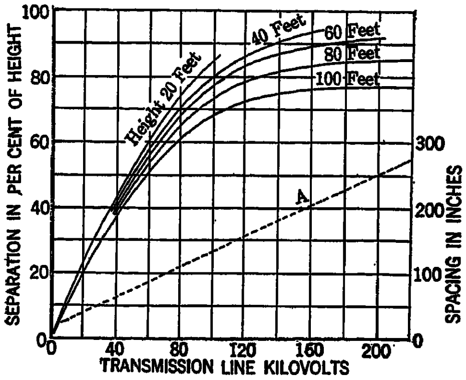


Fig. 3—Separations Required Between Power and Control Lines

These curves show separations necessary to keep the electrostatic induced voltage below 300 volts above ground during normal operation of the power line.

Dotted line shows the spacings used between phases of the transmission line.

Control line strung beneath transmission line.

When the current enters the earth from one electrode, it does not travel in a direct line to the other. It spreads out to a considerable extent in all directions, so that the cross-section of the current path is extremely large. Most of the current flows at a great depth. In order to make the calculations practical, it is necessary to assume a current path confined to a horizontal plane somewhere in the earth, the distance between it and the line being such that the flux included in the theoretical area so formed is of the same value as that actually generated by the transmission line current. This theoretical current-carrying plane is called the equivalent ground plane. It may be simply defined as the surface of a hypothetical earth with negligible resistance and a two-dimensional current distribution, this plane being at a suitably chosen distance below the actual surface of the real earth. There exists a considerable amount of experimental data to aid in the choice of this distance.

It is assumed that the ground current on leaving one ground electrode flows vertically downward to the equivalent ground plane; then follows it. The resistance on the path being assumed negligible, the ground resistance of the earth path is considered concentrated about the ground electrodes.

Having established what the circuit shall be, it is possible to proceed to the methods of calculating the electromagnetic induction. It can be shown under the above assumptions that the e. m. f. induced in a unit length of control line is expressed by this formula:

$$e = 0.00466 \times f \times \log\left(\frac{2h}{d}\right)$$

e is the induced voltage per mile of exposure per ampere of current in the disturbing transmission line.

f = is the disturbing frequency.

h = is the height of the disturbing line above the equivalent ground plane.

d = is the spacing between lines expressed in the same units as h.

The location of the equivalent ground plane below the surface of the earth is a more or less indeterminate quantity depending in the main upon the conductivity of the soil which in turn depends upon the nature of the soil, its moisture content, and the like. If the conductivity is high, or if there is a layer of conductive soil near the earth's surface, the equivalent ground plane will be near the surface. If the conductivity of the earth is poor, the equivalent ground plane will be deeper in the earth. Experience shows that the distance between the earth's surface and the equivalent ground plane may vary from a few feet to 2000 ft. Experience also shows, however, that in most cases the maximum depth that may be expected is approximately 500 ft., so that calculations made with this value for h give in nearly every case values of induced voltage which are amply high. Moreover, as the depth of the equivalent ground plane is increased, the induced voltage increases less rapidly as shown in Fig. 4, which is reproduced from the article by R. Mitsuda and K. Kasai (see bibliography) so that for values of h greater than 500 ft., the results will not be affected as much as may appear off hand. Moreover, there are usually mitigating factors present which, in actual practise, reduce the induction. Consequently we may use the value 500 ft. with safety and write the equation for induced voltage:

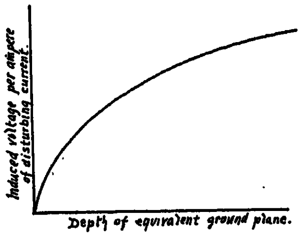


FIG. 4—VARIATION OF INDUCED VOLTAGE WITH DEPTH OF EQUIVALENT GROUND PLANE

$$e = 0.00466 f \left(\log \frac{1000}{d} \right) \text{ volts per ampere mile,}$$

where d is in feet.

For convenience, we may express this equation in terms of 25- and 60-cycle induction:

$$e_{25} = 0.116 \log \frac{1000}{d} \text{ (r. m. s.)}$$

$$e_{60} = .269 \log \frac{1000}{d} \text{ (r. m. s.)}$$

for parallel exposures.

The variation of the induced e. m. f. e, with the spacing d, between the disturbing and disturbed lines is shown in Fig. 5. This curve is not accurate for spacing approaching $2h$. If the equation for e is used, as given, the induction at a spacing of $2h$ would be zero, because in the derivation of e, assumptions have been made which bring this about. Actually, the induction persists to greater distances, theoretically to infinity.

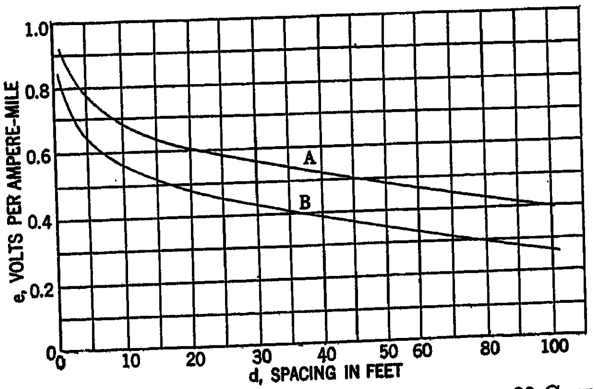


FIG. 5—E. M. F. INDUCED PER AMPERE-MILE AT 60 CYCLES

Curves show variation of e. m. f. with spacing between disturbing and disturbed lines.

Curve A, depth of equivalent ground plane, 1500 ft.

Curve B, depth of equivalent ground plane, 500 ft.

Considering protection against overvoltage, the potential of the control line to ground is the object of interest rather than the e. m. f. acting along the line. It may be well here to point out the differences between electrostatic and electromagnetic induction. In the former case, the induced voltage occurs between the disturbed line and ground. It is independent of the length of the exposure, being determined only by the capacity per unit length and by the disturbing voltage, while the amount of charging current is dependent upon the length of the exposure. In the case of electromagnetic induction, the e. m. f. is induced along the disturbed line and is a function of the length of the exposure and the disturbing current, while the current flowing in the wire is dependent upon the characteristics of the disturbed line. After establishing what the induced e. m. f. is, the control line may be considered as a circuit containing reactance and resistance depending upon the constants of the line, and a source of potential whose e. m. f. is the induced voltage.

In order to determine the potential to ground due to electromagnetic induction, consider the disturbed line by itself with a source of e. m. f, E_B, induced in it. If the control line is totally isolated from ground, the current which flows is chiefly a charging current. Its value is,

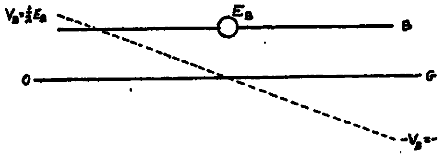


FIG. 6—DISTRIBUTION OF POTENTIAL ON LINE ISOLATED FROM GROUND

$$I = \frac{E_B}{Z = Z'}$$

where Z is the impedance of the line, "the ground impedance being considered zero" and Z' is the impedance between the line and ground consisting of the distributed capacity and resistance of the line to earth. Z is small compared to Z', so it may be neglected.

The voltage between line and ground at any point is dependent upon the capacity between line and ground. If this capacity is evenly distributed, it is apparent that the potential to ground, V_B, will be distributed as in Fig. 6.

The voltage to ground at the middle of the exposure is zero at each end of the line it is $\frac{1}{2} E_B$.

If the control line is grounded at one end but open at the other, the V_B of the grounded end is zero and the total voltage is thrown on the open end. (Fig. 7).

If the line is earthed at the center of the exposure, the same condition exists as in Fig. 6 and a protector at this location will do no good.

If the line is grounded at both ends, a short circuit is thrown on the induced e. m. f., E_B. A current will flow determined by the impedance of the circuit. Assuming zero ground resistance, the current is $\frac{E_B}{Z}$. Supervisory cables are usually made with No. 19 B&S gage wire, having a relatively high ohmic resistance of 44 ohms per mile; hence the reactance may be neglected and

$$I_B = \frac{e}{44} I_{fault}.$$

In this case the voltage to ground of the line is zero,

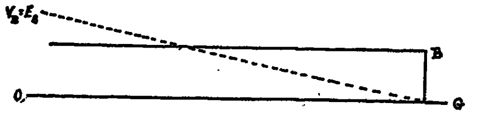


Fig. 7—Distribution of Potential on Line Grounded at One End

and therefore grounding the line at both ends of the exposure is the best safeguard against overvoltage. This can not be done actually, but can be accomplished in effect by connecting protectors between line and ground at or near the ends of the line. If the induced voltage is high enough to discharge the protectors—about 350—500 volts,—the line is effectively grounded for the duration of the fault on the transmission line. If the induction is not sufficient to break down the protectors, the potential on the line will distribute itself as in Fig. 1, but no damage will result. V_B may be of the order of 500 volts. For this reason, a supervisory line should be treated as a 500-volt line.

The current which the protectors may be called upon to pass under fault conditions in the transmission line may reach high values, of the order of 50 amperes, under severe induction on open wire lines. The duration of the discharge may be as much as two seconds, which requires extremely sturdy arresters.

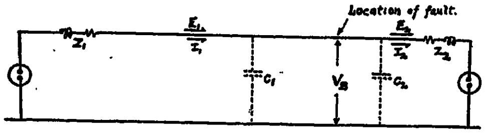


Fig. 8—Diagrams Showing Fault at Intermediate Point

In the foregoing, it is assumed that the fault in the transmission line is at one end of the exposure, so that the direction of the induced voltage is everywhere the same. Should the accidental ground on the power line occur anywhere between the ends—for instance—near the middle of the parallel, current will feed into the fault from both sides. These currents will be in opposite directions; hence the e. m. fs. induced in the sections of the control line on either side of the fault will be in opposition. The current which then flows in the control line ground loop when the protectors at the ends discharge will be a function of the resultant e. m. f. in the line. If the fault should occur midway between power sources of equal capacity, the resultant e. m. f. is zero and no current flows. This does not mean that no voltage to ground appears on the control line. The two induced e. m. fs. oppose each other and the line has distributed capacity to ground. Therefore, a voltage will pile up on the control line at a point opposite to the fault on the transmission line. This voltage may reach a considerable value, and if such is the case, a protector at or near that point is desirable. If a fault occurs somewhere within the parallel, the situation is as pictured in Fig. 8.

Z_1 and Z_2 are the impedances in the lines, I_1 and I_2 the currents, and C_1 and C_2 represent the distributed capacities. The voltage to ground, V_B at the fault location will be $V = E_1 - I_1 Z_1 = E_2 + I_2 Z_2$, *i. e.*, somewhat less than the greater of the induced e. m. fs. but more than the smaller.

The voltage distribution will be as in Fig. 9.

a = is the point opposite the fault in transmission line.
$a\,b = E_1$.
$b\,c = E_2$.
$a\,c = a'\,c' = E_1 - E_2 = I\,Z$ in the line.

When the protectors discharge there can be no potential to ground at a', so the ground potential will take the position $o\,c'$. The voltage to ground at a will then be $V = b\,d$. The ordinates between $o\,b$ and $o\,c'$ represent the distribution of the voltage to ground along one section; those between $b\,c'$ and $o\,c'$ that along the other.

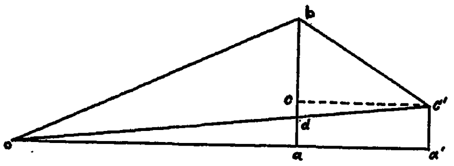


Fig. 9—Voltage Distribution with Fault as in Fig. 5

If there are no protectors on the line, or if they do not discharge, one-half the residual voltage,—that is, one-half the difference between E_1 and E_2—will appear at each end of the line and at the point corresponding to the fault on the transmission line.

Consideration of the case pictured in Fig. 9 shows that if the supervisory line is a cable, it may be well to install protection at intermediate points to safeguard the cable insulation or else use a more highly insulated cable. The number and location of protective equipments necessary to keep the induced voltage below hazardous values may be calculated from the characteristics of the transmission system and the control line. An open wire line will not require intermediate protection because the worst that can happen on the line is an insulator flashover which will probably not cause much damage.

The above discussion has dealt with a single-phase ground return power line, such as a railway trolley. On a three-phase system where the voltages are balanced, there will not be any magnetic field to speak of, and no induction is to be expected on the control line. If the voltages are unbalanced, unbalanced earth currents will flow and an e. m. f. will be electromagnetically induced on the control line.

In the case of a grounded neutral system with an accidental ground on one phase, a heavy ground current will flow. The ground current may be calculated and the induction derived by considering the disturbing line as a single wire with ground return.

Electromagnetic induction from ungrounded systems is usually not severe. So long as the voltage is balanced, no hazardous induction occurs. If an accidental ground occurs on one phase, the line currents will be unbalanced, a high voltage will appear between the ungrounded phases and ground, and a certain ground current will flow. This situation is discussed by Rudenberg (see bibliography) briefly as follows: At the point where the ground occurs, it may be assumed that the phase voltage is neutralized by a fictitious equal and opposite voltage, $-E\,p$, so that the voltage to ground at the fault is zero. This voltage, $-E\,p$, will cause currents to flow superimposed on the normal current caused by $E\,p$. The currents due to $+E\,p$ balance and do not disturb the control line. Then $-E\,p$ urges two currents to flow, one i_1 flowing around the loop consisting of phase 3 on the one hand and 1 and 2 in parallel on the other, and the capacity between them. These currents are indicated by the ordinates of the vertically hatched areas in Fig. 10, i_1 being balanced by $-i_1$ in 1 and 2 will cause no induction. The second current i_2 flows around the loop consisting of 3, the earth, and the capacity between 3 and the earth. It is a ground current and will induce an e. m. f. in the control line. The conditions are similar whether the fault occurs at the sending or the far end.

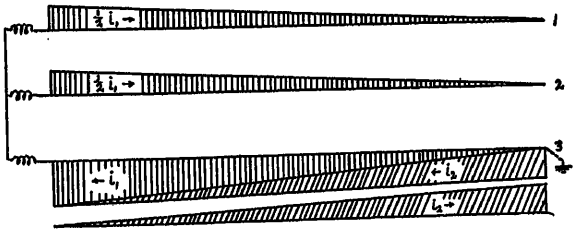


FIG. 10—CURRENTS DUE TO ACCIDENTAL GROUND ON ONE PHASE OF UNGROUNDED LINE

Some figures for the induction from ungrounded systems calculated by Eggeling (see bibliography) are given in Tables I and II. Table II is for simultaneous grounds occurring on two phases at opposite ends in which the ground current is a short circuit current and very heavy. The figures are taken from Eggeling's article and are for 50-cycle induction.

TABLE I

E_A kv.	l	c	i	E_B	E_B in per cent of E_A
35	50	0.00443	2.10	64.2 volts	0.18
35	50	0.00434	2.07	60.5	0.17
60	75	0.00433	5.30	230.	0.32
100	100	0.00463	12.6	774.	0.77
100	100	0.00475	12.9	755.	0.76

TABLE II

E_A kv.	Plant kv-a.	l	i	E_B	E_B in per cent of E_A
35	20,000	50 km.	540	16.5 kv.	47
60	30,000	75	506	22	36.6
100	50,000	100	522	32	32

Here l is the length of the parallel in kilometers, E_A is the line voltage, c is the capacity of the line to earth, i is the disturbing ground current, *i. e.*, $\frac{1}{2}\,i_2$, E_B is the e. m. f. induced in the disturbed line.

For lines with appreciable self-inductance in the path of the current i_2, and low ohmic resistance, resonant conditions may occur which may serve to raise the induction to values 3 to 4 times normal. The figures in the tables take no account of this, and may therefore be too low; on the other hand, mitigating circumstances are usually present, such as overhead ground wires or other grounded conductors; perhaps a cable sheath, which may considerably reduce the induction. In any event, it is apparent that the induction is heavy only for high line voltages and long exposures in such extreme cases as a double fault on two phases. On present-day supervisory control installations, the transmission line voltage and the length of exposure has been moderate. Ordinarily, therefore, the induction on supervisory cables used in conjunction with ungrounded systems will not be hazardous when the fault occurs on one phase only.

However, on systems with a grounded neutral, high voltages may be induced, as heavy fault currents will flow in the case of an accidental ground. As such systems are in the great majority in this country, and since it is difficult to lay down a general solution for the protective problem, each installation consisting of power and control line presenting an individual question, it may be well to work through a hypothetical case in order to clarify the matter. The calculations made will serve as an indication of what may be expected and as a guide to the methods to be applied in the investigation of actual installations.

With this in view, consider a system as depicted in Fig. 11.

In Fig. 11, B and C are two hydroelectric stations operated by supervisory control from a third station, A. These stations feed power into a system at 60 cycles through a 33,000-volt line, with the neutral grounded at each station. The generators and transformers at A, B, C have reactances of 20 per cent and 10 per cent, respectively. The capacities of A, B, and C are

15,000, 5000, and 6000 kv-a. The transmission line is No. 00 B&S gage copper with a triangular spacing of 36 in. B is 4 mi. from A, and C is 5 mi. from B. The supervisory line is a cable spaced 15 ft. from the transmission line. It consists of 10 pairs of No. 19 B&S wires running between A and B and 10 pairs from A to C. The cable between A and B thus contains 20 pairs of wires. At each end of each line, each conductor is connected to earth and cable sheath through a protector.

Then, from Fig. 5, $e = 0.51$ volts per ampere mile.

The problem thus resolves itself into a calculation of the ground current.

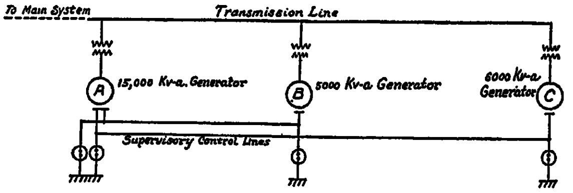


Fig. 11—Example of System with Supervisory Control

Let us first assume the fault, a ground on one phase, to be at C, the end of the line. The calculations are easily made by the phase sequence method, following the procedure described by R. D. Evans (see bibliography). In the calculations made here, the effect of the connected system has been neglected in order to simplify the problem. This is a somewhat violent assumption as the system may feed a considerable amount of additional unbalanced ground current into the fault at C, unless the system is of small capacity. For an exact solution, the connected system should be taken into account and the induction will be higher than calculated here.

Applying the Evans method, the ground currents in the sections $B - C$ and $A - B$ are as follows:

$$I_{B\text{-}C} = 685 \text{ amperes.}$$

$$I_{A\text{-}B} = 382 \text{ amperes.}$$

These calculations neglect the effect of ground resistance which may or may not be a negligible factor, depending upon the individual system.

The induced e. m. fs. will be:

$$E_{AB} = 4 \times 382 \times 0.51 = 780 \text{ volts.}$$

$$E_{BC} = 5 \times 685 \times 0.51 = 1745 \text{ volts.}$$

Both in the same direction.

The voltage induced in the line $A - B$ is thus 780 volts; the voltage induced in the line $A - C$ is 780 + 1745 = 2525 volts.

If the lines were totally isolated from ground and had no protection, the voltage to ground to which the insulation at the ends would be subjected would be:

$$V_{AB} = 390 \text{ r. m. s. or } 550 \text{ peak.}$$

$$V_{AC} = 1260 \text{ r. m. s. or } 1780 \text{ peak.}$$

When the protectors discharge current will flow around the loop $A - B$, and ground; and $A - C$ and ground. These currents will be:

$$I_{AB}' = \frac{780}{4 \times 44} = 4 \text{ amperes,}$$

$$I_{AC}' = \frac{2525}{9 \times 44} = 6.4 \text{ amperes.}$$

Carrying the case further, let us assume the fault at B. We then find the ground currents in the transmission system to be:

$$I_{AB} = 750 \text{ amperes.}$$

$$I_{BC} = 408 \text{ amperes.}$$

These currents are in opposite directions. The induced e. m. fs. are:

$$E_{AB} = 1530 \text{ volts.} \qquad E_{BC} = 1040 \text{ volts.}$$

The line $A - B$ will have induced in it an e. m. f. of 1530 volts. If it were unprotected, a voltage of 765 r. m. s., or 1040 peak, would appear at each end between conductors and sheath.

In the control line $A - C$ the resultant e. m. f. will be $E_{AB} - E_{BC} = 490$ volts, but a high voltage to ground and sheath will appear on this line at the fault point, C.

Suppose the protectors have broken down so that $A - B$ is grounded at A and B and the line from A to C at A and C. Then the currents induced will be:

$$I_{AB}' = \frac{1530}{4 \times 44} = 8.7 \text{ amperes.}$$

$$I_{AC}' = \frac{490}{9 \times 44} = 1.2 \text{ amperes.}$$

Because of the fact that the e. m. fs. induced in $A - B$ and $B - C$ are in opposition, a voltage to ground will exist on the line $A - C$, with a maximum at B. This voltage is indicated by Fig. 12.

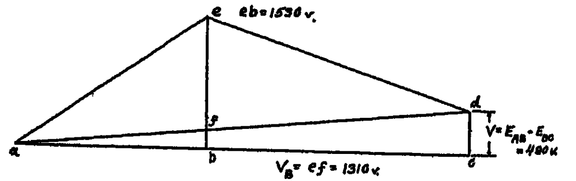


Fig. 12—Voltages Induced in Control Line Shown in Fig. 11 with Power-Line Fault at B

The voltage at B is $e f = 1310$ volts.

Because the line $A - B$ has protection at B, its voltage to ground at B is zero and there may, therefore, be a potential of 1310 volts between the conductors of the line $A B$ and those of the line $A - C$. This is a hazard. Consequently, in this case protectors should be installed on the line $A - C$ at or near B. The voltage $e f$ will then collapse. The protectors on $A - C$ at B will be called upon to discharge a current of

$$I_B' = \frac{1530}{4 \times 44} + \frac{1040}{5 \times 44} = 13.5 \text{ amperes.}$$

Now suppose the fault is at M, a point half-way between A and B. The ground currents in the transmission system are then:

$$I_{AM} = 1095 \text{ amperes.}$$
$$I_{BM} = -690 \text{ amperes.}$$
$$I_{BC} = -316 \text{ amperes.}$$

The e. m. fs. induced in the control line are:

$$E_{AM} = 1120 \text{ volts.}$$
$$E_{BM} = -705 \text{ volts.}$$
$$E_{BC} = -805 \text{ volts.}$$

The resultant e. m. f. in the line $A - B$ will be 1120 − 705 = 415 volts in the direction A to B, causing a current flow in the line $A - B$ of 2.4 ampere when the protectors discharge. If the voltage is of such a low order, the protectors may not discharge and the potential distribution will be as in Fig. 13. If they do discharge, the potential distribution will be as in Fig. 14.

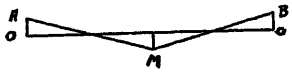


Fig. 13—Voltage on Control Line A B of Fig. 11 with no Protectors

$V_A = V_B = 208$ volts to ground. Power-line fault midway between A and B.

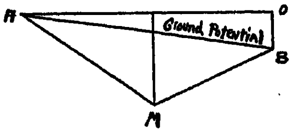


Fig. 14—Voltage on Control Line A B of Fig. 11, Protectors Discharging

Power-line fault midway between A and B.
$O\,M = -1120$ volts, $V_A = V_B = O$. $V_M = 900$ volts to ground.
Residual Voltage, $O\,B = 415$ volts.

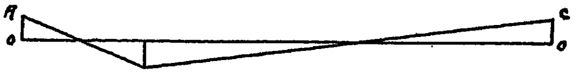


Fig. 15—Voltage on Control Line A C of Fig. 11 with no Protectors

Power-line fault midway between A and B.
$V_A = V_C = 195$ volts to ground.

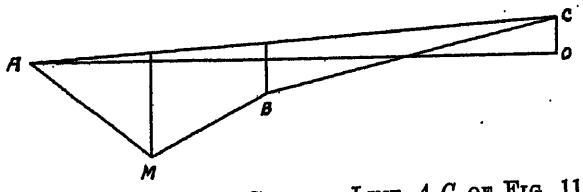


Fig. 16—Voltage on Control Line A C of Fig. 11, Protectors Discharging

Power-line fault midway between A and B.
$V_A = V_C = O$. $V_M = 1210$ volts. Residual voltage = 390 volts.

The resultant e. m. f. in the line $A - C$ will be 1120 − 805 − 705 = − 390 volts; that is, 390 volts in the direction C to A. If there are no protectors on this line, the potential distribution will be as in Fig. 15. With protectors at A and C, these will discharge and the distribution is as indicated in Fig. 16.

From these figures it is apparent that if protectors on both $A\,B$ and $A\,C$ discharge, there may exist appreciable differences of potential between the conductors of $A\,B$ and those of $A\,C$. This may be the cause of failure between conductors. To avoid this, protectors are necessary on $A\,C$ at B. If these are applied, there is no stress between conductors but only between the wires and sheath. Simultaneous protector operation probably does not always occur because if the protectors on one wire discharge, that wire becomes a grounded conductor and acts as a shield for the rest. On the other hand, the cable conductors are fairly high in resistance; hence their shielding action is probably small.

In the case of open wire lines, intermediate protection is not important. When cables are used, protection of cable insulation may be necessary to safeguard it from failure.

On open wire lines, the currents caused by the induction will be much heavier, because the wires used for open construction are much heavier and therefore less in resistance.

While the above described example may serve to indicate the nature of the induced voltages, much higher ones may exist particularly where large connected systems can feed into the ground fault, thus causing the flow of very heavy ground currents. For a system in which considerable energy may feed into the supervised section, all sources of current should be considered. Systems in which the induction may be high should be analyzed to determine the protector requirements if it is desired to safeguard the cable insulation against failures.

On the other hand, there are fortunately certain conditions which act to reduce the electromagnetically induced voltage so that the results obtained for the particular case calculated are possibly pessimistic. The proximity of grounded conductors exerts a shielding effect. The use of an overhead ground wire, or a neutral wire on the transmission line may reduce the induction considerably, since an appreciable part of the fault current will flow through this wire instead of through the earth, whereby the area of the disturbing loop and, therefore, the inducing flux are considerably decreased. The cable sheath itself, if well grounded, may be a potent reducer of induced e. m. f. The cable sheath acts as a short circuited damping turn. The degree of damping which it exerts depends upon the phase angle of the sheath. For effective shielding, it is important that the reactance of the sheath ground loop be as high as possible in comparison to its resistance which means that the sheath must be well grounded, and that the joints in it should be highly conducting. However, where the sheath resistance is appreciable, the shielding effect will be small. In the case of the 20-pair cable between *A* and *B*, in Fig. 13, the sheath resistance is 4.2 ohms per mile while the 60-cycle reactance is 1.2 ohms per mile. Hence, the phase angle and the shielding effect are quite small. For larger cables where the sheath resistance is lower, the shielding effect is much greater. Reductions of 30 per cent in induced voltage have been claimed for such cables. To obtain the best shielding effect, telephone cables have been constructed in which a shell consisting of a number of copper wires is located next to the lead sheath, these wires being solidly grounded at each end of the cable; also, with iron hoops around the sheath to increase the reactance. This has been done to reduce the electromagnetic disturbance of telephone communication. If shielding of this kind is resorted to, it is theoretically possible to forego entirely the use of protectors.

The ground connections to the protectors and the cable sheaths should be as low in resistance as possible. This is important to prevent the building up of high voltages across the ground resistance if heavy currents flow. A transient may be generated in the cable sheath either by lightning or induction from the transmission line. If this transient contains considerable energy, and if the ground resistances of the earth connections to the sheath are high, a high voltage drop may appear between the sheath and the ground. It is possible that this overvoltage may relieve itself from the cable sheath to the conductors of the supervisory line through the cable insulation and from there through the control apparatus to ground. Low ground resistance is important for all lightning arresters. The best lightning arresters hampered by high-resistance grounds, will not give good protection.

It is not to be recommended that cable sheaths be earthed to the station ground where the nature of the soil is such that a low ground resistance is not to be obtained. Under such conditions, if the station ground is used, and the ground fault occurs near one of the stations, a large part of the fault current will take the lower resistance path through the cable sheath in preference to the earth. Thus, the sheath may be called upon to carry very heavy currents which may be sufficient to damage the insulation because of heating, or even to injure the cable sheath. Where the ground resistances are high, the earth connections to the sheath and the protector should be made at a considerable distance from the station ground. As practically the entire voltage drop due to the ground resistance is concentrated within a comparatively short radius of the earth electrode, the cable will not pick up any great voltages if the sheath and station are well separated. The use of an overhead ground wire on the transmission line is of benefit in this connection as it will carry the greater part of the fault current.

A special protector has been developed for use on supervisory control circuits to relieve lightning voltages and with sufficient capacity to discharge the heavy currents of comparatively long duration which may be caused by induction. The essential element of this device is a spark-gap of particular design enclosed in an atmosphere of inert gas at a reduced pressure, to give a low breakdown voltage. The gas used permits of large energy discharges without undue injury to the electrodes which are of arc-resisting material. The breakdown voltage of the gap is of the order of 350 to 500 volts. The protector has four electrodes equally spaced from each other, so that several lines may be protected simultaneously. The most common practise is to connect two of the electrodes to earth, and each of the remaining two to one side of the line.

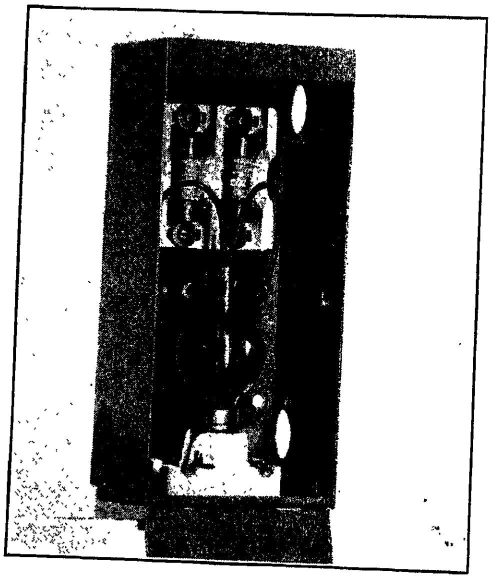


Fig. 17—Supervisory-Control Protector Unit

The complete protector consists of the bulb herein described, mounted in a steel box with a pair of low-voltage, low-capacity fuses, and a terminal block. This device is shown by Fig. 17.

The low-voltage fuses are connected between the protector and the apparatus to clear the apparatus and source of control power from the tube in the case of a sustained discharge or a short circuit of the bulb in the extremity of a very heavy discharge. If the line is open wire, high-voltage fuses are connected between the line and protector to clear the line from the protector and apparatus in the event that it is crossed by the high-voltage wires. In that contingency, the protector discharges due to the line voltage and passes sufficient current to rupture the high-voltage fuses. This isolates the charged control line and prevents high voltage from appearing on the control apparatus. This is usual practise for open wire lines but not for cable.

Where the control line consists of a plurality of conductors, the required number of protective equipments is arranged in rows. The supervisory lines are run through the holes provided in the sides of the boxes and are tapped to the protectors. Each protector is separated from the next by the steel side of the cabinet so that a failure or possible arc in one box will not endanger the others.

The protector bulb is capable of discharging lightning voltages of any order except direct strokes nearby, and will handle many times discharges of 50 amperes of two seconds' duration. It will take care of heavier discharges, but at a sacrifice of length of life. It will discharge several hundred amperes, thereby keeping the voltage on the line below hazardous values even if it is destroyed by so doing.

Bibliography

F. W. Peek, Jr., *Lightning and Other Transients on Transmission Lines*, A. I. E. E. Trans., Vol. 43, 1924, p. 1205.

S. Rump, "Disturbances in Electrical Plants due to Lightning," *Brown Boveri Review*, Nov. 1922, p. 239 and Dec. 1922, p. 254.

F. Schrottko, "Uber den Einfluss der Hochspannungs-Leitungen auf die Betriebs Fernsprech Leitunger," *Etz*, July 11, 1907, p. 685.

G. Eggeling, "Die Berechnung der auf Betriebsfernsprech Leitungen and Drehstrom Hochspannungsgestangen Induzierten Gefaehrdungspannungen," *Etz*, May 12, 1925, p. 368 and Aug. 27, 1925, p. 1297.

Dr. Jaeger, "Beeinflussung von Fernmeldeleitungen durch Hochspannungsanlagen," *Etz*, May 1, 1925, p. 417.

R. Rudenberg, "Elektrische Schaltvorgaenge."

R. Mitsuda and K. Kasai, "The 'KM' Chart, Computation of Electro-Magnetic Induction Between an Aerial Line and Neighboring Circuits," *General Electric Rev.*, May 1925, p. 290.

H. Pleijel, "Induction and Influence Disturbances in Parallel Conducting Systems," Published by A. B. Gunnar Tissells, Stockholm, Sweden.

R. D. Evans, "Finding Single-Phase Short-Circuit Currents," *Electrical World*, April 11, 1925, p. 760.

S. C. Bartholomew, "Power Circuit Interference with Telegraphs and Telephones," *Journal* of the I. E. E., Vol. 62 Oct. 1923, p. 817.

Discussion

C. F. Publow: These comments are contributed as a supplement to those given by:

1. H. C. Don Carlos under *Supervisory Systems*, see page 218, A. I. E. E. Transactions, 1926,
2. Messrs. W. H. Gerrie and C. F. Publow under *Automatic and Supervisory Control*, see pages 1029-34, A. I. E. E. Journal, October 1926, and as corrected in December 1926 A. I. E. E. Journal,

which refer to the supervisory and Automatic installation of the Hydro-Electric Power Commission of Ontario.

Protection is always of vital importance and the following operating record will describe the functioning of the protective transformer equipment which the Commission designed and installed to overcome conditions outlined in the foregoing discussions which had made operation by supervisory control quite unsatisfactory.

The statement was made on page 1032 (see above) predicting the performance of the protective transformers as making the supervisory cable installation immune from:

1. Power lightning-arrester discharge.
2. An insulator spill-over at the control or either of the remote stations.

These protective transformers were installed in July 1926, thus giving up-to-date 1½ years of a service record which I believe is an enviable one. During this time, frequent electric storms have occurred over the district where the system transmission lines run and for this reason it seems safe to assume that the power arresters at all three points have discharged at least several times, and no troubles have been found in the cable.

As to insulator spill-overs at these stations, three have just recently occurred: twice at the control point where, due to ice conditions, an outdoor disconnecting switch, when being operated, flashed over to ground and possibly later developed a three-phase short circuit, the latter one being so severe and relatively long (5 or 6 sec.) that some of the generating stations went out of step; and the other occasion was an oil-switch bushing failure at one of the remote stations.

At the control station the outdoor switch in question had its base grounded to the power lightning-arrester ground located in the forebay a few feet from the substructure foundation. This ground is entirely separated from the station ground proper to which the supervisory equipment is grounded. They are, however, located only about 50 ft. apart, and thus in reference to the ground-voltage gradient which would occur, due to the flow of "fault" current, its effect on the telephone and supervisory cable would be approximately the same as if both grounds had been interconnected electrically. The system neutral is grounded at this station, as well as others, the ground being located approximately 800 ft. away from the station building and has a resistance of approximately 14 ohms. The resistance of the arrester ground is approximately 4 ohms as it forms about half of the total combined ground resistance at this station of approximately 2 ohms.

At the remote station, the oil breaker frame is grounded to the station ground of approximately 2 ohms resistance, it being the one to which the supervisory cable is also grounded.

Connected Capacity. The system had practically its total generated capacity in service at the time these "faults" occurred and it is estimated that a crest voltage of approximately 8000 volts to ground existed at the peak of the spill-overs.

Condition of the Cable. 1. After the first spill-over at the control point—on testing the cable after this "fault" occurred, it was found to be in exactly the same condition as on a previous test some months prior to this and practically the same as when the protecting transformers were installed.

2. After the oil breaker bushing failure at one of the remote stations, no test was made on the cable, but it is thought that one pair of wires failed at this time as described in (3).

3. After the second spill-over at the control station—on testing after this "fault," the two wires referred to in (2) were found short-circuited, one of which was a spare and therefore gave no indication on the equipment that there was cable trouble. However, it happened that one of our meter inspectors was listening in over a pair of wires in the supervisory control cable in connection with testing the remote metering at the time the "fault" occurred, and he reports there was no indication from the telephone of any system trouble. For this reason, the assumption made in (2) has been made.

Owing to the system not being normally connected when the spill-overs at the control point occurred, a number of breakers opened to clear the "fault" and split the system into several groups. Also on both occasions the "Bell" telephone connection (a lead-covered lead) to the control station was disabled and fuses were blown on the system telephone, thus rendering the station operator isolated temporarily from the dispatcher. On the latter occasion he knew from his supervisory indications the condition of the remote stations, and on his own initiative placed four of the six units in service within a few minutes after the "fault" occurred when they had all tripped off on "low" voltage, caused by the "fault," an operation expected by the operator: the other two machines apparently not pulling in due to the low voltage which was approximately 50 per cent normal before the four units were brought on, and about 90 per cent afterwards. This did not require even the replacing of a fuse, an important item under such circumstances.

This performance in a "pinch" is an exceptionally pleasing one from an operating standpoint, and one which is very gratifying to the Commission's engineers. It should be stated in this connection that this supervisory cable, prior to the installation of these protective transformers, suffered very severe damage on a number of occasions, some of them almost severing all the conductors and lead covering and examinations of short lengths of cable close to these damaged portions showed many burnt spots in the insulation of pin-hole size. The cable was only repaired at the time the protective transformers were installed, a number of the conductors was not clear, and thus owing to this condition, the faulty pair referred to in (2) and (3) above does not disturb our confidence in the protective transformers.

We believe Mr. Beck is familiar with the installation referred to, and it would be interesting to have him predict the functioning of the protective equipment described by him under such circumstances.

Mr. Beck refers to "low" and "high" ground resistances; I believe his paper would be more valuable if he were to definitely state the values of these ground resistances rather than use these relative terms.

Chester Lichtenberg: It very early developed that the standards of protection, maintenance, and operation of telephone and telegraph lines were not sufficiently high for supervisory purposes.

With supervisory systems, the lines are predicated to be in continuous service, even though no current may be carried over them. Therefore, if the lines are interrupted for any reason whatever, if they fail, if they are grounded, if they are burned off (as was the case in the experience of the Hydro-Electric Power Commission of Ontario), the entire supervisory system is rendered inoperative, and usually this condition occurs at a time of stress and when it is most needed. Consequently, too much stress cannot be laid upon the necessity for a high standard of maintenance and operation of lines used for supervisory systems.

C. E. Stewart: The paper by Mr. Beck gives a very clear and concise analysis of the abnormal potentials induced in line wires as ordinarily used for supervisory systems. Where the line wires parallel high-tension systems the protection problem must be treated in quite a different manner from that employed in telephone communication. In systems using d-c. impulses and holding or lockout circuits the connections must necessarily be metallic from the dispatcher's office equipment to the substation equipment. This prevents the use of the insulating transformer which is so commonly used as a protective device in wire telephony. It also means that the arrester, used possibly in connection with choke or retardation coils, must provide the entire protection against electromagnetic disturbances which, as the author points out, may under certain conditions be quite severe. Arresters used for supervisory equipment protection must be self-restoring, that is, they must be capable of discharging to ground without fusing or making a permanent connection as is the case of the carbon blocks used in telephone protective equipment.

That portion of the paper which deals with electromagnetic induction should be of special interest to operating companies, for this subject seems to be one which has been generally misunderstood.

One phase of the subject which is a bit outside the scope of this paper, but nevertheless associated and of interest, is the protection of line wires utilizing carrier current for the impulse channel. I refer to a wire circuit installed especially for the use of carrier-current supervisory equipment, such as that on the Chicago, South Shore, and South Bend Railroad. In this installation a wire was strung on the telephone crossarms. For a part of its 60-mile length, it is carried on the same poles as the 33,000-volt power circuits. The use of pellet-type arrestors with high-voltage fuses at each of the eight stations has satisfactorily taken care of lightning, electromagnetic induction, and in one instance direct contact with the high-tension system.

The use of carrier current of course somewhat simplifies the protection problem and permits the use of very positive and reliable protective equipment. In this installation drainage coils eliminate static potentials.

Caesar Antoniono: Mr. Beck in his paper stated that if the control wires are in cable, then unless the lead sheath is fused by the short-circuit power current, high-voltage will be kept from the lines.

This is true, but it is also true that a grounded lead sheath is subject to severe burn-out in the case of power lines coming in contact with the lead sheath, and invariably the lines are damaged by the voltage carried through the cable to the nearest fusible cut-out, and sometimes pass that and damage the equipment, and the restoring of the lines is not a short job.

Another phase which I may touch on is the trouble with bullet holes. This is a trouble which it is difficult to locate and will introduce moisture in the cable lines and affect the control much more than the telephone.

I wish Mr. Beck would make more clear to me the paragraph in which he says, "Hazardous voltages of the order of hundreds of volts are the object of interest. It is therefore not necessary to be as particular in the assumption and calculation as would be the case were telephone or telegraph lines concerned in which very small disturbing voltages might cause false signals or interruption of the service."

If Mr. Beck has in mind that the supervisory control lines are not of equal importance with the telephone or telegraph lines, I must disagree with him, as we find where the control is employed, for switching purposes in particular, the service depending on this switching is affected by its failures. Therefore, our experience is that the lines used for supervisory control must be in better condition than for telephone use, where in carrying out a conversation over the telephone the missing of a word now and then is annoying but does not destroy the sense of the message. But in the case of supervisory control where selectors are used, a reduction of the impulse voltage may cause a failure of the switching contemplated, or an indication loss, or even a wrong switching may result.

In conclusion Mr. Beck covers in very fine style the theory of protecting the supervisory control lines but if these theories are applied in practise they are not going to work out in every detail as given. The highest degree of service can be obtained from the supervisory control lines and system. But we have not yet a control that is nearly 100 per cent reliable when it is most needed. In our experience although we have had various protective devices on our supervisory lines, we have experienced trouble and often the control system has been out of commission at times of power-system trouble when it was most needed.

Summing it up, it seems to me that after having done what Mr. Beck recommends, there is still a desire for improvement in making the supervisory control system trouble-proof against these conditions so as to obtain 100 per cent service during emergencies. Protecting the lines is a step in the right direction, but any disturbance at this time, even the blowing of a fuse during emergencies, can impair and cause inconvenience in the service rendered.

Alfred Herz: (by letter) Mr. Beck's paper draws attention to a condition inherent with physically parallel wire systems and which is very often overlooked. True enough, similar conditions have been thoroughly investigated and experimented upon mostly in the cases of telephone systems physically paralleling power lines. The forces causing the effect are known, precautions and mitigating measures are also known and resorted to in this (telephone) practise.

Committees of engineers have carried on the experiments mutually and satisfactory coordination has been accomplished. The foremost body to work on this coordination is, without doubt, the Inductive Coordination Committee of the National Electric Light Association. I am citing the above just to bring before you the fact that this inductive effect is nothing new and nothing peculiar to supervisory control lines, but is liable to be encountered on pilot-wire relay systems and elsewhere.

To cite an example which came to my attention: The power system involved consisted of single-conductor cables (3-phase) located underground and in parallel ducts to the pilot-wire system which was grounded at both ends. Causes of trouble on the power system beyond the zone protected by the pilot wires promptly caused the pilot-wire relay system to operate at the wrong time. In tracing the matter down we found that extraordinary precautions had been taken to keep the pilot wire at ground potential; this was accomplished by grounding both ends of the system.

When single-phase trouble occurred beyond the protected zone, currents were induced in the pilot wire which were not entirely balanced in the return wire provided, part of the induced current returning through the earth. In this particular case satisfactory operation was obtained by eliminating the ground connection at one end of the pilot system and providing at this point a low-voltage lightning arrester, similar to the one mentioned in Mr. Beck's paper, to guard against the induced voltage at the time of trouble causing insulation breakdown.

From my work with the N. E. L. A. Inductive Coordination Committee I know that in many cases more attention should be paid to the comparatively simple remedial or preventative measures which can often be resorted to more effectively when the installation is being made than afterwards.

Edward Beck: Mr. Publow's discussion is of particular interest in so far as it describes an additional method of cable protection where grounding conditions are troublesome. In my opinion, the same degree of protection would be secured by the use of the arresters described in the paper, provided the cable-sheath ground is well separated from the neutral ground so that fault current is kept out of the cable sheath. It would be interesting to know the resistance of the ground path, including the electrodes, between the stations described by Mr. Publow.

The question of what is a low and what is a high ground resistance always arouses considerable discussion. For lightning arresters on power circuits, a figure of 15 ohms has been mentioned now and again as a maximum. A 15-ohm ground for a distribution type of arrester is probably satisfactory although a lower resistance is better. For the ground-to-cable sheaths and supervisory control protector equipment, the ground resistance should be of the same order. Station and neutral grounds should be as low as possible, of the order of a very few ohms if possible.

Referring to Mr. Antoniono's discussion, possibly I did not express myself very clearly in the paper when speaking of the order of voltages in which we are interested. For the protection of these lines, we should like to know whether the voltages which may be induced between line and ground are a few volts or many. The voltages which are induced between wires are of no particular consequence in this case, but they are in telephone practise. In order to calculate these voltages between wires which give rise to interference, it is necessary to be more exact in the calculations than in the case of supervisory control lines where they are not likely to be high enough to cause trouble. It is quite true that supervisory control lines are as important as telephone lines, perhaps more so, as Mr. Lichtenberg has pointed out, but the possibility of interference with the control is more likely to be an apparatus failure than interference with signals due to disturbing voltage between wires.

With careful protection as outlined in the paper, much can be done toward making the supervisory immune to failures from overvoltage. It is difficult to make general recommendations which will cover all conditions. There are always exceptions and nearly all installations require individual study to achieve the best results. Considerable good can be done by scrutiny of the problem before the installation is made and by installing the lines with the conclusions of this study in mind. Perfection has not been attained. We hope it never will be as it would put a stop to progress. But we are of the opinion that better means of protecting supervisory control systems are available now than formerly, and that by careful application, control failures may be reduced to a minimum.

1926 Lightning Experience on 132-Kv. Transmission Lines

BY PHILIP SPORN[1]

Member, A. I. E. E.

Synopsis.—This paper deals with the interruptions due to lightning on an extensive interconnected 132-kv. system during the year 1926, a similar record covering part of the system having been reported previously.[2] *A layout of the system is shown and details of the type of line and tower construction are given in considerable detail.*

The lightning performance of different sections of the line have been reduced to a common basis and discussed from the point of view material of line structure, height of line, ground wire, and line protective equipment.

Changes in tower design are discussed in the light of experience gained in the past. The work done the past year to reduce the lightning troubles at the stations is described.

The efficacy of the ground wire seems very definitely shown but it is believed further study and attention should be given this matter as a means of reducing the lightning voltage on transmission lines, particularly to reducing the lightning voltage at and near a switching station.

The problem of determining the damage to line conductors, etc., is an important one. Data on this are at present incomplete, but it is planned to obtain a systematic record in the future which will, it is believed, supply valuable design information.

INTRODUCTION

IN the spring of 1926 there was presented before the Institute a paper[2] on the performance of a number of 132-kv. steel tower lines on the system of the American Gas and Electric Company which covered in detail the performance and experience with these lines during the year 1925. Further performance data were promised in that paper, but it has not been found easy to carry out this promise, made with the very best of intentions. In the first place, the lines and system of which these data are a part, have not been operating as a unified system for a very long period. Secondly, the operating personnel has at no time been particularly large. In other words, it has always been down to a strict operating basis. Third, when a line goes out of service, it is undoubtedly the business of the operating man to get it into service with as little delay as possible and afterwards try and determine, if possible, what has happened. When the excitement of the emergency has passed, this often results in a lack of evidence that will withstand critical analysis or give any real information as to what has happened such as would be of real benefit to future design. That there can be worked out an operating set-up which would not overburden the system and yet make it possible to obtain all information necessary, there can be no reason to doubt; but it takes time to work out such a set-up and with the system under discussion that time has been very short. Keeping this in mind it is easier to understand why some of the records presented are in some respects incomplete. They are, however, the best records that could be obtained and perhaps that could be expected.

GENERAL

Fig. 1 shows the 132-kv. transmission network in question. It comprises approximately 910 mi. of actual line, about one-third of which is double-circuit, the actual circuit miles being approximately 1245. The major portion of this network was placed in operation very late in 1925, in 1926, or early in 1927; and one portion, (namely, the Switchback-Logan line), has not yet been placed in operation at the time of writing (November 1927) although it is expected that it will be in operation by the end of 1927.

The nature of the terrain which this network traverses is most varied, ranging from the extremely flat lands in Indiana to the heavy mountainous country in West Virginia.

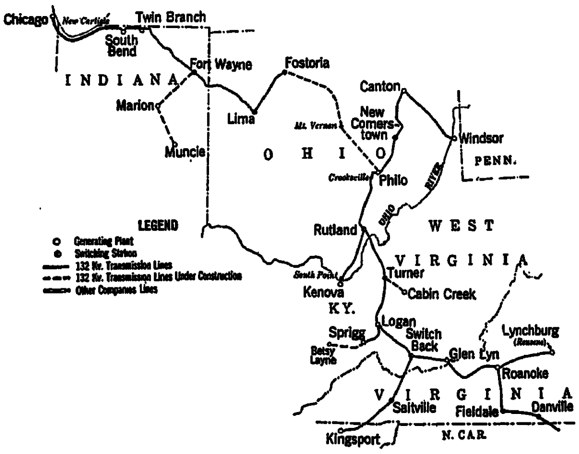


FIG. 1

The system is fed by large generating stations at Twin Branch, Philo, Windsor, Kenova, Cabin Creek, Logan and Glen Lyn. It is tied in with the Chicago system and the systems of the West Penn Power Company, The Ohio River Edison Company, The Ohio Public Service Company, and the Cleveland Electric

---

1. Electrical Engr., Am. Gas & Electric Co., New York, N. Y.

2. *Lightning and Other Experience with 132-Kv. Steel Tower Transmission Lines*, Sindeband and Sporn, A. I. E. E. TRANS., Vol. 45, p. 770.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-18, 1928.

TABLE I

CHARACTERISTICS OF 132-KV. LINES—AMERICAN GAS AND ELECTRIC COMPANY

Column No. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Line designation	Glen Lyn Roanoke	Glen Lyn Switchback	Lima Fostoria	Lima Tw. Branch	Logan Sprigg	Philo Canton	Philo Crooksville	Philo Turner	Roanoke Danville	Roanoke Reusens	Rutland So. Point	Saltville Kingsport	Switchback Logan	Switchback Saltville	Turner Logan	Tw. Branch So. Bend	Windsor Canton	So. Bend New Carlisle
Initially placed in service....	6/13/26	8/13/27	9/3/25(e)	9/18/25	6/28/26	9/28/24	9/28/24	4/22/26	9/16/26	5/5/26	3/15/26	8/25/27	11/27/27	10/16/27	1/30/26	4/5/25	9/–/17	6/24/26
Length of line—(Miles).....	65.0	30.0	45.6	128.5	21.0	73.	15.4	118.7	65.0	43.0	50.3	56.0	50.0	46.0	40.2	4.9	55.0	17.7
Number of circuits..........	2	2	1	1	1	2	1	1	1	1	1	1	2	1	2	2	2	1
Size of conductor wire—(C. M.) A. C. S. R.........	397,500	397,500	336,400	397,500	4/0	336,400	336,400	397,500	336,400	397,500	397,500	397,500	397,500	397,500	336,400	397,500	200,000(a)	397,500
Number of ground wires.....	1	1	1	1	None	1	1	1	1	1	1	1	1	1	None	1	2	1
Size of ground wire—(C. M.) A. C. S. R...............	397,500	397,500	159,000	159,000	None	159,000	159,000	159,000	159,000	197,500	159,000	159,000	397,500	159,000	None	159,000	159,000	159 000
Date ground wire installed..	6/13/26	8/13/27	5/15/26	6/7/26	None	5/6/26	4/9/26	4/22/26	9/16/26	5/5/26	3/15/26	8/25/27	..	10/16/27	None	6/8/26	9/–/17	6/24/26
Line configuration—(Horizontal-Vertical)..........	V	V	V	V	H	V	V	V	V	V	V	V	V	V	V	V	V	V
Protection—Rings and horns	No	Yes	No	Yes	No	Yes	No	No	No	Yes	No	Yes	Yes	Yes	Yes	Yes	No	No
Protection—Horns line side.	Some	..	Some	..	..	..	..	Yes	Some	..	..	..	..	..	..	..	Some	Some
Protection—Horns ground side.....................	Some	Yes	Some	Yes	..	Yes	..	..	Some	Yes	..	Yes	Yes	Yes	Yes	Yes	Few	Some
Nature of terrain...........	Rolling to mountainous	Very mountainous	Very flat	Very flat	Very mountainous	Rolling	Rolling	Rolling to mountainous	Rolling	Rolling	Rolling	Rolling to mountainous	Very mountainous	Mountainous	Very mountainous	Very flat	Rolling	Flat
Length per insulator–(Inches)	4–3/4	5–1/8	4–3/4	5–3/4	6–7/16	4–3/4	4–3/4	4–3/4	5–1/8	5&4–3/4	4–3/4	5–1/8	5–1/8	5–1/8	5&4–3/4	5–3/4	4–3/4	5–3/4
Insulators per string (Suspension) Top....	11	11	12	10	10(b)	11	12	12	11	12	12	10	10	10	10	10	10	10
Middle	11	11	12	10	10×10M	11	12	12	11	12	12	10	10	10	10	10	10	10
Bottom	10	10	10	9	10(b)	10	10	10	10	10	10	10	10	10	10	9	10	9
Length of susp. ins. string (Inches) Top............	52.25	56.38	57.0	57.5	64.4	52.25	57.0	57.0	56.38	57.0	57.0	51.3	51.3	51.3	47.5	57.5	47.5	57.5
Middle..........	52.25	56.38	57.0	57.5	64.4	52.25	57.0	57.0	56.38	57.0	57.0	51.3	51.3	51.3	47.5	57.5	47.5	57.5
Bottom.........	47.50	51.3	47.5	51.75	64.4	47.50	47.5	47.5	51.3	47.5	47.5	51.3	51.3	51.3	47.5	51.75	47.5	51.75
Dead ends–(Single or double)	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
Insulators per string (Dead end) Top....	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	11
Middle	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	11
Bottom	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	11
Type towers...............	A-B-C-E	G-H-K-E	A-B-C	A-B-C	Wd. pole (c)	A-B-C	A-B-C	A-B-C-E	A-B-M-P	A-B-C	A-B-C	AA-BB-CC-E	G-H-K-E	AA-BB-CC-E	B-C-E	A-B-C	A*o*-B*o*	A-B-C-G
Total number of towers.....	269		215	603	189	298	69	425		199	191				155	24	487	82
Number of towers with 10′ extension................	35		78	188	..	71	13	67		19	26				2	5	101	23
Number of towers with 20′ extension................	55		48	166	..	48	12	51		39	38				4	7	23	19
Number of towers with 30′ extension................	..		..	2	..	1	..	..		0	2				0	..	..	2
Number of towers with 40′ extension................	..		..	..	..	..	..	..		0	..				4	..	2–50′	..

(a) Copper conductor
(b) Sides—some with 9 insulators
(c) With steel crossarms
(d) On "E" Towers
(e) Operated 8/3/24 to 9/3/25 @ 66 kv.
(f) Side cond. to pole
(g) Center to outside conductor
(h) To wood pole. 4.0′ deg. to crossarm
(x) "E" Towers

TABLE I—*Continued*

CHARACTERISTICS OF 132-KV. LINES—AMERICAN GAS AND ELECTRIC COMPANY

Column No. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Line designation	Glen Lyn Roanoke	Glen Lyn Switch-back	Lima Fostoria	Lima Tw. Branch	Logan Sprigg	Philo Canton	Philo Crooks-ville	Philo Turner	Roanoke Danville	Roanoke Reusens	Rutland So. Point	Salt-ville Kings-port	Switch-back Logan	Switch-back Salt-ville	Turner Logan	Tw. Branch So. Bend	Windsor Canton	So. Bend New Carlisle
Average length of span—(Feet)	1270		1102	1125	585	1295	1160	1470		1140	1376				1360	1016	595	1125
Maxmium length of span—(Feet)	3321		1400	1775	2220	2665	2200	3050		2135	2858				3770	1375	1550	1876
Minimum length of span—(Feet)	100		745	610	85	360	480	410		434	465				280	520	150	156
Height of std. tower (Gr. to top crossarm (Ft.)	90	92	90	90	35–40	90	90	90	92	90	90	90½	92	90½	90–45(x)	90	74	90
Conductor spacing (vertical) (Ft.) Top—Mid	13	15	13	13	..	13	13	13	15	13	13	13½	15	13½	13	13	12	13
Mid.—Bot.	13	15	13	13	..	13	13	13	13	13	13	13	13	13	13	13	12	13
Conductor spacing—(Horizontal) (Ft.) (For standard towers) Top	21.75–23 21.75(d)	26.75–28 21.75(d)	26.75–23	26.75–23	15.0(g)	26.75–23	26.75–23	26.75–23 21.75(d)	26.75–28	21.75–23	21.75–23	24.75–26 21.75(d)	26.75–28	24.75–26	23 21.75(d)	26.75–23	21.75	26.75–23
Middle	26.75–28 21.75(d)	31.75–33 21.75(d)	26.75–28	26.75–28	..	26.75–28	26.75–28	26.75–28 21.75(d)	31.75–33	26.75–28	26.75–28	29.75–31 21.75(d)	31.75–33	29.75–31	28 21.75(d)	26.75–28	31.75	26.75–28
Bottom	21.75–23	26.75–28	21.75–23	21.75–23	..	21.75–23	21.75–23	21.75–23	26.75–28	21.75–23	21.75–23	24.75–26	26.75–28	24.75–26	23	21.75–23	21.75	21.75–23
Clearance—Line to tower (Normal pos.) At suspension strings-(Ft). Top	6.0	6.5	5.75	5.75	7.0(f)	6.0	5.75	5.75	5.0	5.75	5.75	5.0	7.0	5.0	5.0	5.75	5.25	5.75
Middle	6.5	6.5	6.25	6.25	5.0	6.5	6.25	6.25	6.5	6.25	6.25	6.75	7.0	6.75	7.25	6.25	9.0	6.25
Bottom	7.5	10.0	7.5	7.5	7.0(f)	7.5	7.5	7.5	9.75	7.5	7.5	9.0	10.0	9.0	7.5	7.5	7.75	7.5
Clearance—Line to tower—(45 deg. Pos.) At suspension strings-(Feet) Top	4.8	5.0	5.0	5.0	3.25(h)	4.8	5.0	5.0	4.8	5.0	5.0	4.0	4.0	4.0	3.75	5.0	3.25	5.0
Middle	4.8	5.0	5.0	5.0	..	4.8	5.0	5.0	4.8	5.0	5.0	4.0	4.0	4.0	3.75	5.0	3.25	5.0
Bottom	3.75	4.0	3.75	3.75	3.25(h)	3.75	3.75	3.75	4.0	3.75	3.75	4.0	4.0	4.0	3.75	3.75	3.25	3.75

(a) Copper conductor
(b) Sides—some with 9 insulators
(c) With steel crossarms
(d) On "E" Towers
(e) Operated 8/3/24 to 9/3/25 @ 66 kv.
(f) Side cond. to pole
(g) Center to outside conductor
(h) To wood pole. 4.0 deg. to crossarm
(x) "E" Towers

TABLE II

OPERATING RECORD OF 132-KV. LINES—AMERICAN GAS AND ELECTRIC COMPANY

Column No. 1	2	3	4	5	6	7		8	9	10	11	12	13	14	15	16	17	18	19
Line designation	Glen Lyn Roa-noke	Glen Lyn Switch-back	Lima Fos-toria	Lima Tw. Branch	Logan Sprigg	Philo Canton		Philo Crooks-ville	Philo Turner	Roa-noke Dan-ville	Roa-noke Reusens	Rut-land So. Point	Salt-ville Kings-port	Switch-back Logan	Switch-back Salt-ville	Turner Logan	Tw. Branch So. Bend	Wind-sor Canton	So. Bend New Car-lisle
						1925	1926												
Outages (One circuit only) Due to lightning—1926	16	..	9	23	3		13	3	11	0	13	3	..	..	..	16	0	9	0†
Circuit outages due to lightning—1926	20	..	9	23	3	88	15	3	11	0	13	3	..	..	..	18	0	11	0†
Circuit outages per 100 mi. of line per yr.—1926	48*	..	19.8	17.9	31*	120	20.5	19.5	10.6*	0	36.6*	6.3*	..	..	..	44.8	0	20	0†
Circuit outages per 100 mi. of circuit per yr.—1926	24*	..	19.8	17.9	31*	60	10.3	19.5	10.6*	0	36.6*	6.3*	..	..	..	22.4	0	10	0†

*Corrected for one calendar year on basis of Fig. 3.
†Record incomplete.

Illuminating Company at 132,000 volts. The short-circuit capacity on the system varies from approximately 1750 amperes to approximately 4750 amperes, at 132,000 volts. Values as high as 4000 amperes have been actually measured.[3] From a lightning standpoint, the network as a whole is subjected to thunderstorms considerably in excess of the average over the country as a whole. The isoceraunic lines, *i. e.*, lines connecting points of equal thunderstorm intensity, for the territory in question ranged from 40 to 60 in April, 150 to 200 in June and July, and down to about 60 to 80 in September. All these figures are based on a 20-year period. The average number of storms for the territory as a whole, based on the 20-year data, referred to above, is given in Fig. 2.

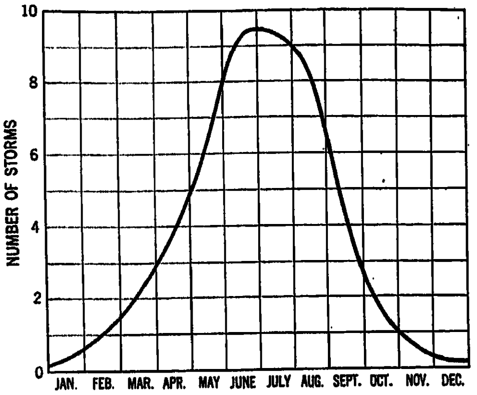


Fig. 2—Average Monthly Distribution of Thunderstorms

In N. Indiana, Ohio, Va., W. Va. and E. Kentucky, based on 20-year average (1904-23)

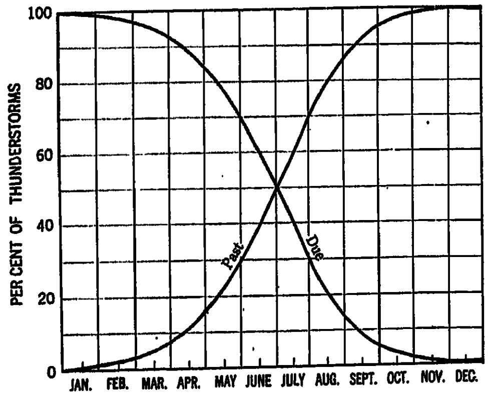


Fig. 3—Per Cent of Thunderstorms During One Year

In N. Indiana, Ohio, Va., W. Va. and Kentucky, based on 20-year average (1904-23)

Description of Lines and 1926 Performance

In Table I there is given a description of the physical characteristics of the various portions of the 132-kv. network under discussion.

The towers referred to in the above table are types A, B, C, A A, B B, C C, E, G, H, K, M, P, A_0, B_0. Towers A, B, C and Towers G, H, and K, have already been described elsewhere.[2] Tower E is a single-circuit tower with conductors arranged in a horizontal plane and utilized especially on long span work. Towers M and P have in general the same dimensions and spacings as towers G and K, with the exception of different factors of safety for the various loadings being employed. Towers A A, B B, and C C, which have been utilized on the very recently constructed lines, are very similar to types A, B, and C, although the tower is quite definitely designed for equal string length on all three phases, this being made possible by the use of the ground wire; and further, clearances are better balanced with respect to lightning voltage. The tower further differs from the previous types in that in its design definite provision has been made for the addition of a second ground wire if operating experience should prove that this is desirable. A sketch of the type A A tower is shown in Fig. 4.

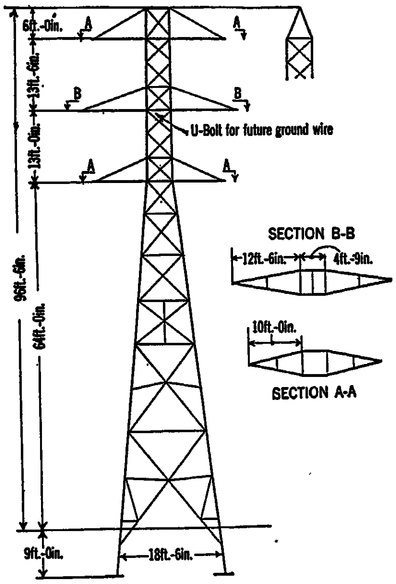


Fig. 4

The performance of the lines during 1926, is shown in Table II. It will be noted that although some of the lines had been in service for only a portion of the year 1926, for comparison purposes, the number of outages has been converted to a yearly basis by the use of Fig. 3. Fig. 3 is merely an integration of the thunderstorm distribution curve shown in Fig. 2.

Examining Table II further, it will be noted, first, that the largest number of lightning outages occurred on the Twin Branch-Lima line. This line was in service for the full year, and has a length of 128½ mi. Based on 100 mi. of circuit, however, it is not at all out of line and checks up with similar performance on the Lima-Fostoria line, and on the Philo-Crooksville line. On the basis of 100 line mi. the largest number of interruptions occurred on the Glen-Lyn-Roanoke line, but this figure is of course subject to the weakness that the line was in service for 6 months only, and there is no way of checking up whether the ratio employed for figuring the yearly number of interruptions was correct for the entire year.

---

[3] *Tests on High- and Low-Voltage Oil Circuit Breakers*, Sporn and St. Clair, Trans., A. I. E. E., Vol. XLVI, 1927, p. 289.

[2] Loc. cit.

[4] See *Monthly Weather Review*, Vol. 52, No. 7, July 1924.

Of the lines that were in service for an entire year the largest number of interruptions per 100 mi. of line was sustained by the Logan-Turner line. This is a line that has no ground wires and, in addition, passes through especially rugged country. The length of the insulator string on this line is particularly short (as shown in Table I) being, in fact, shorter than that of any other line with the exception of the Windsor-Canton line. Expressed in circuit miles, however, the record is rather good. Whether in the case of a double-circuit line, however, the method of dividing the total number of interruptions by two and calling that the interruptions per circuit is questionable. It raises the question whether such a result can be compared with the number of flashovers on a single-circuit line.

To amplify: Consider the two top wires of a double circuit tower line with both circuits subject to the same potential gradient. Upon the occurrence of a lightning discharge, chance will usually determine which one of the two circuits is to flash over first. This assumes, of course, that their arrangement is perfectly symmetrical and that they constitute the same phase wire in their respective circuits. A flashover having occurred on one of them, the question arises whether that flashover does not reduce the energy in the surge sufficiently to lower the head of the wave to such an extent that the second wire will not flash over. The fact that among all the double-circuit lines the maximum number of instances in which both circuits flashed over on any one line was only four against a total number of flashovers of 20 would certainly tend to indicate that some such explanation may be the correct one.

The Roanoke-Reusens line is third in the number of interruptions per 100 mi. of line, but the figure for a whole year was again obtained by a ratio and is open to the same objections as the figures for the Glen Lyn-Roanoke lines. The Philo-Canton and the Windsor-Canton lines, in spite of the fact that the Philo-Canton line generally is exposed to more severe storms than the Windsor-Canton line, have practically the same record for the year 1926, when reduced to a 100-mi. basis. One of these lines, namely the Windsor-Canton line, has two ground wires and is considerably lower than the Philo-Canton line which has only one ground wire. The record during 1926 would seem to show that, from the standpoint of reliability the Philo-Canton line, even though it has only one ground wire, has been put on a plane of reliability equal to that of the Windsor-Canton line. From the data gathered on the Glen-Lyn-Reusens line and Roanoke-Reusens line in 1926, it would seem that the ground wire on these lines was not so effective as it was on the other lines. The ground wire construction being in general the same on all of the lines, a possible explanation for the large number of flashovers may lie in the fact that the tower resistance over the lines as a whole is rather high. Some of the operating data cited by Hemstreet[5] showed that there is a definite relationship between the number of flashovers and the ground resistance in the particular section. This phase of the problem is being given further consideration at the present time.

An interesting line in the series is the Logan-Sprigg line. This line, of wood pole construction, was originally built for 88,000 volts and was operated for a period of about five years at 44,000 volts. At the end of this time, additional insulation was added and the line was cut over to 132,000-volt operation. The majority of the structures are 2-pole A frames with ungrounded steel crossarms. The insulation is unusually liberal. For a while, when the line was first placed in operation, the operating records seemed to indicate an almost lightning-proof line, but detailed investigation has shown that this was not quite so. Considering the shortness of the line,—namely 21 miles,—the record is not particularly good although again the record is open to the criticism that a full year's data were not available.

The rest of the data requires no particular comment.

PERFORMANCE OF PROTECTIVE DEVICES DAMAGE TO CONDUCTORS, INSULATORS, AND HARDWARE

The records of damage to insulators, conductors, and hardware, on the lines is not complete for 1926 so that no attempt will be made to give specific data in this paper. As already pointed out, 8 out of 17 lines now in service were placed in operation during 1926 with the result that the method of inspecting the lines and the reports of damages thereon have not been sufficiently organized to make these data accurate for analysis.

The fact is outstanding, however, that only in one case has a line been out of service due to lightning trouble on it for a period in excess of the time necessary to close the line back in service. The one case referred to occurred on May 18 on the Windsor-Canton line where both circuits tripped automatically due to the breaking of two line wires. It is interesting to note that this occurred on a line that had been very conservatively designed from the standpoint of height above ground, but on the other hand, it had practically no arcing protection. This again substantiates the point that in designing a line both preventive and remedial measures must be considered. Lowering a line, for example, will undoubtedly reduce the lightning voltage which may be induced on the line. Installing a ground wire will do the same thing. Both these measures are preventive. The purpose of the arcing protective devices on the other hand are decidedly remedial.

---

[5] *Recent Investigation of Transmission Line Operation,* J. G. Hemstreet, TRANS., A. I. E. E., Vol. XLVI, 1927, p. 835.

Other cases of trouble, such as burned conductors, burned hardware, and ground wire,—also broken porcelain—were found in moderate amounts by normal patrol and repaired at prearranged times without interruption to normal service. This indicates that from an operating point of view, conditions were very much better than during 1925 on the Philo-Canton line. As typical of the action of the arc on a 132-kv. insulator string with and without arcing protective devices, Figs. 5, 6, and 7 are shown. Fig. 5 illustrates the effect of a heavy arc on a string without protective devices; Fig. 6 shows the action of a less heavy arc on a similar string. In the original photograph, the burns on the top unit are very clearly noticeable. Fig. 7 shows the action of an arc on another string equipped with rings and horns. The effect of the arc on the rings and horns is very clear in the photograph.

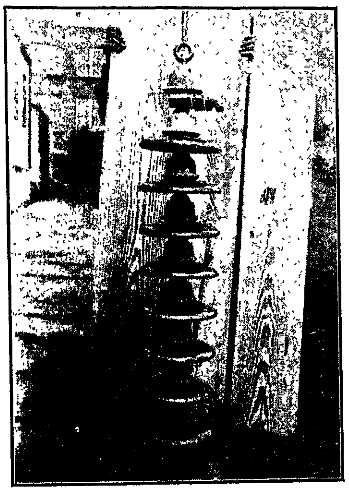


Fig. 5—Result of Typical Flashover of 132-Kv. Insulator String Without Arcing Protection

(Note badly shattered insulator on ground side)

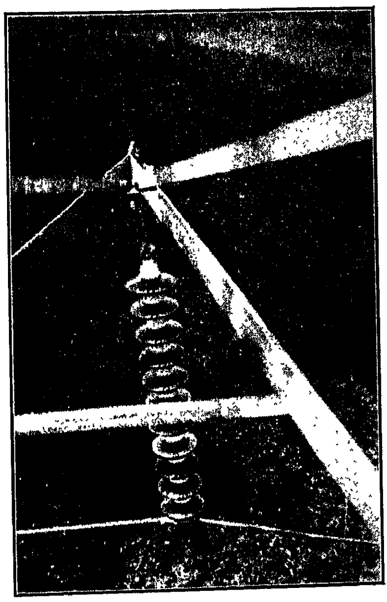


Fig. 6—Result of Typical Flashover of 132-Kv. Insulator String Without Arcing Protection

(Note top insulator still intact but badly burned)

Design Developments During 1927

The experience gained during 1926 was naturally incorporated in some of the design work carried on during 1927 and particularly along the following lines:

1. Whereas in 1926 an extensive program of ground wire installation was carried out, in 1927 much more attention than in the past was given to a phase that had been apparently neglected, and that was the problem of the line entrance. In every case, the ground wire was brought into the station structure and where the station structure design allowed, the number of ground wires brought into the station structure was made larger than the number on the line itself, all with the idea of reducing as much as possible the surge impedance of that portion of the system running from the last tower to the bus.

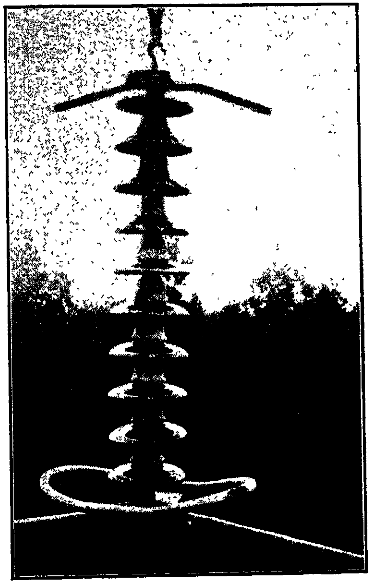


Fig. 7—Result of Typical Flashover of 132-Kv. Insulator String With Stanard Ring and Horn Protection

(Note burned ring above conductor; also horn tip slightly burned. Insulator undamaged)

2. In connection with the work discussed above and along the lines of limiting definitely the amount of overvoltage that can be brought into the station apparatus a reduction in the insulating value on the first mile or so of the line coming into the station was made, all with the idea of making certain that the lightning strength of the apparatus connected to the bus was

equal to, or greater than, the lightning flashover of the insulator string on the first mile of line.

3. In the design of the new lines, where the country did not absolutely demand it, the use of tower extensions has been kept at a minimum so as to keep the effective height of the line at a predetermined level, that level being determined, of course, by the tower design itself. In cases where the topography of the country permitted, the tension to which the conductor was pulled up was considerably reduced, resulting in a greater sag and a closer adherence of the conductor to the contour of the earth, with, of course, a net reduction of the average height of span above earth.

4. More attention was paid to the mechanical aspect of the arcing protection arrangements with a view of minimizing the effects of conductor vibration on the protection assembly itself.

SUMMATION OF EXPERIENCE AND IT'S APPLICATION TO TOWER LINE DESIGN

In the previous paper[2] on these lines, presented in 1926, certain conclusions were drawn with regard to tower line design. Everything that has happened during 1926 would seem to bear out again the conclusions drawn at that time. There is one point only that should be stressed, and that is with regard to the question of ground wire. Previously it was pointed out that unless a line was very low, generally steel tower structures, carrying power conductors of 44,000 volts and over, should be designed and installed with a ground wire. It is now believed that careful consideration should be given to the question of whether or not two ground wires would not in some cases be justifiable with of course particular attention to the location of the ground wires as regarded the main conductors. The experience with lines without ground wires, and particularly the experience obtained during 1926 on the Logan-Sprigg and Turner-Logan lines, would seem to prove definitely that it is very hazardous and extremely unwise to try to operate a line of the type under discussion, without a ground wire. The limit to the voltage which can be picked up on an insulator string being the flashover of the string plus its supporting structure to ground, the likelihood of picking up voltages close to that limit will naturally be much greater without a ground wire than with it. This has been borne out by some of the 1927 experiences with klydonographs, to which reference will be made later. It is also further discussed in the 1927 report of the Transmission Committee of the Institute.[6]

FUTURE DATA

During the past year, much more systematic record was kept of all cases of insulator and conductor damage although even that was not what it should be. In the future, however, an attempt will be made to have these trouble cases, so far as possible, definitely tied up with the cases of lightning trip-outs on the lines. The problem of patrolling 1000 miles of line does not lend itself to making a patrol after each thunderstorm, and it is usually the case that damage on the line can be assigned to some particular storm only in case such damage is sufficiently severe to cause a line outage that does not permit the line being placed back at once. One is faced therefore with the situation that unless an unusually heavy amount is spent on patrolling, it is impossible to tie up definitely each trip-out or to determine the concomitant damage. In this case, as in other cases of the same type, there is undoubtedly a happy compromise, and an attempt will be made to work this out.

KLYDONOGRAPH INVESTIGATION

During the year 1927, in conjunction with the Lightning Subcommittee of the Committee on Transmission and Distribution of the Institute, a comprehensive klydonograph investigation was carried out on a portion of the above 132-kv. system. It is planned to continue this investigation during 1928, and at the end of that time, to summarize it before the Institute.

Acknowledgment of thanks is hereby made to the operating organization of the Appalachian Electric Power Company; to that of the Indiana & Michigan Electric Company and of The Ohio Power Company for their cooperation in gathering together the field data, and to Mr. I. W. Gross for his assistance in correlating it.

Discussion

W. S. Kates: The statement is made that, "The isoceraunic lines, *i. e.*, lines connecting points of equal thunderstorm intensity, for the territory in question ranged from 40 to 60 in April, 150 to 200 in June and July, and down to about 60 to 80 in September."

I should like to know whether these figures refer to monthly periods and if they have been weighted in plotting the curve of Fig. 2.

I note in Table II that reference is made to outages due to lightning. As Mr. Sporn recognizes in his paper, the obtaining of accurate operating data is very difficult, and I should be interested to know what assumptions he made and what classification was made which determined that outages were due to lightning.

Mr. Sporn's paper initiates a type of record which has been lacking in publication to date, yet which is much to be desired. I refer to Mr. Sporn's attempt to correlate storm data with transmission-line performance.

Lack of such correlated data was apparent during a recent investigation of insulator protective-device performance. Proponents of protective devices during such investigation repeatedly stated that certain laboratory test performances are confirmed by operating experience. Usually the operating experience referred to was the statement that operation on a certain line was better after the installation of such a device than previously. Usually such statement was supported by figures of outages per mile during a season. For some of the lines referred to, the towers and insulators were designed for 100- to 150-kv. operation, yet operated only at 44 to 66 kv. In many such cases the protective-device installation was synchronous with the installation of the ground wire and the latter might have had equal or greater effect than the protective device. As Mr. Sporn differentiates, the ground wire is in the nature of a preventive device, whereas the insulator protective device is remedial. In only a very few operating records investigated was storm occurrence recorded or correlated. Yet the number of such storms seems a fundamental factor in the comparison of transmission line performance, because it determines the occurrence of most surges causing discharge.

---

[2] Loc. cit.

[6] Report of Committee on Power Transmission and Distribution, TRANS. A. I. E. E., Vol. XLVI, 1927, p. 783.

A very striking illustration of the extent to which storm occurrence affects line performance when such line performance is stated on interruptions per mile basis is presented by the remark of G. H. Doan, while discussing Mr. Hemstreet's paper presented to the Institute, June 21, 1927 (TRANS. A. I. E. E., 1927, p. 843). Mr. Doan mentions the operation of a 120-kv. system during the year of 1925 without a ground wire and with 114 automatic switch operations resulting. During the winter the line was equipped with a ground wire and the following year, 1926, there were only 7 operations. As Mr. Doan states, the switch operations were reduced 94 per cent.

From these facts as stated, and using outages per mile per year as a basis of comparison, it might be concluded that the ground wire of itself reduced automatic operation 94 per cent. However, Mr. Doan also states further that the storms passing over the transmission line area decreased 47 per cent in number. This latter statement places an entirely different complexion on any quantitative deductions from this operating record.

It is interesting to note also that one large operating company whose interconnected capacity is several hundred thousand kv-a. has, after correlating storm data with outages, decided that the essential factor in the elimination of outages is not remedial but preventive.

Storm intensity seems beyond determination or even approximation at present. Furthermore, transmission lines are economically justified on the basis of operating costs which do not include allowances for use of the lines as laboratories. The mere statement of the number of storms each season, however, would greatly enhance the value of operating records as guides for both designing and operating engineers.

A. O. Austin: As indicated by Mr. Sporn, there are many factors which must be taken into account in comparing the number of flashovers on various systems. While the length of line is usually the best possible base, it is necessary to take into consideration other factors such as difference in clearance, number of insulators or towers per mile, length of insulator string, and the benefit of ground wires. In addition, storms vary widely from year to year or in different localities. As yet we have no direct measure of the storm factor, but in the course of time we shall undoubtedly have improved means of recording and gaging this.

One point to which I should like to call attention is the use of the ground wire under conditions which may change its effectiveness materially. One of the most serious problems for the designer of the insulator or transmission line is to prevent the flashover of an insulator under an oscillation or after the crest of the wave has lowered materially.

Dr. Norinder has shown that while lightning may be aperiodic, severe oscillations are usually set up on the system. Line performance together with a study of the characteristics of the insulator has long indicated the presence of oscillations on many transmission systems which were undoubtedly causing flashovers not at the highest voltages, but rather at values considerably below the flashover voltage of the insulator under impact.

While there is little question as to the reduction of the maximum voltage at the point of disturbance by the use of the ground wire, its effect upon the attenuation or tail of the wave should be given careful consideration; otherwise, flashovers may result where the insulator system has a relatively low flashover voltage for an oscillation or with a wave having a slow rate of decay. Lack of attention to this factor may make it possible for a ground wire to subject the station to higher effective potentials than where none is used and to cause insulators to flash at a distance from the point of maximum disturbance or after the voltage has dropped materially. It is understood that the above applies to disturbances originating out on the system. Where a ground wire is used in the vicinity of the station only, it will result in lower voltages on station equipment due to its absorbing capacity.

Following the release of a bound charge Q, the resulting potential E on the power conductor will be $E = \frac{Q}{C}$ where C is the effective capacitance of the conductor to ground or objects of lower potential. The ground wire keeps the maximum induced voltage down by increasing C, 30 to 40 per cent, but has little effect upon the charge Q. The charge Q upon the power conductor is practically the same with or without the ground wire, particularly where the clearances are large as in the higher-voltage lines. Since the losses on the power conductor are very high at the peak voltage, but drop off very rapidly as the voltage is decreased, it naturally follows that the losses on the conductor will be reduced materially by a reduction of 30 to 40 per cent in the peak voltage. Since the charge on the conductor is practically the same, the natural period will also be reduced owing to the increased capacitance, all of which tends to reduce the absorption of the abnormal voltage.

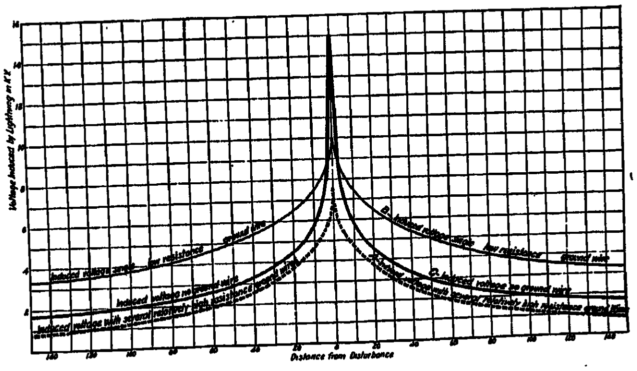


FIG. 1—STUDY SHOWING EFFECT OF GROUND WIRE IN LOWERING INDUCED VOLTAGE AT POINT OF DISTURBANCE AND INCREASE IN VOLTAGE AT THE STATION OR INTERMEDIATE POINTS OVER THAT WHERE NO GROUND WIRE IS USED

LOWER LOSS IN GROUND WIRE WILL DECREASE VOLTAGE AT POINT OF DISTURBANCE BUT TENDS TO REDUCE THE ABSORPTION AND ATTENUATION

This reduction in losses due to the lowering of the maximum voltage may result in a higher voltage at a distance in some cases as in B in the accompanying illustration over that where no ground wire is used as in C. Where the flashover characteristics of the insulator are such that they will withstand the maximum voltage under the steep wave front in B and will not flash over on the tail of the wave or on a resulting oscillation, no flashover will take place on the line, although the station may be subjected to a more severe disturbance than where no ground wire is used. This is particularly true where the ground wire has a low resistance and an absorbing power insufficient to offset the decreased attenuation.

If, instead of a low-resistance ground wire, several high-resistance ground wires are used, the induced voltage and attenuation may be as shown in A. As the number and proximity of the ground wires, rather than their size, determines the increase in the absorbing capacity C, a greater number of high-resistance conductors may be used to reduce the peak voltage at the point of disturbance as in A. The total current induced in the high-resistance ground wires in A will be greater than in B, and, if their resistance is high enough, the $I^2 R$ losses will be greater in A than in B.

Since the energy producing a loss in the ground wire must come from the power conductor, it naturally follows that any loss in the ground wire must result in an absorption of energy in the power conductor which tends to cause an increased attenuation in the voltage on the conductor. This increased attenuation tends to reduce the voltage at a distance and also tends to dampen out an oscillation. Under this condition, the voltage at all points on the system will be lower where the ground wire is used than where none is present. Where a high-resistance ground wire is used, there may be an appreciable rise in voltage between the ground wire and ground at the center of the span. Even though this induced voltage may reach 100 to 200 kv. for severe disturbances, it will probably not reduce the efficiency of the ground wire more than 10 per cent which, I believe, is negligible compared to the absorbing value and increased attenuation due to the resistance.

As the cost of the ground wire is a very appreciable item on any system, its efficiency and effectiveness should be given careful consideration, for if improperly applied, a considerable expense may be incurred without any improvement in the operation of the system, whereas a ground wire of lower cost, but with better absorbing factors, might result in an improvement for all conditions existing on the system, not only out on the line, but in the station as well.

John Murphy: In the last paragraph on the fifth page Mr. Sporn makes a statement regarding some trouble on May 18 on the Windsor-Canton lines. He says both line wires broke on that occasion. In the next sentence he says that that line had practically no arcing protection. I should like to know if I am justified in assuming that the breaks occurred at an insulator.

H. B. Vincent: The author states that the record of damage to insulators, conductors, and hardware on the lines is not complete for 1926 so no attempt will be made to give specific data in the paper. This being the case, it is evident that an accurate analysis cannot be made with respect to attributing a particular outage or interruption to a specific damage. However, it would have been interesting and possibly very valuable at least to know what damage, that is, as a total, has occurred to the insulator, and particularly the damage to individual units with respect to their location in the string.

Assuming that the primary object of the overhead ground wire and insulator protective device is to reduce the number of flashovers, and secondly, to convert extended into momentary interruptions, the ideal protective device should also protect the porcelain from damage.

To illustrate my point, on a recent power test of insulator protective devices, a device installed at both top and bottom of a string of a given number of units functioned satisfactorily, in that no porcelain was damaged. The top portion of the device was then changed to a different type. On test, this new set-up disclosed that 2 units in the string had broken. Yet in both cases under operating conditions, in so far as interruptions to the system are concerned, the records would have been identical, as the damaged string undoubtedly would have held operating voltage.

So without a record of the physical damage, the true efficiency of any particular insulator protective device naturally can be questioned.

The author in Figs. 5 and 7 clearly shows the resulting damage due to lack of arcing protection, so one could infer that with the protection shown in Fig. 7 no damage would occur.

I should like to ask, therefore, if my inference is correct, namely, that in the performance record submitted, in those lines protected, as in Fig. 7, no insulators have been damaged.

E. D. Eby: Mr. Sporn's paper is another example of the increasing attention being given to the line insulation and its relation to connected apparatus. The amount of data collected and its analysis according to the various factors in the problem, must be and are of vital importance in forming conclusions which are expected to influence subsequent practise.

There is a two-fold purpose in these studies: one, the protection of the line, the other, protection of the station apparatus. While statistics on line construction and performance are being collected and made available for study, there are practically no correlated data presented pertaining to the transformers, oil circuit breakers, and protective equipment. Of course, the cases of transformer failure, bushing flashover, etc., are rare compared to insulator trouble; but if there could be presented along with the insulator data a statistical record of connected apparatus as regards general type or design, test voltages applied, flashover voltage in the case of bushings, kind of protective equipment used and how installed, and the nature of failures when such occur, it should become possible to analyze the latter much more intelligently, and to decide upon both line and apparatus design more accurately.

Might I suggest, therefore, that such tables as Mr. Sporn's paper contains be supplemented in this and similar instances with a record of the characteristics and performance of connected apparatus. This would be very useful, for example, in checking the selection of high-voltage bushings for transformers and oil circuit breakers. In the past this selection has been made without reference to the line insulation, being based almost wholly upon the system rating and the test voltage of the apparatus. Such data would serve to check former practise, and guide the designers of high-voltage apparatus at a time like this when the relation of line insulation and apparatus insulation is being closely scrutinized and in some respects undergoing marked revision.

J. H. Cox: (communicated after adjournment) In the solution of any lightning problem a large amount of data is necessary due to the scarcity of lightning strokes in any given spot. For this reason papers like Mr. Sporn's are very desirable; only by the summarizing of a large amount of operating experience can we obtain a check on the performance of various features of lines in connection with lightning.

In taking data on the performance of lines when subjected to lightning, it is quite important that data on the number of storms be included as was done by Mr. Sporn. The United States Weather Bureau has found that the number of thunderstorms in any locality per year is remarkably constant. It is therefore possible that many of the expressions to the effect that the lightning in 1925 was much more severe than in 1926 and 1927, are inspired by the improved performance of many lines during the past two years due to the installation of ground wires following 1925.

Mr. Sporn has mentioned the comparison of the lightning strength of line insulation and of apparatus. Such a comparison is significant when considering the effect on apparatus of impulses which have been relieved by flashing over the adjacent line insulation. It must be remembered, however, that from the point of view of continuity of service the impulse ratio of suspension insulators is unity. That is, when any impulse, with a duration equal to those of lightning surges, is only slightly higher than the 60-cycle crest flashover of the insulator, a flashover will result.

Mr. Sporn has made a practise of continuing the ground wires into the station structure. I have made some calculations on the effect on a traveling wave of discontinuing the ground wire at the last tower out of a station. At such a point there is a change of total surge impedance and there will be a partial reflection of the wave. Calculations indicate that this reflection will cause an increase of the surge of the order of 15 per cent. Therefore, it seems well worth while to bring the ground wires into the station structure, or even to increase the number of ground wires on the last few spans.

Mr. Austin stated in his discussion that a lightning stroke is followed by an oscillation. In klydonograph tests on all types and voltages of lines, at from 10 to 50 locations, mostly the larger number, over four years, not a single record has shown lightning to be followed by an oscillation except where a flashover has resulted, and only on isolated-neutral lines has the oscillation been of more than one or two highly damped cycles. It is recognized that the flashover of porcelain insulators is reduced when subjected to a sustained high-frequency oscillation, but since such oscillations never exist on lines except those with ungrounded neutral, and then only after a flashover has already occurred, one cannot be greatly alarmed at such reduction of flashover.

Mr. Austin states further that a flashover has been experienced when the voltage recorded was less than the 60 cycle crest flashover of the insulation, but he did not state the circumstances. In klydonograph tests, a flashover is frequently experienced and the corresponding voltage recorded very low. It has also been determined, however, that a very high-voltage surge attenuates to a small fraction of its magnitude in a very few miles, and there is absolutely no reason to believe in the above mentioned cases, that the flashover was caused by a low value of voltage. Undoubtedly a higher voltage existed at a distance from the measuring station, and the flashover took place at that point.

It is well known that the amount of protection afforded by ground wires is greater, the greater the geometric mean radius of the ground wires and the less the geometric mean distance of the ground wires from the conductors being protected. It is obvious, therefore, that two small ground wires are more effective than one large ground wire, and any means of increasing the geometric mean radius of the ground wire, without a proportionate increase of the geometric mean distance from the conductors being protected, is an improvement.

Philip Sporn: Mr. Kates has raised some questions with regard to the isoceraunic figures. The figures shown in Fig. 2 were obtained by dividing by 20 the isoceraunic figures as published in the *Monthly Weather Review* referred to. The figures published in the review give the total number of storms per month over a 20-year period. That is, all the storms in that month have been added together, and the figure 80, for example, means that over a 20-year period of Aprils, there were 80 storms. The average for one year, therefore, is of course 4.

The Weather Bureau, incidentally, has stopped gathering the data from which it obtained isoceraunic curves because their 20 years of observation have convinced them that any further data would be merely a further check on the data obtained so far; that over a period of years the data repeat so closely that there is no value in continuing the gathering of further data.

Mr. Kates also brought out the point that data as to storm occurrence are very important in the determination of the operating performance of lines, and they are particularly important in any attempt to draw conclusive information from the operating experience.

I wish to say that I decidedly agree with him, and on our system now we have inaugurated an arrangement for keeping very complete track of this, but I do believe that, in the absence of any other data, the information as given in the *Monthly Weather Review* referred to, can be utilized, and very effectively, in designing. That is, you don't have to keep data of your own, if you examine the information already obtained by the Weather Bureau, to see that it is fallacious to try and apply experience obtained, say, in California to line conditions in Florida or thereabouts.

In California, as I recollect it, the 20-year isoceraunic at the period of maximum storm runs from about 8 to 10. In Florida it runs around 360. Operating experience obtained, as I say, in California cannot be applied directly to design in Florida or design in territories such as those discussed in this paper, where we do not have quite the heavy storms that are experienced in the South, but where we are not far from it.

Another question that has been raised is how we obtain the outages due to lightning. We do not claim that the data are 100 per cent correct, but they have been obtained and checked very, very carefully. Three sources were utilized for obtaining them. In the first place, arrangements had been made for a special record to be kept of those outages that did occur during lightning storms. The log of every station on the system was carefully scrutinized and every reference to a lightning storm checked with relay action. Then the relay records, which are kept independent of any other records, were again gone over, and whenever any one of the three sources showed possibility of lightning action, it was checked against the other two, and if it could be substantiated as a lightning record, it was accepted as such. If the majority of the data based on the three sources indicated that some other cause was responsible for the trip-out that cause was accepted and the lightning rejected as the cause.

In Mr. Austin's discussion, mention was made of a flashover at 400,000 volts on a system that I think could stand a crest voltage of 1,100,000 volts. We have had a very large number of klydonographs this year on our 132-kv. system and the system flashover value of that order, *i. e.*, 1100 kv., yet we have never obtained a record of a flashover at an insulator string where the klydonograph record showed as low a voltage. I think it would be very interesting if these data could be further substantiated.

Another point made by Mr. Austin in connection with the ground wire was that beneficial effects could be obtained on a low-voltage system by skipping the grounding on the poles. We have done that to a considerable extent, but have apparently secured something in addition to what we expected to get. That is, we never did it with the idea in mind of getting better action, but did it to save cost of grounding, and figured that the effectiveness of the ground wire was still sufficiently good even with the skipped grounding points.

A question has been raised as to just where the conductor broke, in the case of the conductor failure cited on the fifth page. Two wires were broken on one circuit and one on the other circuit. The middle wire was cut by lightning on No. 1 circuit at the point of attachment and it was apparently a clear flashover, or lightning break. The dropping of the middle conductor on the bottom phase of No. 1 circuit caused it to burn down and it, too, was broken as well as the bottom phase of circuit No. 2.

The top insulators of the strings on both middle and bottom phases on circuit No. 1 were burned, as well as the top insulators on the string on the bottom phase of the second circuit. The lines at this point were equipped with arcing horns on the line side only, but no damage was found on these horns.

Mr. Vincent in his discussion brought out the fact that he believed the function of the protective device was to hold back the number of flashovers, and later on, when the condition of flashover did occur, to prevent damage to the insulators. At 132,000 volts, I do not believe we get enough effect on the protective devices that have been developed so far, at any rate, to act much in the way of holding back flashovers. That is, we don't get sufficient grading at 132,000 volts to increase the lightning flashover of the string at high voltages, but at 220 kv. it indoubtedly does come into play. I do not think it comes into play at 132,000 volts.

Regarding the performance of the insulator strings with and without protective devices, particularly in Fig. 7, we have qualitative data—it is not quantitative, otherwise we would have published it—to show definitely that in general the trend is as shown by Figs. 5, 6, and 7; that is, we get very little burning, and in many cases none at all, with the protective devices, and we get considerable burning and in many cases heavy breakage where we don't use them.

In conclusion, we plan to continue these studies and hope that in the next few years we shall get much more valuable data.

Vacuum Tube Synchronizing Equipment

BY T. A. E. BELT[1] and N. HOARD[1]
Member, A. I. E. E. Non-member

Synopsis.—Potential from high-voltage circuits is obtained by means of capacitance transformers. This potential is applied to the grid circuit of vacuum-tube amplifiers, the output from which drives a rotating type of synchroscope. Thus the vacuum-tube synchronizing equipment makes unnecessary costly high-voltage transformers, which have been almost universally used for synchronizing in the past.

The paper describes the commercial apparatus, gives equations pertaining to the vacuum-tube amplifier and shows calculated and measured characteristic curves.

THE increasing number of power system interconnections makes synchronizing of high-voltage lines almost a necessity. Potential transformers for 110 kv. and above, which have been almost universally used for synchronizing, are costly. A new method of obtaining transformation from a high voltage to a low voltage by means of a simple capacitance, the amplification of this controlling potential to an appreciable amount of power, the utilization of the output from the amplifier for operating a rotating type of synchroscope makes possible, economically and practically, the almost universal adaptation of high-voltage synchronizing.

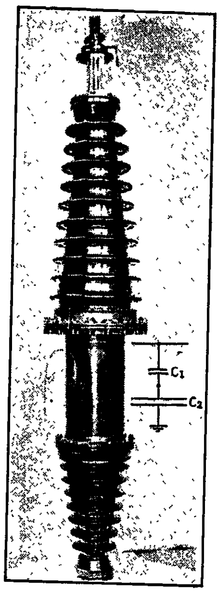


FIG. 1—132,000-VOLT BUSHING EQUIPPED WITH CAPACITANCE TRANSFORMER SHOWING FLEXIBLE LEAD AND SCHEMATIC DIAGRAM OF CONNECTIONS

Description of Units. The vacuum tube synchronizing apparatus as developed for commercial service consists of the following units:

a. Capacitance transformer bushing.
b. Outdoor amplifier.
c. Indoor amplifier and power supply.

Capacitance Transformer Bushing. Bushings equipped with capacitance transformers, (Fig. 1), are of the regular flanged-clamped oil-filled porcelain design, similar in external appearance and construction to the standard line of General Electric Company's oil-filled bushings. This added feature does not in any way affect the normal functions of the bushing when it is used with power apparatus. Where power apparatus bushings are not available and it is desired to use such a bushing, it may be mounted in a separate tank and installed in any convenient place, either in or out of doors (Fig. 2). The capacitance transformer enables these bushings to be used with vacuum-tube equipment in place of instrument potential transformers for synchronizing.

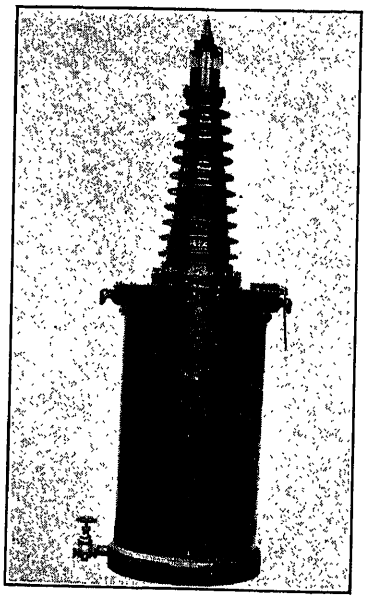


FIG. 2—CAPACITANCE TRANSFORMER BUSHING MOUNTED IN TANK

The necessary voltage for the grid-filament circuit of the amplifying tube is obtained from a high-capacity condenser within the bushing. This capacity is in series with the bushing capacity, the total line-to-ground potential of the system being impressed across both. The voltage obtained across either condenser is inversely proportional to the capacity. The secondary potential is approximately 35 volts. The high-capacity condenser consists of two layers of thin copper sheets separated by a treated paper dielectric and all

---

[1] Both of the General Electric Co., Schenectady, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

wound on a herkolite cylinder. The outer layer of copper is grounded to the ground sleeve and a soldered connection to the inner copper layer is brought out through a terminal, fitted in the ground sleeve. As shown in cross-section drawing, Fig. 3, this terminal is encased by a metal housing to prevent exposure of any live metal.

The outstanding features of this method for obtaining low potential from high-voltage transmission lines are:

a. Since the condenser is mechanically constructed within the ground-sleeve boundaries, the presence of external metal such as power apparatus tank, etc., does not have any effect upon the capacity and therefore the ratio of transformation remains constant.

b. Changes in atmospheric conditions, such as rain, snow, or air impurities deposited on the bushing surface do not affect the ratio of transformation.

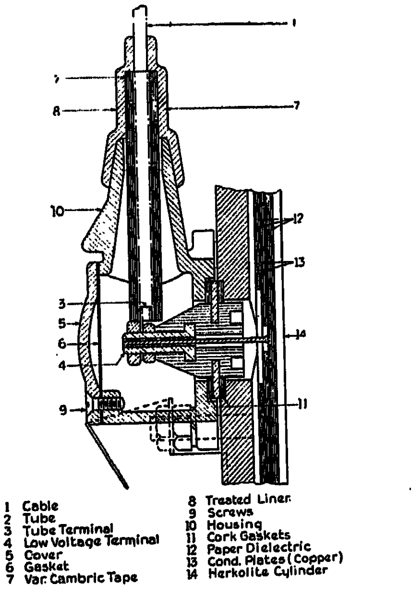


Fig. 3—Cross-Section Drawing Showing Method of Bringing out Terminal from Capacitance Transformer

c. Any normal temperature changes either increase or decrease both series capacities proportionally and, therefore, the ratio remains constant.

d.. Since a very small load for the amplifying equipment is taken off the low-voltage side, the ratio is not affected.

e. The low secondary potential of approximately 35 volts in no way endangers the operation of the bushing or connected apparatus, even at potential which may flash over the bushing.

Outdoor Amplifier. The outdoor amplifier consists of a PT-210 power tube with its associated filament transformer and bias resistors mounted in a steel weatherproof box. This unit is a complete single-stage amplifier. The voltage obtained from the capacitance transformer is impressed on the grid of the tube and the output delivered to the primary of the interstage transformer which is located on the indoor amplifier panel.

The secondary potential from the capacitance transformer is divided by means of a 600,000-ohm potentiometer so that 17½ volts are delivered to the grid of the tube. The amplification constant of the PT-210 tube and the ratio of the interstage transformer are such that the voltage applied to the grid of the second stage of amplification on the indoor panel is 35 volts. The potential from the capacitance transformer cannot be successfully applied directly to the grid of the second-stage amplifier tube since the d-c. grid current of this tube with normal negative bias is sufficient when flowing through the 600,000-ohm resistance across the low-voltage terminals of the capacitance transformer to alter the effective bias

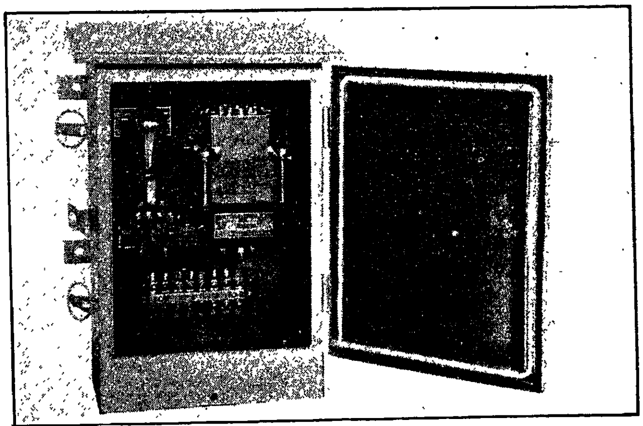


Fig. 4—Single-Unit Outdoor Amplifier

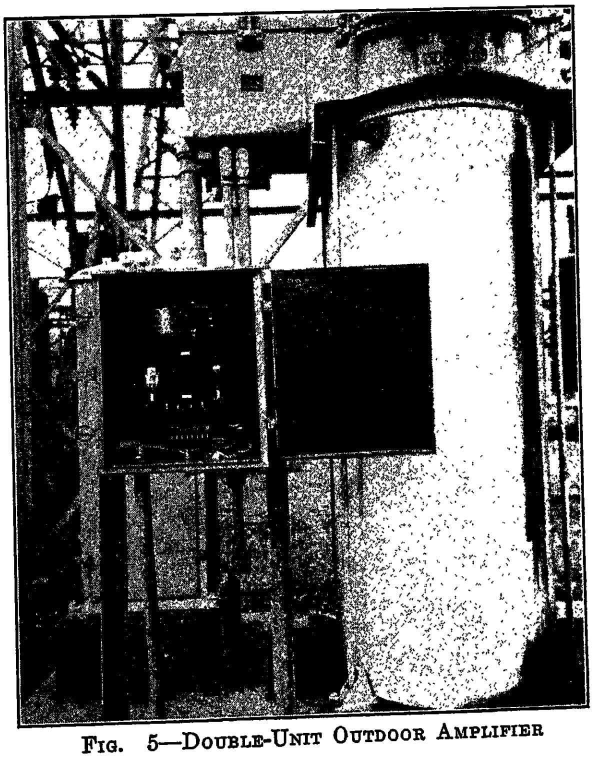


Fig. 5—Double-Unit Outdoor Amplifier

voltage, so that the tube operates on an undesirable portion of its characteristic curve. A single outdoor amplifier unit is shown in Fig. 4.

Two single outdoor amplifiers can be combined so as to make a double outdoor amplifier, as shown in Fig. 5. Double units are usually used when synchronizing directly across oil circuit breakers as this facilitates mounting and reduces the number of necessary connections.

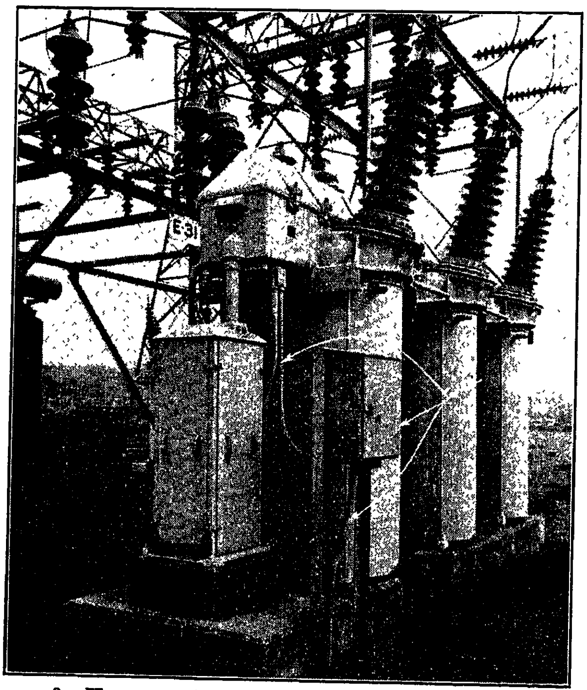


Fig. 6—Vacuum Tube Synchroscope Installation Showing Outdoor Unit and Leads

One capacitance transformer must be connected to the same phase on each side of the oil circuit breaker used for synchronizing, and connections made to the corresponding grids of the amplifier tubes. These units are preferably mounted near the capacitance transformers (Fig. 6) in order to reduce stray capacity effects across the secondary side.

Indoor Amplifier and Power Supply. The indoor amplifier is mounted on a panel of standard switchboard design. It is intended for mounting along side of the main switchboard, although it can be placed in any other convenient location in the station. A voltmeter for indicating the plate voltage to the tubes, a rheostat in the field circuit of the plate and bias-voltage generators, and a fused switch for opening the voltage supply circuit to the motor generator set are located on the front of the panel.

On the back of the panel are mounted two separate single-stage power amplifiers requiring two PR-11A tubes. The undistorted power output which can be obtained from one of these tubes is approximately ten volt-amperes. The grid of each tube receives its excitation through the interstage transformer from the outdoor amplifiers. An output transformer is connected in the plate circuit of each tube and supplies 110 volts for operating the synchroscope. The slight phase shift in voltage through the amplifier and transformer is corrected at 60 cycles by connecting the proper value of capacitance load across the secondary terminals of the output transformer. A metal grill is provided for covering the apparatus mounted on the back of the panel. The plate connections are made under the grill which eliminates any exposed high-voltage circuits.

All power for operating the vacuum-tube equipment is obtained from a small four-unit motor-generator set. The power supply for operating the set can be taken from the station battery or any other suitable voltage source. An automatic starter is located on the indoor panel which puts the motor generator set in operation, as soon as the control circuit is closed, by insertion of the synchronizing plug in the proper receptacle in the switchboard.

The synchroscope is a 7½ in. diameter instrument giving the usual indications of fast, slow, and synchronism. In the experimental installation the synchroscope was mounted on a swinging bracket at the end of the switchboard. On the adjoining bracket was mounted a synchronism indicator operated from potential transformers which in this installation were used to check the operation of the vacuum-tube-operated synchroscope. This installation has been in daily service for over a year proving its simplicity and reliability in actual service.

APPLICATION

The construction and operation of the synchroscope equipment is such that it is easily adapted for any number of incoming lines. When synchronizing is to be accomplished between a number of incoming lines and double busses each incoming line must be equipped with a capacitance transformer but it is only necessary to use one capacitance transformer for each bus. Such an arrangement is shown in Fig. 7. The act of synchronizing, using the vacuum tube equipment; is exactly the same as when potential transformers are used. Thus, the operator inserts the synchronizing plug which

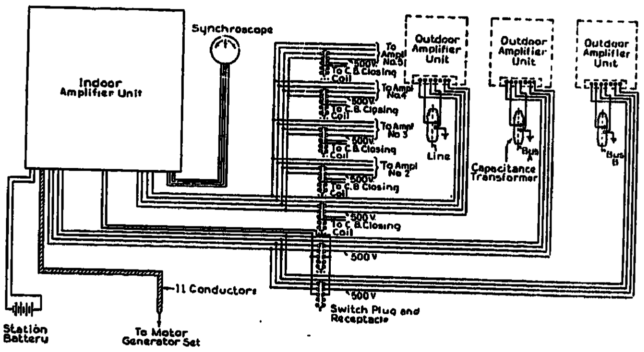


Fig. 7—Schematic Layout for Synchronizing Between Double Buses and a Number of Incoming Lines

automatically places the tube apparatus in operation causing the synchroscope to revolve in the usual manner.

It is of importance to note that one winding of the synchroscope may be excited from a potential transformer and the other winding excited from the vacuum tube amplifier without upsetting the phase relations on the synchroscope. A number of such installations have been made.

The life of vacuum tubes is relatively long and, from records of test data, should be about ten years of service operating on an average of fifteen minutes each day. Thus the maintenance of the apparatus is reduced to a very low degree.

THEORY AND CHARACTERISTICS

The capacitance transformer has a constant ratio and zero phase-angle shift with varying frequency and voltage, provided there is no external circuit connected to it. When the potential of the capacitance transformer is applied to the amplifier, a resistance, R, Fig. 8, and the tube input impedance are connected in parallel with capacity, C_2. The tube impedance at 60 cycles, is very high as compared with the impedance of either C_2 or R and can be neglected. The relation between $\dot{E}_R$ and $\dot{E}$ is expressed by the following equation:

$$\dot{E}_R = \dot{E}\left[\frac{j\,\omega\,C_1\,R}{1 + j\,\omega\,R\,(C_1 + C_2)}\right] \qquad (1)$$

in which $\omega = 2\,\pi\,f$, and the other symbols refer to Fig. 12. Both the voltage ratio and the phase-angle shift of the capacitance transformer and resistance combination, can be obtained from this equation.

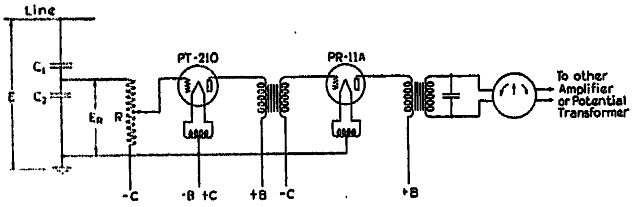


FIG. 8—SCHEMATIC DIAGRAM OF VACUUM TUBE SYNCHROSCOPE EQUIPMENT

This equation shows that the voltage ratio $\frac{E}{E_R}$ is independent of voltage but varies with frequency and resistance. A numerical example will show the mangitude of these variations. A particular 110-kv. (63.5-kv. line-to-neutral) installation has the following constants; $C_1 = 0.000050\ \mu$ f, $C_2 = 0.0772\ \mu$ f, and $R = 600{,}000$ ohms. The variation of voltage ratio with resistance is shown by the curves B and C, Fig. 9. The calculated curve B checks the measured curve C very closely. These curves show that the voltage ratio does not appreciably change with a considerable decrease in resistance. Therefore, the ratio is independent of leakage change within the capacitance transformer, which measurements show to be approximately 100 megohms or more. The ratio of the capacitance transformer mentioned above is 1547 at 60 cycles. At 50 cycles the ratio is increased 0.05 per cent and at 70 cycles it is decreased 0.04 per cent, hence the ratio may be considered independent of frequency.

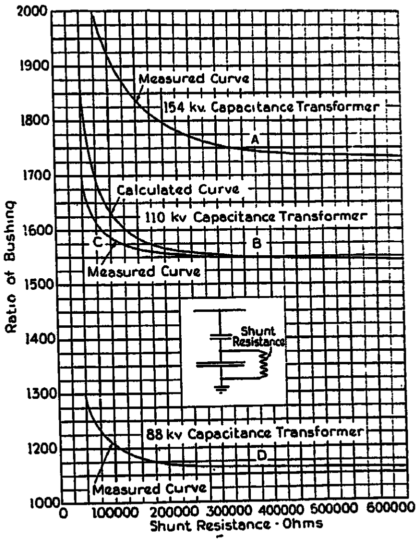


FIG. 9—RATIO CHARACTERISTICS OF CAPACITANCE TRANSFORMERS

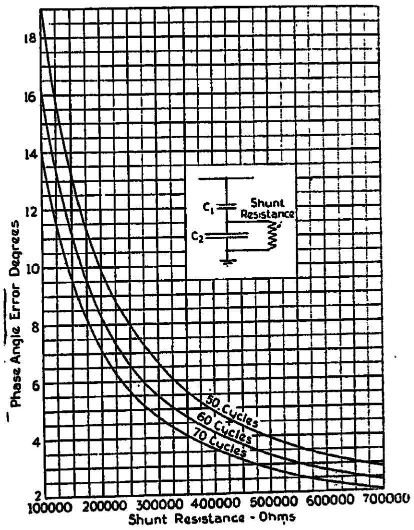


FIG. 10—PHASE-ANGLE CHARACTERISTICS OF CAPACITANCE TRANSFORMERS

There is a phase shift through the capacitance transformer when a resistance is connected across it, and this phase shift is dependent on both resistance and frequency but is independent of voltage. A curve computed from equation (1) showing the variation of this phase-angle change with resistance is given in Fig. 10. Thus 600,000 ohms gives a phase-angle shift of 2.75

deg., 500,000 ohms 3.3 deg., and 700,000 ohms 2.4 deg. at 60 cycles. At 70 cycles and 600,000 ohms the phase-angle shift is 2.4 deg. At 50 cycles the shift is 3.3 deg. That is, the secondary voltage E_R leads the line-to-neutral voltage a small amount, due to resistance load, and changes slightly with change in frequency.

The vacuum tube amplifier, which is used to supply the power to the synchroscope, has characteristics which affect the over-all operation of the equipment. The complete theory of a vacuum tube amplifier is beyond the scope of this paper, but an approximate theory will be developed to show its general operation. The schematic diagram and the equivalent diagram of the a-c. circuits of a single stage of transformer-coupled amplification is shown in Fig. 11. The expression for E_0, the load voltage, can be written

$$\dot{E}_0 = -\mu \dot{E}_g \frac{\dot{Z}_0}{\dot{Z}_T} \qquad (2)$$

where

$\dot{E}_0$ = Load voltage referred to primary of transformer
μ = Amplification constant of tube
$\dot{E}_g$ = Applied grid voltage

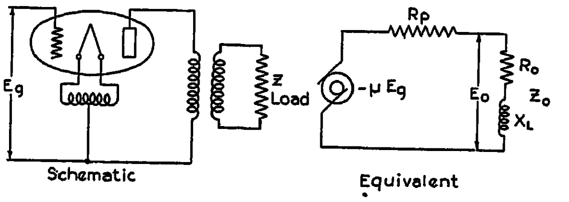


FIG. 11—DIAGRAMS OF SINGLE-STAGE AMPLIFIER

$\dot{Z}_0$ = Load impedance referred to primary of transformer
$\dot{Z}_T$ = Total impedance of tube-plate circuit.

The negative sign of the right side of the equation indicates a phase shift of 180 deg. through the tube itself, but since transformer coupling is used this phase shift can be corrected by proper choice of transformer connections.

Equation (2) shows that the voltage ratio of the amplifier is dependent on the tube-plate impedance and load impedance. The load impedance is independent of applied voltage and in a properly designed amplifier, the tube-plate impedance is independent of applied-grid-voltage. Under these conditions, the voltage ratio through the amplifier is constant for varying voltage input. Also, since these two impedances are independent of voltage, the phase shift through the amplifier is constant with varying voltage.

The effect of frequency changes on the operation of the amplifier can also be predicted from equation (2). This equation may be rewritten in the following form:

$$\dot{E}_0 = -\mu \dot{E}_g \left[\frac{R_0 + j X_L}{R_1 + j X_L} \right] \qquad (3)$$

where $R_1 = R_0 + R_p$ and, hence, is always larger than R_0. As X_L increases with frequency, the fraction on the right side of equation (3) increases but approaches unity as a limit. This shows that the overall amplification of the stage increases with frequency, but approaches a definite limit. The percentage increase in voltage is less, however, than the percentage increase in frequency.

The phase angle between E_0 and E_g can be obtained from equation (3) and is given by the following expression:

$$\theta = \tan^{-1} \frac{X_L}{R_0} - \tan^{-1} \frac{X_L}{R_1} \qquad (4)$$

Now $$R_1 > R_0$$

hence $$\frac{X_L}{R_0} > \frac{X_L}{R_1}$$

and $$\tan^{-1} \frac{X_L}{R_0} > \tan^{-1} \frac{X_L}{R_1}$$

The angle θ is therefore positive, and E_0 leads E_g by this angle. As the frequency increases the two ratios X_L/R_0 and X_L/R_1 increase in proportion and a consideration of the antitangent functions will show that the angle θ decreases with an increase in frequency.

The above discussion explains the general operation of a vacuum tube amplifier with inductive reactance in the load. If, however, the secondary of the transformer is loaded with capacitance until the reactive component of the load impedance is capacitive then the term $j X_L$ in equations (3) and (4) is replaced by $- j X_c$. Under this condition it can be shown in the same manner as before that with an increase in frequency the amplification of the stage decreases, and the phase shift through the stage increases and is always negative.

The commercial apparatus utilizes a two-stage amplifier. Now, if θ_T represents the total phase-angle shift from phase voltage of line to voltage across synchroscope, θ_1, the shift through the capacitance transformer, θ_2 the shift through the first stage of amplification and θ_3 the shift in the second stage then

$$\theta_T = \theta_1 + \theta_2 + \theta_3 \qquad (5)$$

By connecting sufficient capacitance in parallel with the synchroscope, the angle θ_3 can be made negative and equal to θ_1 plus θ_2, and, hence, the total angle shift is zero. With a given capacity, this condition exists for one frequency only and the value of the capacity in the commercial equipment is so chosen as to compensate at the operating frequency.

The curve of Fig. 12 shows the angular error in position of the synchroscope when using the amplifier on one side and a potential transformer on the other. This is equivalent to the shift through the amplifier, as the angular error due to the potential transformer is negligible. The maximum error within the range of 50 to 70 cycles is only 7 deg., or only slightly greater than one min. on the dial of a clock. The device is, therefore, satisfactory when used in combination with a potential transformer.

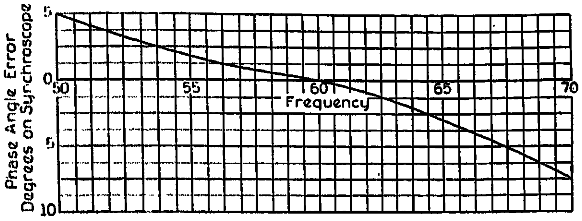


Fig. 12—Synchroscope Characteristic

One winding exciter from potential transformer
One winding exciter from vacuum tube amplifier

Fig. 13 is a curve showing the angular error when two amplifiers are used. In this case the maximum error is reduced to 3.3 deg. which is scarcely discernible on the synchroscope.

Summary

The capacitance transformer in no way affects the electrical characteristics or functioning of the high-voltage bushing in which it is mounted. The ratio is constant under changes in weather conditions. The secondary potential is 35 volts under normal operation, giving relatively low potentials under impulse flash-over of the bushing which in no way is dangerous to the apparatus used in conjunction with the bushing or to the circuits connected to the capacitance transformer. The cost of the capacitance transformer feature when added to a high-voltage oil-filled bushing is nominal.

The outdoor amplifier is mounted close to the capacitance transformer in order to reduce to a minimum the capacity effect introduced by connections. The power amplifier may be located any reasonable distance from the outdoor unit. Synchronizing with the vacuum-tube-operated synchroscope requires no unusual operation on the part of the operator.

The life of the amplifying tubes should be ten years or more, operating for a period of 15 minutes each day.

The ratio of the capacitance transformer is not affected by voltage change of the system; it changes with decrease in shunt resistance but is negligible with a decrease in resistance down to 300,000 ohms. The normal leakage resistance of the capacitance transformer is approximately 100 megohms, and therefore the ratio is independent of internal leakage. The ratio change is 0.05 per cent at 50 cycles and 0.04 per cent at 70 cycles, hence may be considered independent of frequency.

The phase-angle shift changes slightly with change in frequency, being 2.75 deg. at 60 cycles; 2.4 deg. at 70 cycles; and 3.3 deg. at 50 cycles with a 600,000-ohm shunt resistance.

The commercial apparatus utilizes a two-stage amplifier which introduces possible phase-angle errors which are compensated at operating frequency by connecting a capacity load across the output circuit. The maximum angular error in position of the synchroscope hand between 50 and 70 cycles, with one winding excited from a potential transformer and the other winding from the amplifier, is only 7 deg. or slightly more than one min. on the dial of a clock. It is therefore satisfactory when used in combination with a potential transformer.

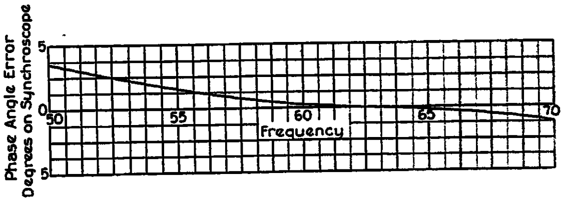


Fig. 13—Synchroscope Characteristic

Excited from vacuum tube amplifier

Acknowledgment

The authors wish to acknowledge the help given by Mr. E. D. Eby and Mr. T. S. Farley who kindly assisted in obtaining the information for this paper.

Discussion

For discussion of this paper see page 696.

Condenser-Type Bushing
Used With Synchronizing Equipment

BY E. E. SPRACKLEN,[1] Member, A. I. E. E.
D. E. MARSHALL,[2] Associate, A. I. E. E.
and
P. O. LANGGUTH[2] Associate, A. I. E. E.

***Synopsis.**—This paper describes apparatus recently developed for utilizing the charging current of a condenser bushing in operating a synchronoscope, thus making a simple and inexpensive method of synchronizing at high voltage. Data are given also covering the operation of the device for different conditions of frequency, voltage, etc., thus assisting in the consideration of the adaptability of the device to other applications. An account of the first installation in the field is also given.*

THE rapid advancement of superpower systems has increased the demand for a simple and economical high-voltage synchronizing device.

In order to meet this demand there has been developed a condenser-bushing potential device which utilizes the inherent capacity potentiometer characteristics of the bushing in providing a source of low voltage which can be used in synchronizing.

This device is relatively inexpensive, is simple, easy to operate, requires small space, and is accurate enough for most synchronizing applications.

The following paper will discuss this device with particular reference to (1) reasons for high-voltage synchronizing, (2) previous methods, (3) general theory of bushing potentiometers, (4) commercial form of the device, (5) performance on test, and (6) application, and finally will give an account of the first installation of the device in the field.

NECESSITY FOR HIGH-VOLTAGE SYNCHRONIZING

The locations where high-voltage synchronizing are advantageous are as follows:

1. At high voltage (110 kv. or over) switching stations where it is necessary to connect two systems and where no low-voltage circuits are available.
2. At high-voltage transformer stations where the high voltage is reduced to a medium value (approximately 66 kv.) and where the high-voltage synchronizer is less expensive than medium voltage potential transformers.
3. In stations where instrument potential circuits are available, but where extensive switching operations are necessary to use these instrument potential circuits for synchronizing.

PREVIOUS METHODS AND THEIR LIMITATIONS

Hitherto there have been in general but two methods of reducing voltages for synchronizing. The first was the use of a potential transformer and the second was the use of a glow tube in connection with an insulator string. The disadvantages of the potential transformer are that it is expensive, bulky, and usually requires quite long high-voltage leads. Also, if used, fuses and current-limiting resistances add complications.

The operation of the glow-tube device depends on the charging and leakage currents of the insulators. Three glow tubes are connected to intermediate points on the insulator strings of the three phases. This connection produces an apparent rotation of the light intensity of the three tubes thus giving an indication of the relative frequency of the two systems.

This system has three inherent defects; the first is that the capacity of the insulators is small, and most of the current is leakage current on the surface of the insulator, subject to variations depending upon the cleanliness and amount of moisture upon the insulators. The second is that the glow of the tubes is faint, thus making the reading of the synchronoscope difficult. Third, the leakage and capacity of the lead wire to the synchronoscope reduce the current available for the tube. The combination of these effects may be such as to render the device inoperative at times.

GENERAL THEORY OF BUSHING POTENTIOMETER

In general, a successful insulator potentiometer must have two qualities—it must be of a relatively high capacity and must be protected from the weather.

These two requirements are met by the condenser bushing. Constructed, as it is, of concentric metal-foil cylinders about the high-voltage conductor, it forms an ideal capacity potentiometer wherein the metal-foil cylinders are plates and the micarta is dielectric of a series of condensers which can be considered as connected in series from the high voltage to ground. By adding a tap to the last step to ground a potentiometer is easily obtained. Its size assures ample capacity for instrument indication and it is completely protected from the weather.

As these bushings are designed for a voltage drop of approximately 4000 volts per layer some means must be provided for lowering the voltage across the last step to the usual instrument voltage of about 110 volts. This may be done either by paralleling the last step with a condenser of sufficient size or by the use of a potential transformer.

The transformer method is the better as it allows a greater current to be drawn from the potentiometer than the condenser method does. However, with

---

[1] Ohio Public Service Co., Cleveland, Ohio.

[2] Westinghouse Elec. & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

the transformer alone, as in Fig. 1, only devices which are specially calibrated can operate with accuracy. Any instrument having resistance in its circuit will cause the instrument voltage to be out of phase with the line voltage. This is shown in Appendix I. However, a modification of this method in which the terminal voltage and phase-angle error are practically independent of the burden of the connected instrument, may be applied.

This modification is effected by the addition of a reactor tuned to resonance with the capacity of the last step of the bushing less the transformer exciting reactance.

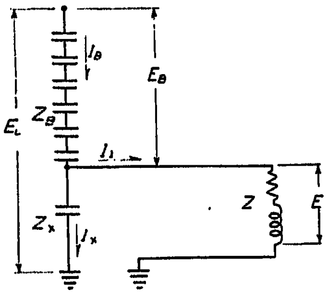


Fig. 1—Theoretical Diagram of Simple Potentiometer Scheme

The theoretical circuit diagram for this connection is shown in Fig. 2. It differs from the circuit of Fig. 1 only in the addition of the reactor Z_1.

From the theoretical discussion of this circuit as given in Appendix II it is seen that the voltage ratio is approximately independent of instrument impedance and the phase angle between line and terminal voltage is approximately zero.

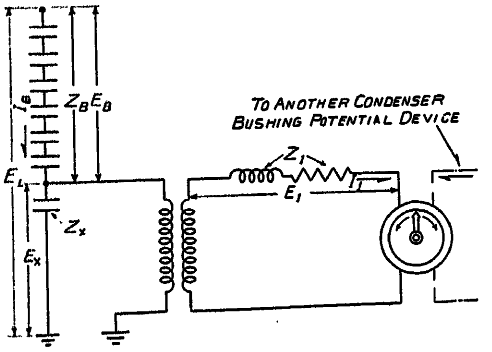


Fig. 2—Theoretical Diagram of Potentiometer with Resonant Circuit

The final form of the device (Fig. 3) is somewhat different from the theoretical circuits so far shown.

A condenser is added to the secondary circuit of the device, and a spark-gap is connected across the primary of the transformer.

The condenser is added as a control of the no-load voltage of the device, and as a protection to the last step of the bushing.

The no-load voltage depends only on the potentiometer ratio of the condenser bushing and the ratio of the transformer. As the voltage across the instrument is approximately constant regardless of the burden, the voltage drop across the reactor and thus the voltage across the last step of the bushing must increase with load. Hence, the device operates with a variable voltage on the last step. It is desirable to keep this voltage just below or slightly above normal operating voltage as a safety measure for the bushing. This is done with the additional condenser as it causes the no-load voltage of the last step to be lower than normal, thus the transformer ratio is lower for a given instrument voltage and thus margin is given for a rise of voltage with instrument load. By this means the addition of the network does not interfere with the normal operation of the bushing.

As the load is increased the voltage across the last step rises. As a limit, at short circuit the lower network becomes a parallel resonant circuit and the only current then flowing in the upper part of the bushing is that necessary to supply losses in the transformer and reactor.

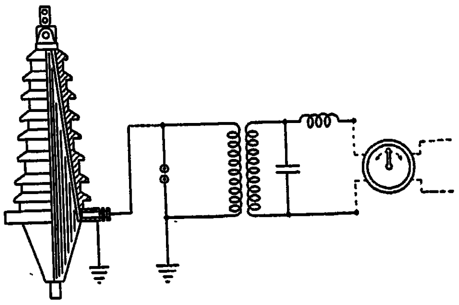


Fig. 3—Complete Schematic Diagram of Condenser Bushing Potential Device

This causes the drop through the bushing to be small; hence the last step voltage must increase considerably. To protect the bushing a spark-gap is placed across the last step set to break down well below the dangerous voltage. This gap also protects the network against abnormal voltages due to lightning or other disturbances on the line.

When allowance is made for transformer iron loss, exciting current and impedance, the equivalent circuit diagram takes the form shown in Fig. 4.

All constants are referred to one side of the transformer. The exciting current is represented by the inductive component, the iron loss by the resistive component, and the last step capacity by the capacitive component of Z_x. The impedance of the transformer is represented by Z_T, placed back of the exciting impedance. The position of this impedance is somewhat in error but its effect on the performance is very small.

The value of the iron loss resistance varies with the frequency and the value of the exciting reactance varies with the saturation. The operating voltage of the transformer is known closely enough to allow these values to be determined by test.

The relation between E_L and E is:

$$E_L = \frac{E}{Z}\left[\frac{1}{Z_c}\left(Z_B + Z_T + \frac{Z_T Z_B}{Z_x}\right)(Z_c + Z + Z_1) + \left(\frac{Z_B}{Z_x} + 1\right)(Z + Z_1)\right] \quad (18)$$

(The derivation of this expression is in Appendix III.)

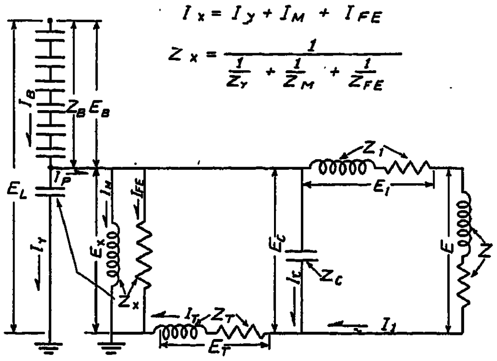


FIG. 4—THEORETICAL DIAGRAM OF COMPLETE POTENTIAL DEVICE

The effect of the various factors in the above equation can be more easily seen by reference to the vector diagram, Fig. 5, which was calculated from the above expression for a 132-kv. (76,200 volts to ground) bushing, carrying a 15-volt-ampere, 80 per cent power-factor load.

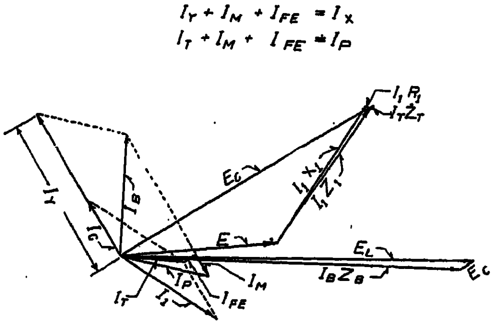


FIG. 5—VECTOR DIAGRAM OF VOLTAGE AND CURRENTS FOR CIRCUIT OF FIG. 6

It will be noticed that the iron loss of the transformer is in such a direction that it plays a considerable part in the amount of the phase angle between the line voltage and the instrument voltage. The effect of the impedance of the transformer, however, is very small.

EFFECT OF CHANGE IN REACTANCE OF REACTOR

It is interesting to see the effect of changing the reactance of the reactor on the voltage and the phase angle of the connected instrument.

The curve, Fig. 6, was calculated from the above formula for the same 132-kv. bushing as the vector diagram Fig. 5. The burden was also 15 volt-ampere, 80 per cent power factor. The no-load voltage = 110. The value of the reactance was changed through the range shown. The load voltage and its phase-angle variation from line voltage was determined from the equation and plotted.

The curve shows that voltage regulation can be improved by decreasing the reactance but at the expense of angular accuracy. The vertical line shows the value of reactance for which resonance with the capacity is obtained. It may be seen here, and, in general, it is found that the best average conditions exist when the reactor is tuned with the condenser.

EFFECT OF CHANGE IN FREQUENCY

A change in frequency affects the characteristics of the circuit chiefly by disturbing the resonance of the condensers and reactor. When the lower circuit is in resonance, the voltage on the instrument is highest. As the reactor is resonant with the condensers at normal frequency, the resonance of the lower portion of the device depends upon the inductance of the load. At unity power factor, the resonant frequency is the normal frequency and for inductive load the resonant frequency is lower. The resonant frequency of an electric circuit, $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{L\,C}}$. From this it may be seen that the resonant frequency must drop with increase of inductance.

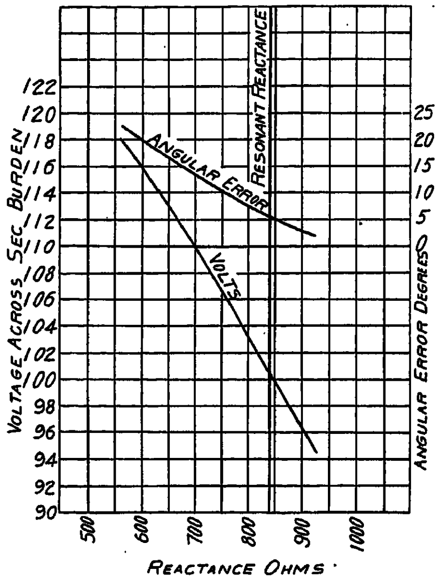


FIG. 6—CHANGE IN TERMINAL VOLTS AND PHASE-ANGLE ERROR WITH CHANGE IN REACTOR FROM NORMAL VALUE

The curve in Fig. 7 shows the variation of voltage for a 110-kv. potentiometer, calculated for 15 volt-ampere at both 80 per cent and 100 per cent power factor. It will be noticed that resonance occurs at normal frequency for 100 per cent power factor, and at a lower value for 80 per cent power factor. As the frequency rises the voltage drops.

INFLUENCE OF LINE-VOLTAGE WAVE SHAPE

If the line voltage is not of sine-wave form the effect of its harmonics on the voltage of the instrument connected to the potentiometer may be seen as follows:

In Fig. 7 the curve of voltage versus frequency shows that at 180 cycles (the third harmonic of 60 cycles) the voltage appearing at the terminals of the device is about 40 volts where the 60-cycle voltage is 108.

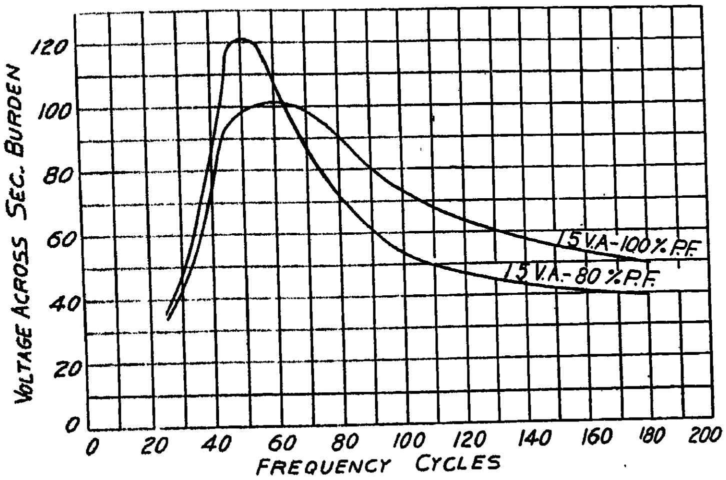


FIG. 7—THEORETICAL VARIATION OF FREQUENCY WITH VOLTAGE

The action of the circuit is such that it will not reproduce the harmonics of the line voltage at the instrument terminals. This causes an inherent error in r. m. s. voltage indication, dependent on the wave shape. The error, however, is slight for most cases. For instance with a 10 per cent third harmonic the error is only 0.3 per cent.

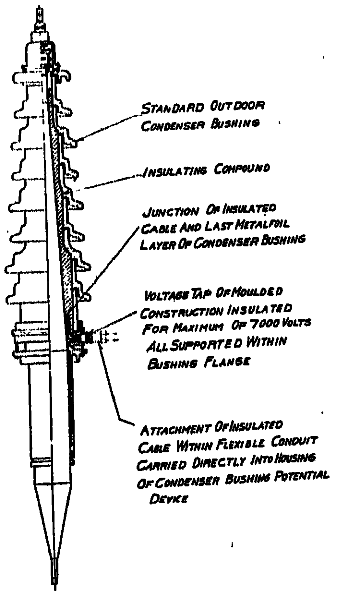

STANDARD OUTDOOR CONDENSER BUSHING
INSULATING COMPOUND
JUNCTION OF INSULATED CABLE AND LAST METALFOIL LAYER OF CONDENSER BUSHING
VOLTAGE TAP OF MOULDED CONSTRUCTION INSULATED FOR MAXIMUM OF 7000 VOLTS ALL SUPPORTED WITHIN BUSHING FLANGE
ATTACHMENT OF INSULATED CABLE WITHIN FLEXIBLE CONDUIT CARRIED DIRECTLY INTO HOUSING OF CONDENSER BUSHING POTENTIAL DEVICE

FIG. 8—CROSS-SECTION THROUGH CONDENSER BUSHING SHOWING METHOD OF MAKING VOLTAGE TAP

COMMERCIAL FORM OF DEVICE

As the name of the device implies, the condenser bushing is used to supply a source of potential. Fig. 8 illustrates in detail the method of tapping the last layer of the condenser bushing. A flexible cable having 7000-volt insulation is fastened to the last metal foil layer of the condenser and extended to a moulded fitting in the bushing flange. This fitting is of the "plug" and "jack" construction, having the tapped lead attached to the "jack." Fig. 9 illustrates the rigidity and simplicity of the tap construction. All material and clearances are designed to retain the insulating properties of the condenser. bushings as at present, thus obviating any possible trouble between tap and ground. When the bushings are not connected to the potential device, a cap is provided for protecting the entrance to the "jack."

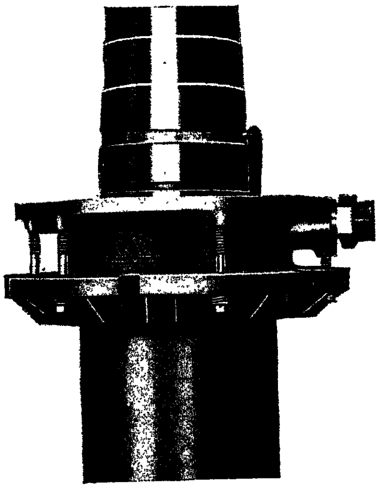


FIG. 9—VOLTAGE TAP CONSTRUCTION ON STANDARD CONDENSER BUSHING

The potential device, itself, excepting the synchronoscope, is assembled in a small protective sheet steel housing as illustrated in Fig. 10. It is suitably adapted for bolting directly to the circuit-breaker tank at a point near the condenser bushing tap. The connection between the tap and housing is approximately two to three feet in length, varying with the size of breaker. This connection consists of a 7000-volt insulated flexible cable protected within a flexible metallic water-proof conduit. Fittings are fastened to each end to permit ease of installation.

The equipment within the housing consists of a spark-gap arrester, low-voltage potential transformer, a reactor, and a condenser. The energy discharge across the gap is small and permits the use of the usual spark gap neutral arrester set to break down at a maximum voltage of 5500 volts. It is assembled within a porcelain casing and installed as shown in the upper left hand corner of the housing. Connection is made to ground through the protective housing of the device.

The low-ratio potential transformer as shown in the lower left hand corner of Fig. 10 is a standard indoor type. It is, however, equipped with a special winding to give the necessary variation required in adjusting the device to resonance. The transformer nominal ratio is 3800 to 200 and is designed to have very small losses, so that all possible energy is available for instrument use. One side of the transformer primary is grounded to the housing of the device.

The network reactor and condenser forming the external network is in the right hand side of the housing. The condenser has a maximum capacity of 4 μ f. and is rated and designed for 220-volt, 60-cycle service. The condenser and reactor are made variable for the purpose of adjusting the resonant condition of the network. The iron-core reactor is designed to have very high reactive factor, low energy loss, and constant impedance over the operating range of current. All wiring on the secondary side of the transformer is interconnected through the terminal block located in the lower right hand corner of the housing. This facilitates adjustment of the secondary circuit to obtain the proper performance of the device. Instrument leads are brought out through a conduit fitting in the bottom of the housing and can be handled the same as secondary leads of a potential transformer.

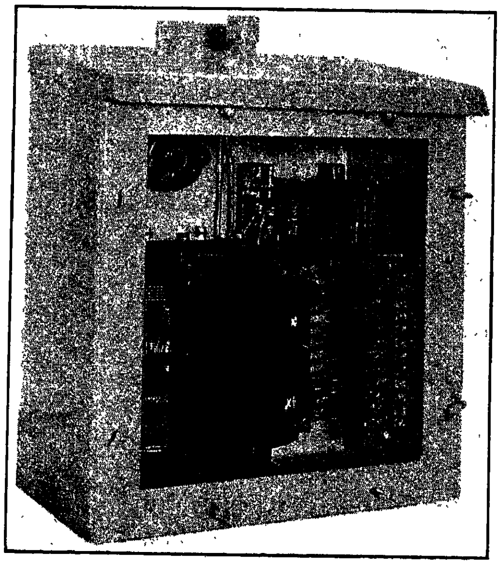


Fig. 10—Housing with Cover Removed Showing Network

The outstanding point to be observed is the simplicity of the device. It is complete in one assembly and can be handled as a unit. The device is on the line at all times and operates continuously without any attention to auxiliaries or secondary potential sources. The actual synchronizing operation is accomplished as easily as with a potential transformer.

PERFORMANCE

Variations in service and burden conditions will cause variations in the performance of the condenser bushing potential device. Extensive tests were made to determine the performance under different conditions and to obtain an idea of the limitations of the device. The scope of this investigation included performance under such conditions as:

1. Variation of line voltage.
2. Variation of line frequency.
3. Variation of secondary burden.
4. Variation of temperature of condenser bushing.
5. Variation of line voltage wave form.

Also conclusive tests were made to demonstrate the adequacy of this device for high-voltage synchronizing.

The tests were performed on a condenser bushing potentiometer designed for service on 110 kv. Voltage across the bushing to ground was measured with a standard potential transformer. The voltages on the secondary side of the device were measured by an electrostatic voltmeter of the Kelvin type, whose low burden permitted accurate measurement. (Burden of electrostatic voltmeter is approximately 0.5 volt-amperes.)

The device under test was adjusted to give minimum phase-angle error between the voltage across the bushing and the secondary voltage of the device when carrying rated burden.[3] (12 volt-amperes at 80 per cent power factor, based on 110 volts at 60 cycles.) This was obtained by using the transformer tap to give the desired no-load voltage and adjusting the condenser and reactor to give proper full-load voltage and phase-angle error.

Effect of Line-Voltage Variation. The effect of the variation of line voltage at 60 cycles is shown in Figs. 11 and 12. The voltage characteristic is practically a straight line regardless of burden showing practically constant ratio. These curves can very easily be adapted for use as calibrations when the device is used for obtaining voltage indications from high-voltage systems. Phase-angle error is practically constant with variation in line voltage.

Effect of Line-Frequency Variation. The general effect of frequency variation over the commercial range is shown in Figs. 13 and 14. Here the voltage characteristics are illustrated for different frequencies for a 15-volt-ampere burden. From inspection of both figures it appears that a practically constant ratio is obtained for a given frequency. Further frequency effects are shown in Figs. 15 and 16. Here it is evident that the resonant condition of the device is somewhat affected by frequency variation. Similar sets of curves may be drawn at other than 100 per cent voltage point. It is found that the frequency characteristic becomes flatter and thus more constant at lower values of line voltage. Another interesting performance is the effect of the power factor of the burden on the resonant condition of the network. The more resistive secondary burden flattens out the voltage frequency performance.

---

[3] Note: All burdens given in the paper are in volt-amperes based on 110 volts at 60 cycles.

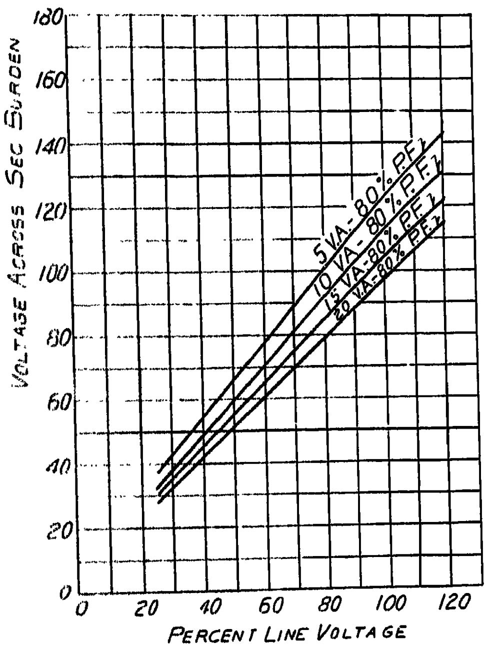


Fig. 11—Variation of Burden Voltage with Line Voltage at 60 Cycles. Power Factor of Secondary Burden—80 Per Cent (Test Data)

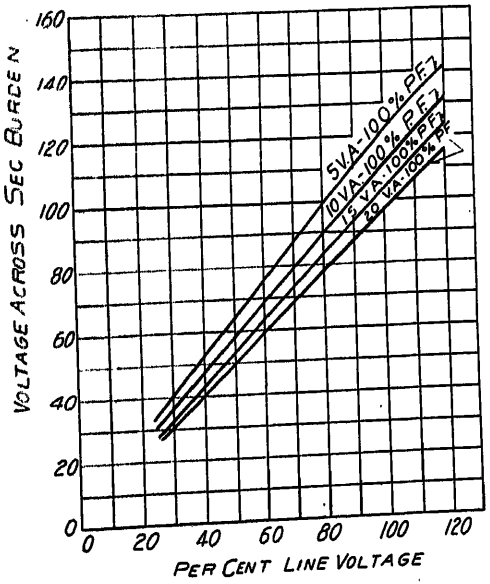


Fig. 12—Variation of Burden Voltage with Line Voltage of 60 Cycles. Power Factor of Secondary Burden—100 Per Cent (Test Data)

The phase-angle variation between the voltage applied to the bushing and the voltage of the secondary burden is illustrated in Fig. 17. Here is shown the effect of both burden and frequency. This same set of curves holds practically the same regardless of voltage variation; that is, at 60 cycles the phase-angle error will be 6 deg. regardless of the line voltage.

Effect of Secondary Burden Variation. The characteristics illustrating the effect of burden are included in all of the performances mentioned above. The voltage regulation over the range of burden given is very satisfactory considering the rated burden of the device. Although the burden is changed, the voltage characteristic remains a straight line for a given frequency. The power factor of the burden appears to have little effect on voltage performance as shown in Figs. 11 and 12. However, power-factor change affects the frequency-voltage characteristic and causes

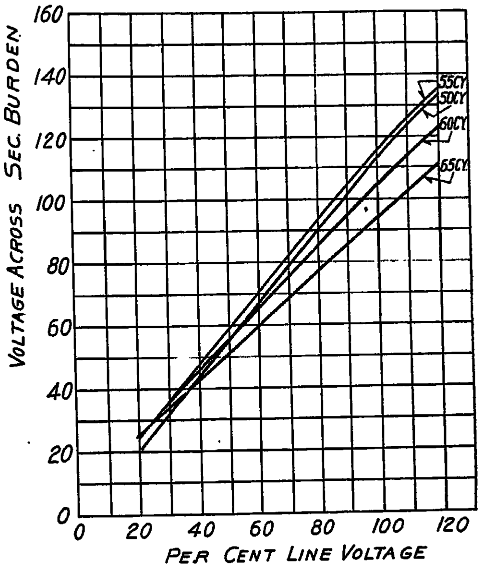


Fig. 13—Variation of Burden Voltage with Line Voltage at 15 Volt-amperes—80 Per Cent Power Factor Burden, for Various Frequencies (Test Data)

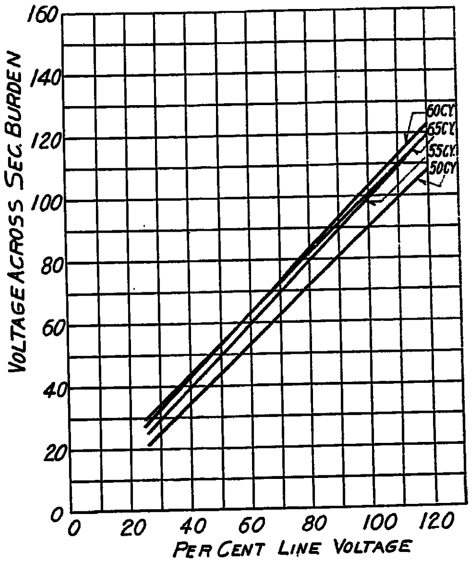


Fig. 14—Variation of Burden Voltage with Line Voltage at 15 Volt-amperes—100 Per Cent Power Factor Burden, for Various Frequencies (Test Data)

the resonant point of the device to be slightly shifted resulting in curves as shown in Figs. 15 and 16.

Effect of Temperature Variation. Data were obtained regarding this condition by heating a condenser bushing up to 140 deg. fahr. While cooling, the values of secondary voltage and phase angle were obtained at different secondary burdens down to 60 deg. fahr. The phase-angle variation was negligible while the voltage variation amounted to two and one-half per cent for a burden of 15 volt-amperes at 50 and 60 cycles. It is evident that the heating of the condenser bushing in service will cause little effect on the operation of the device.

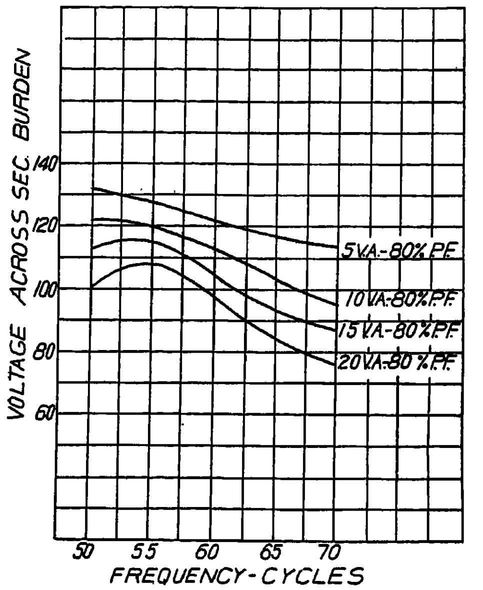


FIG. 15—VARIATION OF BURDEN VOLTAGE WITH FREQUENCY FOR 80 PER CENT POWER FACTOR BURDENS. LINE VOLTAGE 100 PER CENT (TEST DATA)

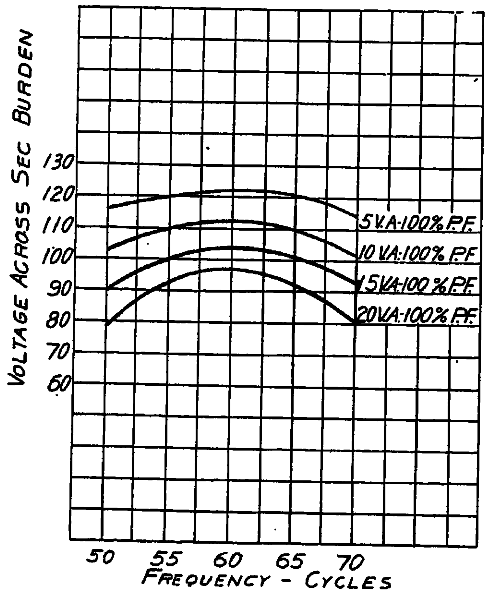


FIG. 16—VARIATION OF BURDEN VOLTAGE WITH FREQUENCY FOR 100 PER CENT POWER FACTOR BURDENS. LINE VOLTAGE 100 PER CENT (TEST DATA)

Effect of Wave Form. All of the above curves and information are based on sine-wave form of line voltage. In Fig. 18 is shown an oscillogram illustrating the wave form of current and voltage within different parts of the device. The supply voltage, according to analysis, is practically a sine wave having only three per cent each of the third and fifth harmonics.

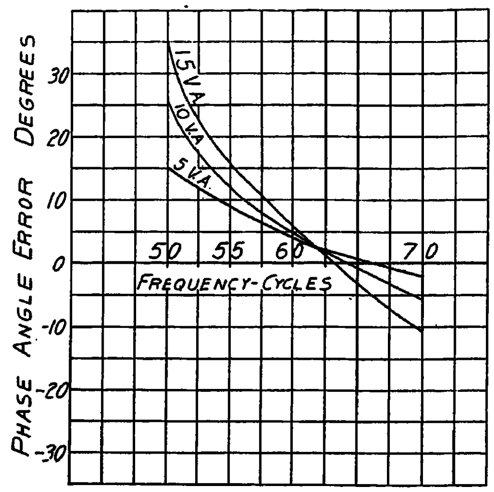


FIG. 17—VARIATION OF ANGULAR ERROR WITH FREQUENCY AT 100 PER CENT LINE VOLTAGE FOR DIFFERENT SECONDARY BURDENS

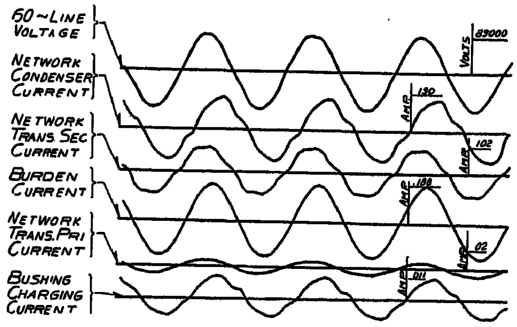


FIG. 18—OSCILLOGRAPHIC RECORD OF CURRENTS AND VOLTAGE IN CONDENSER BUSHING POTENTIAL DEVICE. SLIGHTLY DISTORTED VOLTAGE WAVE

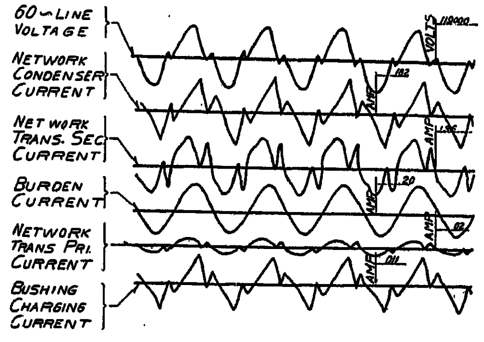


FIG. 19—OSCILLOGRAPHIC RECORD OF CURRENTS AND VOLTAGE IN CONDENSER BUSHING POTENTIAL DEVICE. BADLY DISTORTED VOLTAGE WAVE

In Fig. 19 oscillographic performance is given. The same conditions of voltage, frequency, and burden exist as before except a poor wave form of voltage was arbitrarily caused by inserting a regulator in the supply and caused to operate at high saturation. Analysis of the supply voltage gave:

1st Harmonic—100 per cent of first.
3rd " —22.2 " " " "
5th " —12.0 " " " "
7th " — 5.3 " " " "
9th " — 0 " " " "
11th " — 0 " " " "

$E_{r.m.s.} = \sqrt{(100)^2 + (22.2)^2 + (12)^2 + (5.3)^2} = 103.2$ per cent.

This was checked by actual measurement of the secondary voltage under the two conditions of wave form. In general, considering the above performance, commercial variation of wave form will cause very little affect in the operation of the device.

It is interesting to note the wave form of secondary burden obtained in both cases. Although considerable distortion is obtained in the capacity circuits, the reactor smooths out all variations and results in a very good wave form of secondary current through the burden.

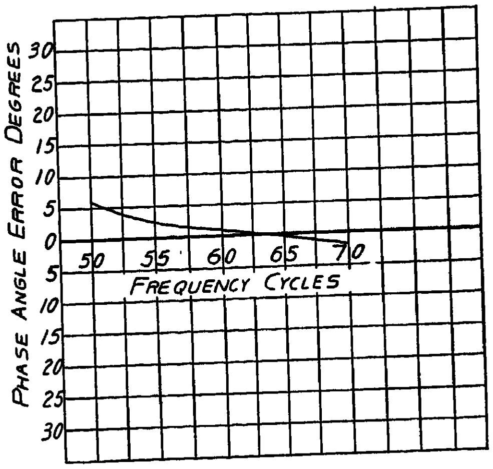


Fig. 20—Phase-Angle Error When Synchronizing with Two Condenser Bushing Potential Devices (Test Data)

Performance Obtained with Synchronoscope. A set-up was made to determine the operation of the synchronoscope with this device subjected to various conditions. Fig. 20 shows the phase-angle performance that may be obtained with variation in frequency of the line voltage. This information was obtained using similar condenser bushing potential devices on either side of the synchronoscope operated from the same line voltage source. The angular error is the deflection of the synchronoscope needle. This curve shows that the phase angle error is negligible when using two potential devices for synchronizing.

In Fig. 21 is shown the phase-angle performance when an attempt is made to synchronize with a condenser bushing potential device on one of the lines to be synchronized and a potential transformer on the other. The phase-angle error is permissible for synchronizing in the region of 60 cycles, but for greater ranges of frequency the device should not be used when balanced against a potential transformer. The variation of phase angle in either case is practically unaffected by variation in line voltage. The voltage performances obtained with a synchronoscope burden are illustrated in Fig. 22. Here, again, a constant voltage ratio is obtained for a given frequency. The performance with variable frequency is comparable to that discussed under effects of frequency. It appears, therefore, that the frequency variation will cause very little change in

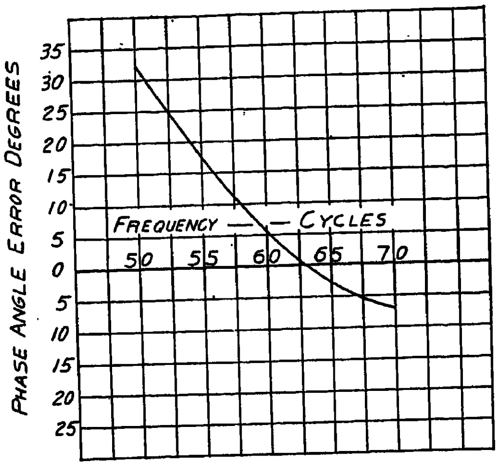


Fig. 21—Phase-Angle Error When Synchronizing with Condenser Bushing Potential Device and Potential Transformer (Test Data)

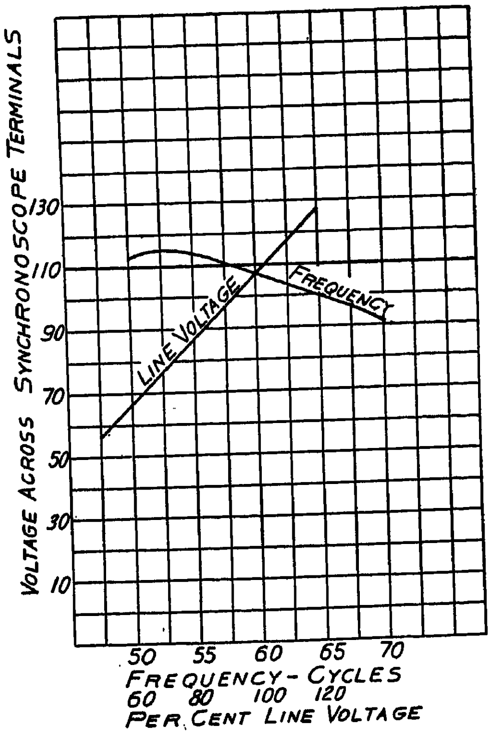


Fig. 22—Voltage Across Synchronoscope Terminals for Variations in Line Voltage and Frequency (Test Data)

the voltage at the synchronoscope terminals. In general, these performance data indicate that synchronizing with the potential device on both sources to be synchronized will be just as reliable as is obtained in potential transformer practise.

APPLICATION

This device may be applied to the standard condenser bushings assembled on circuit breakers, power transformers or in separate containers with the following limitations in burden depending on primary voltage:

1. 88-kv. Line Voltage—10 volt-ampere based on 110 volts.
2. 110-kv. Line Voltage—12 volt-ampere, based on 110 volts.
3. 132-kv. Line Voltage—15 volt-ampere, based on 110 volts.
4. 154-kv. Line Voltage—20 volt-ampere, based on 110 volts.
5. 220-kv. Line Voltage—30 volt-ampere, based on 110 volts.

These limitations are given for 60-cycle service. For 50-cycle service the burdens will be reduced to five-sixths of the above values. For 25-cycle service, the device should only be applied to the higher voltage bushings since the maximum burden permissible is but 40 per cent of the 60-cycle limits.

In case greater secondary burdens than those above are to be used, two potential devices on the same breaker can be used in parallel per phase thus allowing double the burden given above. This will apply mostly to relay applications where it will be permissible to open the paralleled secondaries when the breaker is in the open position. This limitation is made because the condenser bushing potential device on the line side of the breaker will feed back into the device on the other side.

If it is desired to synchronize around the breaker with the same two equipments used in parallel for relaying, arrangements must be provided for opening the secondaries when the breaker is open and transferring the voltage to the synchronoscope terminals. This all can be readily accomplished by use of auxiliary switches operated by the breaker mechanism.

The most favorable application of this device is for synchronizing at high voltages. The question of voltage regulation is not very important when operating a synchronoscope and several degrees phase angle error is permissible in the synchronizing operation. The voltage ratio as obtained with the condenser bushing device is practically constant as discussed above. In addition when synchronizing with the use of two identical potential devices, the phase-angle error cancels out.

The use of a potential transformer on one line to be synchronized and a condenser bushing potential device on the other is not recommended. Station layouts should be made to anticipate the use of a condenser bushing device on each of the lines to be synchronized. Under this condition the maximum phase-angle differential that may be expected between any two condenser bushing devices throughout the commercial frequency range (50 to 70 cycles) is six degrees. This error is due to possible manufacturing variations within the potential device itself and is designed to be practically zero at 60 cycles. This small variation in phase-angle error will not interfere with the usual synchronizing operation.

When it is proposed to synchronize between a potential transformer connected between phase on one system and a potential device connected on a breaker bushing of another system, the conditions are changed considerably. There are two points to be considered:

1. When the potential transformer is connected between lines, the secondary voltage will be 30 deg. out of phase with the secondary voltage of the condenser bushing potential device, assuming zero phase-angle error in the potential device as well as in the potential transformer. This is due to the fact that the transformer measures the line-to-line voltage, and the condenser bushing potential device measures the voltage to ground. This difference can be readily taken care of by arbitrarily setting the needle on the shaft of the synchronoscope 30 deg. from its usual zero, in a direction depending on the relative connection to the potential sources.

2. The phase-angle error of a condenser bushing potential device may be as high as 6 deg. when carrying a burden of a synchronoscope at 60 cycles. However, if it is contemplated synchronizing at system frequencies as low as 50 cycles, phase-angle error as high as 30 deg. may be expected. This means that the synchronoscope would point 30 deg. off zero at instant of synchronism.

It may be feasible to synchronize under this condition provided particular care is taken by the operator. An alternative would be to mark additional synchronizing points on the synchronoscope dial corresponding to the different frequencies. This requires that the operator keep the frequency in mind when synchronizing.

The usual method of synchronizing with phase voltage instead of wire to neutral voltage eliminates any possibility of incorrect synchronizing due to "floating neutral." Where phase-to-neutral voltage is used a floating neutral, as is experienced in poorly grounded systems, may give false indication on the synchronoscope. If such a system condition is anticipated, then synchronizing should be accomplished by means of line to line voltage rather than line to neutral voltage. This would require the use of two condenser bushing potential devices, one on each of the two phases, with interconnection of the secondary of the network so as to obtain the equivalent of a line-to-line voltage.

OTHER APPLICATIONS

There are several other uses of the condenser bushing potential device in addition to synchronizing. The same limitations with regard to phase-angle and voltage performances, as above mentioned will apply here also. In general on applying this device, the maximum permissible burden should not be exceeded. Otherwise the condenser bushing itself will be affected by increasing unduly the voltage across the tapped layer. Additional applications may be listed under three general headings:

1. Voltage indication
2. Frequency indication
3. Relay operation.

The potential device gives a very satisfactory indication of voltage although not accurate enough for power measurements. Voltage indication may be obtained between phase and ground or between phases depending on the relative connection of the secondary of the devices on the different phases. Standard switchboard voltmeters can be used with this device for giving voltage indication and connected the same as when using potential transformers. Curves similar to those given under performance can be used as a calibration if necessary.

Frequency indication of a high-voltage line is also easily accomplished by use of the potential device. Inasmuch as there is no frequency change within the device, a true indication of frequency will be given. The standard switchboard frequency meter can be used within the burden limitations of the device and handled the same as when using the potential transformer.

Probably one of the most important future applications of the condenser bushing potential device will be in the relay field. Generally, the device may be used in place of potential transformers for operation of potential relays provided the phase-angle and voltage performance characteristics are satisfactory for proper functioning. In view of the fact that the device operates continuously, a potential source is always present. Due to simplicity and economy as compared to potential transformers, the use of relay schemes requiring potential should become more general. It will be particularly useful where protection is obtained by voltage indication between phase and ground, since the secondary voltage of the device is proportional to the voltage from phase-to-ground rather than between phases. This type of protection is in popular use on low-voltage circuits and its use has been hampered on high-voltage circuits due largely to the expense of potential transformers. Protection requiring the use of phase-to-phase voltage may also be obtained by proper interconnection of the secondary phase leads.

First Installation

The first installation of this high-voltage synchronizing device has been made at the Bluebell Substation (Alliance, Ohio) of the Ohio Public Service Company with very satisfactory results.

Bluebell Substation is located near the center of a 132-kv. transmission line 45 miles long, between Canton, Ohio and Warren, Ohio. Since no other load is tapped from the line between Canton and Warren, synchronizing at either terminal point is possible without affecting the Bluebell Substation supply. This, however, requires that synchronizing be accomplished using low voltage circuits and necessitates considerable switching operation. At the time the station was built it was considered too expensive to install 132-kv. potential transformers for synchronizing purposes only.

During the spring of 1926 an Electrostatic Glow Meter was installed, to provide a means of synchronizing more rapidly, and with fewer switching operations. While this installation was quite economical, it was reliable only under favorable weather conditions.

In order to obtain a device operative under all conditions, the condenser bushing potential device was installed in August, 1927. The cost was but slightly in excess of the electrostatic glow meter installation cost and upon investigation it was found to be less than one-sixth the cost of a potential transformer synchronizing installation. This new scheme, although more simplified, has all the advantages of the standard potential tranformer scheme but none of the irregularities of the glow meter scheme.

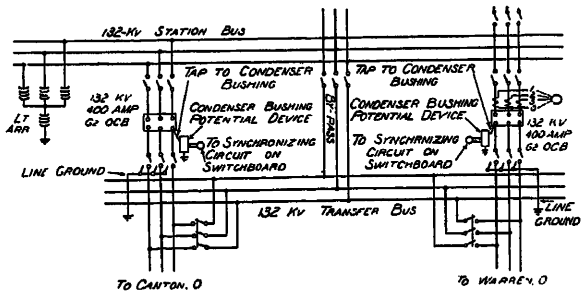


Fig. 23—Structure Diagram—Bluebell Substation—Ohio Public Service Company, Alliance, Ohio

In Fig. 23 is shown a schematic diagram of the 132-kv. busses in the Bluebell Substation, indicating particularly the relative connections of the condenser bushing potential devices. A detailed wiring diagram of the potential device connections is shown in Fig. 24. Here is illustrated the method of handling the secondary circuit. Similar procedure is followed as in potential transformer practise.

Fig. 25 shows one of the potential devices, with cover in place and with cover removed as installed on the 132-kv.—"G-2" oil circuit breakers. The units are mounted on pedestals at one tank of each circuit breaker. The field installation of the complete potential device consisted of the following:

1. Attachment of housing to pedestal and installation of pedestal. (For future installations, lugs will be provided on the circuit breaker tanks for bolting.)

2. Attachment of protected flexible cable to the tap on the condenser bushing and to the unit.

3. Attachment of the secondary leads from the

device to the synchronizing switch located on the station switchboard.

In Fig. 25 the tap from the condenser bushing may be seen entering the top of the potential device and the conduit carrying the low-voltage lead from the unit to the switchboard may be seen entering the ground at the base of the supporting pedestal. From the unit to the switchboard the insulation through the station control ducts is 600 volts. The wiring diagram in Fig. 24 shows clearly the simplicity of the apparatus and the connections as applied to this installation. A standard synchronism indicator, except lamps and standard synchronizing plugs or switches are used on the switchboard.

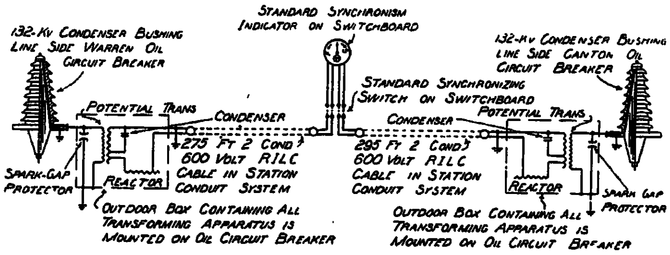

STANDARD SYNCHRONISM INDICATOR ON SWITCHBOARD
132-KV CONDENSER BUSHING LINE SIDE WARREN OIL CIRCUIT BREAKER
132-KV CONDENSER BUSHING LINE SIDE CANTON OIL CIRCUIT BREAKER
STANDARD SYNCHRONIZING SWITCH ON SWITCHBOARD
POTENTIAL TRANS
CONDENSER
CONDENSER
POTENTIAL TRANS
275 FT 2 COND 600 VOLT RILC CABLE IN STATION CONDUIT SYSTEM
295 FT 2 COND 600 VOLT RILC CABLE IN STATION CONDUIT SYSTEM
SPARK-GAP PROTECTOR
REACTOR
REACTOR
SPARK GAP PROTECTOR
OUTDOOR BOX CONTAINING ALL TRANSFORMING APPARATUS IS MOUNTED ON OIL CIRCUIT BREAKER
OUTDOOR BOX CONTAINING ALL TRANSFORMING APPARATUS IS MOUNTED ON OIL CIRCUIT BREAKER

FIG. 24—SCHEMATIC DIAGRAM OF OIL CIRCUIT BREAKER—CONDENSER BUSHING POTENTIAL DEVICE AND CONNECTIONS FOR SYNCHRONIZING

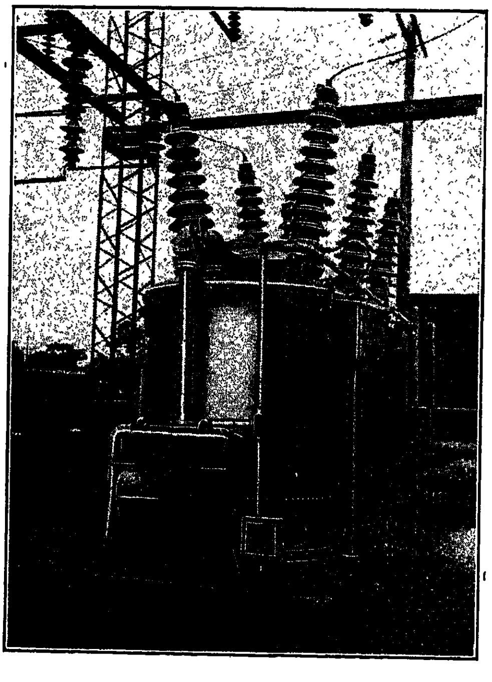


FIG. 25—CONDENSER BUSHING POTENTIAL DEVICE AS INSTALLED ON BREAKER TANK

It is the intention on future installations to synchronize across one pole of each breaker rather than from breaker to breaker in order to permit a greater flexibility in the switching set-ups and simplify switchboard connections. To date, 50 synchronizing operations have been made using the two condenser bushing potential devices with no incorrect indications.

Appendix I

In the diagram, Fig. 1, the transformer is neglected and the constants are referred to one side of the transformer. The symbols used are defined by reference to the diagram.

$$I_x = \frac{I_1 Z}{Z_x} \tag{1}$$

$$I_B = I_x + I_1 = I_1\left(1 + \frac{Z}{Z_x}\right) \tag{2}$$

$$E_B = Z_B I_B = Z_B I_1\left(1 + \frac{Z}{Z_x}\right) \tag{3}$$

$$E = I_1 Z \tag{4}$$

$$E_L = E_B + E = I_1\left(Z_B + \frac{Z Z_B}{Z_x} + Z\right) \tag{5}$$

$$E_L = \frac{E}{Z}\left(Z_B + \frac{Z Z_B}{Z_x} + Z\right)$$

$$= E\left[Z_B\left(\frac{1}{Z} + \frac{1}{Z_x}\right) + 1\right] \tag{6}$$

Then the ratio of the potentiometer

$$= \frac{E_L}{E} = \left[Z_B\left(\frac{1}{Z} + \frac{1}{Z_x}\right) + 1\right] \tag{7}$$

It is interesting to substitute $Z = \infty$ in equation (7) as a check on its accuracy. Then

$$\frac{E_L}{E} = \left[Z_B\left(\frac{1}{\infty} + \frac{1}{Z_x}\right) + 1\right] \tag{8}$$

$$= \left[\frac{Z_B}{Z_x} + 1\right] = \frac{Z_B + Z_x}{Z_x}$$

This gives the ratio of the potentiometer at open circuit as proportional to the total impedance of the bushing divided by the impedance of the last step.

The effect of load may be seen as follows:

Substitute the following terms in equation No. (7).

$Z_B = -j X_B$ (capacity reactance only)

$Z_x = -j X_x$ (capacity reactance only)

$Z = R \pm j X$ (Resistive with either inductive or capacitive reactance)

Then the ratio

$$\frac{E_L}{E} = \left[-j X_B\left(\frac{1}{R \pm j X} + \frac{1}{-j X_x}\right) + 1\right]$$

$$= \left[\frac{-j X_B}{R \pm j X} + \frac{X_B}{X_x} + 1\right] \tag{9}$$

In order that the ratio $\frac{E_L}{E}$ be a numerical quantity, that is that there be no phase angle between E_L and E, all j quantities in the above equation must cancel. By inspection it may be seen that for any value of load resistance, R, except zero the j terms will not cancel and there will be an angular difference in phase between the line voltage E_L and the instrument voltage E. If an instrument having negligible resistance such as an electrostatic voltmeter, or some highly inductive circuit, were connected to the terminals of the above device the value $R_1 = 0$, and equation (9) becomes:

$$\frac{E_L}{E} = \left[\frac{-j X_B}{\pm j X} + \frac{X_B}{X_x} + 1 \right]$$

$$= \left[X_B \left[\frac{1}{\pm X} + \frac{1}{+ X_x} \right] + 1 \right] \qquad \textbf{(9a)}$$

here all the j terms have canceled out, $\frac{E_L}{E}$ is a numerical value and hence the phase between E_L and E is zero.

It is interesting to note that if $\pm X = - X_x$, or if the load is highly inductive with negligible resistance and is resonant with the last step of the bushing X_x, the ratio $\frac{E_L}{E}$ would equal 1 and full voltage would be impressed across the transformer primary thus causing its ruin. This case is practically never attained completely in practise but it can be approached closely enough to be dangerous.

Appendix II

A working knowledge of the action of the circuit in Fig. 2 can best be gained by dividing it into two parts, considering each separately and then uniting the action of the two to approximate the action of the whole circuit.

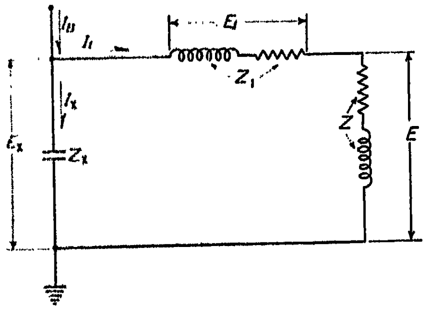


FIG. 26—THEORETICAL DIAGRAM OF LOWER PORTION OF POTENTIOMETER

Let the first part be the impedance of the bushing from the inner conductor to the tap Z_B (Fig. 2). The second part will then be the rest of the circuit including the last step of the bushing Z_x, the transformer, the reactor Z_1, and the burden of the synchroscope.

Assuming a bushing of 21 steps, the impedance of the first part Z_B will be 20 times the impedance of the last step Z_x. The impedance of the network is designed so that its total impedance, including the last step is never much higher than the original impedance of the last step Z_x. With change of load the network impedance will fluctuate but can be considered constant and equal to Z_x with a relatively small error. Hence, under all load conditions, within the rating of the device, the charging current can be considered as equal to:

$$I_B = \frac{E_L}{(Z_B + Z_x)} \qquad \textbf{(10)}$$

Substituting for Z_B and Z_x their values $-j X_B$ and $-j X_0$

$$I_B = \frac{E_L}{-j (X_B + X_0)} \text{ or } E_L = -j I_B (X_B + X_0)$$

$$\text{approximately} \qquad \textbf{(10a)}$$

The second portion of the circuit as shown in Fig. 26 is the same as the lower portion of Fig. 2 except that the constants of the circuit have been referred to one side of the transformer, and the transformer itself has been neglected. Analyzing the relation between E and I_B:

$$I_B = I_1 + I_x; \qquad \textbf{(11)}$$

$$I_x = I_1 \frac{(Z_1 + Z)}{Z_x}; \qquad \textbf{(12)}$$

$$I_B = I_1 \left[1 + \left(\frac{Z_1 + Z}{Z_x} \right) \right] \text{ Substituting (12) in (13)}$$

$$\textbf{(13)}$$

Substituting $I_1 = \frac{E}{Z}$:

$$I_B = \frac{E}{Z} \left[1 + \left(\frac{Z_1 + Z}{Z_x} \right) \right]$$

$$= E \left[\frac{1}{Z} + \frac{Z_1}{Z Z_x} + \frac{1}{Z_x} \right] \qquad \textbf{(14)}$$

Now if Z_1 is pure inductance and equal to $j X_0$, and if

$$Z_x = -j X_0, \qquad \textbf{(15)}$$

the relation between I_B and E becomes:

$$I_B = E \left[\frac{1}{Z} + \frac{j X_0}{-j X_0 Z} + \frac{1}{-j X_0} \right]$$

$$= \frac{E}{-j X_0} \text{ or } E = -j X_0 I_B \qquad \textbf{(16)}$$

Thus it is seen that the relation between E and I_B is independent of the load impedance, Z. From (10) and (16) the ratio E_L becomes:

$$\frac{E_L}{E} = \frac{-j I_B (X_B + X_0)}{-j I_B X_0} = \frac{X_B + X_0}{X_0} \qquad \textbf{(17)}$$

Hence, the voltage ratio is approximately independent of instrument impedance and the phase angle between E_T and E is approximately zero. The greater the number of steps on the bushing the better will be the approximation.

A more exact solution for the circuit of Fig. 2 is not difficult, but any additional information it would give is of little value because the iron loss of the transformer plays an important part in the characteristics of the device.

Appendix III

The derivation of Formula No. 18 is as follows:

From Fig. 4:

$$\text{Instrument Voltage} = E = I_1 Z \qquad (1)$$

$$\text{Reactor Voltage} = E_1 = I_1 Z_1 \qquad (2)$$

$$\text{Condenser Voltage} = E_c = E + E_1 = I_1 (Z + Z_1) \qquad (3)$$

$$\text{Condenser Current} = I_c = \frac{E_c}{Z_c} = I_1 \frac{(Z + Z_1)}{Z_c} \qquad (4)$$

Current Through Transformer Impedance $= I_T$

$$I_T = I_c + I_1 = I_1 + I_1 \frac{(Z + Z_1)}{Z_c} = I_1 \frac{(Z_c + Z + Z_1)}{Z_c} \qquad (5)$$

Transformer Drop $= E_T$

$$E_T = Z_T I_T = Z_T I_T \frac{(Z_c + Z + Z_1)}{Z_c} \qquad (6)$$

Volts on Lower Step of Bushing $= E_x$

$$E_x = E_T + E_c = \left[Z_T I_1 \frac{(Z_c + Z + Z_1)}{Z_c} + I_1 (Z + Z_1) \right] \qquad (7)$$

$$E_x = I_1 \left[\frac{Z_T}{Z_c} (Z_c + Z + Z_1) + (Z + Z_1) \right] \qquad (8)$$

Current into Combined Impedance $Z_x = I_x$

$$I_x = \frac{E_x}{Z_x} = \frac{I_1}{Z_x} \left[\frac{Z_T}{Z_c} (Z_c + Z + Z_1) + (Z + Z_1) \right] \qquad (9)$$

Bushing Current $= I_B = I_x + I_T$

$$I_B = I_1 \left[\frac{Z_T}{Z_c Z_x} (Z_c + Z + Z_1) + \left(\frac{Z + Z_1}{Z_x} \right) + \frac{Z_c + Z + Z_1}{Z_c} \right] \qquad (10)$$

Bushing Drop – Net Work to Line $= E_B$

$$E_B = I_1 Z_B \left[\frac{Z_T}{Z_c Z_x} (Z_c + Z + Z_1) + \frac{(Z + Z_1)}{Z_x} + \frac{Z_c + Z + Z_1}{Z_c} \right] \qquad (11)$$

Line Volts to Neutral $= E_L = E_B + E_x$

Adding (11) + (8) and substituting (1) =

$$E_L = \frac{E}{Z} \left[\frac{1}{Z_c} \left(Z_B + Z_T + \frac{Z_T Z_B}{Z_x} \right) (Z_c + Z + Z_1) + \left(\frac{Z_B}{Z_x} + 1 \right) (Z + Z_1) \right] \qquad (12)$$

Discussion

VACUUM TUBE SYNCHRONIZING EQUIPMENT

(Belt and Hoard)

USE OF CONDENSER TYPE BUSHING IN SYNCHRONIZING

(Spracklen, Marshall, and Langguth)

New York, N. Y., February 17, 1928

E. D. Eby: In connection with Fig. 8, it is mentioned that the value of the shunt resistance R may vary appreciably from the normal value of 600,000 ohms without seriously affecting the ratio. If this shunt resistance should become very small, however, the ratio would be affected. It is interesting to note what effect upon the ratio would result from a low insulation resistance in the condenser C_2. This insulation resistance may be considered a part of the total shunt resistance since it is in parallel with R.

The insulation resistance of the condenser C_2 itself is normally of the order of 100 megohms, but this resistance may fall as low as one megohm without seriously affecting the ratio. With one megohm in parallel with a resistance R of 600,000 ohms, the combined value would become 375,000 ohms, which from Fig. 9 is seen to affect the ratio to a negligible degree.

The phase-angle error is also dependent upon the value of R as shown in Fig. 10. With the insulation resistance of condenser C_2 and connected leads as low as one megohm, and with R equal to 600,000 ohms, the phase-angle error increases from 2.75 deg. to 4.4 deg. which is an unimportant change in the phase angle.

Thus it is seen that such a low value of insulation resistance of condenser C_2 and connected leads cannot seriously affect either the ratio of this capacitance transformer or its phase angle for synchronizing purposes.

The statement is made that the ratio of this capacitance transformer is independent of the presence of external metal structures. This has been demonstrated by testing capacitance-transformer bushings with large metal cylinders surrounding the porcelain shells so that the capacitance between bushing conductor and external objects is greatly increased. This was found to have no effect upon the transformer ratio. This is the result of complete shielding of the condenser C_2 by the ground sleeve of the bushing which extends beyond the ends of the condenser plates. That is, changes in the external capacitance of the bushing do not affect that part of its internal capacitance which determines the ratio.

Likewise, changes in the external leakage resistance of the bushing cannot affect the ratio because this resistance is in parallel with the external capacitance which, as shown above, may change without affecting the ratio. For this reason, the presence of rain, snow, dirt, etc., on the external surface of the bushing merely increases the leakage current from line to ground but does not change the current through the capacitance transformer.

Tests have also been made at varying temperatures which have shown no change in ratio. This is to be expected since the insulating materials in the condenser C_2 are of the same character as those used in the puncture structure of the bushing shown in Fig. 8 as C_1. Any changes in the permittivities of the insulating materials, therefore, affect both capacitances similarly.

As a further check on the ratio of the capacitance transformer under actual operating conditions, there is being set up on a 110,000-volt power system a parallel installation of capacitance transformer and standard potential transformer. Simultaneous readings of system voltage will be taken over a considerable period of time and under varying operating conditions. It is believed these observations will help to confirm the calculations and laboratory tests which indicate that the capacitance transformer may be used for metering as well as for synchronizing purposes.

A high degree of operating safety in this device is assured by the high safety factors employed in the design and construction of the capacitance transformer. At 60 cycles, the flashover voltage of the bushing is about 5½ times its operating voltage from line to ground. The puncture strength of condenser C_2 is about 100 times its operating voltage. This gives a safety factor of 100 at operating stress, and 18 above 60-cycle flashover. With impulse voltages, the safety factor is correspondingly high. This design of oil-filled bushing will withstand impulse flashover indefinitely, and the capacitance-transformer insulation is correspondingly strong. This is of the greatest importance to the operation of the synchroscope which may be called upon to synchronize lines during lightning storms.

Each capacitance-transformer bushing receives a 60-cycle flashover test as well as the usual one-minute high-potential test. The capacitance transformer receives a test of 1000 volts for one minute. The capacitance of each transformer is measured and the low-tension voltage calculated from the measured capacitance. A variation of ± 5 volts is allowed. Each capacitance transformer is calibrated with its vacuum-tube equipment.

Capacitance-transformer bushings of this type have been built for voltages from 88,000 to 220,000 volts. They can be supplied for lower voltages if desired. The additional cost of capacitance transformers in oil-circuit-breaker bushings is so small that they have been specified on some orders in anticipation of a possible installation of the synchronizing equipment at a later date.

H. L. Crumley: I have had some very good operating experience with the vacuum-tube synchroscope.

We have installed on the Metropolitan Edison system one outfit at the present time. We had no misgivings as to the reliability of the equipment. The only thing we were afraid of was what would happen with impulse voltages.

We had one experience that convinced us of the reliability of the outfit. One evening during a severe lightning storm, the line to which this synchroscope was connected was knocked out and in a few minutes it was resynchronized at the station using the vacuum-tube synchroscope. An inspection showed that the bushing of the capacitance transformer had been cracked, due to a flashover. This flashover was evidently the cause of the line trip-out.

Since we have seen what may occur from impulse voltages and even to the flashover of the bushing itself, we have no fear as to the future operations of this type of equipment.

Philip Sporn: (by letter) It has always been realized that in a high-voltage station consisting of a half dozen lines or so, and perhaps a series of transformer banks utilized for obtaining a distribution voltage, it would be highly desirable to be able to synchronize every line. However, cost considerations of potential transformers primarily, although there were others, very often dictated the elimination of direct synchronizing facilities especially if it was possible to figure out a sequence of switching under which synchronizing could be done at lower potentials at some other point. Now this arrangement, while it often looked very beautiful on paper, did not work quite as smoothly when put into the hands of a group of substation operators. The result has been that many high-tension stations that were originally designed without such synchronizing facilities, later had them put in at a very much greater expense because operating experience showed their almost absolute necessity.

These two papers point out means of doing this at very much less expense than was thought possible a few years ago. Of the two, the straight condenser method is apparently simpler but it appears to me that vacuum-tube method has more than enough other advantages to compensate. For one thing, at no point outside the breaker does any high potential exist. Again, it is highly desirable to get operating men familiar with, and used to operating and relying upon vacuum-tube devices. Today, it is possible to synchronize, to communicate, and to supervise operation with vacuum tubes; but ultimately we will surely use them for voltage regulators. Other uses are bound to follow. The power engineer therefore must make up his mind that the vacuum tube is going to be a tool and a piece of apparatus without which, in the years to come, he will not be able to get along on his power system, and that the sooner he gets the operating people thoroughly versed with it the better off everybody will be.

Messrs. Spracklen, Marshall, and Langguth point out that it is not recommended to use a potential transformer on one side to be synchronized against a condenser bushing on the other. Where, however, it is desired to meter at the high voltage and a set of potential transformers is utilized for that purpose connected to the bus, it would be most logical in that case to synchronize against the potential transformers. In such a case the vacuum-tube arrangement would have a slight advantage. On the other hand it is conceivable that the cost of the straight condenser arrangement may be sufficiently lower to make it possible to install, under such a set of conditions, an additional condenser bushing connected on the bus and yet not increase the cost over the vacuum-tube arrangement.

Neither paper mentions any costs. I think these might be of interest. In a 132,000-volt switching station that was cut into service within the last six months we had occasion to synchronize six circuits against either one of two buses. The cost of doing it by vacuum-tube arrangement ran close to $8400. The estimated cost of accomplishing the same thing by standard 132,000-volt potential transformers was $34,000.

J. H. Cox: (communicated after adjournment) The authors of this paper have confined themselves to normal-frequency applications of the condenser-bushing potentiometer. However, in addition to being useful for those applications it forms an ideal potentiometer for making measurements of transient voltages on transmission lines. In this application it is merely necessary to connect the measuring device between the tap and ground. Since the potentiometer is a capacity coupling the ratio of the voltages is constant regardless of magnitude or wave shape, so long as the measuring devise is also a capacity. Both the klydonograph and the cathode-ray oscillograph come in this

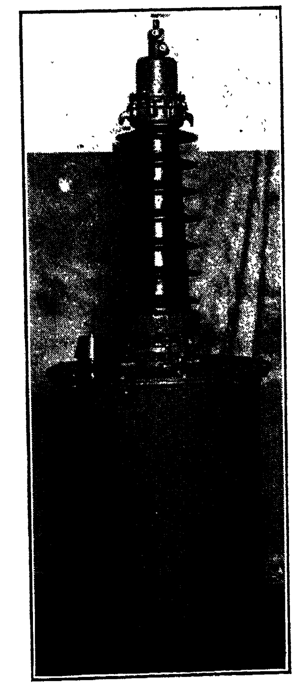


Fig. 1

category as the energy consumed by either is entirely negligible. The normal voltage on the tap, being 4000 volts, is above that required as the normal voltage on either of the above instruments. This may be reduced to any desired value by connecting a suitable external capacity in parallel with the instrument. This, of course, causes a voltage equal to the amount of the reduction to be impressed across the remainder of the bushing, but this should be negligible.

The three principal objections to the capacity potentiometers as commonly used in klydonograph tests are cost, space, and the change in ratio with varying weather conditions. Often a major part of the expense of a klydonograph investigation has been in the potentiometers; when the breaker, or transformer, bushings are provided with a tap there is little added expense, and even when installed specially they can later be used as spare bushings. Due to the mutual effect between adjacent air potentiometers they must be separated by considerable distance; the bushing potentiometers do not affect each other and may be spaced as close as safety permits. Of course, when apparatus bushings are used little additional space is required. The supporting insulators of the air potentiometers were subject to dirt and moisture, with a consequent change of ratio when the leakage became large enough to be important; the capacities in the bushing potentiometer are enclosed.

When used as transient-voltage potentiometers these bushings are not limited to the higher-voltage classes as they are for normal-frequency applications. They have been applied on systems with voltages as low as 6600 volts. Even these bushings have a capacity of about 300 micro-microfarads per step, which is ample to supply the klydonograph, which has a capacity of about 10 micro-microfarads. Also, for voltages of 25,000 and less the cost of condenser-bushing potentiometers is not too high to prevent their being built for special tests. Fig. 1 herewith shows a potentiometer, with a special tank, for 66,000 volts.

T. A. E. Belt: (communicated after adjournment) Since nothing is of more importance in high-tension apparatus than the bushing, I wish to bring out several points.

Assuming that the condenser bushing has a 60-cycle flashover of 5½ times its normal line-to-ground potential, and that the impulse ratio of the bushing is 2, it can be shown that approximately 60,000 volts may be obtained on the tap at the time of an impulse sufficient to arc over the bushing. If the tap is left isolated, adequate insulation and clearance must be provided for 60,000 volts. On the other hand, if the tap is grounded, the dielectric of a 110,000-volt bushing will be overstressed by approximately 6 per cent, that is, the bushing insulation will be operating at a reduced factor of safety. This reduction in dielectric strength, however, may be compensated by increasing the size of the bushing.

When the tap is used to supply potential for relays, a momentary flashover of the protective gap will cause momentary loss of voltage, which seemingly may cause incorrect relay operation.

A circuit described in an article in the *Siemans-Zeitschrift*, Vol. 6, Oct. and Nov. 1926, makes use of the condenser bushing. It is reported here that temperature affects the bushing characteristic. One case of trouble mentioned was caused by one bushing operating continuously in the shade while the other bushing at times was exposed to the sun.

T. S. Farley: (communicated after adjournment) The capacitance transformer described in the paper by Belt and Hoard produces a secondary voltage E_R which has practically the same wave shape as the line voltage but reduced in magnitude. Eq. (1) shows that as the frequency increases, the ratio E/E_R approaches $\dfrac{C_1 + C_2}{C_1}$ as a limit, which for the capacitance transformer mentioned equals 1545. The ratio for 60 cycles is 1547. Hence it is evident that the magnitude of the fundamental and all harmonics are reduced in the same proportion. Also from Eq. (1) it can be shown that as the frequency increases the phase shift decreases to zero as a limit. The phase shift for 60 cycles is only 3.3 deg. and is less for any of the harmonics. Therefore it is seen that the secondary voltage of the capacitance transformer is a true representation of the line voltage including all harmonics.

Because of these characteristics of the capacitance transformer, it is very well adapted for use in conjunction with properly designed vacuum-tube circuits to supply potentials from high-voltage lines for accurate measurement work.

T. A. E. Belt: In addition to using the amplifying equipment for synchronizing, potential for other purposes may be obtained simply by energizing the outdoor amplifier independently of the indoor unit, thus obtaining the potential from a separate output transformer connected in the plate circuit of the amplifier tube.

Mr. Eby pointed out that in the design of the capacitance transformer, a high factor of safety was used to avoid dielectric failure under lightning impulses. It is gratifying to know that these precautions which were thought necessary in the design have proved to be worth while in operation; for Mr. Crumley tells us that even though the porcelain of a bushing was split by lightning and part of the oil gone, the capacitance transformer continued to function.

E. E. Spracklen: Although our experience with the equipment installed at Bluebell Substation has not covered a long period of time, we feel that its operation is quite dependable and that the voltage of the secondary circuit offers no serious hazard if the installation is properly made.

The simplicity of the complete circuit and the use of standard switchboard apparatus highly recommends this hook-up in the considerations of installation, operation, and maintenance costs.

Since the installation was made last August nearly one hundred operations of the synchronizing device at Bluebell Substation have been performed with very satisfactory results, and entirely without any need for adjustments or replacements of apparatus.

As brought out in our paper, the installation at Bluebell Substation was made partially with the idea of obtaining field experience with this synchronizing scheme and of obtaining data on the possibilities of the various applications for condenser bushings which may be accomplished economically, simply, and dependably. Up to this time nothing has occurred to change opinions regarding these applications and it is hoped that substantial data may be assembled in the near future.

P. O. Langguth: In connection with the condenser-bushing potential device, it is well to note the advantages that may be obtained with regard to use of auxiliaries on high-voltage circuit breakers. It is now possible to obtain both a current indication and a voltage indication from circuit breakers or other power apparatus which are equipped with condenser bushings.

This latest development which uses the charging current from a condenser bushing for operating synchronoscopes or other meters depending on voltage indication is very interesting. The device is rather simple and much less expensive than the usual potential-transformer installation. The condenser-bushing device is complete in one unit as shown in Fig. 10 of the paper. This comprises the complete equipment for use with the tapped condenser bushing. No separate source of power or vacuum tubes is required for use with this device. It is well to note that the secondary leads from the outdoor housing are handled in the same manner as the secondary leads from a potential-transformer installation. This device is sturdy, and requires no maintenance. Potential is available across the secondaries continuously as it is not necessary to disconnect the device from the primary when not in use.

There are four broad uses of this device. Due to the fact that potential is available in the secondary circuit at all times (*i. e.*, as long as the bushing is connected to the line) whether in a circuit breaker or power transformer, this device is not limited to synchronizing.

1. For operation of a synchronoscope for synchronizing high-tension lines between 88 kv. and 220 kv.

2. For operation of frequency meters. Since there is no frequency change in the potential device the frequency indicated on the secondary will be the same as the frequency of the line.

3. For operation of voltmeters. An indication of line voltage can be obtained since the voltage characteristic is practically constant.

4. For operation of relays.

The application of this device for relay protection of systems is more or less in the development stage. However, the future looks very bright in regard to use of this device for protection of systems against phase-to-phase and phase-to-ground faults. In fact, several large power systems are at present installing these devices for such protection. Reference should be made here to the paper[1] on "Relay Protection" given by Messrs. Crichton and Graves at the Regional Meeting in November. Relaying, of course, is made possible due to the fact that the secondary voltage of the condenser-bushing potential device is available at all times without any attention to auxiliaries, such as vacuum tubes.

I should like to say a word about the application of this device to relaying and synchronizing. In general, it has been the practise when using potential transformers to synchronize between lines or between phases. However, when using the potential device with the condenser bushing, the voltage obtained at the secondary device is in phase with the line-to-ground voltage and, of course, in that case synchronizing is based on the phase relation between line and ground. If it is desired to synchronize with a potential transformer which is connected to the bus for metering, it is necessary to install two condenser-bushing potential devices. The two devices would be installed on the phase to be synchronized and the secondaries connected in series, thus resulting in a secondary voltage, in phase with the phase-to-phase system voltage. In relaying between line and ground or between phases, it should be noted that each condenser bushing on the circuit breaker can be equipped with a potential device. Then with proper secondary interconnections all six devices may be used for synchronizing, relaying, voltage indication, or frequency indication without overloading the device.

In the previous discussion some mention has been made of the voltage tapped from the bushing. For example, the normal tap voltage is about 4000 volts for a 132-kv. condenser-bushing device. The synchronoscope or other potential instruments are insulated from this high voltage by the voltage transformer in the device. It is well to keep in mind that the impedance of the upper part of the bushing is in the neighborhood of 11,000,000 ohms; thus if the primary were short-circuited the maximum current that would flow is about 12 milliamperes. If the secondary of the device were short-circuited the maximum current would be approximately 250 milliamperes, which is too low to cause any serious damage.

If overvoltage occurs on the last step of the bushing the gap discharges with the above mentioned 11,000,000 ohms in series. The gap is set for 5000 volts and its discharge is limited to the above primary current, and thus when the voltage returns to normal the gap automatically ceases to discharge. The high capacity of the condenser bushing protects it from impulse voltages.

The high-voltage circuit of the device (4000 volts approximately) is completely enclosed in a grounded metallic housing and insulated within for 7000 volts. The danger of any one's coming in contact with this circuit is very remote.

The condenser-bushing potential device is relatively inexpensive and can be applied at a nominal cost for either one or two breakers.

In regard to Mr. Belt's comments I should like to point out that the insulation of the plug tap to the bushing, for impulse voltages is as good as the insulation of the bushing itself. Thus the insulating qualities of the bushing are in no wise impaired by the addition of the tap and therefore no more trouble due to impulse voltages will be experienced in the future than in the past.

In regard to the 6 per cent increase in stress on the upper portion of the bushing when the gap is discharging, I should like to emphasize the fact that the discharge of the gap is not an arc but a glow. For this reason the gap acts not as a short circuit but as an overflow to excess voltages. Hence the increase in voltage stress is not 6 per cent but lower—approaching 6 per cent as a maximum. The factor of safety of the bushing is great enough to allow this overvoltage indefinitely. Therefore the danger to the bushing caused by the almost infinitesimal time duration of an impulse is very slight.

The same gap-discharge characteristic mentioned above prevents sufficient loss of potential to influence relay operation during impulse flashovers even if they were of sufficient time interval which is unlikely. Tests made since the writing of the paper have shown that the secondary voltage drops only a few per cent, thus substantiating the fact that the normal primary voltage is maintained practically constant during disturbances. Repeated tests also demonstrate that the gap operation is automatic in that, when an impulse or surge "passes over," the gap ceases to function. It operates only in case of overvoltage and tends to hold the primary voltage at normal value.

Temperature variations of the condenser-bushing device are taken up in detail in the original paper. The effects are so small as to be negligible on most applications of the device.

All of the above is confirmed by field experience which has demonstrated satisfactory operation of this device in service, and its inherent automatic compensation for overvoltage conditions. The following points may be considered as being of especial importance from the standpoint of installation and operation. The apparatus is entirely self-contained. The 4000-volt circuit is short, thoroughly insulated and protected from mechanical injury or accidental contact by grounded metal housings. No other circuits have a voltage in excess of approximately 110 volts. There is no rotating apparatus of any kind used. There is no apparatus used which is subject to deterioration. In short, the initial installation and the succeeding apparatus is on a par with that of a standard low-voltage instrument transformer.

---

[1] L. N. Crichton and H. C. Graves, Jr., TRANS. A. I. E. E., 1928, Vol. 47, No. 1, p. 259.

Metallic Arc Welding Electrodes

A Study of the Effects of Surface Materials

BY J. B. GREEN[1]
Non-member

Synopsis.—*To understand the effect of surface materials on metallic arc welding electrodes, something must be known as regards the fundamental theory of arc heat distribution, arc vapor resistance, and similar matters which are modified by the presence of surface materials. The character of the arc may be changed by chemical and structural variations in the surface materials. Stability is one such arc characteristic.*

Two types of stability are recognized,—chemical and electrical. Surface materials have a significant influence on the brittleness or ductility of weld metal. They also can be used to control the arc type, four such types being recognized. The influence of surface materials on the heating of electrodes is another phase of the matter of practical significance. Surface materials may be employed to control both the operating characteristics of the electrode and the resultant weld properties. Among the operating characteristics might be mentioned stability and rate of melting. Among the weld properties are included freedom from blow-holes and tensile strength. These and other practical applications of a comprehensive knowledge of surface materials are cited in this paper.

* * * * *

BEFORE the effects produced by surface materials on metallic arc welding electrodes, can be understood, some familiarity with the arc itself must be gained, and an understanding of many of the fundamentals of welding acquired. Much of the subject matter presented is still in the controversial stage of development, but, to avoid numerous modifying remarks throughout the discussion, all reference will be assumed for the time being to be fact.

The arc itself appears to be a double thermocouple, the two couples being the junctions of the electrode tips with the opposite ends of the arc vapor stream. In Fig. 1 the welding arc is shown with the direction of thermocouple voltages common in mild steel welding indicated by arrows. The work is usually connected to the positive terminal of the welding generator. A voltage rise takes place at the filler rod end of the arc stream, the heat energy of the arc being changed to electrical energy while at the deposit end, the welding current suffers a corresponding voltage drop, electrical energy being changed to heat. There are eight possible combinations of thermocouple voltage directions and welding current directions. These are shown in Fig. 2, the eight combinations resulting from the possibility of running the welding current in either direction.

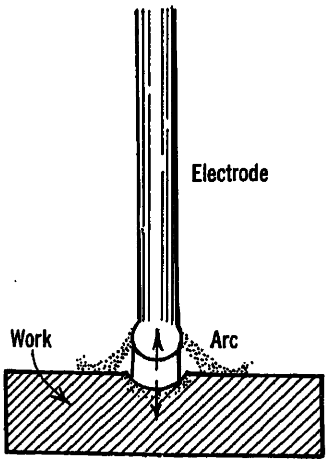


FIG. 1—THE METALLIC ARC SHOWING BY ARROWS THE DIRECTION OF THE THERMOCOUPLE VOLTAGES MOST COMMON IN MILD STEEL WELDING.

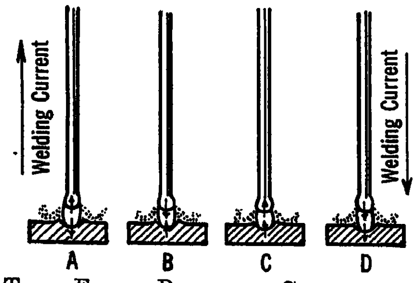


FIG. 2—THE FOUR POSSIBLE COMBINATIONS OF ARC THERMOCOUPLE VOLTAGE WITH THE TWO POSSIBLE DIRECTIONS OF WELDING CURRENT.

Two possibilities of varying the character of the arc are presented by this theory of arc heat distribution. One is the variation of directions of the thermocouple and impressed welding current combination, and the other is the e. m. f. of the thermocouples and impressed current.

The resistance of the arc vapor has little effect on arc heat distribution but it practically controls the total arc heat. Besides the arc vapor resistance, considered as a whole, the relative resistance of the core and sheath of the arc stream affect the character of the arc.

In studying both the total heat and its distribution in the metallic arc in connection with welding, two phases of the subject must be considered,—weld properties and operating characteristics. To bring the matter of these two phases under control the electrode designer makes use of varying combinations involving the four fundamentals of design. These are chemical composition and structure of the filler rod itself together with the chemical composition and structure of the surface materials.

---

[1] President, Fusion Welding Corporation, Chicago.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-17, 1928.

Heat is not the only factor to be studied. The electrical energy supplied by the welding generator is also converted into magnetic energy and light energy. Chemical energy, mechanical energy, and other forms are also resultants. The metallic arc and arc welding are so complex that to try to grasp all the variables and their interrelations as a whole baffles even the best trained scientific mind. For an understanding it is necessary to analyze and synthesize until the field has been covered, always remembering that the maintenance of the arc depends on the equilibrium of many variables, the change of any one usually resulting in a compensating shift of all the others. This is true not only of the arc but of the whole welding operation. These remarks are introduced to indicate that within the scope of a discussion confined essentially to the effect of surface materials on metallic arc welding electrodes completeness is impossible.

These surface materials fundamentally may vary chemically or structurally. Chemically, their variation is almost infinite, and structurally they may be coarsely or finely powdered or may be fibrous; they may be crystalline or amorphous; they may be applied evenly or unevenly, more on one side than the other, more at one end than the other or in single or multiple layers. Also, the relative mass of surface to electrode material may be varied. This relative mass has given rise to five classes of coatings. From the lightest to the heaviest they are commonly designated as the "inappreciable" coatings of Elliott, the residual coatings of Churchward, the surfacing of the author, the flux coatings of Hollup and Holslag, and the slag coatings of Kjellberg and Jones. All except the extremes are in quite general commercial use in this country, while the slag coatings seem preferred in Europe where alternating current is in common use for welding.

Taking up first the chemistry of the surface materials, it is found that both the arc heat distribution and total arc heat may be influenced through this fundamental. The electrodes themselves are metallic: hence they are first-class electrical conductors. The surface materials are most commonly non-metallic and are therefore second-class conductors. When the vapor of a second-class conductor predominates in the arc stream, the higher the temperature, the lower the resistance. This state of affairs tends to increase arc stability, since a given variation in arc length produces a lesser variation in resistance. The arc is complex and most observed tendencies are usually the algebraic sum of several tendencies. For example, an absolutely clean iron electrode operates with greatest stability when made the positive terminal of a d-c. generator because the thermo-couple action is such that the preponderance of heat appears at the negative end of the arc stream. By applying an "inappreciable" coating of calcium hydroxide, this thermocouple action is reversed and the arc becomes less stable, although the surface material is a second class conductor. Reverse the direction of the welding current and the stability is restored. Increase the quantity of the calcium hydroxide and the stability increases until the arc stream becomes saturated with the resultant vapor, after which further increase of stability ceases regardless of additions of the surface materials.

A welding rod was prepared to show the significance of surface materials as just indicated. The bottom or starting end of this electrode possessed no surface materials, the center had the residual wire-mill lubricants, which became essentially calcium carbonate by the time the electrode was used, and the top coated with the usual commercial flux composed of calcium carbonate together with siliceous materials. As such an electrode is run with direct current, (regular polarity), the first third gives a very unstable arc, the middle third, an arc which is commercially acceptable, and the top third, a very stable arc. All this seems due to a combination of arc heat distribution and arc vapor resistance.

To better understand the effect of surface materials on arc stability something must be said of stability itself. This matter is quite complex, but broadly, there is electrical and chemical stability. As the arc length increases, its resistance goes up, the arc voltage rises and the current falls. It is inherently unstable. Therefore ballast resistance must be introduced in a constant potential source of supply current, or other stabilizing means be provided in a variable potential supply. Rasch has developed the equation for stability with a constant potential supply and the equation also applies at any one instant with variable voltage supply. This equation is

$$S = \frac{E_0}{I} - \frac{K_s L}{I^2}$$

where

S = What might be called "Stability Units."

E_0 = Overvoltage or generator voltage minus arc voltage. It is the voltage required to force the current through the ballast resistance.

I = Current in amperes flowing in the welding circuit.

L = Arc length in inches.

K_s = A constant of the electrode called "Stability Constant."

In Fig. 3, it is plotted for extreme values of the constants as they appear in welding. It will be noted that as the current increases, a variation in the constants has little effect on the stability, all curves falling between 0.1 and 0.2 stability units at 500 amperes.

Chemical stability is no doubt due to violent chemical reactions taking place in the arc, usually between the electrode materials and surrounding atmosphere. It is well known that as the current is increased from zero, the arc is at first silent, then the voltage drops, making a volt—ampere curve of seemingly hyperbolic form. This is shown in Fig. 4 from *A* to *B*. At *B* it drops suddenly and continues on in much the same form from B^1 to C^1. The arc, after passing the critical point B, hisses. This drop appears to be due to the introduction of the vapor of a second-class conductor a product of chemical reaction, and this same chemical reaction produces the "hiss." If a hissing arc is surrounded with a chemically inert atmosphere, the hiss ceases and the arc voltage rises to values indicated on the line B-C.

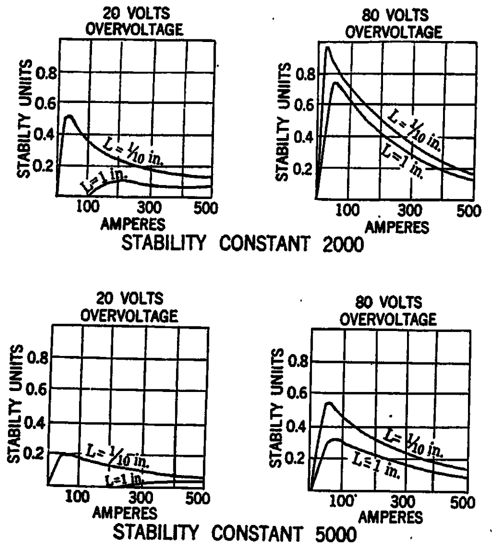


FIG. 3—RASCH'S EQUATION FOR STABILITY PLOTTED AS CURVES, USING EXTREME VALUES OF ARC LENGTH, BALLAST RESISTANCE, AND STABILITY CONSTANT

Most surface materials affect the stability constant of the electrode because they are vaporized and enter the arc stream. In this manner they affect the electrical stability of the arc according to Rasch's equation. In Fig. 5, is shown a picture of the welding arc taken with infra-red light showing three stages of the transfer of a single drop of metal in the metallic welding arc. In taking these photographs and others shown in this paper, a light filter was interposed between the arc and the plate, and this allowed substantially infra-red light only to reach the plate. At the left, in Fig. 5 the drop has reached maximum size and is about ready to capillary across to the deposit. Note that the drop is almost completely surrounded by the arc flame and, just prior, it was so surrounded. In the center, the drop has just touched the deposit metal. Note that the flame has disappeared and that a white hot, but still solid, collar is left on the end of the electrode. At the right the drop is about to neck off re-establishing the arc. The collar is still in evidence. This collar quite evidently represents the material receiving maximum contact with the surrounding atmosphere while at temperatures most apt to combine with the oxygen and nitrogen. An inert surface material which is fluid and tends to wet this white hot collar obviously protects it from contact with the surrounding atmosphere. Thorium oxide is such a material and a mild steel electrode coated with it gives an arc practically as slient as that between carbons.

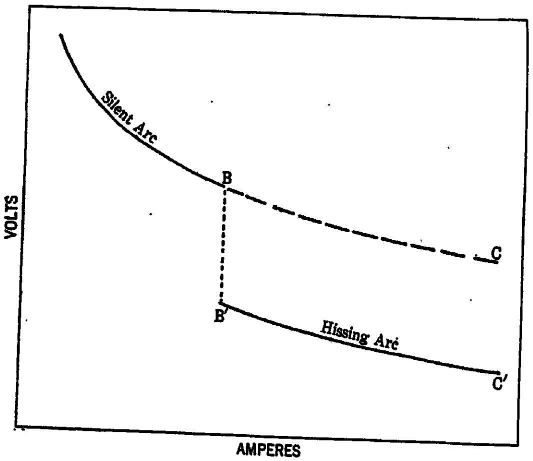


FIG. 4—VOLT-AMPERE CURVES OF THE "HISSING" AND "SILENT" ARCS

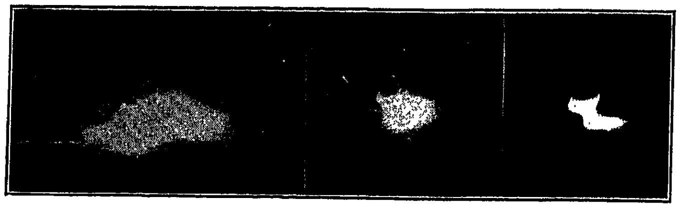


FIG. 5—THREE STAGES OF THE TRANSFER OF A SINGLE DROP OF MOLTEN METAL AS IT TAKES PLACE IN METALLIC ARC WELDING

In Fig. 6 this arc is shown at the left. Note the large size of the drop as compared with the one at the center from the same electrode uncoated. The electrode with thorium oxide permits the drop to form without being literally blown off before reaching maximum size by violent chemical action. At the right is an arc from the same electrode coated with berylium oxide. Note that this is smaller than that from the uncoated electrode at the center. The chemical action seems more violent and the drops are detached very rapidly. Each of these views was selected from several thousand pictures taken as representing the largest drop in each case.

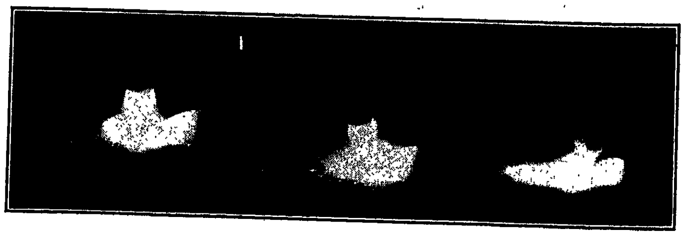


FIG. 6—ARCS WITH THREE TYPES OF ELECTRONS
Left coated with thorium oxide
Center uncoated
Right coated with berylium oxide

These pictures lead to a few remarks on arc types, all of which may be produced by surface materials.

At any certain current, every electrode tends to give some one of the four recognizable arc types. The one at the left is a steady core type while the one at the right is a sheath type; the hollow center of which cannot well be represented by photography. In the steady core type the arc tends to pull off the bottom center evading the upper edge of the drop. In Fig. 7 the reverse is true, an erratic core type arc being shown which tends to pull off only from the upper edge of the drop. This is a very difficult arc for the welder to handle. A saturated arc is shown in Fig. 8, the arc stream pulling off from the entire cross-section of the electrode. This is probably the most satisfactory arc for the welder to handle. The sheath type is probably a special form of the erratic core type, or vice versa, the arc merely pulling off from all points of the upper edge of the drop at once. This leads to the conclusion that the different arc types are the resultant of the absolute resistance of the arc vapor stream and the relative resistance as between the central and outer portions. A steady core type arc often may be changed to a saturated type by merely increasing the current density. Some electrodes melt in their entirety on account of resistance heating before the current density is raised to the point that a saturated arc results.

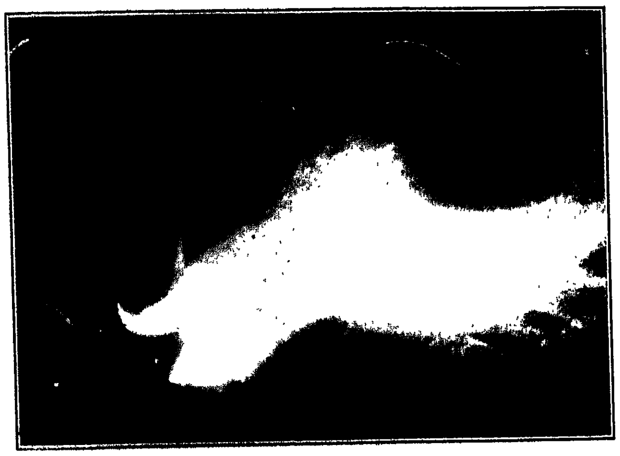


FIG. 7—ERRATIC CORE TYPE ARC

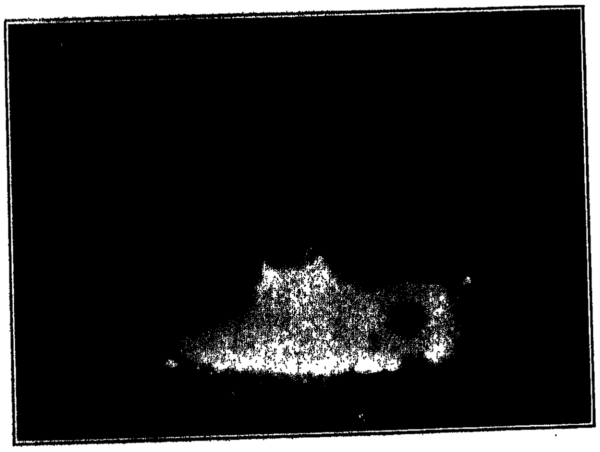


FIG. 8—SATURATED TYPE ARC

The influence of surface materials on the heating of electrodes is another phase of the matter of practical significance. Most electrode heating is due to poor holder contact and so perfect contact must be assured in studying the effect of surface materials themselves in this respect. In Fig. 9 are shown the curves obtained by allowing the same electrode to reach equilibrium temperatures in the air at various currents, the surface only being varied for each curve. It will be noted that less current is required to heat what is known as a bare electrode as shown in Curve *A*, possessing only the residual wire mill lubricants as a coating, than the heavier coated Green Surface shown in Curve *B*, or the polished rod shown in Curve *C* which is free from all surface materials. Conversely, the Green Surface and polished rods will carry higher welding currents than the so-called bare finish.

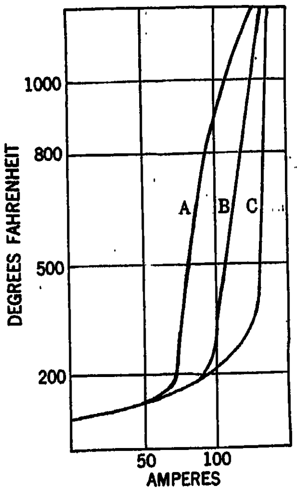


FIG. 9—EFFECT OF ELECTRODE SURFACE

A. Bare
B. Green surfaced
C. Polished

To most people the practical application of all this fundamental theory is the interesting thing. Both operating characteristics and weld properties may be controlled through the intelligent employment of surface materials applied to metallic arc welding electrodes. Possession of the handbook data on the subject permits deliberately designing an electrode for a specific purpose. Not many years ago, electrodes were developed by hit and miss experimenting and a use found for them after they were developed. Handbook data cannot very well be developed without a knowledge of the fundamental theory. It would be like trying to design structural members without a knowledge of moment of inertia, for example, which requires calculus to compute. For obvious reasons, most of the electrode designer's handbook data of which the author has knowledge must, for the time being, be kept a trade secret of the research laboratory of the company where it originated. No attempt, therefore, will be made to be specific or give reasons, but merely a few of the practical applications involving surface materials on welding electrodes will be cited.

One of the earliest applications was in Europe where alternating current is largely used for welding. The bare electrode seldom gives sufficient stability for a-c. welding as a commercial process and so slag or flux coated electrodes were developed. The welding of mild steel has become a production process subject to all the competition of other such processes. The electrode, therefore, must exactly fit the job. Every different set-up represents a different set of conditions to be met. Sometimes quality is involved. The weld must be free from blow-holes. These are the result of a race between the escaping gas and the solidifying metal in which the gas loses and is imprisoned in the weld. Fig. 10 shows the blow-holes in a piece of stainless steel. Such steel offers a splendid opportunity to study the subject of blow-holes because they are generally elongate and thus indicate their origin, their extent of travel, and direction of travel. Through the proper use of surface materials on the electrode the distribution of arc heat may be controlled and the blow-holes held at the bottom of the weld allowed to reach the center or the top, or to escape entirely leaving a sound weld.

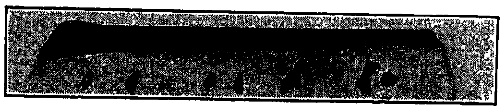


Fig. 10—Longitudinal Cross-Section of a Chromium Steel Metallic Arc Weld Deposit used to Study Blow-Holes in Welds

The significance of blow-hole control through the use of surface materials is indicated in the report of a 46-mi. all metallic arc welded pipe line. The coated electrodes which happened to be developed in the author's laboratory were used for welding the 46 miles. When completed, the entire line was tested at 650 lb. per sq. in. pressure without showing a single blow hole leak.

Often oxides and nitrides are found in arc welds representing a content of 1 per cent in mild steel, while all other elements, other than iron, are not over ½ per cent. Thus in determining the physical properties of the weld in so far as such properties depend on the chemical analysis of the deposited metal, these oxides and nitrides are often the predominating factor. Surface materials may be used to protect the transferred metal so that the brittleness incident to the presence of oxides and nitrides is reduced or eliminated.

Again, competition may demand that welds be made at high rates of speed. For example, ½-in. plate welding has been done by beveling the edges and running three layers of weld metal, usually brushing and often caulking each layer as laid. For the highest quality work of this sort, three feet per hour per operator is good practise. By using the surfaced electrode developed in the author's laboratory, which alters the arc heat distribution, sound welds fuzing clear through a ½-in. plate have been made at a rate of over 20 ft. per hour on plates butted together without beveling and with one passage of the electrode.

Some feats which are at least impractical in any other way may be accomplished through the use of surface materials on electrodes. For example, safes and vaults are often lined with stainless steel because the oxygen cutting torch will not successfully cut it. In forming stainless steel into tanks and other articles it becomes desirable to cut it, (say, for a manhole), where shearing is impractical. The author's laboratory has developed a metallic cutting electrode which, on account of the structure of its surface materials, readily cuts stainless steels. It also cuts and welds under many extreme conditions. It has even been used to lay down a weld deposit under water.

These are only a few out of the very many practical applications of a knowledge of surface materials on metallic arc electrodes. Furthermore, the effect of surface materials is only one consideration of arc welding. Metallic arc phenomena are extremely complex and when used for welding, the complexity is tremendously increased. Knowledge of the subject is very far from complete and probably the greatest opportunity for future fundamental research lies in discovering adequate means for measuring the arc thermocouple voltages.

Discussion

(Editor's Note: The following discussion by Mr. Green is in answer to questions asked after presentation of the paper. The questions are not repeated here.)

J. B. Green: The diameter of all the electrodes shown in the pictures was $^3/_{16}$ in. and we used a lower current value than would be considered proper in practical welding, about 125 or 150 amperes.

The low current was used to give slower melting of the electrodes so that the moving pictures could be taken more satisfactorily. If the currents had been different the pictures would probably be different. We were searching along broader lines. We were trying to find out, first, whether or not these four fundamentals had any effect at all, whether the chemical composition and structure of the rod and surface materials really did affect the welding rod. If we go into the subject still deeper such matters will certainly be taken into account.

Thorium oxide and berylium oxide were used as coatings but in practise those materials are too expensive and they are not suitable for other reasons. For example, thorium oxide will permit the production of an extreme example of a steady core-type arc. It protects the neck of the drop of molten metal from air contact. In those two respects it might be considered an excellent surface material for electrodes, but unfortunately it forms a very bad slag on the weld deposit. This is a coky slag which is very adherent and difficult to remove in the case of multiple-layer welding. The thorium oxide used for these experiments costs $48 a pound which would prohibit its commercial use.

For these pictures we have endeavored to use extreme cases, not picking necessarily the commercial case, but endeavoring to go as far as we could in any one direction.

The question was raised as to why the arc sometimes passed to the top of the puddle and sometimes to the bottom.

I have done a great deal of speculating on that point and up to the present have been unable to devise any experiments either to prove or disprove a hypothesis. I believe that to be a matter of relative, and what I might call absolute resistance of the arc stream. The relative resistance refers to the core and the sheath, if you can imagine the space between the electrode ends and the weld deposit as having a core and a sheath. If the core has a relatively high electrical resistance, the arc will pull off from the upper rim, because the outside sheath is of relatively low resistance. If there is not sufficient current flowing so that the arc can pull off from all points at once, it naturally will jump around from one point to another, forming the erratic core-type arc. However, if there is enough current flowing so that it pulls off all points at once, it forms a sheath-type arc. At any one current density, it might form either an erratic core-type or a sheath-type, depending on the absolute resistance of the vapor.

All of these pictures are of hand welding. We didn't find the automatic welding so favorable for taking these pictures. It may be we didn't have the proper control devices.

The same source of current was used for all the pictures, a constant-potential source with a large excess of ballast resistance.

Fluxes used in gas welding, I think, are employed very largely to dissolve the oxide, while in electric welding (and this is more or less my theory) I believe that they protect the welding rod from air contact and therefore formation of embrittling oxides and nitrides, but they have also another function, and that is the distribution of heat. I think in metallic arc welding the solution of the oxides is probably their least function.

E. Herzog: (by letter) Mr. Green has done pioneering work of great importance in his arc photographs. In his theoretical work, however, the assumption of various thermocouple combinations requires further explanation. Thermocouples as usually defined in physics do not seem to be able to generate the voltages or convert the quantities of energy appearing in the arc, even at arc temperatures. Can Mr. Green explain just why he has cast overboard all accepted explanations of arc phenomena in favor of his theory?

J. B. Green: Mr. Herzog refers to accepted explanations of arc phenomena, which I infer he feels apply to arc-heat distribution, and asks why I have cast these explanations overboard. While probably I am familiar with much of the explanatory matter which has been published concerning arc phenomena as it relates to arc-heat distribution, I do not know that this has been accepted. Furthermore, I did not intend to leave the impression that I have cast overboard these explanations. My thermocouple theory of arc-heat distribution is merely an effort to add another explanation and, if possible, make it more rational. From the days of the classic investigation of Hertha Ayrton, investigators have observed arc phenomena which could be interpreted as voltage drop or elevation at the arc terminals. The theory of thermocouple action seems to be a rational explanation of these voltage changes, except on one count, and that is the apparent magnitude of the voltage change. It is an established fact that the voltage change occurring in thermocouples composed of solid metals is quite small. In the metallic arc, new conditions are introduced. The junctions are between molten metal and a vapor. Usually they are also operating at extremely high temperatures. Until someone actually measures the thermocouple voltages in the metallic arc, who can tell just what effect these factors have on the action? My attitude towards my own thermocouple theory is neither to accept nor reject it until someone has actually measured the voltage changes.

My theory has been deduced by reasoning from incomplete experimental data but, so far as I know, the same applies to all other theories covering the same subject. I am inclined to the opinion that when the matter is completely worked out and proved, the theories previously advanced will be found useful in explaining all thermocouple action, while thermocouple action itself will be found to be the true explanation of arc-heat distribution.

The Stability of the Welding Arc

BY P. ALEXANDER*

Member, A. I. E. E.

Synopsis.—The stability of the iron welding arc is influenced by a number of factors, such as the nature and the aerodynamic state of the surrounding atmosphere, the strength and distribution of the magnetic field, the composition of the electrode, the nature and the amount of gases occluded in the electrode, etc. In this paper, however, only the first item has been discussed in an attempt to show that the cooling of the electrode and the arc core by dissociation of the surrounding gas has the preponderant influence on the stability of the arc.

* * * * *

INTRODUCTION

THE term "stability" of the welding arc can be interpreted in various ways and in accordance with conditions under which the welding arc is used.

In this paper, only one phase of this problem is discussed, namely, the influence of the various gases on the arc conduction of the current and on the voltage necessary to maintain the arc. The stability of the arc is judged here by the average voltage drop across the short iron welding arc.

ARC CONDUCTION

The metallic welding arc is a combination of two factors, distinct, yet dependent upon one another. The conduction of the electric current is one factor and the transfer of the material across the arc stream is another factor. Both these factors are greatly influenced by the gas surrounding the arc. The first factor, perhaps, is more directly affected; therefore, only the first one will be discussed in this paper.

It is a well established fact that the conduction of the electric current in the arc is an ionic phenomenon. At any instant in a given volume of the arc core, there is about an equal number of electrons and positive ions. These carriers of the electric charges travel under the influence of the electrostatic field in opposite directions. The electrons migrate towards the anode, while the positive massive ions bombard the surface of the cathode spot.

It is this bombardment by the positive ions that keeps the temperature of the cathode sufficiently high to permit the thermionic emission of the electrons.

Since the electrons are moving at a much higher velocity than the positive ions, the relative number of these carriers is not the same. The number of electrons greatly exceeds the number of positive ions passed through the arc in a unit of time, so that practically the total current is carried by the electrons.[1] Yet without the continual production of the positive ions, the arc phenomena would be impossible; hence both factors play equally important roles and any condition affecting one of these factors (or both at the same time) will have great influence on the stability of the arc.

THERMIONIC EMISSION OF ELECTRONS FROM THE CATHODE

The first factor,—that is, the thermionic emission of the electrons from the cathode,—is a function of temperature and is expressed by the well-known Richardson's formula† $I = A\,\theta^{\frac{1}{2}}\,\epsilon^{-\frac{Q}{2\theta}}$. The temperature is the first and the most important factor.

The second factor is the nature of the material of the cathode. Various substances at the same temperature will emit very different electronic currents. Among the substances giving the largest thermionic emissions, calcium oxide holds a marked place. If calcium oxide be present in sufficient amount on the surface of the electrode of a vacuum tube the thermionic emission may increase several hundred times over that produced by an electrode made, for example, of pure iron. In the case of the arc conduction of the current, it can be expected that CaO present on the surface of the electrode will also greatly facilitate the maintenance of a stable arc.

The third factor affecting the thermionic emission from the cathode is the gas surrounding the cathode. Various gases influence this emission to different degrees. The gas that has the most marked effect is hydrogen. Owing to its strong electropositive nature the first gaseous layer next to the cathode creates such a steep potential gradient, that the electrons will escape the surface of the cathode with great facility.

Taking, again, the example of calcium oxide, it is established that the thermionic emission from that compound surrounded with hydrogen increases very considerably. The thermionic current from incan-

---

*Research Engr., General Electric, West Lynn, Mass.

[1] For numerical references see bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., February 13-17, 1928.

†. $I = A\,\theta^{\frac{1}{2}}\,\epsilon^{-\frac{Q}{2\theta}}$ represents the general law of thermionic emission and is applicable to the various substances.

I represents current in amperes

θ represents absolute temperature which is given by $\theta = 273 +$ deg. cent.

ϵ represents 2.718

A and Q are constants which have various values for different substances. Thus for thermionic emission in air at low pressure:

Substance	Q	A
Platinum......	131×10^3	6.9×10^7
Carbon........	196×10^3	10^{15}
Sodium........	63×10^3	10^2
Baryta........	83×10^3	7×10^7
Lime..........	80×10^3	5×10^7
Magnesia......	79×10^3	4

descent calcium oxide in air at low pressure is 0.05 amperes per sq. cm. If hydrogen replaces the atmospheric gases, the thermionic current at the same temperature will be 1000 amperes per sq. cm. of the cathode spot. Even a trace of hydrogen is sufficient to increase the thermionic emission.

IONIZATION

Passing now to the consideration of ionization, the first factor to be analyzed is the ionizing potentials of the materials present in the state of gas or vapor in the space between the electrodes.

The ionizing potential expressed in equivalent volts is the potential through which the electron must fall to acquire a sufficient energy to ionize the gaseous atom or molecule. The amount of energy necessary to produce this effect is proportional to the stability of the atomic structure. Since the helium atom possesses the most stable atomic structure, the ionizing potential for that gas is the highest. After helium comes neon, and so on, until calcium, and finally caesium (which requires the least energy for ionization) is reached.

TABLE I
TABLE OF IONIZING POTENTIALS[3]

Name	Atomic number	Symbol	Ionizing potentials volts
Helium	2	He	24.5
Neon	10	Ne	21.5
Argon	18	A	15.3
Oxygen	8	O	13.56–(15.5)
Hydrogen	1	H	13.53–(15.9)
Phosphorus	15	P	(13.3)
Nitrogen	7	N	10.8–(16.9)
Mercury	80	Hg	10.39
Sulphur	16	S	(10.31)
Zinc	30	Zn	9.35
Silicon	14	Si	8.50
Copper	29	Cu	7.69
Silver	47	Ag	7.54
Iron	26	Fe	7.4
Manganese	25	Mn	7.4
Lead	82	Pb	7.39
Molybdenum	42	Mo	7.1
Chromium	24	Cr	6.7
Titanium	22	Ti	6.5
Calcium	20	Ca	6.09
Aluminum	13	Al	5.96
Barium	56	Ba	5.19
Sodium	11	Na	5.12
Potassium	19	K	4.10
Caesium	55	Cs	3.90
Acetylene		C_2H_2	(12.3)
Carbon Monoxide		CO	(14.3)
Carbon Dioxide		CO_2	(14.3)

NOTE: Values in parentheses refer to molecules.

From examination of Table I it may be concluded that if the vapors of alkaline earths or alkali metals be present in the space between the electrodes striking and maintaining the arc would be much easier than in air. The voltage required to strike the arc and the voltage across the established arc will be lower. This conclusion is quite in accord with the experimental data.[4]

From Table I it may also be seen that the ionizing potentials of certain monatomic gases are very much higher than those of molecular gases. Helium, for instance, has the highest ionizing potential—namely, 24.5 volts—whereas the ionizing potential of nitrogen is only 16.9 volts.

At first glance it would seem to be much easier to strike and maintain the welding arc in nitrogen than in helium; yet experiments indicate that the contrary is true; in fact, it requires much lower open-circuit voltage to strike the arc in helium than in nitrogen. With certain precautions, 20 volts are sufficient to strike the arc in helium, whereas at least 30 volts are necessary to strike the arc in nitrogen.[5]

This apparent paradox can be explained by the fact that in monatomic gases the electron impacts below the ionizing or radiating potentials are elastic. In helium, for instance, the electron impacts below 20 volts (minimum radiating potential) are elastic. The electron does not lose its energy at the first or second collision, but rebounds and proceeds in a zigzag course until it falls through the potential giving it sufficient energy to bring the helium atom into an excited state at the next impact. If an excited atom which already had absorbed 20 volts is again hit with an electron of sufficient energy, then the helium atom will be ionized.

This preservation of the energy of the bombarding electron during the successive impacts explains the possibility of drawing the arc in helium with lower open circuit voltage than in nitrogen.

In nitrogen as in all diatomic gases the electron impacts are not elastic. In that case the accumulation of energy by an electron falling through the potential should be fast enough so that in spite of continual losses during the encounter with the molecules, it will be able to acquire energy sufficient to ionize. Therefore the striking voltage in molecular gases must be considerably higher than in monatomic gases.

COOLING OF THE ARC

The next factor affecting the ionization is the dissipation of energy and therefore the cooling of the crater and the arc core. It is a well established fact that it requires much greater voltage to strike the arc with water cooled electrodes than with electrodes allowed to become incandescent.[6]

The gases surrounding the electrodes and the arc core in certain cases cause such effective cooling that the arc voltage or the voltage necessary to strike the arc is very much higher. If the open-circuit voltage of the welding generator is not increased the arc becomes very unstable.

From an examination of the tables giving the physical constants of the gases, it may be seen that helium has the highest coefficient of heat conductivity: namely, 0.000339 calories per deg. cent. Yet the cooling effect of that gas on the arc is not greatly different than that of air of which the coefficient of heat conductivity is only one-sixth as great; namely, 0.0000568. On the other hand, if helium be compared with hydrogen, which has about the same coefficient of heat conductivity (0.000327) it will be found that the cooling effect of hydrogen is many times greater than that of helium. This indicates that the cooling effect of the gas is a function of some other much more powerful factor than the mere dissipation of heat by conduction.[7]

The well-known experiments conducted by Dr. Langmuir on heat losses from incandescent tungsten filaments placed in hydrogen established that the powerful cooling effect of that gas is due to the absorption of energy by dissociation at high temperature of the molecular hydrogen into the atomic state.[8] However, the mere fact of the dissociation of the molecules into the atomic state is not sufficient to produce the cooling effect. To produce that, the dissociation products should diffuse away from the arc core so that the recombination of the atoms into molecules with the inevitable restoration of the absorbed energy will occur at a sufficient distance from the arc core. Otherwise there will be an immediate restitution of the absorbed energy which excludes the possibility of the cooling effect. The most energetic cooling effect produced by hydrogen is due not only to the fact that the whole mass of that gas in the first gaseous layers next to the arc core is dissociated into atomic state, but also to the fact that the atomic hydrogen rapidly diffuses away from the arc core.

The most common gas met in welding is, of course, nitrogen, since it constitutes 78 per cent of the atmospheric air. This gas also dissociates into the atomic state in the first gaseous layers next to the arc core. The heat of dissociation of nitrogen is 274,000 calories per mol, that is, almost three times as great as that of hydrogen, (98,000 cal. per mol), so that it has the greatest capacity for absorbing energy from the arc. If the products of dissociation, that is, the atomic nitrogen, could be blown away from the arc in the same way as the atomic hydrogen is blown away from the arc, the atomic nitrogen would be three times as efficient a heat transmission medium as the atomic hydrogen. However, the dissociation of nitrogen into the atomic state does not begin until the temperature is 3500 deg. cent., whereas the dissociation of hydrogen is appreciable at 1800 deg. cent. A small percentage of nitrogen is dissociated in the first gaseous layer next to the arc core, but as soon as it leaves that region it recombines into the molecular state, with a complete restoration of the absorbed energy. Therefore the cooling effect of this reaction is neglible.

Carbon monoxide presents an interesting subject for study because, in spite of a very high heat of dissociation it is the gas in which the welding arc can be maintained with the greatest ease.

The heat of dissociation of carbon monoxide as calculated from the band spectra is 258,000 calories per mol.[9] If, in spite of this enormous heat of dissociation, this gas does not produce any appreciable cooling of the arc, it either is not dissociated by the arc or the products of dissociation cannot be blown off the arc, but recombine in the immediate vicinity of the arc core with full restoration of the absorbed energy.

For purposes of elucidation if possible on this interesting point, the writer made the following series of experiments:

After preparing and purifying a sufficient amount of carbon monoxide, a steam of that gas was passed as shown in Fig. 1 through a cylindrical clear fused quartz vessel in the middle of which were disposed two tungsten electrodes. The flow of the gas was adjusted so as to duplicate as nearly as possible the conditions of an actual welding operation, if such be made with carbon monoxide as a shielding gas. The out-flowing gas, after passing through a vessel filled with an aqueous solution of NaOH, was conducted to a quartz nozzle and burned in contact with the atmosphere. By bringing into contact and then withdrawing the tungsten electrodes, an arc of 30 amperes and 28 volts was established in the stream of gas for one minute. At the end of 60 sec. the arc was hardly visible through the black deposit on the inside walls of the vessel.

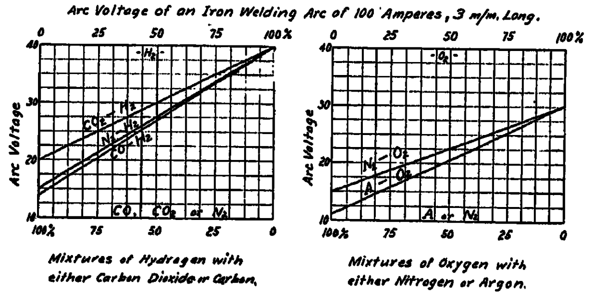


FIG. 1

The color of the flame of the outflowing gas was changed from the characteristic pale blue of carbon monoxide to yellow.

The following conclusions may be drawn from the above observations. Since the three known oxides of carbon—namely, CO, CO_2, and C_2O_3—are not solids at room temperature, the black deposit could be either pure carbon, a mixture of carbon and condensed tungsten vapor, or tungsten carbide. The chemical analysis of the deposit indicated, however, that the black deposit was carbon.

The experiments repeated with various mixtures of H_2 and CO indicated that the presence of H_2 does not prevent the formation of the black deposit. The dilution of CO merely reduces the amount of the deposit per unit time. However, when the experiment was repeated in pure argon, the walls of the vessel remained perfectly clear.

It may be concluded that carbon monoxide is dissociated by the arc and that one of the products of dissociation was diffused away.

This is an apparent contradiction to the well-known fact that the dissociation of carbon monoxide begins only at an extremely high temperature. Therefore, the gas should behave in this respect like nitrogen. This contradiction, however, is apparent only, since that reaction took place in the presence of readily oxidizable materials, and the heat of formation of the secondary reactions of the liberated oxygen (and probably a part of carbon) compensates for the cooling effect of the first reaction.

However, since the gas has a smoky appearance in the immediate vicinity of the arc core it may be inferred that the liberated carbon was condensed into the solid state in the region next to the arc core. Since the heat of sublimation of carbon is quite accurately known, it is possible to represent the above reaction by the equation:[9]

$$C \text{ (gaseous)} = C\text{(solid)} + 141{,}000 \text{ calories}$$

so that even in the absence of other secondary reactions of oxidation and carburation of the electrodes, the condensation of carbon alone restores the greatest part of the energy previously absorbed from the arc core.

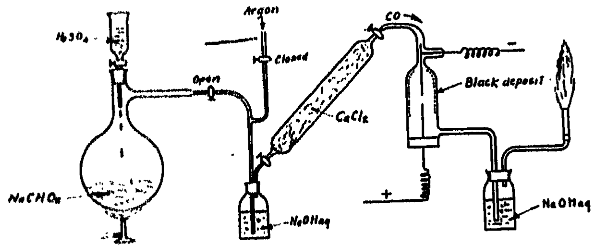


FIG. 2

The oxidation of tungsten, that is formation of WO_2 or WO_3 give respectively 131,400 and 196,300 calories per mol.

In the case of iron the formation of Fe_2O_3 or Fe_3O_4 produces 197,700 and 270,800 calories, respectively. In other words, the condensation of the gaseous carbon and the oxidation reaction supply an amount of heat almost sufficient to neutralize the cooling effect of the first reaction. The as yet unpublished results obtained by other experimentators, communicated to the writer, indicate that in the arc maintained in CO there are formed products of polymerization such as C_xO_y or $(CO)_x$.

If this is correct, then the mechanism of dissociation of CO may be explained in a different way, namely, the products of polymerization diffuse away from the arc core without absorption of energy, and being unstable, break up in the cooler regions of the arc. This explanation of the mechanism of dissociation seems to the author to be the most plausible.

The most remarkable property of carbon monoxide is its stabilizing influence of the welding arc burning in any gas to which carbon monoxide is added in sufficient amount.

This property is quite astonishing because the ionizing potential of this gas is high; namely, 14.3 volts. This fact, however, can be reconciled with the stability of the arc in that gas, by observing that only a certain percentage will be dissociated. The rest of the gas will be in molecular form and that with regard to the ionization by successive electron impacts carbon monoxide behaves in an astonishingly similar manner to mercury vapor.[11] Owing to the existence of several critical potentials both these gases may accumulate the energy necessary for ionization by steps, and as in mercury vapor, so in carbon monoxide gas it is possible to maintain an arc with a very low voltage.

Carbon dioxide gas is dissociated in the arc into oxygen and carbon monoxide. A part of the produced carbon monoxide will undergo further dissociation into carbon and oxygen. However, in the presence of an excess of carbon dioxide, the amount of dissociated carbon monoxide will be very small.

The heat of dissociation of carbon dioxide into oxygen and carbon monoxide is 68,000 calories per mol. If the products of dissociation are blown off the arc, the arc will be always burning in carbon dioxide, which through its dissociation will produce a very strong cooling effect causing instability of the arc.

The voltage drop may be as high as 30-35 volts and the arc will be very unstable. If, however, the arc is maintained in a more or less stationary atmosphere of the dissociated carbon dioxide, it will be stable, and the arc voltage of an arc of say 175 amperes will be only 19 volts.

Fig. 2 shows the relationship between the composition of the gaseous atmosphere and the arc voltage at an iron welding arc of 100 amperes, three millimeters long.

CONCLUSION

The stability of the iron welding arc is influenced by a number of factors, such as the nature and the aerodynamic state of the surrounding atmosphere, the strength and distribution of the magnetic field, the composition of the electrode, the nature and the amount of gases included in the electrode, etc. In this paper however, only the first item has been discussed in an attempt to show that the cooling of the electrode and the arc core has the preponderent influence on the stability of the arc.

References.

1. D. C. Prince, *Mercury Arc Rectifier Phenomena*, JL. A. I. E. E., July, 1927.
2. J. A. Crowther, Ions, Electrons and Ionizing Radiations, p. 120.
3. L. Rolla and G. Piccardi, *Chimie et Industrie*, October, 1926.
4. T. C. Hebb, "Low-Voltage Arcs," *Phys. Rev.* 12, p. 482.
5. K. T. Compton and P. S. Olmstead, "Low Voltage Arc in Helium," *Phys. Rev.* 15, p. 545.
6. C. D. Child, *Phys. Rev.* 1910, Vol. 30, p. 311.
7. P. Alexander, *The Electric Arc*, JL. A. I. E. E., TRANS. Vol. 47, p. 334 1927.
8. I. Langmuir, "Flames of Atomic Hydrogen," *G. E. Rev.*, March, 1926.
9. R. T. Brige and M. Sponer, "Heat of Dissociation of Non-Polar Molecules," *Phys. Rev.*, August, 1926.
10. H. Kohn and M. Guckel, *Naturwiss* 12, (1924). p. 139.
11. F. L. Mohler and P. D. Foote, *Phys. Rev.*, January, 1927.

Discussion

Elihu Thomson: The paper is one which sums up in a brief way the result of a lot of study that has been given to the subject by Mr. Alexander.

Mr. Alexander, of course, has made a great point of dissociation. Dissociation in the way that he deals with it means simply that a molecule by the delivery of electric energy is broken up into atoms. He deals with hydrogen dissociation, which forms the basis of the Langmuir atomic-hydrogen process of welding, where an arc is formed and is fed with hydrogen, which passing through the arc is dissociated into atomic hydrogen. Consequently, the process is called the atomic-hydrogen method. It is known that a very large amount of energy is required to separate hydrogen from hydrogen, and as the atoms leave the arc and the gas falls in temperature slightly they recombine and there is enormous restoration of the energy which was demanded to dissociate them.

Mr. Alexander deals with such a process and applies it to other gases and compounds. Let us take the case of carbon monoxide. Carbon monoxide, as he shows, requires for its dissociation a large amount of energy. This would indicate rather high voltage in the arc. But he also studies it in this way. Does the energy come back, and where does it come back? It comes back at once close to the locus of dissociation. He shows that the gases do not diffuse away, and calls attention to that property of carbon itself, which being vaporized, is immediately condensed into the solid form, in the form of what we might call "soot," if we could see it. He shows that in the condensation of the vaporized carbon we have also an enormous output of energy. The carbon might be visible, if we could separate it from the oxygen, but the oxygen dissociated from the carbon monoxide is there to recombine, and it also represents a store of energy given out when it recombines with the carbon. Mr. Alexander so accounts for the properties of the carbon-monixide arc, which has been found to be one of the best stabilizers for arcs drawn in other gases. A mixture of carbon monoxide with hydrogen, for example, gives an excellently stabilized arc as compared with what you would get with hydrogen alone.

Eugene Herzog: It is only in one minor point that I feel obliged to disagree with the author, *i.e.*, in his application of Langmuir's dissociation theory to carbon monoxide. It does not seem logical to me to assume that the fact of no energy absorption would explain the stabilizing influence of carbon monoxide on the arc; a more active participation of carbon monoxide through easier ionization probably due to resonance potentials seems a necessary assumption. A hasty search of literature did not disclose the existence of any resonance potential, but data seem entirely lacking on the subject.

P. Alexander: Referring to Mr. Herzog's question, I will say that in Table I, I give the ionizing potentials not only of atoms but also of molecules. For instance, if we take oxygen, the ionizing potential of atomic oxygen is given as 13.56. When oxygen is in a molecular state it requires a slightly higher energy to ionize the molecule. The value given is 15.5. And so on. Most of the given values refer to the atoms, because when these elements are vaporized they are in an atomic state, not in a molecular state. The molecular state of these elements is not stable.

Undoubtedly certain molecules are what most of the physicists in this country call in an excited state. That is, they have already absorbed a certain amount of energy. Ionization of these excited molecules is easier, so that energy necessary for a bombarding electron to ionize them will be less.

The absence of absorption of energy does not explain the stabilizing properties of carbon monoxide. It explains only why in spite of the observed dissociation of a certain amount of carbon monoxide the arc core is not cooled by this reaction. In certain cases it may happen that the subsequent reactions of the dissociation products with other materials present in the arc may liberate more energy than the amount absorbed from the arc core. In such a case the arc core will be heated instead of cooled; thus the dissociation of one mol that is one gram-molecule of carbon monoxide absorbs 248,445 calories which can be expressed by the equation:

$$\mathrm{CO} = \mathrm{C}_g + \mathrm{O} - 248{,}445 \text{ calories}$$

The (C_g) gaseous carbon condenses immediately into solid carbon thus:

$$\mathrm{C}\text{ (gaseous)} = \mathrm{C}\text{ (solid) plus } 141{,}000 \text{ calories}$$

Also the atomic oxygen recombines into molecular state with evolution of energy thus:

$$\tfrac{1}{2}(\mathrm{O} + \mathrm{O}) = \tfrac{1}{2}\mathrm{O}_2 + 81{,}000 \text{ calories.}$$

And finally if the molecular oxygen combines with tungsten vapor that reaction will yield:

$$\tfrac{1}{2}\mathrm{W} + \tfrac{1}{2}\mathrm{O}_2 = \tfrac{1}{2}\mathrm{WO}_2 + 65{,}700 \text{ calories}$$

so that the total amount of the liberated energy will be 141,000 + 81,000 + 65,700 which equals 287,700. However, the stabilizing property of carbon monoxide depends not on the absorption or evolution energy, but as I stated "owing to the existence of several critical potentials."

With regard to the literature on this subject, I would refer to the reference (11) given at the end of my paper.

C. H. Keel: I think that two years ago Mr. Alexander presented a paper describing the use of the metallic electric welding process in an atmosphere of hydrogen. I wonder if there has been as much development in that process as there has been in the case of tungsten electric welding in hydrogen, and if there are any practical limitations to that process.

P. Alexander: The development of the shielded-arc process is going on in our works, but as is the case with all the new processes it is not sufficient to establish the fundamental facts about the process before it is commercialized. It is necessary also to evolve the technic of handling that process. The present work which we are doing in our factories consists of the development of the technic.

We also find that there are certain limitations to the process. For certain work, this process is not suitable. We find that in other fields, unexpected fields, this process is very suitable and gives better results than we expected it would.

A. P. Wood, in his paper[1] tells of the application of this process to large generators.

J. W. Sheffer: Does the atomic-hydrogen process produce welds free from oxides?

P. Alexander: It does. The resistance to corrosion of atomic-hydrogen welds is not definitely ascertained as yet, but I believe that it should give higher resistance to corrosion than welds with oxides and other impurities.

Experimentally, atomic hydrogen was used on welding plates up to 2 in. thick, but in practical work I believe much lower thicknesses should be used. If you will notice, in the pamphlet issued by the General Electric Co., thicknesses up to ¼ in. only are recommended.

---

[1] *Welding and Manufacture of Large Electrical Apparatus*, A. P. Wood, see p. 717.

Arc-Welded Structures and Bridges

BY A. M. CANDY[1]

Member, A. I. E. E.

Synopsis.—*The use of arc welding for constructing buildings, machinery, bridges, etc., has come into prominence during the past eighteen months, due to the merits of the process becoming recognized by a number of designers. Undoubtedly within the next few years the applications of the process will be extended rapidly as our available data are corroborated and amplified and as the limitations of the process become well defined.*

The arc welding process of joining steel members to produce a fabricated structure, either to replace riveting or to take the place of castings, will produce results not obtainable in any other way known at present. Among the advantages of arc welding are the following:

1. *The weight of materials required in a member or structure can be reduced by as much as 18 per cent.*
2. *No material is removed from members for welded connections such as rivet holes for riveted construction.*
3. *The weight of connecting material in some cases can be reduced 50 per cent to 90 per cent.*
4. *Absolute fixation of one member to another can be easily and economically obtained.*
5. *Continuity of members intersecting others can be readily obtained.*
6. *Greater rigidity can be obtained in a structure.*
7. *Overlapping members can be completely sealed by weld metal, thereby excluding moisture, preventing corrosion, and making a structure easier to paint.*
8. *Changes in or additions to structures can be made with a great saving in both time and expense.*
9. *More pleasing designs and in some cases, designs impossible by other processes, can be produced.*
10. *The welding process is practically noiseless.*

IN the past when beams or columns were required of section greater than the largest commercial rolled shapes available, they were constructed of angles for flanges and stiffeners and a plate for a web as shown in Fig. 1. Today, by using the arc weld process, a similar beam can be made from plate material exclusively (See Fig. 2). The former girder weighs 798 lb., and carried a maximum central load of 68,900 lb. The latter weighed 18 per cent less, or 656 lb., and carried a maximum load 13 per cent greater, or 78,000 lb. The load deflection diagrams of these two girders are shown by Fig. 3, as 20 and 20-A, for riveted construction using angles and plates whereas 21 and 21-A are designed for welding using plate material only. This latter design was used for the large second floor girders in the Sharon arc welded building (Fig. 4.) These occurred in pairs over the crane runway illustrated in Fig. 5. These girders weighed 9 tons each, whereas girders made of angles for flanges and stiffeners for riveted design had an estimated weight of 11 tons which shows 18 per cent saving. Since there are 22 of these girders in the building, a total of 44 tons of steel is saved by the welded design.

For the first arc welded girder type railroad bridge, this same construction is used for the main side girders, each weighing 6 tons, (Fig. 6). This bridge has a span of 53 ft. 9 in. and is 62 ft. 4 in. long over all, requiring 20 tons of steel.

One of the advantages observed in the design and construction of this bridge was the ease of connecting the inside sketch stiffeners to the main girder webs and top faces of the floor beams as compared with the connection details which would be required for riveted construction.

The advantages of not punching holes in members for welded construction are almost too obvious to mention as indicated by the following example:

To show the advantages gained by simply substituting arc welding for riveting in tension members, two specimens were prepared as shown by Nos. 17 and 18 in Fig. 7. Specimen No. 17 comprises two angles 2½ in. by 2 in. by 3/16 in. by 4 ft. long with their 2½ in. legs located against two shank plates ¾ in. by 5 in. by 1 ft. long, with the angles overlapping the plate ends four inches. The heel and toe of each angle was welded along the 4-in. length and across the ends of each angle to the shank plates. Specimen No. 18 was slightly different in that the angles were 4 ft. 2½ in. long, overlapping the shank plates by 6½ in., and were riveted to the shank plates with 6 rivets of ⅝ in. diameter, power driven in 11/16 in. diameter holes.

The comparative failures of these specimens are shown by Fig. 8. The welded specimen first failed through the 2½-in. leg followed by tearing through the 2-in. leg adjacent to the end of the shank plates. The riveted specimen failed through the first rivet hole at the end of the shank plate.

The following tabulation gives the pertinent data:

Specimen number	Lb. elastic limit	Lb. ultimate load	Ultimate load in lb. per sq. in. of sectional area of the 2 angles
17	70,000	94,000	58,000
18	60,000	72,000	44,500

In addition to the fact that the welded specimen was more than 30 per cent stronger than the riveted specimen, it is interesting to note, in passing, that the elongation of the welded specimen was 168 per cent greater than the riveted specimen, the values being 1.12 in. over a length of 4 ft. 3½ in. for the riveted specimen, and 3 in. for the welded specimen measured over a length of 40 in.

---

[1] Welding Engineer, Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-17, 1928.

The weight of connecting material for welded joints is very much less than for equivalent riveted connections. For example, the 9-in. *I*-beam, (Fig. 9), required only two pounds of weld metal to attach it to the columns and it withstood a maximum central load of 67,200 lb. In the case of the specimen, Fig. 10, the connecting angles and rivets weighed 22 lb. and the

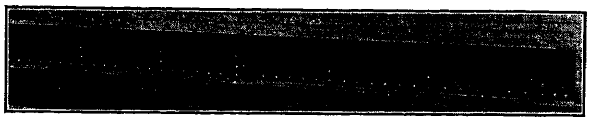


Fig. 1—Riveted Girder

Girder 15 ft. long, 14-15/16 in. deep, weight 798 lb. Four flange angles 4 by 3 by 5/16 in. 16 web stiffener angles 3 by 2 by 1/4 in. 12 filler plates 2-1/2 by 5/16 in. and two 5 by 5/16 in. Web plate 14 in. by 5/16 in. Performance is shown in Fig. 3, curve No. 20.

Fig. 2—Arc-Welded Girder

Girder 15 ft. long, 14-3/4 in. deep, weight 656 lb. Two flange plates 10 in. by 3/8 in. 16 web stiffoner plates 3 by 1/4 in. Web plate 14 by 5/16 in. Performance is shown in Fig. 3, curve No. 21.

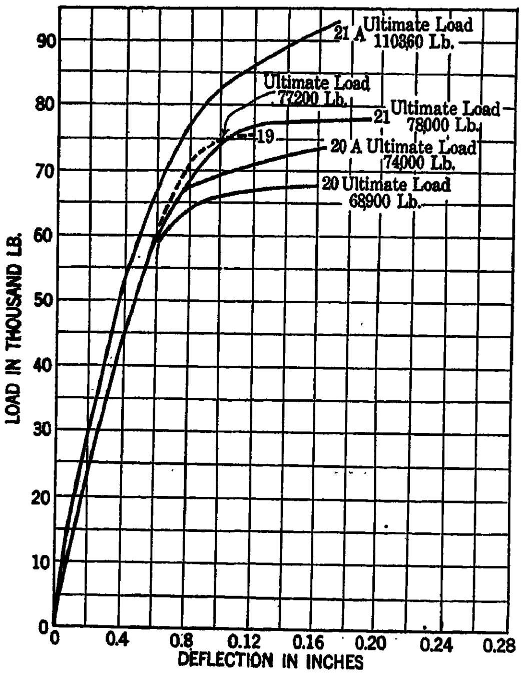


Fig. 3—Load-Deflection Curves of Riveted and Welded Girders Shown in Figs. 1 and 2

Curve 20A is a specimen duplicating the riveted girder in Fig. 1. Curve No. 19 is a specimen using identical material except that the connections were made with weld metal instead of rivets which reduced the weight of complete specimen to 785 lb. instead of 798 lb. Curve No. 21A is a specimen similar to arc-welded girder shown in Fig. 2, except that 4 additional web stiffener plates were used and all stiffeners were made 4-1/2 in. wide; also a top cover plate 9 in. by 5/16 in. by 6 ft. long was added, making the total weight 795 lb.

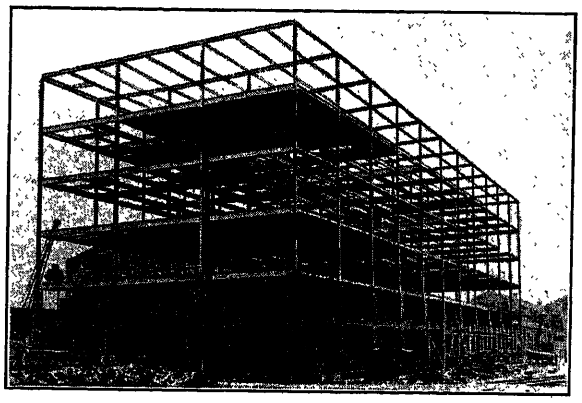


Fig. 4—Largest Arc-Welded Building Produced to Date

Five stories and mezzanine floor 70 ft. by 220 ft. by 80 ft. high, 790 tons of steel; saving 95 tons of steel since a riveted building of same design would require 885 tons of steel. Location, Sharon, Pa.

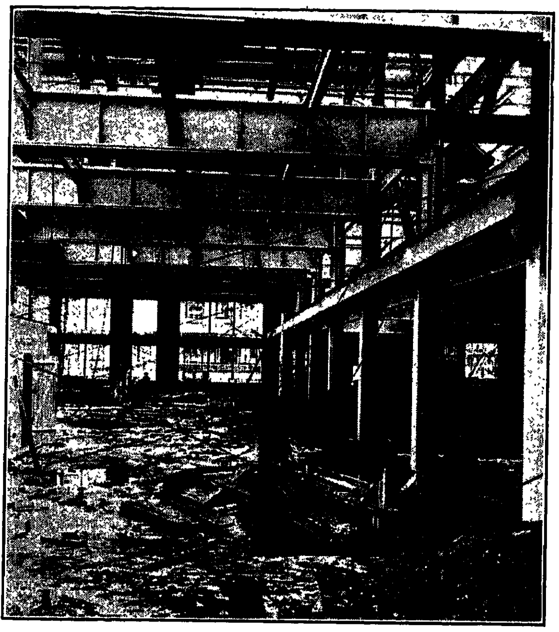


Fig. 5—View Down Crane Runway of 790-Ton Arc-Welded Building

Showing 9-ton girders made up of plate material, only saving 2 tons of steel as each would require 11 tons of steel for riveted construction.

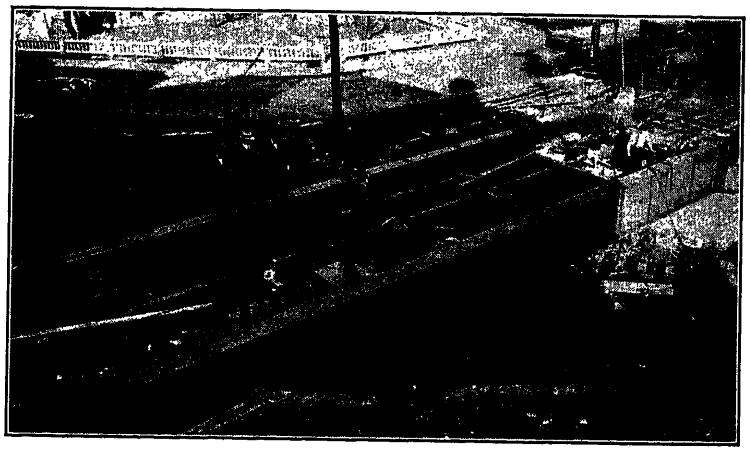


Fig. 6—All Arc-Welded Railroad Bridge of Through-Girder Type

specimen carried only a central load of 58,700 lb. or 12½ per cent less capacity with 11 times the weight of connecting material.

The load deflection diagrams of these specimens are shown by Fig. 11 curves No. 8 and No. 9, respectively.

These same specimens illustrate the fact that complete fixation of the 9-in. *I*-beam to the column was obtained in case of the welded specimen as all failure was due to buckling of the beam top flange and crippling of the web. In the case of the riveted specimen the first failure was the bending of the top angles followed by

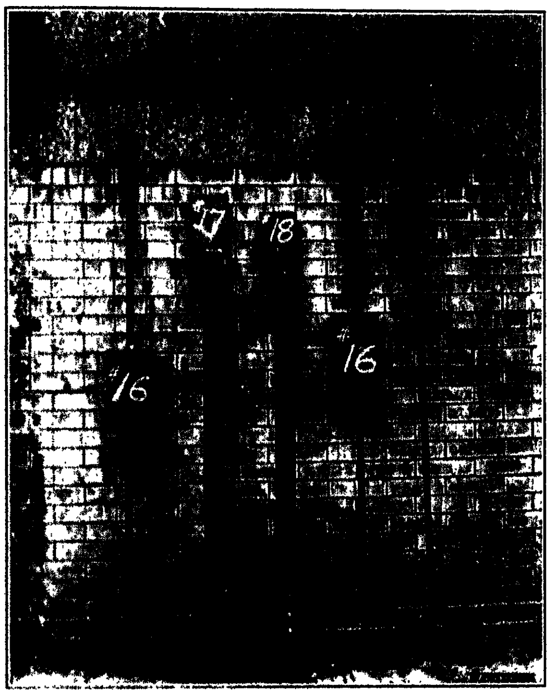


FIG. 7—TENSION-TEST SPECIMENS WELDED AND RIVETED

No. 17 is arc-welded. No. 18 is riveted. Deflection curves are shown in Fig. 8.

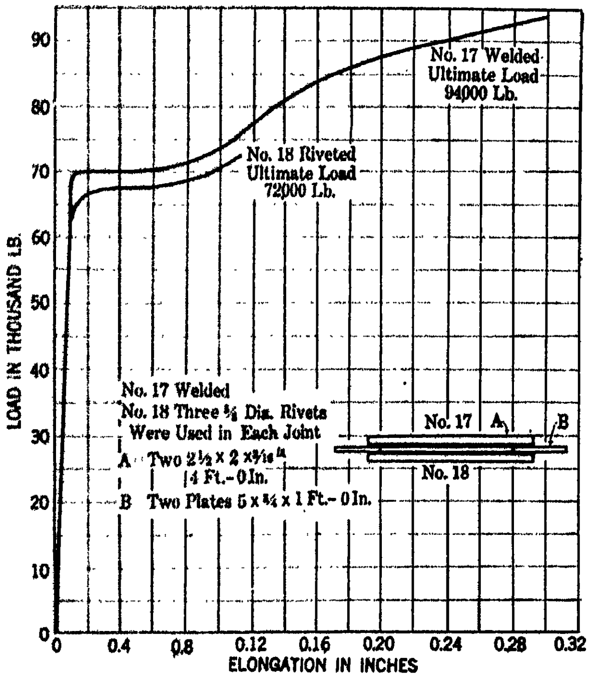


FIG. 8—LOAD-DEFLECTION CURVES OF SPECIMENS 17 AND 18

The welded specimen No. 17 failed just outside the shank plate through the angles at 94,000 lb. and elongation of 3 in. on 3 ft-4in. centers. Riveted specimen No. 18 failed through first rivet hole at 72,000 lb. and elongation of only 1.12 in. on 4 ft 3-1/2 in centers.

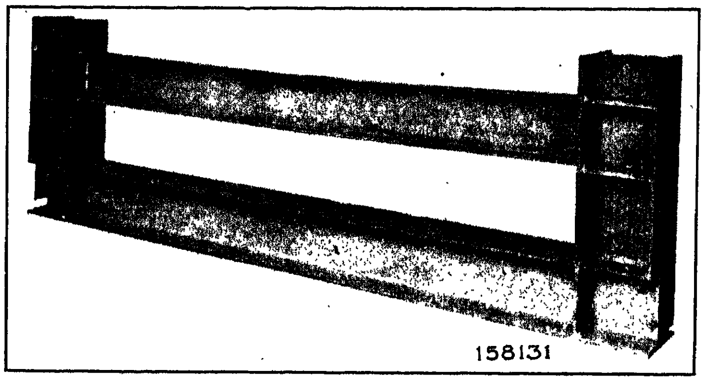


FIG. 9—ARC-WELDED BEAM

Beam member is a 9 in. 21.8 lb. 7 ft-11-3/8 in. long welded at ends to 10 in. H columns 49-1/2 lb. per foot. Performance shown by Fig. 9 curve No. 8.

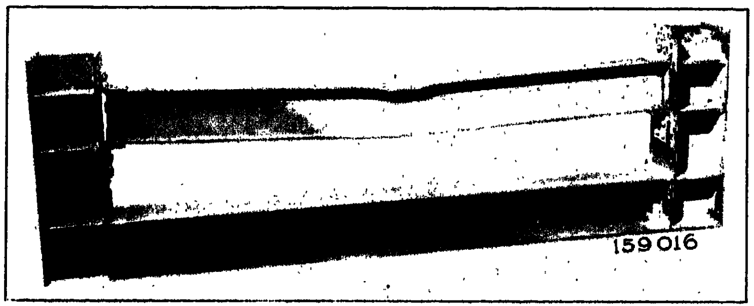


FIG. 10—RIVETED BEAM

Specimen same as shown in Fig. 6, except that beam is secured to column by 1/2 in. thick angles and 20 rivets 3/4 in. diameter. Performance shown by Fig. 9 curve No. 9.

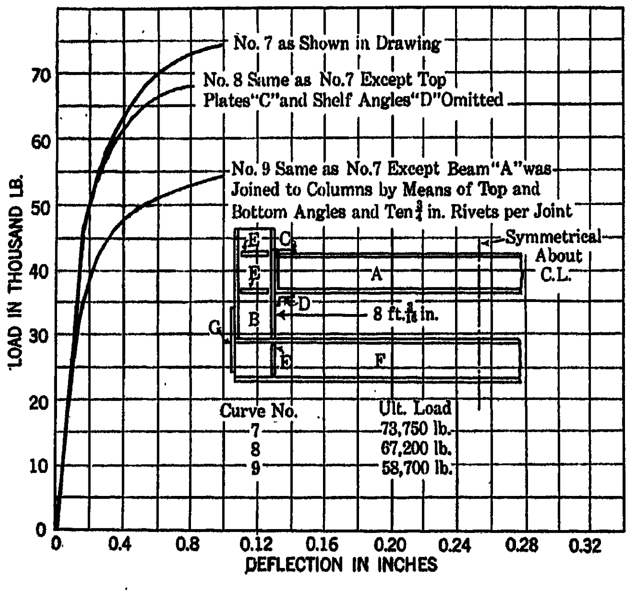


FIG. 11—LOAD-DEFLECTION CURVES OF BEAMS IN FIGS. 7 AND 8

Also Curve 7 shows a specimen similar to Fig. 7 except that seat angles *D* 4 in. by 3 in. by 3/8 in. by 5 in. long and top plates *C* 4 by 7/16 by 5 in. long were used as shown in sketch.

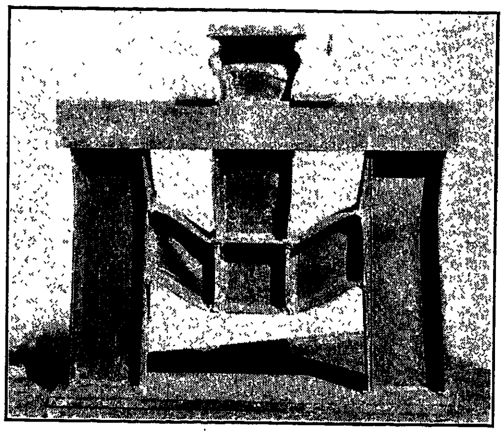


FIG. 12—WELDED BEAM CONSTRUCTION

Beam member 6 in. 1-12-1/2 lb. 18 in. long arc welded to columns. Beam deflected 3-5/8 in. with 100-ton load.

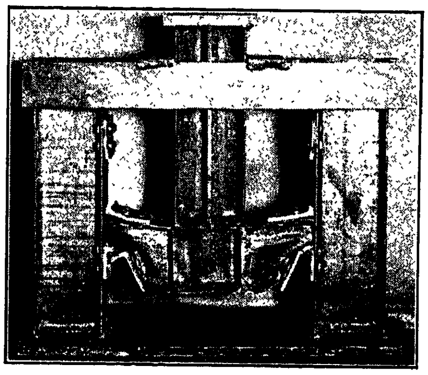


FIG. 13—RIVETED BEAM CONSTRUCTION

Beam member same as for Fig. 8 but secured to column by 1/2 in. angles and 32 rivets 5/8 in. diameter. Beam deflected 1 in. with 60-ton load when top angles failed.

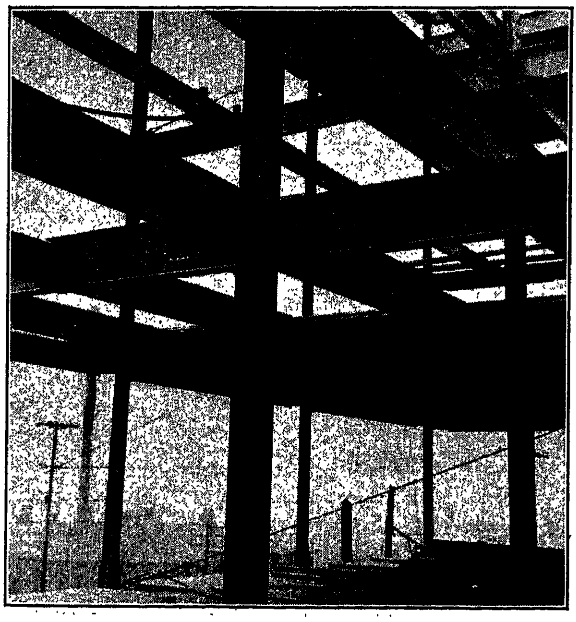


FIG. 14—TYPICAL COLUMN CONNECTIONS IN 790-TON ARC-WELDED SHARON BUILDING

Continuity of 20 in. 165 lb. header girders secured by plates in column just above the 15 in. I floor beams. Continuity at bottom of 20 in. I secured through seat angles under 15 in. I floor beams.

flange buckling and web crippling. These characteristics are shown possibly even better in Fig. 12, which specimen carried a maximum load of 100 tons whereas specimen Fig. 13 carried only 60 tons when the top angles failed. In these cases, the connecting material ratio is ten to one in favor of welded connections which involved 1½ lb. of weld metal and 15 lb. of angles and rivets, with improved performance for the welded structure.

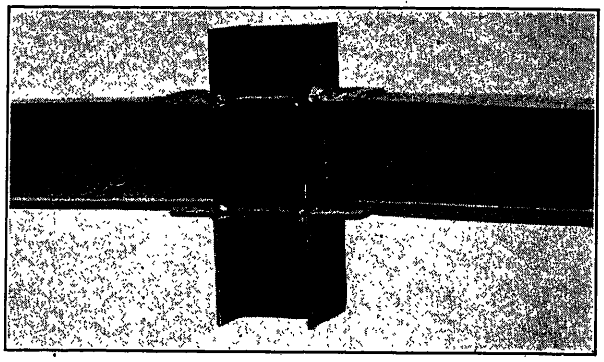


FIG. 15—WELDED BEAM CONSTRUCTION

Two cantilever 9 in. Bethlehem I-beams 20-1/2 lb. secured to 8 in. H column 32 lb. using seat angles 4 in. by 3 in. by 3/8 by 5 in. long, top reenforcing plates 4 in. by 1/2 in. by 4-1/2 in. long. Continuity plates, 3 in. by 5/8 in. by 6-5/8 in. long located in column, give the effect of making beams continuous through the column. Failure took place by crippling of beam webs. Performance shown in Fig. 13 Curve No. 2.

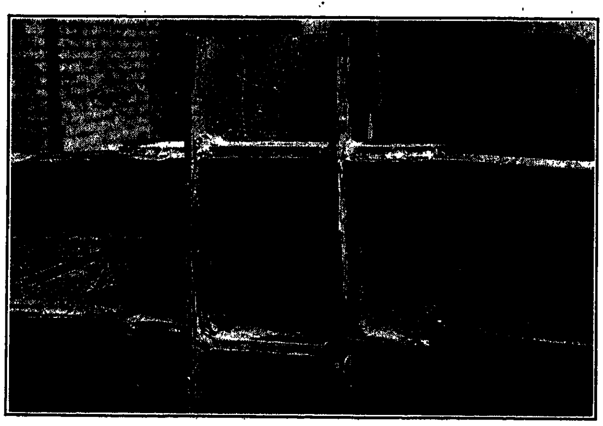


FIG. 16—CLOSE-UP OF SPECIMEN DUPLICATE OF FIG. 11

Shows stress lines on beam webs and no failure of welded connections. This specimen was coated with cement to accentuate the stress lines. Performance shown by Curve 2 of Fig. 13.

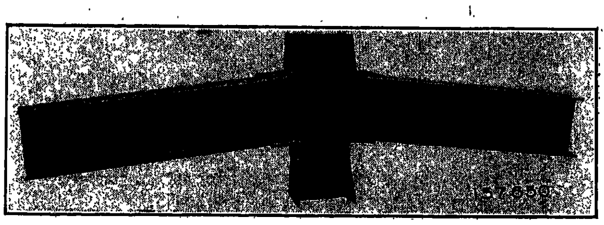


FIG. 17—SPECIMEN SIMILAR TO FIG. 11 EXCEPT CONTINUITY PLATES WERE OMITTED IN COLUMN

As a result of omitting continuity plates the column web buckled. Performance shown in Fig. 13 Curve No. 4.

The advantage of obtaining continuity of lines of beams or girders by welded connections lies in the fact

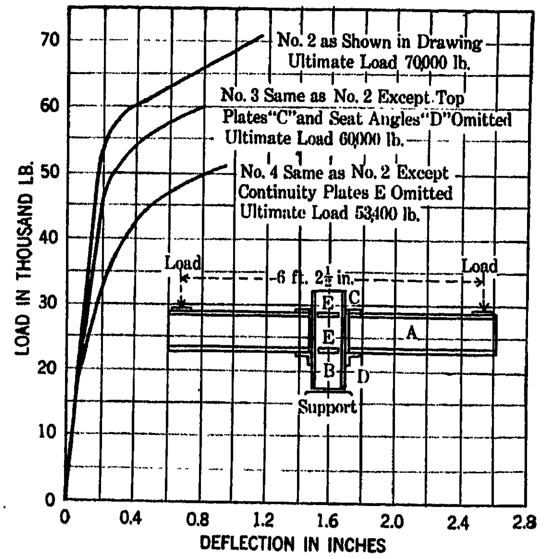


FIG. 18—LOAD-DEFLECTION CURVES OF SPECIMENS IN FIGS. 10 AND 12

Also Curve 3 shows a specimen similar to Fig. 10 except that no seat angles or top reenforcing plates were used.

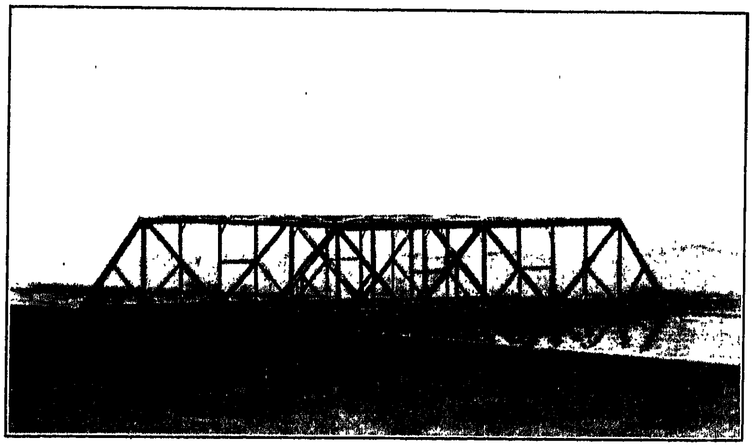


FIG. 19—FIRST TRUSS-TYPE ARC-WELDED RAILROAD BRIDGE AT CHICOPEE FALLS, MASS.

Welded bridge uses 80 tons of steel. Riveted bridge design specified 120 tons of steel.

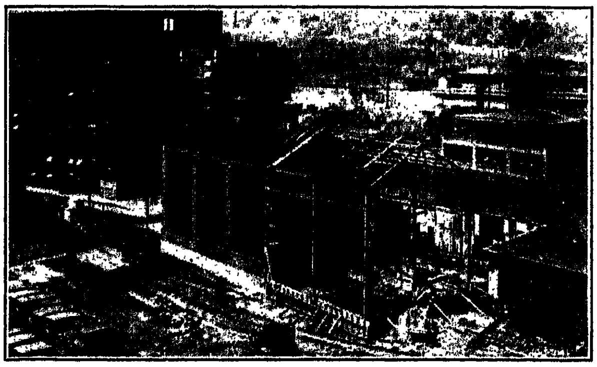


FIG. 20—TWO BAYS BEING ADDED BY ARC WELDING TO A SMALL ARC-WELDED BUILDING

Making structure 40 ft. by 100 ft. by 40 ft. to tops of columns. A saving of over 10 per cent was made compared with estimated riveted cost.

that the designer can save weight in the structure. For example, in the 790-ton Sharon building, 51 tons of steel was saved by designing floor beams and header girders for continuity. The continuity of the girders through the columns was obtained by welding continuity plates in the columns in line with the top flanges of the girders as per Fig. 14. That this construction will produce continuity is illustrated by Figs. 15 and 16, which show failure by crippling of the *I*-beam web without any sign of weld failure whereas if the plates in the column are omitted as per Fig. 17, the column web will fail as indicated. The loads in these cases being 70,000 lb. for the former and 53,400 lb. for the latter. The load deflection curves for Figs. 16 and 17 are shown by No. 2 and No. 4, respectively, of Fig. 18.

The application of continuity and reduction of connection details for the design of the first arc welded truss-type railroad bridge Fig. 19 resulted in a saving of 33½ per cent by weight in the steel required in the structure. The welded bridge involves 80 tons of steel whereas the riveted design requires 120 tons of steel. In the case of the welded design, the connection details are only about 5 per cent of the weight of the structure, whereas the connection details for the riveted design are about 30 per cent of the weight of the structure.

FIG. 21—CLOSE-UP VIEW OF ADDITION, FIG. 15

Shows small amount of disturbance in masonry where members are added to an existing structure by arc welding.

Where corrections in, or additions to, existing structures are to be made, the arc process will show tremendous savings, and as a result, much work of this type, such as the reenforcing of the Great Western Railroad Bridge, at Leavenworth, Kansas, over the Missouri River, has been performed recently. In this case, over

110 tons of plate material, etc., were added to the structure at a figure considerably below the riveting cost.

In the case of the 3-bay welded building, Fig. 20, it was decided to add two bays. To do this, only a relatively small amount of outside masonry was removed to permit of the welding being done as illustrated by Fig. 21.

The major saving in this class of work is in the elimination of drilling holes for rivets and tearing away masonry on the rear side of the members to permit backing up the rivets while being driven.

Due to its many merits, such as lower costs, absence of noise, etc., the arc welding process will undoubtedly come into greater and greater use. The information which is accumulating from its use on relatively simple buildings will make it available in combination with riveting for larger and more complicated structures, and should lead eventually to the possibility of its use almost exclusively on the largest and most complicated structures built of structural steel.

Discussion

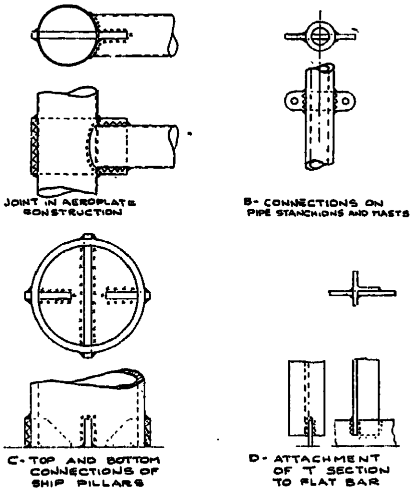
J. W. Owens: Mr. Candy brought up two points to which I should like to refer:

The first one is that he partially admitted the advantage, or so-called advantage, claimed for the riveted joint, namely, the possibility of its adjustment by slippage under load as against the non-slippage of the welded joint. This is a point which welding engineers have not conceded.

In the first place, if a structural engineer could produce an integral building or one without a joint, in other words the most perfect building that could be constructed, he would do it. He does not do so because of mechanical and commercial considerations in the manufacture, shipment, fabrication, and erection of the steel. The nearer then that we can approach the ideal of a homogeneous structure, the more perfect will be our buildings, and welded construction approaches this ideal very closely.

Again, a riveted joint is known from tests to slip at 15 to 20 per cent of its ultimate strength, and once a riveted joint starts to slip, you do not know what results such slippage will have on the loads which other parts of the structure will be required to stand and for which such parts were probably not designed. In dealing with naval architects I have found that they have a tendency to stress the so-called advantage of joint slippage because of the complex stresses that the ship's hull is subjected to in service. My point of view is that if welded joints have a working strength greater than the elastic limit of the material welded, the elasticity of the base metal will take care of such stresses. Any stress beyond the elastic limit of the material welded will permanently deform the structure regardless of whether it is welded or riveted. Practically, then, the most perfect structure that can be made is one in which all the joints are welded. However, it should be designed specifically for welding.

The second point I should like to call attention to is Mr. Candy's remarks about the slotted joint. This joint was, I think, first introduced in airplane construction and is today used extensively in this class of work, the tubing being slotted for an insert plate as shown in the accompanying Fig. 1 at A. The Newport News Shipyard has recently extensively employed the slotted joint in the construction of the hull structure of the S. S. *Virginia*. This ship is a sister ship of the *California*, and the largest merchant ship to be built in the United States to date. In this case, the tops and bottoms of her tubular pillars, some of which were 22 in. in diameter and of ¾ in. material, were slotted for plates as shown in C, thus making possible a joint which was twice as strong as the original riveted joint obtained by means of a forged angle-ring riveted to the pillar. This welded connection was made at about one-fourth of the cost of the riveted one. Two of the other applications to which I have applied this joint are shown in B and D. This type of joint will be found very useful in the development of various structural designs. This was called to the attention of the American Welding Society at its fall meeting in Detroit.


A- JOINT IN AEROPLANE CONSTRUCTION

B- CONNECTIONS ON PIPE STANCHIONS AND MASTS

C- TOP AND BOTTOM CONNECTIONS OF SHIP PILLARS

D- ATTACHMENT OF T SECTION TO FLAT BAR

FIG. 1—VARIOUS APPLICATIONS OF THE SLOTTED JOINT

I should like to emphasize that, in all my experience with practically every type of welded structure, I have rarely come across a case in which the structure would not be cheaper if welded instead of riveted, provided it had been specifically designed for welding. In my opinion, the time is not far distant (perhaps it will be within the next seven or ten years) when we shall have completely welded ships. The Newport News Shipyard is now welding the major portion of non-strength parts of ships constructed at this yard and a number of the important strength members, *e. g.*, pillars, rudders, frames, etc. Before long, it is expected, we shall be completely welding bulkheads.

A. M. Candy: Referring to Mr. Owen's remarks relative to flexibility in structural steel connections, I wish to make the following comments. In riveted designs for beam-to-column connections as the beam is loaded it will be deflected to a certain point, depending upon the loading on the beam, which results in a certain amount of bending in the connection angles which secure the beam end to the column. The very act of bending of the connecting angles relieves the stresses which would otherwise be imposed on the connecting material. In designing steel structures for welding the amount of weld metal which may secure the beam to the column in so far as handling the shear load on the connection, may be entirely inadequate to take care of the stresses which may result at the connection if this amount of metal is placed where it will restrain the end of the beam from separating from the column by the amount which it would have separated had a standard riveted connection been employed. If, for example, the top and bottom flanges of a given beam are to be welded to a column the amount of welding used must be sufficient to carry all of the stresses which will be thrown into the connection point when the beam is loaded, said stresses to be calculated upon the basis that the beam end is fixed to the column and cannot move with the deflection of the beam under load. These secondary stresses which are produced through the connection due to the fixation of one member to the other may in some cases be several times the normal loading which would result on a standard riveted connection. I believe that if Mr. Owens gives this point some little thought, he will readily see the idea which the speaker had in mind when briefly mentioning this very characteristic.

Relative to the slotted construction used in airplane-fuselage manufacture, I believe that Mr. Owens will find that the gusset plates used at the connection points are not slotted but that the tubular members are slotted so that they will slip on over the gusset plates, which are solid, welding being performed around the edges of the tubes where they come in contact with the gusset plates. As a matter of fact, slotting the gusset plates in this type of joint would defeat the very purpose which it is desired to accomplish, namely, to have a solid continuous web member throughout the tubing sections, which are joined by the gusset plate.

Welding and Manufacturing of Large Electrical Apparatus

BY A. P. WOOD[1]
Associate, A. I. E. E.

Synopsis.—*The object of this paper is to show how extensively electric welding is being used in the fabrication of large electrical apparatus. It outlines the use and advantages of welding for supplanting the casting and machinery of parts and describes briefly the welding equipment developed for this purpose and some of the electrical machines produced by this method.*

THE steps required in producing an electrical machine of iron or steel castings consist chiefly of engineering, drafting, making patterns and castings, machining, assembling, and finally, filling and painting.

The tendency of the present day, however, is to produce machines, particularly the larger electrical machinery, of welded steel plates. It is possible now by following the new modern process of fabricating a machine practically to eliminate the pattern shop as well as the foundry.

Welding passed beyond the experimental stage several years ago. Previously, it was principally used for repair work. Automatic welding equipment has recently been developed, which guarantees a much better weld, thus greatly reducing the possibility of personal error present in hand welding and insuring uniformity throughout the length of the weld. Considerable time and money is saved by resorting to automatic welding wherever possible, as it is more rapid than hand welding and there is no loss of welding wire as in the case in hand welding.

A drafting department can quickly produce simplified drawings calling for steel plates to be cut by flame, some of which are easily rolled circular, and then mostly automatically welded together, thus producing a stator or rotor.

By making these large structures of fabricated plates, no expensive patterns are required. Valuable pattern storage space, which had to be increased as time went by is thereby released and can be used for other manufacturing purposes. Foundry space is reduced and the substitution of welded forms for castings results in a great saving of time and eliminates the possibility of a defective casting due to cores shifting, blow holes, cold shots, and pouring gates, which require time to burn and machine off.

The work of the machine shop is reduced to a minimum, as the plates arc-welded together have approximately the finished dimensions, while in a casting there is of necessity a great amount of excess metal which is never twice alike and which requires excessive machining.

The first use of welding in a-c. machine construction was spot welding the armature-core air-duct spacers to the laminations instead of riveting them on. The next development was the adoption about 11 years ago of welded clamping flanges for holding the stator laminations. These flanges consist of sheared-off steel fingers arc-welded to steel retaining plates. About this time, arc-welded air deflectors made of sheet iron and angle iron were introduced.

In 1924, the General Electric Co. designed and built a 1400-kv-a. machine having the stator frame of welded steel plates. This stator consisted of two side plates having the supporting feet as part of the plate. The outside wrapper plate, foot plates, and ribs for retaining stator punchings, were then hand-welded in place. It was lighter than a similar cast machine, and deflec-

---

[1] A-c. Engg. Dept., General Electric Co., Schenectady, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Feb. 13-17, 1928.

tion tests proved it to be much stronger. The chance of breakage due to handling while in the course of manufacture, or to short circuits after being installed, was greatly reduced. During the three intervening years, practically all stator frames of our machines have been changed over to this new form of construction.

When the steel plate frame is ready for painting it is of nice appearance, as any scale attached to the plates has been removed by sand blasting. No filler being required, the surface is quickly finished with a paint gun.

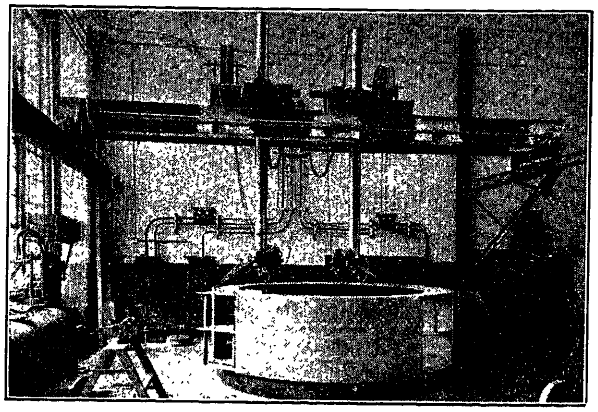


FIG. 1—STATOR-FRAME AUTOMATIC WELDER FOR FRAMES UP TO 14 FT. DIAMETER

These years of experience have justified the making of arc welded steel-plate machines as large as 40 ft. in diameter for slow speed, and as high as 10 ft. for high speed, and up to the present writing, of capacity up to 100,000 kv-a.

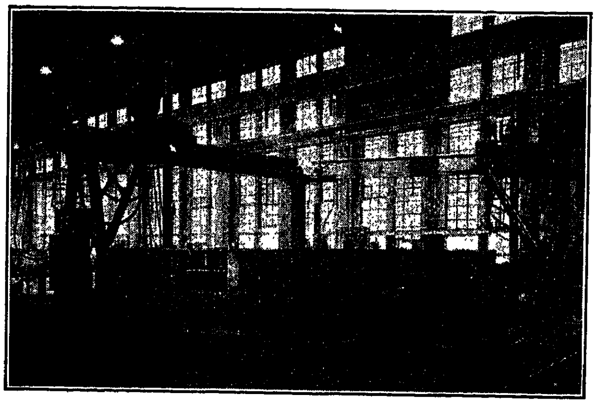


FIG. 2—STATOR-FRAME AUTOMATIC WELDER FOR FRAMES EXCEEDING 14 FT. DIAMETER

Very extensive tests have been made on samples of plates welded together with single-, double-, and triple-layer welding—both hand and automatic—so that there is no question about the sort of welding to use for any particular design.

Wherever possible, on vertical or horizontal stators, rotors, bearing brackets, etc., hand welding has been superseded by automatic welding. On large diameter stators two or more circular plates made of segments, when in the same plane, are welded together by a semi-automatic hydrogen-enveloped arc. This results in a ductile weld that is easy to machine or grind and will not break if accidently stressed by being dropped in handling.

For general work, three principal types of automatic welding machines have been developed. On the first type of welding machine, the welding heads are stationary and the piece to be welded is rotated past the heads. It is, however, only practical to rotate the work up to a certain diameter (see Fig. 1). With the second type of automatic welding, used for larger diameters, the radial arms of the automatic welding machine are made to rotate by a motor drive, Fig. 2. The third type of automatic welding is a straight-line welder in which the work is stationary and the welding head is moved.

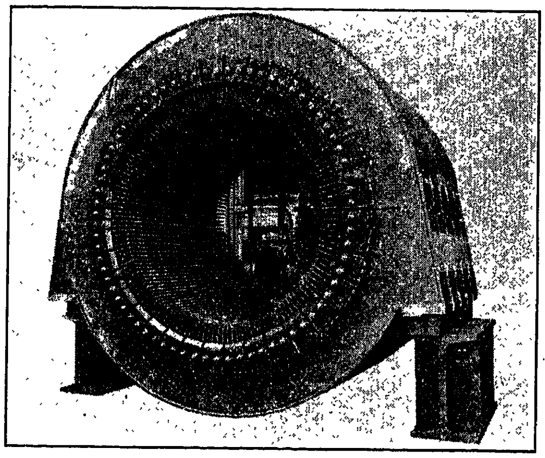


FIG. 3—STATOR WITH PUNCHINGS FOR 50,000-KV-A. SYNCHRONOUS CONDENSER

The large automatic welding machines shown in Figs. 1 and 2 weld from 6 to 8 ft. per minute. The current used is between 300 and 400 amperes at 20 to 30 volts, depending on the gap between the pieces to be welded. Direct current is used.

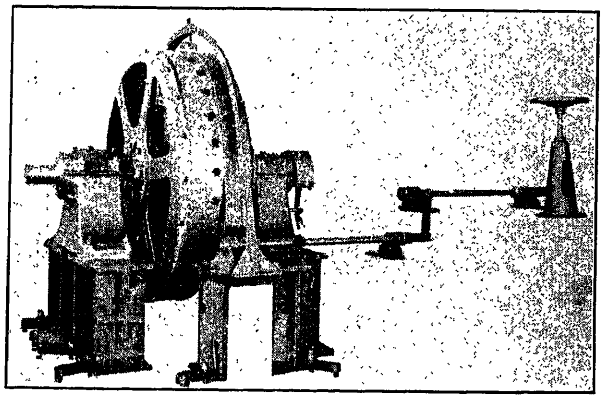


FIG. 4—A WELDED ROTATING STATOR FRAME FOR SUPER-SYNCHRONOUS MOTOR

The semi-automatic hydrogen-enveloped welding uses approximately 250 amperes at 40 to 60 volts, as it requires a higher voltage to maintain the arc.

The welding rod, or reel, on the automatic machines is of G. E. Type-B shielded wire, which consists of a steel core and outer steel layer between which is the flux. A high speed of welding is possible with this wire.

A large horizontal stator of 50,000 kv-a. is shown, Fig. 3. You will note the supporting feet are cut out as part of the side plates and welded to a heavy plate which is drilled for the holding down and adjusting bolts. The stator is split horizontally to aid in shipping, the two halves being held together by bolts and dowels inserted in the rectangular blocks, which were completed as a unit before welding in the stator. As a result of arc welding, we have produced large stator frames in 90 deg., 120 deg. and 180 deg., sections that require no setting up on expensive iron floors for machining or dowelling at the splits, as this operation is entirely eliminated. The ventilating air openings are located in the circular outer apron, or wrapper sheet, and are covered with punched metal covers. The laminations are mounted on the welded ribs and clamped tightly together by welded non-magnetic steel clamping flanges.

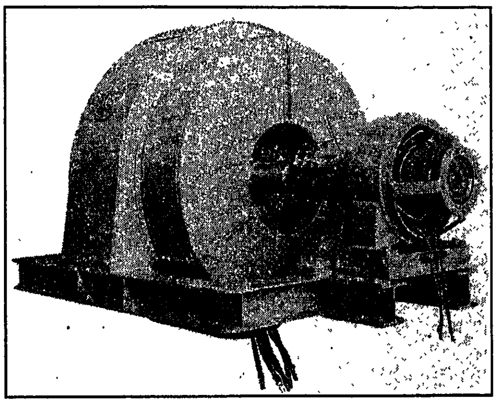


Fig. 5—Welded Beam Base for 2500-Kv-a. Synchronous Condenser

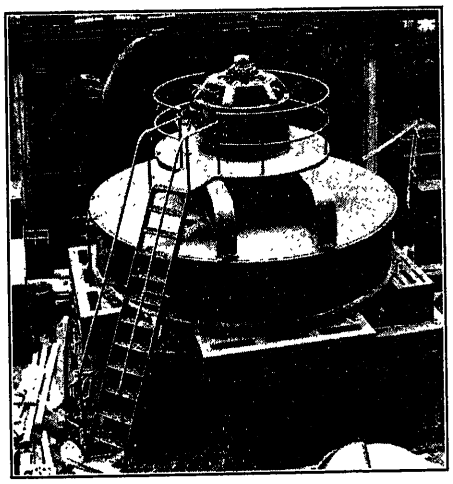


Fig. 6—Vertical Water-wheel Driven Generator Practically Entirely Welded

A rotating stator frame for a G. E. super-synchronous motor built up of arc welded rolled plates is shown in Fig. 4.

The next step in the elimination of castings on the horizontal machine was to replace the cast iron base with one of welded beams. (See Fig. 5.) This design has been developed and built for machines up to 50,000 kv-a.

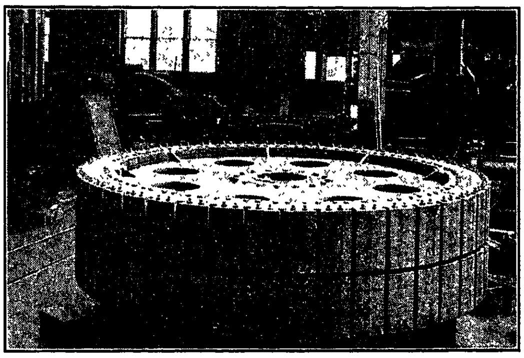


Fig. 7—Welded Rotor Spider for 19,700-Kv-a., Rev. per Min. Generator

Fig. 6 shows a medium sized vertical alternator, the mechanical parts of which are almost entirely of welded construction. The stator frame is similar in construction to the horizontal stator previously described, except that the bottom plate is of larger diameter, to permit of fastening to the foundation plates. A heavy rolled ring is welded to the top of the outside wrapper plate for supporting the top bearing bracket.

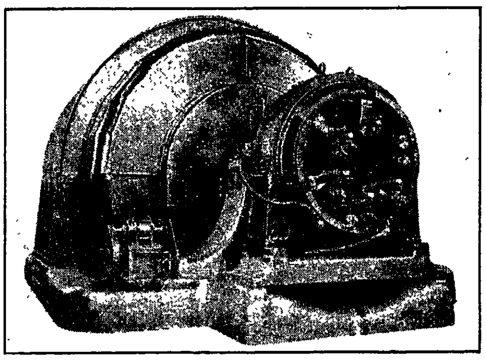


Fig. 8—Welded Air Deflector

A structural steel welded upper bearing bracket consisting of two rolled I-beam girders between which is bolted a welded housing carries the upper guide bearing and supports the thrust bearing which carries the weight of the rotor, waterwheel runner, and thrust. The ends of these girders are cut off to a rather large radius and a steel plate is rolled to this radius and welded to the web of the beam. This construction does not appreciably affect the strength of the beam and gives the structure a more pleasing appearance. The platform around the exciter is made of plates, welded together, and the exciter, itself, has a frame made of a steel slab rolled to shape. Where the load on the upper bearing bracket is so large that rolled steel beams of sufficient size cannot be obtained, these brackets are made up by plates welded together. We have recently finished two such fabricated beams similar in section to the rolled I-beam which are 30½ ft. long and 6 ft. high. The flanges and webs are welded together throughout their entire length. Between these beams is carried a welded thrust bearing housing designed to support a load of 350 tons.

A large number of cast rotor spiders has been replaced by various designs of welded plate construction. On the smaller spiders the hub is cut from thick steel plate or made from casting stock, to which is welded a center web-cut out of heavy rolled plate. The rotor rim is steel plate rolled to the proper diameter and welded to the center web. This outside rim may be either a single plate with a butt welded joint or made of two rolled plates having the joints staggered and welded together completely around the outer edges. Since the rim plate is welded to both sides of the web plate around the complete periphery, the welded joint of the rim does not have to take the entire load of the centrifugal force of the poles.

For the larger class of rotors a welded-plate spider is used to support a rim built up of thin laminations. Such a design for a 19,700-kv-a. generator is shown in Fig. 7. The center spider consists of circular plates for the top and bottom members which are welded to a center hub. Plates are spaced around the outer periphery of the plates which serve as a backing support for the rim laminations and are slotted for keys which transmit the torque from the shaft to the rim. Bolted to the bottom circular plate are segments of heavy plate which act as a braking surface and in addition support the weight of the rim. This type of design has been quite generally adopted as standard for the larger slow-speed vertical generators.

Welded structural-steel bearing brackets for supporting the lower guide bearing have been standard on large diameter vertical machines since about 1922. Ordinarily, these brackets are built of rolled shapes fabricated by flame cutting and welding. Mounted on the lower bracket are combined brakes and jacks which are also fabricated of parts welded together.

The original cast iron enclosing shields for positive ventilation on horizontal machines have been replaced by a combination welded and riveted sheet iron shield. Fig. 8. clearly shows the method of obtaining a large radius to the shields which is pleasing to the eye and seems so necessary when castings were used. An open-type spot-welded shield has been standard for a good many years.

The development of the art of welding and flame cutting has marked a new era in the design of electrical machines, particularly those which are either very large or special. The advantages of obtaining a structure which is homogeneous, free from possible casting defects, long and costly machining, and which lends itself so admirably to changes in design without any more expense than the change of a drawing, are, in the writer's opinion, so great that they cannot be ignored.

Discussion

F. D. Newbury: I wish to endorse and, if I can, emphasize what Mr. Wood has just said about the fundamental importance of this change in our machinery construction.

The introduction of welding and the use of rolled sections, instead of castings, has given the designer an entirely new basis for his design. Instead of the foundry and the pattern shop we have substituted the steel rolling mill, the gas torch, and welding equipment. As one man has recently expressed it, a fabricating shop for electrical machinery now is something like a carpenter's shop: it contains a stock of standard shapes, something to cut them to the proper size, and something to nail them together into the shape desired.

I would also emphasize the fact that to obtain the advantage of this new construction, the designer must design his structure for welding rather than for casting.

Mr. Wood also pointed out, I believe, that unless the designer does so take advantage of the new method, there is apt to be little saving in cost in the structural parts over corresponding cast parts. The great saving, and the obvious saving, is in the elimination of patterns. So this new method is applicable generally to special designs for larger machines that are seldom duplicated.

To take full advantage of this new method, machine operations as such must be eliminated as much as possible, and in our own experience we have found it possible already to build frames for machines of very large size with no machining operations except drilling the holes for certain through-bolts.

But, while there are obvious advantages from a cost and delivery standpoint in the elimination of patterns and in the elimination of castings, this very fact has introduced rather serious problems for the factory administrator. It is not an easy thing for the factory manager to become reconciled to the fact that he is losing his foundry load, and it is a real problem for the manager to face, to scrap millions of dollars of factory equipment due to the introduction of a new method. But American managers have always boasted of their willingness to scrap obsolete equipment and they are now faced with the fact that a great deal of the foundry equipment of the electrical manufacfacturers and undoubtedly other manufacturers, as they take up this new work, must be scrapped in favor of a newer and less expensive method of construction.

We have also found an incidental advantage in the changed attitude of mind of our foundry friends. Since they have experienced this new competition there has been less thought about tonnage and more attention has been given to methods of reducing foundry costs, as for example, through use of thinner sections.

Of course, this new method will not completely eliminate castings by any means. The casting method will show lower costs, for parts of smaller diameters, and for parts that require considerable fabrication work, and particularly where the size and volume of the output warrant machine-foundry methods.

There is just one more point I should like to call to your attention, that the advantages of structural materials in our electrical machinery have been recognized for many years. Some 28 years ago the 5000-kw. engine-type generators that were built for the Interborough elevated and subway plants in New York City had a completely fabricated rotating part, with riveted built-up spiders. About 18 years ago we attempted to design a line of d-c. machines with entirely structural parts, but that plan was abandoned because there was no satisfactory low-cost method of assembling available at that time.

About 15 years ago I was interested in a structural frame design that we gave up because of cost. There riveting was contemplated instead of welding.

So it should be placed to the credit of the welding process that we are now able to make these savings and achieve these very desirable results in electrical machinery design.

B. L. Barns: A very important advantage of welded construction is the greater freedom of the designer in using dimensions and proportions best suited for the electrical characteristics that are desired. When large and expensive patterns were used the electrical designs were often compromises. When a particular rating was required the proportions of the machine were often largely determined by existing patterns with the result that the active material was not always used to the best advantage or some characteristic was not as good as it might have been. With this new construction the designer has greater freedom in choosing the proportions of the active material and as a result better machines are being built.

Another advantage that has been experienced in the factory with which I am connected is a material reduction in delays in manufacture due to defective castings. Even though great care may be exercised in the inspection of castings in the foundry, occasionally defects or flaws are discovered after the machine work is partly done. If the defect is of a sufficiently serious nature it is necessary to scrap the casting and order a new one. This, of course, upsets the whole schedule of production and a delay in shipment probably results. On the other hand the welded construction is free from hidden defects and manufacturing delays are correspondingly reduced. The absence of delays from these causes is an economy that is incidental to this type of construction.

H. M. Hobart: In spite of all the nice things said about these machines, I think the designs they have been turning out recently are quite unbeautiful. I wonder if they can't, without abandoning all these advantages, bring in a little less unloveliness.

F. D. Newbury: We have often had this point about appearance of the machines brought up. My answer has always been that a thing is good-looking if it is designed according to correct mechanical principles, if there is no imitation about it. A straightforward design is always good-looking to an engineer.

B. L. Barns: Our company just recently received an order for several large low-speed machines. The standard which was set on the floor was flat, like a pancake, and had a large diameter. My assistant, Mr. Sills, got the happy idea of building the frame with flat sides, a polygon. The purpose of that was to add vertical lines to the frame and make it look higher. The upper bearing bracket for supporting the thrust-bearing, we intend to build of the girder type, two girders side by side with the thrust bearing support in between. So we carried out the same idea and made all the lines on the girder straight. We built a small model which I took to the customer's office and exhibited, and the enthusiasm for the general appearance was unbounded. They didn't want a round machine after that.

A. P. Wood: I might answer Mr. Hobart by saying that the new generation of designers has probably been influenced by modern cubist tendencies in art.

In order to help beautify the welded steel construction you will note by photographs in the paper that the effect of rounded corners is obtained by flanging the thinner sheet-metal enclosing hoods. In some cases a curve as large as 6 in. is used which eliminates the square appearance. But from a manufacturing point of view, the heavy plates which are passed through large rolls, cold, must remain squared, as in this shape they best lend themselves to automatic welding.

I believe that, as many others have, the more you see of machines of this construction the better you will like them.